

# **Automating the implementation of an equilibrium profile model for glacier reconstruction and Equilibrium Line Altitude calculation in a GIS environment**

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## **1. Introduction**

Reconstruction of glacier equilibrium line altitudes (ELAs) associated with advance stages of former ice masses is widely used as a tool for palaeoclimatic studies. This requires an accurate reconstruction of palaeo-glacier surface hypsometry, based on geomorphological evidence. Classically, the approach used to define ice-surface elevations follows the ‘cartographic method’, whereby contours are estimated based on an ‘understanding’ of the typical surface form of contemporary ice masses. This method introduces inherent uncertainties in the palaeoclimatic interpretation of reconstructed ELAs, especially where the upper limits of glaciation are less well constrained and/or the age of such features in relation to terminal moraine sequences is unknown. An alternative approach is to use equilibrium profile models to define ice surface elevations. Such models are tuned, generally using basal shear stress, in order to generate an ice surface that reaches ‘target elevations’ defined by geomorphology. In areas where there are no geomorphological constraints for the former ice surface, the reconstruction is undertaken using glaciologically representative values for basal shear stress. Numerical reconstructions have been shown to produce glaciologically “realistic” ice surface geometries, allowing for more objective and robust comparative studies at local to regional scales.

User-friendly tools for the calculation of equilibrium profiles are presently available in the literature. Despite this, their use is not yet widespread, perhaps owing to the difficult and time consuming nature of acquiring the necessary inputs from contour maps or digital elevation models. Here we describe a tool for automatically reconstructing palaeo-glacier surface geometry and ELA calculation from the resulting DEM.

## **2. Palaeoglacier reconstruction**

The approach presented here for palaeoglacier reconstruction is based on Benn and Hulton

(2010) using Nye's (1952) method, which assumes a perfectly plastic ice rheology. This method assumes that a glacier's thickness on a horizontal bed at any distance from a known margin is set by the formulas:

(1)

$$h = \sqrt{2h_0s}$$

(2)

$$h_0 = \frac{\tau}{\rho g}$$

Where 'h' is the ice thickness on a 0 bed surface (m),  $\tau$  is the basal shear stress in hpa (hectopascals), 's' is the horizontal distance to the margin (m),  $\rho$  is the density of the ice (set to a constant of 900 Tons x m<sup>-3</sup>) and 'g' (9.81 m x s<sup>-2</sup>) is the gravitational acceleration. Benn and Hulton (2010) presented an Excel spreadsheet that applies this formula in iterative steps along a glacier flowline, allowing an irregular bed topography to be considered. The resulting formula for each step along the glacier profile is:

(3)

$$h_{i+1}^2 - h_{i+1}(B_i + B_{i+1}) + h_i(B_{i+1} - H_i) - \frac{2\Delta x \bar{\tau}_Y}{\rho g} = 0$$

This is solved as a quadratic equation:

(4)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where:

$$a = 1$$

$$b = -(B_i + B_{i+1})$$

$$c = h_i(B_{i+1} - H_i) - \frac{2\Delta x \bar{\tau}_Y}{\rho g}$$

Being:

'h' the ice surface elevation and 'B' the bed elevation.

The Benn & Hulton (2010) spreadsheet applied this tool with two levels of user inputs and therefore two degrees of resultant accuracy. The first level assumes a constant, user defined basal shear stress (between 50000 and 150000 hpa) along the glacier flowline. This approach is sufficient for reconstructing the palaeoglacier surface of small, simple glaciers, but will fail for complex glaciers and ice caps due to several reasons:

- Constrictions in the valley topography reduce the cross sectional area available for ice flux, resulting in an increase in ice thickness along the flowline.
- The model will continue to artificially thicken glaciers in flat ice caps.
- It does not allow for the ‘tuning’ of glacier thickness to available landform evidence which indicates the former glacier surface.
- It neglects variations in basal shear stress due to the increased/reduced influence of basal sliding as a component of ice flux or the variability of yield stress over soft sediments.

