Informal Labor Markets and Macroeconomic Fluctuations

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Abstract: This paper examines the adjustment of developing country labor markets to macroeconomic shocks. It models a two sector labor market: a formal salaried (tradable) sector that may or may not be affected by union or legislation induced wage rigidities, and an unregulated (nontradable) self-employment sector facing liquidity constraints to entry. This is embedded in a standard small economy macro model that permits the derivation of patterns of comovement among relative salaried/self-employed incomes, salaried/self-employed sector sizes and the real exchange rate with respect to different types of shocks in contexts with and without wage rigidities. The paper then explores time series data from Argentina, Brazil, Colombia and Mexico to test for cointegrating relationships corresponding to the patterns predicted by theory. We identify two types of regime. The first corresponds to periods where demand shocks to the nontradable sector offer new opportunities to (informal) entrepreneurs, the informal sector expands “procyclically,” and the exchange rate overshoots toward appreciation in the short run, or remains at its productivity determined levels. The second corresponds to periods of negative shocks to the formal salaried sector in the presence of wage rigidities where the sector plays a more traditional “buffer” role during downturns.

JEL: Informality, Labor market dynamics, Self-employment, Real exchange rates.

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1. Introduction

The debate over the role of informal workers - those unprotected by labor legislation - in the developing country labor market goes back almost half a century. A prominent stream of the literature has intellectual roots perhaps best distilled in Harris and Todaro’s (1970) vision of markets segmented by wage setting in the formal sector that leaves the traditional sector rationed out of modern salaried employment.\(^1\) The view of the informal sector as the inferior segment of a dual labor market, expanding during downturns to absorb increased unemployment, became highly influential in the International Labor Organization, its Latin America affiliate, the Latin America Regional Employment Program (PREALC), and the World Bank.\(^2\)

However, dating at least from Hart’s (1973) work in Africa, a parallel stream has stressed the sector’s dynamism and the likely voluntary nature of much of the entry into informal self-employment, analogous to the mainstream literature such as Jovanovic (1982), and Evans and Jovanovic (1989), and Evans and Leighton (1989).\(^3\) Recent work has called into question the value of the conditional income comparisons, commonly used to demonstrate segmentation, both on conceptual and empirical grounds.\(^4\) Further, a first look at time series for Mexico suggests more nuanced cyclical behavior than that of a shock absorber during downturns. Figure 1 plots the evolution of the relative salaried/informal self-employed sector sizes and respective earnings and shows that during the recovery of 1987-1991 both the relative size of the informal self-employed sector relative to the formal sector, and the relative earnings of the self-employed rise, consistent with a procyclical expansion of that sector. Since roughly 75% of the informal self-employed are found in services, transportation or construction, it is plausible that the boom in real estate and other non-tradable industries across this period created new opportunities for entrepreneurs who, for whatever reason, chose to be informal.\(^5\)

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1. In fact, in Harris and Todaro’s model, the “traditional” sector was the rural sector disposed to migrate. However, it represents perhaps the first analytically worked out view of the dual labor market and remains highly relevant to the debate over the segmented rural sector.

2. For early statements, see Sethuraman (1981), Tokman (1978), Mazumdar (1975), respectively.


4. See Maloney 1999, Pratap and Quintin 2006

5. The self-employed are concentrated in nontradables: Brazil 92%, Colombia 87%, Mexico 83%.
statistical anomaly is suggested by Loayza and Rigolini’s (2006) recent finding of pro-cyclical movement in the informal sector in several of their sample of 42 countries. However, it is also the case that the subsequent period leading up to the crisis of 1995, the countercyclical movements envisaged by more traditional segmentation views appear, manifested as a negative comovement of earnings and labor market sector sizes.

These distinct patterns suggest that the pro- or countercyclicality of the two labor market sectors may depend on the sectoral origin of the shocks, and the presence of binding wage rigidities. They also suggest that time series data on these series may offer potentially useful labor market diagnostics, for instance, in identifying the roots of expansion of the informal sector across a given period. However, for this to be the case, we need to understand the drivers of the very large observed movements in relative wages which in a textbook world, would be forced to equivalence. Three effects in principle may be at play: barriers to the arbitrage of labor earnings due to barriers to entry to either sector, barriers to arbitraging of returns to capital of the self-employed which are generally not separable in labor market surveys from earnings of labor per se, and changes in the skills composition of the sectoral work forces.

