A Gentle Introduction to Dynamical Systems Theory for Researchers in Speech, Language, and Music.

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[1] Dynamical Systems Theory (DST) is the lingua franca of Physics (both Newtonian and modern), Biology, Chemistry, and many other sciences and non-sciences, such as Economics.

To employ the tools of DST is to take an *explanatory stance* with respect to observed phenomena. DST is thus not just another tool in the box. Its use is a different way of doing science.

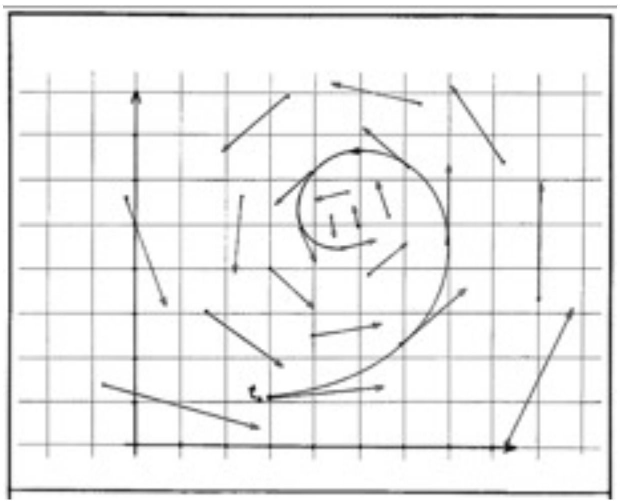
DST is increasingly used in non-computational, non-representational, non-cognitivist approaches to understanding behavior (and perhaps brains). (Embodied, embedded, ecological, enactive theories within cognitive science.)

[2] DST originates in the science of mechanics, developed by the (co-)inventor of the calculus: Isaac Newton. This revolutionary science gave us the seductive concept of the mechanism.

Mechanics seeks to provide a deterministic account of the relation between the *motions* of *massive bodies* and the *forces* that act upon them.

A dynamical system comprises
• A state description that indexes the components at time *t*, and
• A dynamic, which is a rule governing state change over time

The choice of variables defines the state space. The dynamic associates an instantaneous rate of change with each point in the state space.



Any specific instance of a dynamical system will trace out a single trajectory in state space. (This is often, misleadingly, called a *solution* to the underlying equations.)

Description of a specific system therefore also requires specification of the *initial conditions.*

In the domain of mechanics, where we seek to account for the motion of massive bodies, we know which variables to choose (position and velocity).

The description of the lawful change of state over time extends *far beyond the domain of mechanics.*

In most domains, and for most systems, we will not have an explicit dynamic available to us. But the language of dynamics continues to be of use in describing systems and their interactions.

[3] Example: A Tornado

A tornado exists in the domain of atmospheric phenomena, and its components are ultimately the molecules in the atmosphere.

It comes into being (or *self-organizes*) under just the right conditions (= *constraints*).

While it persists, elements within the system are non-independent. Together, they constitute a transient *identity* that can be distinguished within its domain.

We will never have a set of explicit equations describing the motions of the molecules, but we can still ask important questions:

• Is the system stable?
• Which constraints allow the system to come into being, and when does it cease to exist?
• How does it respond to perturbations?
• How will two or more systems interact?

[4] Of Systems and Components: No level is privileged

We must distinguish between the level of the system, and its components.

WIthin a system, the degrees of freedom of the components are drastically reduced, due to their mutual non-independence.

In complex systems, no single level has priority. A system formed at one level out of may components may, itself, function as a component in a superordinate system.

If the variables we choose to observe do not constitute a system in this formal sense, then models will fail. This is why economic models fail. Always.

[5] A Well-worked Out Example: The Haken-Kelso Bunz model (1985)

In the HKB model, the components are the two hands/fingers. We define the domain as the hands/fingers constrained to wag periodically. We set a task constraint of wagging the hands/fingers at identical frequencies.

Under these constraints, the hands become non-independent, and there are only two stable patterns observed.

Pattern 1 (in phase; relative phase = 0): the hands go through their cycles in lock step, each starting the cycle (say at maximal extension towards the body midline) at the same time.

Pattern 2 (anti-phase; relative phase = π): As one hand starts its cycle, the other is precisely half way through its cycle. This is akin to a syncopated rhythm, where the drums and the guitar may be half a cycle out of step with each other.

The superordinate system (2-hands-together) has less degrees of freedom than the components considered individually. The mathematical exegesis presented in the famous 1985 paper (not for the faint hearted) shows how 2 components, each with 2 degrees of freedom (position & velocity of each hand considered separately) collapses to an equivalent system with a single degree of freedom (relative phase).