A second level in the model allows users to define a variable shear stress and apply a scaling factor which accounts for topographic constrictions. These changes are entirely user-defined, making the procedure time consuming for large ice masses. Additionally, there is little guidance as to the magnitude and location of changes to basal shear stress required to fit the model to landform evidence. The iterative process in the model means that varying combinations of shear stress variability along the flow-line can be sufficient to match the model to the landform evidence, although with different resultant geometries across the length of the glacier.

## **2.1. Palaeoglacier automatic reconstruction**

Here we present a tool which implements the Benn & Hulton (2010) model in ArcGIS. The only necessary inputs for this tool are 1) a relevant digital terrain model (DTM), 2) mapped outlines of the former glacier terminus position (usually a frontal moraine system) and any relevant geomorphological constraints on ice surface elevation (e.g. lateral moraines, trimlines etc.) and 3) a line feature class defining the likely flow lines of the former glacier. This is only necessary if the glacier developed based on more than one source area (for example several cirques from where ice tongues developed and finally joined to a main one). These flow lines have to be drawn from the glacier terminus up valley, lateral tributary flow lines must go from the coalescence point to the top, and be contained as a different feature within the same feature class (no multipart features). A fourth 4) element can be optionally added: transect lines across observable topographic constrictions.

The procedure for glacier surface reconstruction is stepwise. The first step calculates the glacier surface without shear stress and  $f$  factor modulation. This first step is based on the fact that a glacier occupies a single watershed. The lowest point along the frontal moraine system is defined as the pour point for the glacier watershed. If a flow line has not been defined, the trunk stream defined by the flow accumulation raster is used as an input; otherwise this is user-defined. Next, the script creates regular and ordered points along the flow line from bottom to top. Thirdly, distance between points, total distance to the terminus, bed altitude, the coefficients in the Equation 4 and the solution of that formula for each point is calculated, providing the reconstructed glacier thickness at each given point. Fourthly, a DEM is created using the calculated point altitudes (basal altitude + ice thickness) as an input and the

watershed as a limits as the DEM. Finally the reconstructed glacier DEM is cut against the terrain DTM, providing a surface where the reconstructed ice thickness along the flowline is above the surrounding terrain (Figure 1).

