Objectively Assessed Demand

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Abstract: The 2012 National Planning Policy Framework requires housing needs assessments to take into account market indicators such as affordability; however little guidance is provided on how this information should be incorporated in practice. This paper shows the effects on needs assessments of taking affordability seriously, using formal models that lead to a simple performance metric that can be readily applied by all local authorities; the implications are radical. The paper also shows how ideas of self-organisation can be used to promote consistent behaviour across local authorities, bearing in mind that authorities are currently required to collaborate within a housing market area. The paper concludes with a policy discussion and suggests that the proposed indicators could be used to supplement existing planning practices.

Keywords: National Planning Policy Framework; housing need; affordability; housing market areas.

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**Policy Summary**

- The National Planning Policy Framework requires local authorities to take into account market signals in their assessments of future housing needs; furthermore they are expected to work in conjunction with authorities that are part of their housing market areas (HMA). The 2015 PPG provides some clarification of the indicators that are likely to be of value, such as affordability, but provides no indication of how the measures should be used in practice to moderate assessments based on household projections. Neither document sets out a common methodology to be used by all authorities for the calculation of objectively assessed need. This was recognised in the 2017 White Paper since it leads to inconsistencies across local authorities and the Government has proposed consulting on an appropriate methodology that could be adopted by all authorities.

- Existing guidance is unclear about the difference between need and demand, but the distinction is important when market indicators are taken into account. This paper considers the implications for local plans when affordability is incorporated seriously rather than as an *ad hoc* adjustment.

- Attempts to improve affordability have relied primarily on ways to increase housing supply; in fact work dating back to the 2004 Barker Review of Housing Supply has consistently shown that to have any significant effect on national or regional affordability, supply increases have to be very large and long-lasting. In practice, we suggest that it is impossible to *stabilise* affordability (measured by house price to earnings ratios) even at current levels by supply policies alone; rather supply policies need to be one element of a wider package. Nevertheless, it is still possible to introduce supply measures that improve consistency and “fairness” across local authorities, using the requirement that authorities should collaborate with districts within the HMA. Perhaps surprisingly, there are currently big differences across even neighbouring authorities and reducing the dispersion between neighbours actually improves national affordability.

- This requires the development of performance metrics: we suggest that metrics require three characteristics – they have to be consistent with economic theory and the empirical evidence; they have to be relatively simple with an intuitive interpretation for both the public and planners and; they need to be easily constructible using modest amounts of data that are regularly published. Two related indicators meet these criteria; the first is the ratio of the local population to the housing stock weighted by average earnings and the second is the ratio of employment to the housing stock, again weighted by average earnings. Neither measure requires household projections and, therefore, avoids some of the problems that arise from this source.

- The two measures have very different implications for the optimal spatial distribution of the housing stock; population-based measures suggest that housing should be expanded primarily in wealthy suburban and rural areas. Elmbridge in Surrey tops the rankings in terms of additional housing requirements and since this has the highest house prices outside London, its position is, perhaps, unsurprising. However, although the majority of highly-ranked authorities are in the South, this is not exclusively the case, because the assessments are based on the ratios in each authority relative to those of contiguous authorities. So, Rushcliffe in the East Midlands ranks third – although its house price is not dissimilar to the national average, it has experienced good educational results and ranks highly on liveability indices. Trafford in the North West also rates highly. By contrast, employment-based measures lead to a more concentrated distribution of housing requirements, largely in major urban areas.

- Local plans based on affordability alone – even though the two metrics have a clear relationship to current approaches - imply major changes to the distribution of the housing stock and would lead, no doubt, to considerable opposition given that we have to start from where we are.
Nevertheless, we would argue that the measures provide valuable information on relative priority areas even if the absolute increases implied by the metrics could not easily be implemented. There is a good case for requiring that these measures should be one element of local plans that could be easily constructed by all authorities and provide a basis for negotiations. If performance metrics differ considerably between neighbouring authorities, there is at least a case to be explained.
Objectively Assessed Demand

1. Introduction

In 2012 a new National Planning Policy Framework (NPPF) was introduced in England (Department of Communities and Local Government 2012); the NPPF provides the basis for the preparation of local plans, including assessments of housing needs and sets out twelve core land-use principles underpinning plan-making. These include the requirement to:

... ‘proactively drive and support sustainable economic development to deliver the homes, business and industrial units, infrastructure and thriving local places that the country needs. Every effort should be made objectively to identify and then meet the housing, business and other development needs of an area, and respond positively to wider opportunities for growth. Plans should take account of market signals, such as land prices and housing affordability, and set out a clear strategy for allocating sufficient land which is suitable for development in their area, taking account of the needs of the residential and business communities’ (NPPF, page 5).

Two elements in the quotation are particularly relevant here: first, the responsibility objectively to identify need and, second, the requirement to take account of market signals. The NPPF (pages 38, 39) also outlines the nature of the evidence base required to construct the plans: it stresses the need to prepare a Strategic Housing Market Assessment; a requirement to work with neighbouring local authorities where there is a common housing market area; the need to identify the mix of housing, including affordable housing and; the use of household and population projections.

Further information on the methodologies to be used was published in the 2015 Planning Practice Guidance (PPG), *Housing and Economic Development Needs Assessments* (DCLG 2015), although the PPG stresses that no single approach is definitive; this has proved important since it has meant a lack of uniformity in approach across local authorities. Nevertheless, PPG indicates that government household projections should provide the starting point, but can be adjusted for local conditions, for example expected changes in employment. PPG also states that projections of housing need derived from household projections should be adjusted for market indicators, such as changes in house and land prices, rents, affordability, the development rate and overcrowding. The value of incorporating market indicators into planning decisions had first been recognised in the 2004 Barker Review of Housing Supply, but PPG provides no information on how the information should be used to adjust needs projections; if house prices in a local authority are growing by, say, 10% faster than neighbouring authorities, by how much should construction be increased? In practice authorities have adopted *ad hoc* approaches not based on any formal modelling.
The 2017 Housing White Paper, Fixing our Broken Housing Market (Department of Communities and Local Government 2017) noted that only a third of local authorities had adopted a plan since the publication of the NPPF. It highlighted that the construction of plans was slow and expensive, hampered by the fact that there was not a standard methodology for assessing housing requirements. The White Paper, therefore, committed the Government to consult on the options for introducing a standardised approach.

In addition to slowness in production and the absence of standardisation, there are a number of methodological shortcomings in current needs assessments. First, as noted above, there is no formal method of incorporating market indicators; second, the distinction between housing need and demand remains confusing; third, the method still heavily relies on official, trend-based household projections, which as discussed below have important weaknesses; fourth the appropriate definition of housing market areas is always likely to be controversial. Each of these issues is discussed further in the next sections.