Note: the components are never atomistic. Each component is, itself a complex beast made out of many smaller components. Turtles all the way down.

[6] Synergies

One of the most remarkable findings in the study of behavior is that bodies (or bodies + tools) often act as if they were much simpler systems - as if they were special purpose mechanisms designed precisely for the skilled task at hand.

These as-if mechanisms are sometimes called Coordinative Systems, or Synergies.

Bernstein (1930, reported in Latash, 2008) studied variability in the movements of skilled blacksmiths repeatedly hitting an anvil. He found that the diverse body parts involved worked together as a unit, or mechanism, and not as an externally controlled system. Variability at the hammer tip was *less* than variability of individual joints that are linked in a segment chain from shoulder to hand. Thus no part of this on-the-fly mechanism is independent of the other.

Kelso et al found something similar in the yoking together of the articulators in speech. A perturbation (downward kick) administered to the jaw during the final segment of either
/baz/ or /bab/ produced an almost instantaneous compensatory reaction. The reaction was in the upper lip for /bab/ and in the tongue for /baz/. This shows that the articulators are transiently yoked together into a synergy.

[7] The Embodied Task Dynamic model (Simko & Cummins, 2010, 2011) implemented such synergies in a simplified model of the vocal tract. The starting point was the gestural score, as found in Articulatory Phonology (Browman & Goldstein, many refs). This prior model produces nice fluent yoked movement of the articulators based on the gestural score. An outstanding problem has been how to generate the score (on and off times for individual gestures).

We found that by embodying the articulators (treating them as massive bodies with inertial properties), we could constrain their movement to be optimal in a well-defined sense (minimize energy, maximize communicative efficiency). This served to allow timing to be an emergent property of the embodied system.
[7] Some Useful Dynamical Concepts

Identity

Not all observables can be characterized as dynamical systems. They must exhibit mutual non-independence, greatly reduced degrees of freedom, and lawful change over time. A handful of stock prices, for example, will not display this lawfulness.

A dynamical system has a transient identity. It comes about under certain conditions (constraints).

Stability

Is an observed behavior stable?

Will the system return to that behavior after a perturbation?

How many stable behaviors does the system exhibit?

What are the hallmarks of a shift from one stable behavior to another?

Attractors

If, over the long term, the system occupies only a limited part of its state space, and if, when perturbed to another part, it returns to the former, that part is called an *attractor*.

There are exactly three kinds of attractors in deterministic dynamical systems.

Point Attractors: trajectories converge to a single point in state space. Example: a pendulum will come to rest at the bottom of its arc.

A point attractor in state space does not mean the components are unchanging. The fixed relative phase observed in the HKB finger wagging task is a point attractor in state space (relative phase), but the components are still wagging.

Limit Cycle Attractors: An invariant trajectory is repeatedly traced out in state space. Any system displaying a limit cycle behavior may be called an oscillator. Phase is straightforward to define for a limit cycle. (See *Terminology* below).

Chaotic Attractors: The system occupies a finite subset of state space, but without repetition. Two trajectories that start very close together will diverge rapidly, losing all mutual predictability.

Parametric variation and structural stability

Change in a system is always a function of the state of the system *and* the constraints that bring it into being. Those factors that remain unchanged while we observe the system are its *parameters*. Examples: Pendulum (gravity, mass, length); HKB (rate of oscillation).

If we change the parameters, we may change the qualitative characteristics of the system. E.g., in the HKB model, at fast rates there is only one attractor (relative phase = 0).

Attractors may appear, disappear, or may change from one form into another.

These kinds of qualitative change in dynamical organization within complex systems are accompanied by entirely generic signatures: Increase in variability (critical fluctuations), and a temporary *increase* in the degrees of freedom.

These hallmarks do not depend on the physical instantiation of the system. Many characteristics of dynamical systems are entirely generic, and are found in domains from matter to social organization and beyond.

If small parameter changes alter the qualitative dynamics, we say the system is *structurally unstable*.

Coupling/Entrainment

If two dynamical systems interact with each other, and the composite system is simpler than the sum of the two systems, then they can be said to be coupled, or entrained.

The mathematics of entrainment is best worked out for oscillatory (limit cycle) systems, but the phenomenon is *not* restricted to such systems (e.g. flocking and shoaling behavior in birds and fish).

Coupled oscillator models are currently popular, but they should be interpreted with a degree of caution: Are the systems actually systems in the above sense? Do they each display intrinsic limit cycle dynamics? Are they causally influencing one another?

Entrainment can be mutual, or one-way, as when your endogenous body rhythm of 23-25 hours is entrained by the sunlight and your activity levels to a fixed 24 hour cycle. (Pro tip: jetlag is best fought by exposing yourself to sunlight.)