To capture these effects, we begin by constructing a model of developing country labor markets that is firmly rooted in the established advanced country literature. We postulate two sectors: a salaried (tradable) sector where workers receive a wage and are covered by labor legislation or unions; and a nontradable self-employed sector of the kind postulated by Lucas (1978) with heterogeneity in level of entrepreneurial ability and where, following Evans and Jovanovic (1989), credit constraints can constitute a barrier to entry from salaried work. Self-employed workers receive a variable return to invested capital and their labor which, due to capital adjustment costs arising from credit constraints, may deviate from long run equilibrium levels.

We locate this labor market in a standard macroeconomic framework (Obstfeld and Rogoff 1996) that allows us to capture additional information on the sectoral origin of the shocks through the real exchange rate - a measure of relative prices of tradables.
and nontradables. This allows us to move beyond simply defining cyclical movements as a deviation from trend and to characterize the nature of the shocks driving it. Given the high concentration of the informal self-employed in the nontradables sector, we are able to derive patterns of comovement between the relative returns and relative sizes of salaried and self-employed sectors, and the real exchange rate.

Finally, we introduce potential wage rigidities in the salaried tradable sector. As in the classic Harris-Todaro formulation, formalized in Rauch (1991), the labor market can become segmented with workers rationed out of salaried/tradable employment and being forced into the self-employed/nontradables sector where earnings adjust to equate labor supply and demand. This segmentation gives rise to distinct patterns of comovement of the three series in response to productivity or demand shocks.

Thus, we provide an integrated model of LDC labor markets that permits developing a typology of comovements of macroeconomic time series that, once identified, can help identify the source of shocks and the presence or absence of formal sector segmenting distortions. The latter offers an alternative to unreliable conditional income comparisons. Empirically, we employ multivariate cointegration techniques to establish these predicted patterns of comovement and their evolution over the last two decades in Argentina, Brazil, Colombia and Mexico. These countries all have large informal self-employed sectors, and have experienced very large movements in levels of economic activity, the relative sizes of the two labor market sectors, and real exchange rates.

We confirm episodes of expansion of informal self-employment consistent with the traditional segmentation views. However, we also identify episodes consistent with

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6 Total returns to informal self employment and salaried employment incorporate differences in taxes, risk premia, flexibility, etc all of which will lead to incomes not being equated, even in the absence of segmentation. See Maloney (1999).

7 In Mexico from 1988-1995, Argentina 1990-1995, and Brazil beginning in 1992, the exchange rate appreciated, often dramatically, following stabilization policies that fixed the nominal exchange rate, liberalized capital markets, and implemented other reforms.
the sectoral expansion being driven by a positive demand shock to the nontradables sector and “procyclical” behavior of the informal self-employed sector.

2. A Model

We consider the case of a small economy that produces two composite goods, tradables and nontradables. The salaried sector is assumed to produce tradables (T), the numeraire, while the production of nontradables is concentrated in the self-employed sector (N)\(^8\). All workers are homogenous when salaried. However, following Lucas (1978), self-employed sector individuals (j) differ in terms of entrepreneurial capability, \(\phi_j\) distributed uniformly on \([0,1]\). For simplicity, we also normalize the labor force to unity so that, provided that the economy is not in a corner solution, the value of entrepreneurial ability of individual \(m\), who is indifferent between salaried work and self- employment, also corresponds to the size of the salaried labor force. That is, \(\phi_m = \phi^* = L_T\) where \(\phi^*\) is the ability of the individual who is indifferent between self-employment and wage work. Thus, we preserve the overall labor supply constraint while building in a decrease in the marginal entrepreneurial ability as labor shifts toward self-employment.

 Tradable output \(Y_T\) is CRS in capital \(K_T\) and labor \(L_T\):

\[
Y_T = A_T F(K_T, L_T) = A_T K_T^{\alpha_T} L_T^{1-\alpha_T}.
\]

Production of individual \(j\) in the self-employed sector is given by \(y_j = A_N \phi_j k_j^{\alpha_N}\).