However, the main focus of the paper is on the first issue - how to introduce market indicators into local housing assessments, taking account of theoretical and empirical evidence on housing demand and house price formation. In addition, the paper considers the implications for housing requirements across local authorities; if taken at face value, the conclusions are radical and, therefore, some caution is urged in using the results alone as the basis for local plans. Nevertheless, there are still significant advantages of the proposed method; in addition to being consistent with the evidence, the derived indicators are straightforward to construct using regularly published information (and so can be updated), they have an intuitive meaning and can be related directly to existing household-based projections, even without using such projections. Because we are concerned with market indicators and affordability, demand rather than need is the appropriate concept and, so, the title of the paper becomes, ‘Objectively Assessed Demand’ rather than the more usual ‘Objectively Assessed Need’.

The approach is based on three central ideas; first, methods should have strong theoretical and empirical foundations; second, the methods should imply fairness and consistency in the treatment of local authorities (and we use ideas from the self-organisation literature to achieve this) and; third, planning should not (and cannot) be used to offset short-run housing cycles and, so, a long-run approach is required.
Section 2 briefly considers further three of the four problems noted above with existing needs assessments. Section 3, then, considers the fourth problem – formal methods for incorporating market indicators - and sets out the theoretical and empirical models underlying our market-indicator led approach. Section 4 presents empirical estimates of housing requirements for the English local authorities, including sensitivity tests. Section 5 considers the policy implications and caveats to the results. Section 6 draws conclusions.

2. Conceptual Issues in Existing Objective Needs Assessments

2.1 Need or Demand?

The distinction between demand and need still appears to cause confusion in housing assessments; to economists the former is a clear concept and refers to effective demand, i.e. the level of housing consumption that can be supported by the ability to pay; therefore, a rise in house prices, for example, is expected to reduce housing demand, for a given level of incomes. Worsening affordability is an indicator of housing shortages and remaining with parents for longer or sharing is a market response, which may or may not case social problems and externalities for the wider economy. Need, however, is arguably less clearly defined, but generally has a broader scope, reflecting the nature of housing as a merit good. So, in addition to providing affordable housing for lower-income groups, PPG refers to the necessity of taking into account homeless households, those with social or physical impairments, households that lack basic facilities or are overcrowded in their current residences, and those escaping harassment. Since many of these households will not be able to access market housing, need is likely to exceed effective demand. Nevertheless, the PPG defines need as:

‘Need for housing in the context of guidance refers to the scale and mix of housing and the range of tenures that is likely to be needed in the housing market area over the plan period – and should cater for the housing demand of the area and identify the scale of housing supply necessary to meet that demand’ (italics added).

So need is defined in terms of need, but also refers to market concepts of demand and supply. Given the lack of clarity, it is, perhaps, unsurprising that plans have struggled to incorporate market indicators in a consistent manner; an increase in those that cannot access the market, for example, because of disability, may be imperfectly reflected in market prices.
2.2 Household Projections

Official household projections still remain the starting point for local housing assessments and it is easy to see why their use remains pervasive, despite the fact that their production is a large exercise undertaken approximately every two years. Authorities are unlikely to be able to conduct the exercise individually and, indeed, the population flows across authorities through migration imply that, for consistency, the production has to be centralised. But the idea that net additions to the housing stock should at least equal the expected increase in households is appealing as a starting point for plans. However, there are problems; first Meen and Andrew (2008) demonstrate that matching increases in household numbers and net additions will not be sufficient to stabilise affordability, because of increasing demands for housing services by existing households, which are not taken into account. Therefore, if affordability is considered important, household projections by themselves are an insufficient basis for plans.

Second, household projections are trend-based, relying on information from censuses and the Labour Force Surveys. Implicitly, they suggest that the past economic trends that influenced household representative rates will continue in the future. But, in recent years, household representative rates amongst younger households have been negatively affected by worsening affordability, particularly since the incomes of younger groups have grown at a slower rate than older groups. Those constructing household projections have to make a judgement whether these rates will recover if and when affordability improves. This is an example of a more general problem; household growth is endogenous to the state of the economy; if incomes are growing, household formation is likely to expand and if affordability worsens, formation is expected to fall. Some of these changes will be cyclical and can be ignored in trend household projections, but there is no easy way to distinguish between cyclical and trend changes in this context.

2.3 Housing Market Areas

There is an acceptance in the NPPF that individual local authorities rarely constitute a housing market area (HMA) because of interlinkages caused by residential mobility and commuting flows and PPG states that needs assessments should be conducted in conjunction with other local authorities. In addition to migration flows and travel to work boundaries, PPG recommends the use of house price information as an aid to identifying HMAs. Similar house prices in levels or changes may be one
indicator of common areas since migration flows generate convergence in price growth. There is now a very large international literature on the most appropriate way to define HMAs (see, for example, Jones 2002, Jones et al 2004 and Watkins 2001) and the most comprehensive assessment for the country as a whole can be found in Jones et al (2010). In principle, there is no reason why local authorities comprising an HMA should be physically adjacent, particularly where house prices are taken into account. For example, Brighton and Hove (GL Hearn 2014) is part of Sussex Coast Housing Market Area and includes, additionally, the local authorities of Adur, Chichester, Lewes and Worthing; but only Adur and Lewes are physically contiguous to Brighton. Nevertheless, in many cases, since migration and commuting flows decline with distance, contiguity is expected to be a feature; furthermore Local Enterprise Partnerships, which are likely to be related to HMAs are also frequently contiguous and this characteristic is exploited in the next section.

3. Objectively Assessed Demand: The Analytical Framework

3.1 The Basic Model

Estimates of objectively assessed demand require a strong theoretical and empirical evidence base for the determinants of house prices and affordability; importantly, the parameters of the housing demand functions that underlie the price estimates have to be time invariant. These have been studied extensively in the international literature and we rely on the parameters derived from a series of consistent studies built up by the author over the last twenty-five years (see Meen 1990, 2000, 2002, 2013, Meen and Andrew 1998), which demonstrate that the values have been remarkably constant since the late 1960s, despite the structural changes that have taken place in the UK housing market, for example, deregulation of mortgage markets in the early 1980s. Critically, there have been only limited changes to the central income and price elasticities of owner-occupier housing demand.