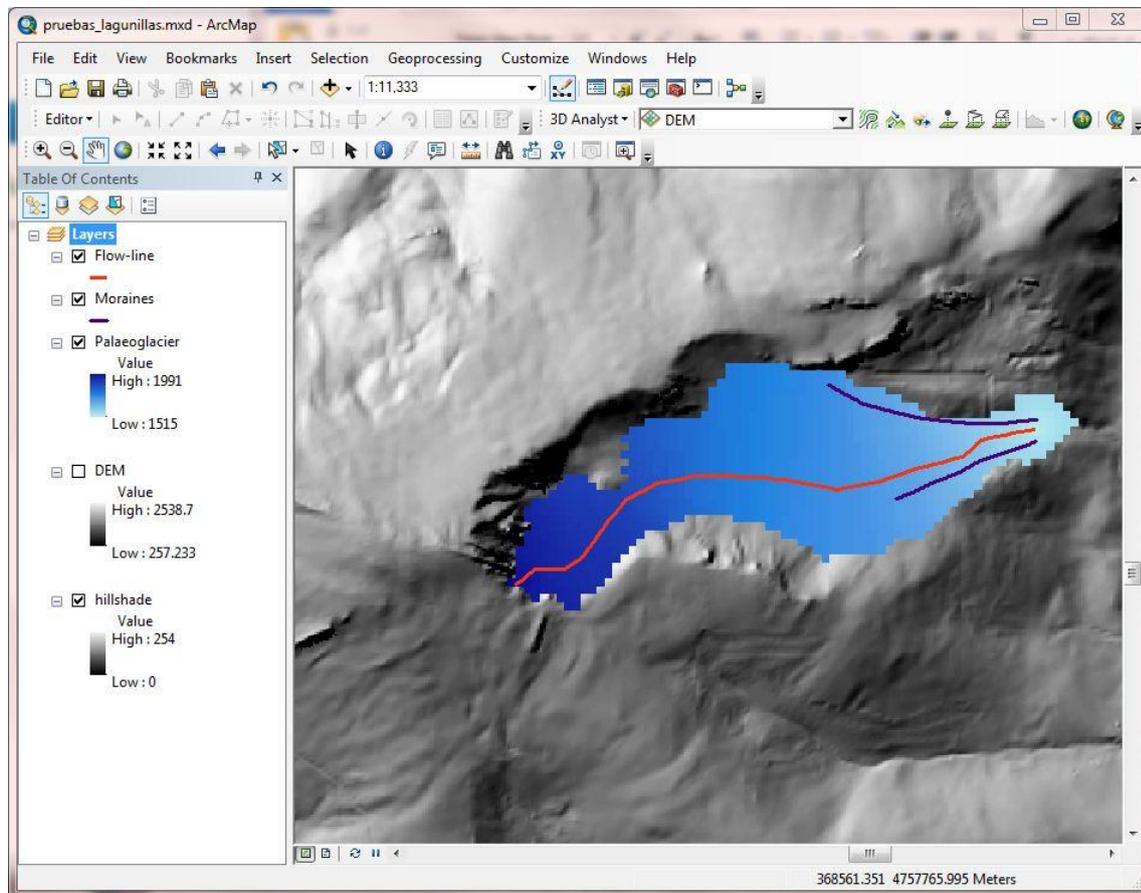


Figure 1. Palaeoglacier surface reconstructed from 1) flow-line, 2) moraines and 3) DEM.

The second step calculates the  $f$  factor along user defined transects and applies this to the surface reconstructed in step 1. The third step, currently under development, runs a Monte Carlo simulation of basal shear stress with a uniform probability distribution for changes to both the magnitude of basal shear stress and the location of change. An additional output is provided: the distribution of basal shear for the resultant reconstructed surfaces that best match the available landform evidence. This can be compared to any evidence on the valley floor that suggests a significant change in the conditions controlling basal shear stress.

### 3. ELA calculation from palaeoglacier DEM

Additionally, we present a tool for automatically calculating ELA, using either the Accumulation Area Ratio (AAR) or Area Altitude Balance Ratio (AABR) method and the newly generated palaeoglacier surface as an input. Both AAR and AABR are the most used methods for ELA calculation on palaeoglaciers, as they take into account glaciers mass balance (Osmaston 2005) and have been successfully tested against contemporary glaciers (Rea 2009, Kern and Lazlo 2010). This tool follows the Osmaston (2005) calculation procedure for ELA AABR and has been inspired by Gonzalez-Trueba and Serrano (2004) paper for AAR calculation.

The AAR method assumes that the accumulation area occupies a given ratio of the total glacier surface area which largely depends on climatic regime and the mass balance gradient of the glacier itself. Steady state glaciers typically range between 0.5 and 0.8 (González-Trueba and Serrano 2004), and a 0.67 ratio is considered a realistic value for alpine glaciers (Kern and Laszlo 2010).

The AABR method is considered more exact than AAR for it also accounts for glacier surface hypsometry throughout the glacier's surface and its contribution to the mass balance, so further areas to ELA contribute more to mass gain or loss than nearer ones (Osmaston 2005, Rea 2009). The mass balance contribution in relation to the ELA is weighted through a ratio, which also varies depending on the glacier's environment. Rea (2009) empirically suggested some ratios, giving a 1.69 general ratio as the least error generating for all glaciers. In any case both AAR and AABR use accumulation-area ratios which are chosen by the user, relying on previous experience or empirically built suggestions.