Labor is mobile across sectors, but entrepreneurs planning to switch sectors must accumulate or decumulate their capital before doing so. Because we appear to observe non-arbitraged wage differentials, we assume that, though capital is mobile both internationally and across sectors, there are adjustment costs that prevent this from happening instantaneously. For the self-employed sector, capital markets are not perfect and, as Evans and Jovanovic (1989) demonstrated for the US, entrepreneurs are often

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\(^8\) As usually assumed, one unit of tradables can be transformed into a unit of capital at no cost. The reverse is also true. Nontradables can be used only for consumption. Capital can be used for production and then consumed (as a tradable) at the end of the same period.
credit constrained. We capture this by assuming that those entering self-employment must install some capital the period before producing and pay a standard deadweight installation cost (paid in terms of tradables) of \( \frac{\chi I_j^2}{2h(k_j)} \), where \( I_j \) represents the change in capital stock between two successive periods for self-employed individual \( j \) and \( \chi \) is inversely related to the speed of adjustment. \( h(k_j) \), a linear function of capital accumulated by the self-employed individual \( j \). We further assume that individuals willing to leave self-employment must dispose of all the capital they have in place before they become employed in the salaried sector.\(^9\) This specification ensures that the labor market will not adjust fully in one period and that differentials in net remuneration among sectors are not instantly arbitraged by labor flows. This permits us to analyze both steady state movements in relative wages, relative sector sizes and exchange rates, but, also transitional dynamics.

### 2.1 The firm

The representative tradable sector firm maximizes

\[
\max \sum_{t=1}^{\infty} \left( \frac{1}{1+r} \right)^{t-1} \left[ A_{T,s} F(K_{T,s}, L_{T,s}) - w_{T,s} L_{T,s} - I_{T,s} \right], \quad \text{subject to:} \quad I_t = K_{s+t} - K_s
\]

where \( w_{T,s} \) is the wage (gross) prevailing in the tradables sector at time \( t=s \). The world interest rate \( r \), expressed in terms of tradables, is assumed to be constant. The first order conditions are standard:

\[
A_T f'(k_T) = r \\
A_T [f(k_T) - f'(k_T)k_T] = w_T
\]  

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\(^9\) This specification ensures that (de)installation costs are always finite. Further, since marginal costs of capital (de)installation are increasing, capital adjustment will not happen instantaneously.
Because $r$ is the world interest rate expressed in terms of tradables, it must correspond to the marginal product of capital in the salaried/tradable sector. The wage prevailing in the sector is equal to labor’s marginal productivity. Because both factors do not shift instantaneously across sectors, these two conditions may fail to hold ex-post in the event of unanticipated shocks.

In the self-employed sector, individual $j$ maximizes

$$\sum_{s=1}^{\infty} \left( \frac{1}{1+r} \right)^{s-1} \left[ p_j A_N \phi_j k_j^{\alpha_N} - \frac{\chi}{2} \left( \frac{I_{j,s}^2}{h(k_{j,s})} \right) - I_{j,s} \right]$$

subject to: $I_{j,s} = k_{j,s+1} - k_{j,s}$.

The first order condition is given by

$$I_{j,s} = \frac{q_s - 1}{\chi} h(k_{j,s})$$

(3)

$$q_{s+1} - q_s = r q_s - p_{s+1} A_N \phi_j \alpha_j k_{j,s+1}^{\alpha_j - 1} - \frac{1}{2\chi} (q_{s+1} - 1)^2$$

(4)

where $q$ denotes the shadow price of installed capital in nontradables and $p$ denotes the price of nontradables relative to the price of tradables. In other words, $p$ is simply the inverse of the real exchange rate defined as the relative price of traded goods in terms of non-traded goods. Equation (3) indicates that investment is positive only for values of $q$ larger than 1. Equation (4) is a standard investment Euler equation. In the long run, it must also be true for all self-employed individuals that returns to capital equal the market rate of interest:

$$p A_N \phi_j \alpha_j k_j^{\alpha_j - 1} = r$$

(4')

and that the pivotal individual is indifferent between wage work and self-employment:

$$(1 - \alpha_j) p A_N \phi_j k_j^{\alpha_j} = w_r.$$. 
2.2 The Consumer

As is standard, we assume that the economy is inhabited by an infinitely-lived representative consumer whose demand and asset holdings are identified with aggregate national counterparts and who maximizes a lifetime utility function of the form

\[ U_t = \sum_{s=t}^{\infty} \beta^{s-t} u(\Phi(C_T, C_N)). \]