The standard approach in the literature derives models of housing demand and prices from inter-temporal optimising models of household behaviour; the appendix shows how the aggregate housing demand function (1) is derived. Housing demand is simply a function of per household income, the number of households and the price of housing services captured by the real house price multiplied by the cost of capital; the product is known as the user cost of capital and measures the per period price of a unit of housing services. Compared with the standard demand function
employed for non-durable goods, the main difference is that the cost of capital includes an allowance for housing depreciation and also takes account of any expected capital gain, measured as a negative cost, since housing is tradeable as an asset.

\[
\ln(H^D(t)) = \alpha_0 + \alpha_1 \ln[RY(t)/HH(t)] - \alpha_2 \ln[g(t) \cdot CC(t)] + \alpha_3 \ln(HH(t)) + \varepsilon_1(t)
\]

(1)

where:

\[
CC(t) = (1 - \theta(t))i(t) - \pi(t) + \delta(t) - \dot{g}^e(t)/g(t)
\]

(2)

\(H^D(t)\) stock demand for housing;
\(g(t)\) real purchase price of dwellings;
\(\theta(t)\) household marginal tax rate;
\(RY(t)\) real household disposable income;
\(i(t)\) market interest rate;
\(\delta(t)\) depreciation rate on housing;
\(\pi(t)\) general inflation rate;
\(HH(t)\) total number of households;
\(CC(t)\) cost of capital
\(\dot{x}(t) \equiv \frac{dx(t)}{dt}\) denotes the time derivative for any variable \(x(t)\).
\(\dot{g}^e(t)/g(t)\) is the expected real capital gain.
\(t\) is a time indicator
\(\varepsilon_1(t)\) error term.

The cost of capital (equation 2) is graphed in Figure 1 from 1969; this is the operational version used in estimation below; in addition to the variables covered in (2), this version allows for property taxes, maintenance expenditures and the mortgage rationing that took place prior to the early 1980s and since the Global Financial Crisis. Credit shortages raise the cost of capital and further details are given in Meen et al. (2016). Importantly for the later analysis, there is no evidence of a long-run trend in the cost of capital since the late 1960s, (ADF(4)=−4.3); because the cost of capital is a modified real interest rate, stationarity is in line with expectations.
Conditional on the supply of the housing stock, the equilibrium (log) real house price is given by (3).

\[
\ln(g(t)) = (\alpha_0/\alpha_2) + (\alpha_1/\alpha_2) \ln[RY(t)/HH(t)] + (\alpha_3/\alpha_2) \ln[HH(t)] - (1/\alpha_2) \ln[H^S(t)] - \ln[CC(t)] + \varepsilon_2(t)
\]  

(3)

\(H^S(t)\) stock supply of owner-occupied housing.

From (3), real house prices are dependent on the demand and supply indicators and the equation provides insights into the long-run drivers of affordability; the income and price elasticities of housing demand, \(\alpha_1\) and \(\alpha_2\), are particularly important below since, from (3), they determine the responsiveness of house prices to an income change. Nevertheless, although our primary interest is in (3) as the long-run solution, it is helpful for estimation to embed the equation in an autoregressive distributed lag model (ARDL), (4):

\[
\Delta \ln[g(t)] = \sum \beta_i \Delta \ln[g(t - i - 1)] + \sum \gamma_i \Delta \ln[X(t - i)] + \theta_1 \ln[(g(t - 1)] \\
+ \theta_2 \ln[X(t - 1)] + \varepsilon_3(t)
\]  

(4)

\(X' = [RY/HH, RW, H^S/HH, CC, WSH]\) and \(\varepsilon\) is an error term;

\(RW\) real household wealth;

\(WSH\) ratio of post-tax wage income to personal disposable income;

\(\Delta\) (quarterly) first difference operator.
The vector of regressors \( (X') \) includes two additional variables to (3), real household wealth and the share of wages and salaries in household income; both were found to be significant in Meen (2013). The latter reflects changes in the distribution between wage and investment income over time and is important if the income elasticity of demand for housing differs between the two sources. The earlier paper found that housing demand and prices are more sensitive to changes in wage income. Note also that in \( X' \), \( H^p \) and \( HH \) are expressed as a ratio to ensure price homogeneity with respect to equal changes in the housing stock and number of households or \( \alpha_3 = 1 \) in equation (3).

Table 1 shows the results from estimating equation (4), excluding insignificant variables, on UK quarterly data between 1970Q1 and 2012Q4\(^2\); the long-run solution is shown in (5) – this is the equivalent to equation (3) - and yields a long-run income elasticity of house prices of 2.5, a housing stock elasticity of -1.4 and a cost of capital semi-elasticity of -0.05. The diagnostic statistics show little evidence of model misspecification; there is no evidence of heteroscedastic disturbances and the residuals are normally distributed, although the Lagrange Multiplier test indicates some residual autocorrelation.

\[
\ln[g(t)] = -6.93 + 2.49 \ln[RY(t)/HH(t)] - 1.37 \ln[H^S(t)/HH(t)] - 0.05[CC(t)] + \ldots \quad (5)^3
\]

As noted above, it is important that the long-run parameters show limited variability over time; this could be shown from sub-sample tests on the equation in Table 1, but an alternative is to examine the parameters from a series of papers that have been published since 1990, using similar functional forms. The results from four papers (including the current study), over different time periods, are summarised in Table 2; these show a considerable degree of similarity in the key parameters and, importantly, in all cases the income elasticity of house prices is well in excess of one. From equation (3), this implies that the income elasticity of housing demand exceeds the price elasticity. This is a feature also in the UK time-series study by Muellbauer and Murphy (1997), whereas UK hedonic studies also support the finding that the demand for housing characteristics is income elastic (see, for example, Cheshire and Sheppard 1998).

\(^2\) Note that the user cost is not expressed in logs since Figure 1 shows that the variable may take negative values temporarily on this definition.

\(^3\) The remaining terms from the long-run solution to Table 1 are excluded in equation (5) since they are not central to the subsequent argument.
Table 1. UK House Prices, Dependent Variable: $\Delta \ln(g)$

<table>
<thead>
<tr>
<th>Estimation period</th>
<th>1970Q1-2012Q4</th>
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<tbody>
<tr>
<td>Constant</td>
<td>-0.707 (4.9)</td>
</tr>
<tr>
<td>$\ln (g)_{t-1}$</td>
<td>-0.102 (6.9)</td>
</tr>
<tr>
<td>$\ln (RW)_{t-1}$</td>
<td>0.022 (2.2)</td>
</tr>
<tr>
<td>$\ln (HS/HH)_{t-1}$</td>
<td>-0.140 (3.0)</td>
</tr>
<tr>
<td>$\ln (RY/HH)_{t-1}$</td>
<td>0.254 (4.9)</td>
</tr>
<tr>
<td>$CC_{t-1}$</td>
<td>-0.005 (12.3)</td>
</tr>
<tr>
<td>$WSH_{t-1}$</td>
<td>0.340 (2.4)</td>
</tr>
<tr>
<td>$\Delta \ln(RY/HH)_{t}$</td>
<td>0.257 (3.1)</td>
</tr>
<tr>
<td>$\Delta (CC)_{t}$</td>
<td>-0.006 (5.7)</td>
</tr>
</tbody>
</table>

Adj $R^2$ 0.73
Equation standard error 0.0160
Lagrange Multiplier (serial correlation) Prob($F_{4,153}$) = 0.02
ARCH (heteroscedasticity) Prob($F_{1,169}$) = 0.14
Ramsey RESET Prob($F_{1,156}$) = 0.61
Jarque-Bera (Residual Normality) Prob = 0.15

The equation includes seasonal dummies and dummies to reflect the abolition of double mortgage tax relief in 1988. t-values in brackets.