The toolbox uses as an input one (or several) DEM(s) representing the glacier(s) surface, which can be of any supported raster datasets in ArcGIS. An existing folder is required to store the results. Two parameters are needed in addition to the DEM and the folder. The first key parameter is the interval altitude for glacier surface area measurements. The interval has been set automatically to 50 meters, but the user can reduce or increase it as needed. A larger interval will reduce final accuracy, as the minimum error is equal to the measurement interval. The second key parameter is the ratio. This is user defined taking into account the explanations given above. Default values of 0.67 for AAR and 1.75 for AABR have been set following the aforementioned literature.

The script mainly relies on the results of the SurfaceVolume\_3D tool, which is embedded in the 3D Analyst package. This tool calculates the real surface ABOVE or BELOW a given threshold. Results from this tool are stored in a .txt file. The script first runs the tool to calculate the surface ABOVE (for AAR) and BELOW (for AA and AABR) thresholds as many times as needed given the altitude range and the interval, stores the results in a .txt whose name is the same as the glacier surface DEM (so user can check the result performance) and reads the resulting text file to get the relevant information for ELA calculation. Two values are used for calculation: the reference altitude and the surface value for that altitude.

For AAR both values are stored in a dictionary, and then the key surface is calculated from the total surface and the purposed ratio. Finally the script goes through the dictionary and finds the altitude whose surface value is immediately higher than the key surface. This yields an altitude value whose maximum error is the range of altitudes used.

The AABR method does not calculate the glacier as a whole, but splits it into strips, so that the relation of area/altitude is taken into account. First, the script calculates the surface value BELOW each reference altitude, and then a list of altitudes situated at the mean altitude value for each strip is retrieved. Then the real surface for each strip is calculated, and the result is multiplied by the mean altitude of the strip. All the results are added and divided by the total surface of the glacier. The result is the AA value, which is equivalent to the AABR value with the Balance Ratio set to 1.

In order to model differences in Balance Ratio the script performs some further calculations: a trial altitude is created and is iterative subtracted to the mean belt altitude of each belt and multiplied to each belt's area. If the result of the operation is negative it is multiplied to the given Balance Ratio. All multiplications are added yielding a result which is initially positive, but decreases on iterating along different trial altitudes. When for the first time the result is negative, the ELA is situated in the belt below the trial height. The script will show the lower limit of that belt as the ELA, so again an error equal at most to the range interval is committed. Results are directly shown in the dialog box alongside some information about the script's performance (Figure 2). The scripts have been conveniently commented so users can follow their rationale and eventually adapt it to their needs.