Where \( C_T \) and \( C_N \) stand for consumption in the tradables and nontradables sectors. \( \Phi(C_T, C_N) \) is a linear homogenous function of its arguments and \( u(. \) is isoelastic with intertemporal substitution elasticity \( \sigma \). The \( \beta \) element is the standard time-preference factor which is exogenously given. We assume that the representative consumer owns a share equal to one of the representative tradable firm and in each entrepreneurial activity, and receives dividends.\(^{10}\)

The representative consumer faces a lifetime budget constraint

\[ \sum_{s=t}^{\infty} \left( \frac{1}{1+r} \right)^{s-t} (C_{T,s} + pC_{N,s}) = (1+r)Q_t + \sum_{s=t}^{\infty} \left( \frac{1}{1+r} \right)^{s-t} \left( w_{T,s}L_{T,s} + (1-\alpha_N) \frac{1}{\phi^*} p_sA_{N,s} \Phi_j(k_{j,s})d\phi_j - \frac{\chi}{2} \left( \frac{I_{N,s}^2}{h(K_{N,s})} \right) \right) \]

where national financial wealth \( Q_t = B_t + K_{N,t} + K_{T,t} \) is measured in terms of tradables and \( B \) stands for net aggregate holdings of foreign assets. \( I_{N,s} \) represents total investment and \( K_{N,s} \) total capital accumulated in the self-employed sector at date \( s \).

For the general case of a CES utility function\(^{11}\)

\[ \frac{C_T}{C_N} = \frac{\gamma}{(1-\gamma)} p^\theta \quad (5) \]

\(^{10}\) It would be equivalent to consider the case where producers directly borrow capital from the representative consumer and the latter is the one who would take the investment decisions as shown in Obstfeld and Rogoff (1996).
relative intratemporal consumption depends only on the relative price $p$ and not upon consumer's spending level where $\gamma$ indicates the weight of the traded good in the utility function and $\theta$ represents the constant (and strictly positive) elasticity of substitution between tradable and non-tradable goods.

Moreover,

$$\frac{C_{T,s+1}}{C_{N,s+1}} = \left(\frac{p_{s+1}}{p_s}\right)^\theta \frac{C_{T,s}}{C_{N,s}}.$$  \hspace{1cm} (6)

A rise in the relative price of nontradables causes growth in tradables consumption growth relative to nontradables consumption.$^{12}$

Since, by assumption nontradables can only be consumed, in equilibrium consumption equals production in the self-employed sector. Substitution and the combination of the Euler equation for tradables consumption with the lifetime budget constraint of the representative consumer yield an expression for the optimal consumption of tradables:

$$C_{T,s} = \frac{(1 + r)B_t + \sum_{t=s}^{\infty} \left( \frac{1}{1 + r} \right)^{t-s} \left( Y_{T,s} - I_s - \frac{\chi}{2} \left( \frac{I_{N,s}}{h(K_{N,s})} \right) \right)}{\sum_{t=s}^{\infty} \left( \frac{1}{1 + r} \right)^{t-s} \left( \frac{P_t}{P_s} \right)^{\gamma-\theta}},$$  \hspace{1cm} (7)

where $P$ is the price index $P = [\gamma + (1 - \gamma)p^{1-\theta}]^{1/(1-\theta)}$ which is increasing in $p$.

$^{11}$ See Obstfeld and Rogoff (1996, pp 226-235) for a full derivation.

$^{12}$ Note that if $\sigma = \theta$, tradables consumption remains constant along the perfect foresight paths.
2.3 Properties of the Model

Before turning to the dynamics of the economy, we first describe its steady state equilibrium and assess the impact of permanent productivity and consumption shocks. We then introduce a wage rigidity in the salaried sector. The results of all exercises are tabulated in Table 1.

2.3.1 Shocks in the Long Run

Productivity shocks are represented by a permanent variation in the $A$ productivity scale coefficients and demand shocks by a permanent variation in the $\gamma$ parameter. In the following, variables with hats refer to rates of change ($\dot{x} = \frac{\Delta x}{x}$). Log differentiation leads to the following results, assuming that initial $p = 1$ and initial $\gamma$ is equal to one half.