Table 2 Dependent Variable: $\ln(g)$ – Long-Run Solutions

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>Equation standard error</td>
<td>0.016</td>
<td>0.015</td>
<td>0.016</td>
<td>0.016</td>
</tr>
<tr>
<td>$\ln (RY/HH)$</td>
<td>3.000</td>
<td>2.401*</td>
<td>2.614</td>
<td>2.490</td>
</tr>
<tr>
<td>CC</td>
<td>-0.054</td>
<td>-0.037</td>
<td>-0.061</td>
<td>-0.049</td>
</tr>
<tr>
<td>$\ln(HS/HH)$</td>
<td>-1.809</td>
<td>-1.744*</td>
<td>-1.545</td>
<td>-1.373</td>
</tr>
</tbody>
</table>

* Specification is slightly different because neither variable is divided by HH.

A high income elasticity has significant policy implications; if a policy objective is to stabilise or at least improve affordability by increases in housing supply, and (as is commonly the case), affordability is measured by the ratio of house prices to incomes, the necessary conditions are easily derived. Rewriting (5) as equation (6), for a given cost of capital, two crucial conditions arise. First, changes in affordability (measured on the left-hand side as house prices divided by per household
income) depend on the growth of income relative to the housing stock, but, in the UK, whereas income grows in the long run at approximately 2.5% per annum, the housing stock increases at less than 1% per annum. Second, the rate at which affordability worsens depends on an estimated parameter of approximately 1.5, derived from the income and price elasticities of housing demand⁴. The higher the income elasticity of demand, the faster affordability worsens for given values of the income-housing stock ratio. Since the income elasticity of demand is high in the UK, affordability worsens rapidly. In principle, this could be offset by an increase in the cost of capital, but, in practice, from Figure 1 and its time-series properties, there is no evidence that it has played such a role, although variations in the cost of capital are important in explaining short-run variations in house prices.

\[
d \ln \left( \frac{g(t)}{RY(t)/HH(t)} \right) \approx 1.5 d \ln \left( \frac{RY(t)}{H^S(t)} \right) - 0.05 d [CC] + \cdots (6)
\]

Equation (6), therefore, demonstrates the difficulty in improving affordability by increases in housing supply alone – the required permanent increases would have to be larger than have ever been achieved consistently in the past. Nevertheless, the ratio of income to the housing stock is a key indicator of long-run affordability and can be exploited for land-use planning purposes. Notice that the number of households plays no role; increases in households only have an influence if accompanied by a rise in household incomes.

### 3.2 Local Indicators for Land-Use Planning

Even if affordability cannot be fully stabilised by supply increases alone, fairness would suggest that the income-housing stock ratio should not significantly differ between local authorities; those authorities that have high ratios would be expected to suffer the most extreme deterioration in affordability. In practice, to operationalise the concept, regularly published data need to be available at the local level. Now consider the income-housing stock ratio in more detail; in Table 1, \( RY(t) \) is measured as household disposable income and \( H^S(t) \) is the owner-occupier housing stock. However, the empirical results also suggest that wage income has a bigger effect on housing demand than investment or self-employment incomes. This is a useful result since wages and salaries can be

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⁴ Note that the parameters are derived from a national house price equation. It may be the case that the key parameters vary locally and this is potentially important for land-use planning. This issue is considered in Section 5.
derived at the local level, whereas $RY(t)$ cannot. Wage income is the product of average earnings and the number of employees, where local information on average earnings can be obtained from the Annual Survey of Hours and Earnings, measured on either a residence or workplace basis. In addition, at the local level, we use the total housing stock rather than the owner-occupied stock, since, in the published data, the private stock is measured as a residual category. Therefore, the first local metric, measuring the earnings to housing stock ratio is $\left[ YHS = \frac{ER*EE}{HST^s} \right]$, where $ER$ = average earnings; $EE$ = number of employees; $HST^s$ = the total housing stock. Each variable is measured at the local authority level.

However, there is a possible additional indicator; rather than employment, the total population ($POP$) in the local authority might be used, $\left[ PHS = \frac{ER*POP}{HST^s} \right]$. Although the numerator is no longer an exact measure of wage income, the use of population, rather than employment, has the attraction that it is less likely to be endogenously determined within the area and, so, needs assessments would no longer be required to predict future employment growth. Furthermore, comparing this indicator with the more conventional planning approach, where the required number of homes is matched by the expected increase in households, there are two key differences; first, the new measure is defined in terms of stocks of dwellings and population, rather than flows. Second, the new measure can be interpreted as the ratio of demographics to the housing stock, weighted by average earnings in each area. Since household formation is influenced by both earnings and population size (see Andrew and Meen 2003), the measure introduces two of the main variables without the need to construct separate complex household projections.

Since this second measure has a looser relationship to the original concept, its ability to predict local variations in affordability needs to be tested. Equation 7 shows the results, where the dependent variable ($AFFORD$) uses the ONS measure of affordability, constructed as the ratio of median house prices to median gross annual residence-based earnings in each local authority in 2015. The independent variable, ($PHS$), averages the ratio between 2011 and 2015 in order to reduce local volatility in earnings between the time periods. The equation shows a strong relationship with approximately half of the local variation explained by this one variable. Although there is no necessary reason why the elasticity constructed on cross-section local authority data should be the same as that constructed on national, time-series information (1.5 in equation 6), the elasticity is similar at 1.1. Nevertheless, the equation standard error is high at 23%, reflecting the variety of additional local factors that affect affordability; Copeland in the North West, where prices in 2015
were only three times earnings, compared with an English average of almost eight, is the most affordable authority, but includes the Sellafield nuclear reprocessing plant. (PHS) is the main indicator used in the simulations in the next section, supplemented by the alternative indicator (YHS).

\[
\ln (AFFORD) = -3.194 + 1.129 \ln (PHS) \\
(10.8) \quad (17.7)
\]

\[R^2(\text{adj.}) = 0.493; \quad \text{Equation standard error} = 0.23; \quad N = 323; \quad \text{t-values in brackets}\]