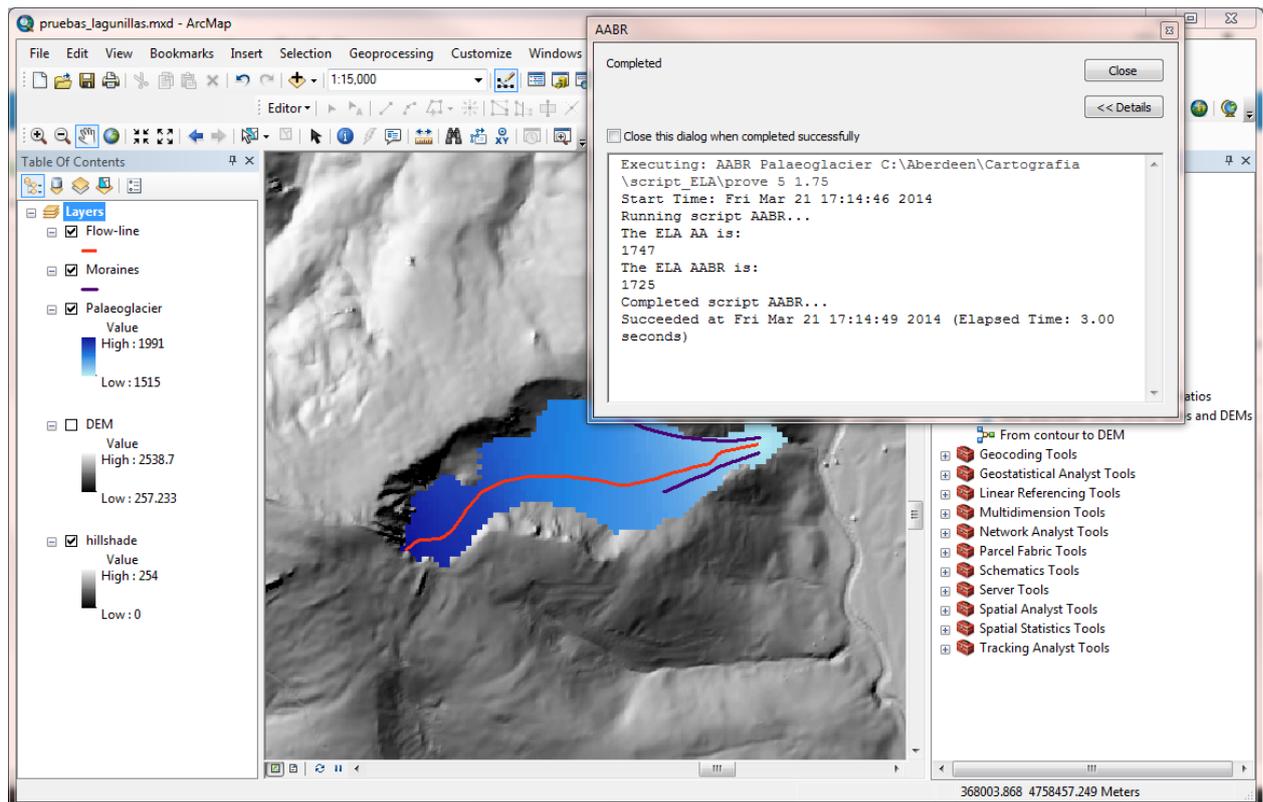


Figure 2. ELA calculation for the reconstructed palaeoglacier with the AA method and the AABR method at a 1.75 ratio.

#### 4. Tool performance and conclusions

The tools described above are being tested on a large set of glaciers. Palaeoglacier reconstruction works well on simple valley glaciers, where the difference between well constrained manual reconstructions ELA and the modelled reconstructions is negligible. More extensive glaciers are sensitive to shear stress variability in terms of the resultant reconstructed ELA. Therefore it is essential to develop a way to reliably model shear stress change along the glacier. The tool will be tested against contemporary glaciers, using bed topography derived from ground penetrating radar transects as an input. The ELA measurement tool is quite simple and has been fully tested. It works neatly and its error is always subject to the interval calculation chosen.

This toolset provides a standardised method for glacier reconstruction that can be quickly and

systematically applied to large geomorphological datasets. Potential applications include the calibration of currently used ratios for ELA calculation and the systematic reconstruction of palaeo-climates from palaeo-glaciological data, an approach which is currently being implemented for the Younger Dryas and Little Ice Age.

## 5. Acknowledgements

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## Biography

*Ramón Pellitero is a postdoctoral researcher at the University of Aberdeen. He completed his PhD with a thesis on Quaternary glacial and periglacial geomorphology in N Spain, as well as its climatic significance. Now he is engaged in a UoA led international project that investigates glacier-climate relationships and atmospheric circulation during the Younger Dryas (YD) (~12,000) across Europe.*

*Craig Frew is a PhD student at the University of Aberdeen, currently involved with mapping variations in the timing and magnitude of 'Little Ice Age' (LIA) glacier advances throughout the European-North Atlantic Region. To do this he has been working closely with colleagues in the Aberdeen Cryosphere and Climate Change research group to develop programmatic tools that enable the systematic calculation of palaeo-glacier Equilibrium Line Altitude (ELA) from geomorphological data.*