An example of non-periodic entrainment is found when two speakers read a text in synchrony with one another. The degree of synchrony observed speaks of tight entrainment. The superordinate dyadic system is brittle, and speech errors can cause the complete cessation of speaking. (Cummins, 2011) The system is best understood as a superordinate dyadic system with no central locus of control. This contrasts strongly with received models within the cognitivist/information processing/computational tradition.

Further Reading

Abraham, R. and Shaw, C. (1983). *Dynamics, The Geometry of Behavior, Part 1*. Aerial Press, Santa Cruz, CA. Great introduction from the very basics to advanced topics, all done through lavish illustration and no equations.

Chemero, Anthony (2010) *Radical Embodied Cognitive Science*, MIT Press. This is not an introductory text, but it provides a compelling case that dynamical systems thinking is essential in modern cognitive science.

Kelso, J. A. S. (1995). *Dynamic Patterns: The Self-Organization of Brain and Behavior*. MIT Press, Cambridge, MA. This book summarizes one of the best worked-out examples of the application of dynamical systems modeling to human behavior. Relatively little on speech, but provides a good foundation with which to tackle subsequent work in dynamical modeling of speech.

Norton, A. (1995). *Dynamics: an introduction*. In Port, R. F. and van Gelder, T., editors, *Mind as Motion: Explorations in the Dynamics of Cognition*, chapter 1, pages 45–68. Bradford Books/MIT Press, Cambridge, MA. This is a brief chapter that covers the basic mathematics. The book itself is a widely eclectic set of papers employing dynamical concepts, but without a unified approach.

Some Terminological Clarification

I found the following terms confusing when I first met them. This brief note may help.

Dynamics vs Kinematics

In the study of bodily movement, a distinction is often made between dynamics and kinematics. Kinematics describes the surface form of the movement - recording change of position and velocity over time. A dynamical account relates such observations to the underlying forces that give rise to them in accord with natural law. So there is no dynamical description available for the implausible motion of a cartoon character, but there is for any embodied being.

Phase

Phase refers to the position of a variable as it moves through a cycle. That is simple enough, but various conventions conspire to confuse the unwary. Many authors describe position in the cycle with a number in the range 0..2*pi, so that the halfway point is at pi radians. Others choose to place 0 in the middle of the range, and so phase ranges from -pi..pi. Yet others choose to describe it with numbers in the range 0..1. It makes no difference which convention you adopt, but try to be explicit and consistent.

Matters are not helped by two entirely different senses of the term phase. Firstly, in mechanics, where observed variables naturally come in pairs of position and velocity, the state space is, by convention, called a phase space. This usage has nothing to do with the previous sense, and is just an unfortunate historical accident. Engineers and physicists will often use the term phase-space for what I have called state space.

A third sense (confused yet?) is in the term “phase transition”, which comes from material physics, and refers to a global qualitative change in the properties of a system, as when ice changes to water. The term “phase transition” is sometimes used in an extended sense to indicate a change in the qualitative dynamics of a system as some control parameter is varied, e.g. rate in the HKB model.

Parameters and Variables

There is little consistency in the field here. In an ideal world, the term ‘variable’ would be reserved for the observed quantities that change over the timescale of observation. Position and velocity are common variables. Parameters are (relatively) fixed quantities that do not vary over the time scale of observation. Thus, for some purposes, interest rate might be considered a parameter (if I seek to understand the daily depreciation of my savings, for example) whereas for other purposes, it might be a variable (in observing the long-term behavior of the economy).

But usage is inconsistent, and you may need to attend to the context to figure out which is meant. Often, we are interested in the way that a dynamical system is affected as a parameter is varied (pendulum: change the mass; HKB: change the rate of oscillation). A parameter that is experimentally controlled like this is usefully called a control parameter.

Order parameters and collective variables

In the study of complex systems, a single variable that is observed is typically itself a complex index of many lower level components. Your heart rate, for example, is a single observable quantity that arises from the interaction of many bodily processes and components. The terms ‘order parameter’ and ‘collective variable’ are used interchangeably, and confusingly, to refer to any such quantity. In the HKB parlance, the relative phase that captures the macroscopic state of the entire system is often referred to by either name.

References

Cummins, F. (2012) Periodic and aperiodic synchronization in skilled action.
Frontiers in Human Neuroscience: **5**:170; doi: 10.3389/fnhum.2011.00170

Latash, M. (2008). *Synergy*. Oxford University Press, USA.

Simko, J. and Cummins, F. (2011). Sequencing and optimization within an embodied task dynamic model. *Cognitive Science*, 35(3):527-562.

Simko, J. and Cummins, F. (2010). Embodied task dynamics. *Psychological Review*. 117(4):1229-1246.