### 3.3 Interactions between Local Authorities

The National Planning Policy Framework stresses the need for local authorities within a common housing market area to act in unison in assessing housing supply requirements. In fact, a duty to take into account outcomes in neighbouring authorities may both improve affordability nationally on average and reduce the variance of affordability across authorities. This can be demonstrated using ideas taken from the literature on self-organisation and complex systems. One well-known example outside of economics comes from the murmurations of starlings; in this case, flocks have no identifiable leader, but each starling reacts to be behaviour of its nearest neighbours. As a result, beautiful patterns of aggregate behaviour emerge in the sky as the flock suddenly changes direction in response to external shocks. There are many other cases in the natural world, for example fish shoals or ant colonies that follow pheromone trails, but the point is that apparently random individual behaviour can lead to well-ordered aggregate systems dynamics when the social interactions between neighbours are taken into account. In economics, the best-known example is the model of segregation developed by Schelling (1971), which subsequently led to an extensive literature examining the properties of dynamic spatial systems when individual behaviour is affected by the decisions of neighbours.

Section 2.3 noted that collaboration across local authorities was common, but not necessarily only with neighbouring authorities. But, as a modelling simplification, we assume that each authority takes into account (PHS) or (YHS) in all contiguous authorities in its own housing supply decisions; the interactions are captured in a 326 by 326 spatial weights matrix, where values equal one for contiguous areas and zero otherwise. It is conceptually straightforward to amend the matrix to
reflect actual partnerships rather than contiguity. The simple rule adopted is that each authority must set a target for \( PHS \) or \( YHS \) – and so improve affordability from \( 7 \) - equal to the average in the contiguous areas (where contiguous outcomes are measured between 2011 and 2015). This is achieved by increasing its own housing supply where the income-housing stock ratio is currently above the average, but no supply increase is required where the indicator is below the average. Importantly, note that the rule does not necessarily set an absolute target for the required increase in the number of new homes nationally, but establishes a ranking of priority areas for expansion, where the performance metric is currently at its worst. Indeed, as shown in the next section, the metric, by definition, implies that the required increase in supply is zero in some areas, but this just reflects the need for the worst-performing areas to catch up; in order to prevent affordability worsening further in the future as incomes grow, more homes (or other policy initiatives) are, in fact, required in all areas.

The nature of the weights matrix has three further consequences; first, most authorities are linked indirectly in addition to any direct linkages through contiguity. If area \( a \) is contiguous to \( b \) and \( b \) is contiguous to \( c \), then supply changes in \( c \) affect requirements in \( a \). Second, it is not necessarily the case that the majority of required new supply needs to be concentrated in the South on this measure – requirements are judged purely on the relative positions of nearest neighbours and since affordability is uniformly poor across the south, the simulations do not always show that the largest increases should be in the South. To stress the point, the question is not how to equalise affordability across the country by supply increases (although the simulations show that convergence occurs to some extent); instead, our primary concern is with the consistency of behaviour in nearby authorities, where a degree of uniformity might be expected. Third, the solution to the model simulations is iterative and we demonstrate in the next section that the convergence towards the final outcomes reduces both national house prices and the spatial dispersion of affordability. The iterative solution arises from the nature of the spatial linkages.

---

\(^5\) Models of complex systems generally rely on simple rules of behaviour capturing the nature of the interactions.
4. Rankings of Local Housing Requirements

The system can be set up as a spreadsheet model, where starting values in each of the 326 English local authorities for the variables that comprise \((PHS)\) or \((YHS)\) are required; these are total population, the total housing stock, average earnings and the number of employees, which are all averaged over 2011-2015 in order to avoid short-run variations and, in a small number of cases, missing observations. These are readily available and published annually. Therefore, the data requirements are modest. The method can be described using the population-based measure \((PHS)\); the method for \((YHS)\) is identical.

On the first iteration (which might be considered as a form of contracting), the 2011-2015 starting values for \((PHS)\) are multiplied by the spatial weights matrix to obtain the average values in contiguous areas for each local authority, \(\overline{PHS}\); these are compared with the outturn values for each local authority also over 2011-2015. If \(\overline{PHS} < (PHS)\), then, the required target value becomes \(\overline{PHS}\), but remains at \((PHS)\) otherwise. In order words, if the ratio is above the average of contiguous authorities, then the authority is required to meet the average.

On the second iteration (or recontraction), the starting values are the new target values for each authority, which, using \(\overline{PHS}\) are, typically, lower than at the start of the previous iteration. The process is, then, repeated over a number of successive iterations. Table 3 shows the results across three iterations for both \((PHS)\) and for estimated affordability, using equation (7); since the difference in results between the iterations is gradually decreasing, the outcomes converge towards a solution. The table shows both the means and standard deviations over the local authorities; \((PHS)\) is measured as an index. The important point is, although the rules are only set between contiguous authorities, there are national improvements in affordability, both in terms of the mean and the dispersion across authorities. However, the indirect interactions across the spatial weights matrix also implies that the iterative convergence towards the full equilibrium is extremely slow and three iterations does not give the final solution. Over ten iterations, for example, mean affordability reaches a value of 6.7 (standard deviation = 1.1), compared with 7.1 after 3 iterations (1.4). If all authorities are linked to all others either directly or indirectly, then eventually affordability will converge on a common value, but, arguably, this is not a helpful concept since the number of times each local authority would need to recontract would be impractically large. However, this highlights
the importance of the structure of housing market areas. The areas with which local authorities choose to interact affect outcomes.

Table 3. Affordability Using (PHS)

<table>
<thead>
<tr>
<th></th>
<th>Base (estimated)</th>
<th>Iteration 1</th>
<th>Iteration 2</th>
<th>Iteration 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PHS</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>103.2</td>
<td>99.2</td>
<td>97.4</td>
<td>96.1</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>21.3</td>
<td>18.1</td>
<td>16.9</td>
<td>16.2</td>
</tr>
<tr>
<td><strong>Affordability</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>7.7</td>
<td>7.4</td>
<td>7.2</td>
<td>7.1</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>1.8</td>
<td>1.5</td>
<td>1.4</td>
<td>1.4</td>
</tr>
</tbody>
</table>

Nevertheless, it might be argued that an improvement in affordability after three iterations from 7.7 to 7.1 is modest, particularly since the simulation implies that the required national increase in net additions to the housing stock from the 2015 level is approximately 1.3 million units (2.5 million after ten iterations to improve affordability to 6.7) and would, clearly, take a significant number of years to be achieved. This reflects the empirical results in Table 1 and the findings in the Barker Review of Housing Supply – large increases in supply are required to have a major impact on affordability. For comparison, net additions achieved between 2011 and 2015 were only 700,000. It should also be remembered that the estimates do not allow for future population or earnings growth – they are simply the requirements for a more equal distribution across authorities.