**Relative Prices:** Differentiating (4') and aggregating across all $j$ gives

$$\dot{p} + \dot{A}_N + \phi^* - (1 - \alpha_N) \dot{k}_s = \dot{r} = 0$$

Although individual ability remains constant by assumption, $\phi_j = 0$ and hence the capital growth rate is the same for everyone, the labor reallocation after a shock results in a change in the pivotal individual so that $\phi^*$ is no longer equal to zero for the labor force as a whole. By the same logic

$$\dot{p} + \dot{A}_N + \phi^* + \alpha_N \dot{k}_s = \dot{w}_T$$

where $\dot{k}_s = \dot{k}_j$ and is given by equation (4'). Defining $\eta_{LT} = \frac{w_L L_T}{Y_T}$, labors’ share in tradables output, $\dot{w}_T = \frac{1}{\eta_{LT}} \dot{A}_T$, and then

$$\dot{p} = \frac{1 - \alpha_N}{\eta_{LT}} \dot{A}_T - \dot{A}_N.$$ 

This simply restates the Balassa Samuleson result that, for values of $\frac{1 - \alpha_N}{\eta_{LT}}$ close to 1, the real exchange rate is determined by the relative rates of productivity growth.
**Relative Sector Size:** Demand for tradables and nontradables can be re-written as,

\[
C_T = \frac{\gamma Z}{\gamma + (1 - \gamma) p^{1-\delta}} \quad \text{and} \quad C_N = \frac{p^{\delta} (1 - \gamma) Z}{\gamma + (1 - \gamma) p^{1-\delta}},
\]

where \( Z = w_T L_T + (1 - \alpha_N) \int (p A_N \phi_j k_{j N}^s) d\phi_j + r \tilde{Q} \).

In order to simplify the analysis we assume that total financial wealth remains constant across steady states. We assume implicitly that any variation in the total level of physical capital is fully offset by an equal, but opposite variation in foreign assets holdings. That is, with international borrowing, a rise in the stock of physical capital for instance, can be financed by an equal fall in \( B \) without affecting the level of total financial wealth\(^{13}\). This allows us to write

\[
\hat{Z} = \phi_{LT} \left[ \hat{w}_T + \hat{L}_T \right] + \phi_{se} \left[ \frac{1}{1 - \alpha_N} \hat{A}_N + \frac{1}{1 - \alpha_N} \hat{\rho} - \hat{\phi} * \Psi \right]
\]

where \( \phi_{LT} = \frac{w_T L_T}{Z} \), \( \phi_{se} = \frac{(1 - \alpha_N) \int (p A_N \phi_j k_{j N}^s) d\phi_j}{Z} \), and \( \Psi = \frac{2 - \alpha_N}{1 - \alpha_N} \left[ (\phi *)^{2-\alpha_N} \right] \).

Changes in nontradables consumption can be written as

\[
\hat{C}_N = -\gamma + \hat{\theta} \gamma - (1 - \gamma) \hat{\rho}
\]

and changes in total production in the self-employment sector (expressed in units of tradables) by

\[
pY_N = \frac{1}{1 - \alpha_N} \left[ \hat{A}_N + \hat{\rho} \right] - \Psi \hat{\phi} *.
\]

Since nontradable goods market equilibrium requires that \( \hat{C}_N = \hat{Y}_N \), the entrepreneurial ability of the pivotal worker, and implicitly, the share of the workforce in tradables, can be written as:

\(^{13}\) See Obstfeld and Rogoff (1996, Chap. 4) for an application.
\[ \dot{\phi}^* = -\Omega_1 \left[ -\gamma + \hat{\phi} \frac{\hat{A}_r}{\eta_{LT}} [\phi_{LT} + \phi_{se} - 1 + (1 - \alpha_N) (\gamma (1 - \theta) - 1)] + \hat{\phi}_N (1 - \gamma (1 - \theta)) \right], \]

where \( \Omega_1 = [(1 - \phi_{se}) \psi + \phi_{LT}]^{-1} \).