Table 4 shows the top twenty authorities, where the required net additions are largest on this measure after three iterations, expressed as a proportion of the total housing stock in order to standardise for the different sizes of the authorities; fifteen of these authorities lie in the South of the country i.e. in the South East, London and the East. Three authorities lie in the Midlands and only two in the North. In terms of affordability, most, but not all, experienced affordability worse than the English average in 2015 of approximately 7.9. The rankings after ten iterations are similar, but not identical – fifteen authorities remain in the top twenty.

Elmbridge, which is at the top of the rankings, lies in a wealthy area of Surrey within the M25 orbital motorway around London; its median house price was approximately £0.5 million and is the most
expensive authority in England outside London. Both St Albans and Chiltern are also in the top five most expensive districts outside London. Nevertheless, the median price in the London borough of Kensington and Chelsea was £1.2 million and is the most expensive borough in the country; however the northern part of the district includes Grenfell Tower and is an area of high deprivation, dominated by social housing, illustrating the contrasts even within a borough.

Rushcliffe in the East Midlands rates highly on the list even though its affordability is better than the national average and its median house price is less than £250,000, but, as noted above, authorities are judged in terms of the position relative to neighbours, not the national position. In the case of Rushcliffe, (PHS) is noticeably higher than in any of its eight contiguous authorities. This is unsurprising since it has particularly good educational results and appears high on liveability indices, such as that produced by the Halifax.

Perhaps the feature that stands out in Table 4, however, is the very large required increases in the housing stock for these authorities – at least 20% (and these become even larger after ten iterations at lower affordability values) - if the rules were to be met; it reinforces the conclusion above that, for these authorities, equalisation would take many years, bearing in mind that, nationally, increases in the housing stock are under 1% per annum. But it should be borne in mind that equalisation implies that approximately 200 of the authorities require no increase or an increase of less than 5% under the rule.

Table 4. Required Net Additions (% of the Housing Stock)

<table>
<thead>
<tr>
<th>Local Authority</th>
<th>Region</th>
<th>% of Housing Stock</th>
<th>Affordability (Outturn 2015)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elmbridge</td>
<td>South East</td>
<td>37.9</td>
<td>12.9</td>
</tr>
<tr>
<td>St Albans</td>
<td>East</td>
<td>35.5</td>
<td>11.7</td>
</tr>
<tr>
<td>Rushcliffe</td>
<td>East Midlands</td>
<td>35.4</td>
<td>7.2</td>
</tr>
<tr>
<td>Kensington &amp; Chelsea</td>
<td>London</td>
<td>33.5</td>
<td>29.6</td>
</tr>
<tr>
<td>Trafford</td>
<td>North West</td>
<td>31.9</td>
<td>7.0</td>
</tr>
<tr>
<td>E. Cambridgeshire</td>
<td>East</td>
<td>30.1</td>
<td>7.8</td>
</tr>
<tr>
<td>Sevenoaks</td>
<td>South East</td>
<td>28.3</td>
<td>11.4</td>
</tr>
<tr>
<td>Redbridge</td>
<td>London</td>
<td>28.2</td>
<td>9.5</td>
</tr>
<tr>
<td>Ribble Valley</td>
<td>North West</td>
<td>27.1</td>
<td>7.2</td>
</tr>
<tr>
<td>Waverley</td>
<td>South East</td>
<td>27.0</td>
<td>10.4</td>
</tr>
<tr>
<td>Tunbridge Wells</td>
<td>South East</td>
<td>26.2</td>
<td>8.5</td>
</tr>
<tr>
<td>Winchester</td>
<td>South East</td>
<td>25.5</td>
<td>10.5</td>
</tr>
<tr>
<td>S. Norfolk</td>
<td>East</td>
<td>24.7</td>
<td>7.0</td>
</tr>
<tr>
<td>Area</td>
<td>Region</td>
<td>Earnings</td>
<td>Price</td>
</tr>
<tr>
<td>------------------</td>
<td>-------------</td>
<td>----------</td>
<td>--------</td>
</tr>
<tr>
<td>Brentwood</td>
<td>East</td>
<td>24.4</td>
<td>8.2</td>
</tr>
<tr>
<td>Hart</td>
<td>South East</td>
<td>24.1</td>
<td>9.9</td>
</tr>
<tr>
<td>E. Hertfordshire</td>
<td>East</td>
<td>23.9</td>
<td>9.4</td>
</tr>
<tr>
<td>Canterbury</td>
<td>South East</td>
<td>23.3</td>
<td>7.5</td>
</tr>
<tr>
<td>Solihull</td>
<td>West Midlands</td>
<td>23.1</td>
<td>7.5</td>
</tr>
<tr>
<td>Chiltern</td>
<td>South East</td>
<td>23.1</td>
<td>11.8</td>
</tr>
<tr>
<td>Harborough</td>
<td>East Midlands</td>
<td>22.8</td>
<td>7.7</td>
</tr>
</tbody>
</table>

The rankings are, however, affected by the choice of measure. In Table 4, those areas that have high average residence-based earnings will rank highly and, so, it is unsurprising that wealthy areas, usually outside large city centres, feature in the table. In addition, those areas where population is large relative to the housing stock – suggesting shortages – are likely to be higher in the rankings. But the choice of \((YHS)\), which uses employment rather than population, biases the results more towards employment centres and, indeed, spatially concentrates the required stock; the use of work-place rather than residence-based earnings will have a similar effect.

Given the importance of London as an employment centre, under an employment-based measure, the capital dominates the results, notably in the City, Westminster and Kensington and, therefore, the London boroughs are not shown in Table 5. Furthermore, because \((YHS)\) is much larger in the three districts, they would have disproportionate effects on the overall results and, therefore, zero weights are attached to them in the spatial weights matrix. But there is still a concentration of employment in large cities more generally and, as a result, in two-thirds of local authorities the optimal housing stock under the employment-based measure is lower than under the population-based measure\(^6\). However, for the third of (mainly urban) authorities where the stock is higher, the required increases are very large indeed. The table shows the top twenty authorities where requirements are largest, measured after three model iterations. The second column refers to the difference in the optimal stocks between the income and population-based cases; Manchester and other large cities lead the table. The third column standardises by the size of the housing stock and, on this measure, the special case of Crawley, which includes Gatwick Airport, is evident.

Tables 4 and 5, therefore, demonstrate the importance of the choice of affordability measure; population-based measures lead to a wider dispersion covering suburban areas, whereas the

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\(^6\) However, in many cases, the former would imply a reduction in the housing stock from current levels in suburban and rural areas, but, in practice, since population and earnings will grow over the future this is unlikely to be necessary.
employment-based metric generates greater concentration in large urban areas, although questions remain over the feasibility of the very large increases required.

Table 5. The Difference between Income and Population Based Measures

<table>
<thead>
<tr>
<th>Area</th>
<th>Differences in the Stocks (Nos.)</th>
<th>% of stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manchester</td>
<td>132571</td>
<td>60.8</td>
</tr>
<tr>
<td>Leeds</td>
<td>117387</td>
<td>34.6</td>
</tr>
<tr>
<td>Nottingham</td>
<td>85132</td>
<td>63.6</td>
</tr>
<tr>
<td>Newcastle upon Tyne</td>
<td>77960</td>
<td>63.2</td>
</tr>
<tr>
<td>Milton Keynes</td>
<td>60904</td>
<td>56.6</td>
</tr>
<tr>
<td>Crawley</td>
<td>58004</td>
<td>131.4</td>
</tr>
<tr>
<td>Bristol</td>
<td>54604</td>
<td>28.2</td>
</tr>
<tr>
<td>Trafford</td>
<td>51824</td>
<td>52.9</td>
</tr>
<tr>
<td>Exeter</td>
<td>50549</td>
<td>95.7</td>
</tr>
<tr>
<td>Warrington</td>
<td>46596</td>
<td>51.4</td>
</tr>
<tr>
<td>Cambridge</td>
<td>46146</td>
<td>90.3</td>
</tr>
<tr>
<td>Northampton</td>
<td>37048</td>
<td>39.4</td>
</tr>
<tr>
<td>Oxford</td>
<td>35668</td>
<td>61.5</td>
</tr>
<tr>
<td>South Gloucestershire</td>
<td>29765</td>
<td>26.1</td>
</tr>
<tr>
<td>Winchester</td>
<td>28772</td>
<td>57.1</td>
</tr>
<tr>
<td>Reading</td>
<td>27704</td>
<td>41.2</td>
</tr>
<tr>
<td>West Berkshire</td>
<td>27162</td>
<td>41.0</td>
</tr>
<tr>
<td>Peterborough</td>
<td>26820</td>
<td>33.3</td>
</tr>
<tr>
<td>Welwyn Hatfield</td>
<td>26655</td>
<td>57.3</td>
</tr>
<tr>
<td>Warwick</td>
<td>26240</td>
<td>42.6</td>
</tr>
</tbody>
</table>
5. Caveats and Policy Implications

Simple rules require simplifications; the question is whether the simplifications introduced by the modelling invalidate the central insights. First, the model assumes rules based on the contiguity of local authorities; as a first approximation, we would argue that this is a reasonable assumption since housing market areas typically are closely related, but it is not the case (an example was given in Section 2.3) that local authorities always co-operate fully with neighbouring districts. There is no conceptual difficulty in adjusting the spatial weights matrix to reflect actual co-operation, but there would be effects on the results; one of the characteristics of contiguity is that all authorities are connected either directly or indirectly because of the overlaps – the Isle of Wight and the Isles of Scilly are exceptions (and are omitted from the analysis above) - and, so, eventually, affordability converges, although only after an implausibly large number of iterations. Other configurations of housing market areas are likely to limit the indirect linkages and, so, reduce the degree of convergence and iterations. A related issue is that some authorities have larger numbers of contiguous authorities than others; for the latter, the rule implies that averages are calculated over a relatively small number of authorities, which may introduce a higher degree of volatility.

Second, the coefficients underlying the rule defined by equation (6) are derived from a national house price equation and there is some evidence, e.g. Meen (1999), that the coefficients, in fact, exhibit a spatial pattern; the elasticities in house price equations are, typically, higher in the South of the country than the North. However, from equation (5), as long as the elasticities of house prices with respect to income and the housing stock remain in the same proportions, the rule is unaffected; it is the relative rather than the absolute value of the coefficients that matter, although different absolute coefficients can affect the speed of adjustment parameter (1.5 in equation (6)).

Third, standard residential location theory indicates that prices often fall with distance from the centre of cities; since we are concerned with local authority averages and examine affordability only relative to contiguous authorities, arguably, this may not be too important quantitatively, but it is nevertheless the case that the rule does not allow for the fact that affordability is expected to be worse in major cities, reflecting the lower travel to work times and commuting costs. Fourth, the housing stock is measured in units; this is a data constraint that affects current projection methods as well as those examined here, but the more appropriate concept is the stock of housing services, which recognises that houses are not homogeneous and differ in sizes and characteristics. Fifth, it
might be argued that local authorities could collude under the rule in order to reduce their housing requirements; in fact, since the rules are based only on (publicly available) historical information, rather than future expectations, collusion is not possible, except, perhaps, in the choice of partner authorities. Sixth, the model makes no allowance for either legal constraints, such as Green Belt designation or physical constraints, for example, high proportions of rivers or mountains in an area. In fact, current housing assessments do not take into account Green Belt and, of course, the physical environment affects population as well as the desired housing stock.

Finally, and most importantly, the metrics need careful interpretation; \( PHS \) and \( YHS \) define the necessary increase (or decrease) in the housing stock in each district in order to move towards more consistent behaviour; this in turn improves national affordability and reduces the dispersion. But the ratios are defined for current values of the population (and earnings) in each area; in terms of the ratios, implicitly, this means that headship rates will rise – and households increase - as supply expands. In practice, since migration is strong across local areas (although limited within a housing market area), population is potentially endogenous and, so, changes in the housing stock could induce new population flows in addition to changes in headship rates. The metrics, as with current projection methods, do not allow for the induced population flows. This would require a wider, more complex, model in which migration was endogenised.

A related issue is that the calculations are concerned with changes in the current housing distribution in order to encourage greater consistency and equality, rather than with the conventional assessment of housing requirements for the future. Given future projections of population and earnings, future demand can certainly be calculated by these methods, but it is not the primary aim of this paper. Nevertheless, the paper has significant policy implications concerning the way in which affordability should be incorporated into housing projections. Any performance metric to be used in planning needs, we would suggest, three properties: (i) to be consistent with the theoretical and empirical evidence; (ii) to be easily and intuitively understandable by planners, government and the public; (iii) to be easily constructed and readily updatable. The proposed measures possess the three properties. Although slightly more loosely based on theory, our own preference would be the ratio of population to the dwelling stock, weighted by average earnings \( PHS \). This has some relationship to current methods, but uses population rather than households and concentrates on stocks instead of flows and explicitly allows for income differences, which are a
key driver of housing demand. But it makes no judgment about the number of households or the desired household size which, as noted above, are difficult to predict and removes unclear concepts.

However the results are inevitably controversial; the earnings weights and the reliance on population imply that more housing is required in the wealthier areas and these are likely to be on green field sites. The use of employment or workplace-based earnings will, instead, lead to greater concentration on urban sites, but the required redistribution is large in this case and there are questions of feasibility. Although not examined in this paper, a slightly less radical approach (but less aligned to the theory) might be to attach a lower weight to the spatial earnings differentials – in the above changes in earnings have the same weight as changes in employment or population. Nevertheless, the main point is that even using a more rigorous approach to account for affordability differences, there remains scope for judgement, which has a major impact on the results, notably between urban and rural development.

In practice, we would not recommend that these performance metrics should be the only basis for planning decisions – the outcomes are far too radical to achieve common acceptance. But there is a good case that they should become one of the measures that should be included and discussed as part of local authority housing plans, allowing each authority to nominate the members of its housing market area. If a district’s measures differ substantially from those of its neighbours, there is at least a case to be answered and this provides a starting point for negotiations. Furthermore, they can be quickly constructed and do not add to planning delays.

6. Conclusions

The National Planning Policy Framework requires local authorities to take account of market indicators such as affordability and, yet, the Framework and subsequent Planning Practice Guidance provide only limited help on how the policy should be implemented. This paper formalises the concepts and demonstrates what they might mean in practice for housing requirements through the development of performance metrics. These metrics have three key properties – they are consistent with theory and the empirical evidence; they are fairly easily understandable and are intuitively plausible with a relationship to current planning practice and; rely on a small number of easily available indicators, which are regularly published. The paper concentrates on two indicators; the
first is the ratio of population to the housing stock in each district weighted by average earnings and, so, captures both demographic and economic influences. The second is the ratio of employment to the housing stock again weighted by average earnings. The two measures have very different implications for the required distribution of the housing stock, but neither rely on conventional household projections; the latter leads to a greater concentration of the housing stock in large urban areas, whereas the former indicates greater requirements in suburban (largely wealthy) areas.

The paper suggests that, at the aggregate level, increases in housing supply alone are unlikely to stabilise affordability in the long run and, so, the paper concentrates instead on the distribution of the housing stock and its fairness, based on the relative performance of neighbouring authorities; in general, it might be expected that contiguous authorities should exhibit similar values for the metrics. Allowing for contiguity, in fact, improves national affordability and reduces the spatial dispersion, but it is important to understand what the results actually imply; they are not traditional long-run future projections of housing requirements, although these can be constructed by this method. Rather the rules and simulations show the required spatial distribution of the housing stock on the basis of current earnings and population, which can be compared with the current actual distribution. The paper also shows the differences in the distribution under the two metrics.

In practice, the proposals, if implemented in their strictest form, would be radical and are unlikely to be acceptable to the general public even though the intuition is evident. The changes are simply too great. Nevertheless, the results show the implications of taking affordability seriously and consistently; they are not a small adjustment to existing plans, but fundamentally change their nature. However, even if the metrics do not become the dominant force in planning, we would suggest that they provide a valuable additional indicator, as part of the planning process, which local authorities can quickly and easily calculate.

References


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Appendix: The Basic Life-Cycle Housing Model

The standard housing life-cycle model assumes two goods, housing services and a composite consumption good \((C)\). If, for simplicity, the flow of housing services is proportional to the demand for the housing stock \((H^D)\) and, given an assumed constant real discount rate \((r)\), lifetime utility is described in an infinite horizon continuous time by equation (1a):

\[
\int_0^\infty e^{-rt} \mu(H^D(t), C(t)) \, dt \tag{1a}
\]

\(\mu(H^D, C)\) denotes the instantaneous utility of the representative household. (1a) is maximised with respect to the budget constraint (2a) and technical constraints (3a) and (4a) which describe changes to real asset stocks (housing and financial, respectively) over time.

\[
g(t)X(t) + S(t) + C(t) = (1 - \theta(t))RY(t) + (1 - \theta(t))i(t)A(t) \tag{2a}
\]

\[
\dot{H}^D(t) = X(t) - \delta(t)H^D(t) \tag{3a}
\]

\[
\dot{A}(t) = S(t) - \pi(t)A(t) \tag{4a}
\]

where:

- \(H^D(t)\) stock demand for housing;
- \(g(t)\) real purchase price of dwellings;
- \(X(t)\) new purchases of dwellings;
- \(S(t)\) real savings net of real new loans;
- \(\theta(t)\) household marginal tax rate;
- \(RY(t)\) real household income;
- \(i(t)\) market interest rate;
- \(A(t)\) real net non-housing assets;
- \(\delta(t)\) depreciation rate on housing;
- \(\pi(t)\) general inflation rate;
- \(\dot{x}(t) \equiv \frac{dx(t)}{dt}\) denotes the time derivative for any variable \(x(t)\).

From the first-order conditions, the marginal rate of substitution between housing and the consumption good, \(\mu_H / \mu_C\), is given by (5a):

\[
\mu_H(t)/\mu_C(t) = g(t)\left[(1 - \theta(t))i(t) - \pi(t) + \delta(t) - \dot{g}(t)/g(t)\right] \tag{5a}
\]
The right-hand side in (5a) is the widely-used standard definition of the real housing user cost of capital (UCC) and represents the real price of housing services, where \( (\dot{g}^e(t)/g(t)) \) is the expected real capital gain.

If, for illustration, instantaneous utility takes the popular additive CRRA isoelastic form, \( \mu(C(t), H^D(t)) = [C(t)^{1-\gamma} + H^D(t)^{1-\gamma}]/(1-\gamma) \), then, \( H^D(t)/C(t) = UCC(t)^{(1/\gamma)} \), where UCC is equal to the right-hand side of (5a).

Aggregating, under CRRA utility, total demand for housing is given by the log-linear housing demand function (6a), which assumes that aggregate non-housing consumption is related to permanent income, RPY. In practice, in the main text, we use actual rather than permanent income on the grounds that in the long run (our main concern) actual income should proxy permanent income.

\[
\ln(H^D(t)) = \alpha_0 + \alpha_1 \ln[RPY(t)/HH(t)] - \alpha_2 \ln[g(t) \times CC(t)] + \alpha_3 \ln[HH(t)] + \varepsilon_1(t) \quad (6a)
\]

- \( HH(t) \) total number of households;
- \( RPY(t)/HH(t) \) per household permanent income;
- \( \varepsilon_1(t) \) error term.

\[
CC(t) = (1 - \theta(t))i(t) - \pi(t) + \delta(t) - \dot{g}^e(t)/g(t) \quad (7a)
\]

\[ \alpha_1 = 1.0; \quad \alpha_2 = \frac{1}{\gamma}; \quad \alpha_3 = 1.0. \]