**Relative Earnings:** The change in self-employment production expressed in tradables units is now:

\[ pY_N = \frac{\hat{A}_r}{\eta_{LT}} - \psi \hat{\phi}^* = \frac{\hat{A}_r}{\eta_{LT}} + \Omega_2 \left[ -\gamma + \frac{\hat{A}_r}{\eta_{LT}} [\phi_{LT} + \phi_{se} - 1 + (1 - \alpha_N) (\gamma (1 - \theta) - 1)] + \hat{\phi}_N (1 - \gamma (1 - \theta)) \right] \]

where \( \Omega_2 = \frac{\psi}{(1 - \phi_{se}) \psi + \phi_{LT}} \). The relative change in total production also corresponds to the relative variation in entrepreneurs’ earnings, \( w_{Nj} \), as the latter is a constant proportion of the former. The change in average self-employment production expressed in terms of tradables units can be written as:

\[ E(pY_N) = \frac{\hat{A}_r}{\eta_{LT}} - \psi \hat{\phi}^* + \frac{\phi^*}{1 - \phi^*} \hat{\phi}^* \]

\[ = \frac{\hat{A}_r}{\eta_{LT}} + \Omega_3 \left[ -\gamma + \frac{\hat{A}_r}{\eta_{LT}} [\phi_{LT} + \phi_{se} - 1 + (1 - \alpha_N) (\gamma (1 - \theta) - 1)] + \hat{\phi}_N (1 - \gamma (1 - \theta)) \right] \]

where

\[ \Omega_3 = \frac{\psi - \phi^*}{(1 - \phi_{se}) \psi + \phi_{LT}} > 0. \] It is straightforward to verify that \( \Omega_3 < \Omega_2 \).

### 2.3.2 Dynamics

In order to qualify the dynamics of the model in the event of a shock, we linearize the first order conditions for profit maximization by the self-employed around the steady
state. The latter being characterized by \( q = 1 \) (\( q \) denotes the shadow price of installed capital in nontradables) and, \( \bar{k}_j \) we obtain

\[
k_{j,t+1} - k_{j,t} = \frac{q_t - 1}{\chi} h(\bar{k}_j)
\]

\[
q_{t+1} - q_t = r \left[ (1-\alpha_N) \frac{h(\bar{k}_j)}{\chi} \bar{k}_j + 1 \right] (q_t - 1) + r \left[ (1-\alpha_N) \bar{k}_j \right] (k_{j,t} - \bar{k}_j).
\]

The equations \( \Delta k_j = 0 \) and \( \Delta q_j = 0 \) characterize the equilibrium dynamics. They are depicted in a two-equation phase diagram in \( q \) and \( k_j \) that shows the dynamics of the investment decisions of self-employed individuals (figure 2). The line denoted by SS indicates the perfect foresight path.

As the steady state level of investment chosen by each individual is not identical, we expect to observe that a common shock affects heterogeneous individuals differently. When a shock leads to a contraction of the self-employment sector, for workers whose entrepreneurial ability falls below the threshold steady state value of \( \phi^* \) (those who would be better off in the wage work sector), the perfect foresight path leads to zero capital and zero capital shadow value at steady state, as depicted in figure 3. Should self-employment expand, new-entrants invest initially \( I_0 = \frac{1-r}{r\chi} a \) - independent of the wage prevailing in the salaried sector since the initial shadow value of their capital is above 1 \( (q_0=1/r) \). Due to heterogeneous entrepreneurial ability, workers will not all move across sectors in the same period. For instance, in the case of a shock leading to a rise in returns to self-employment, more able entrepreneurs in the salaried sector would move first. A detailed analysis is presented in Appendix 2.

The adjustment to the steady state depends on the relative values of \( \sigma \) and \( \theta \). Indeed, \( C_{T,t} \) is given by (7) which suggests that the level of tradables consumption along the saddle path is affected by variations in \( p \) in a manner that could either reinforce or offset the impact of a shock. The impact of a rise in \( p \) on consumption is dampened by
consumers’ inter-temporal choices if $\sigma > \theta$, and amplified if $\sigma < \theta$. If $\sigma > \theta$, consumption of nontradables declines slower than consumption of tradables. The opposite occurs if $\sigma < \theta$. This implies that migration takes longer in a situation when inter-temporal substitution prevails over intra-temporal substitution.

2.4 Responses to Productivity and Demand Shocks

In order to define short/medium term properties we need to qualify "on-impact" effects of various shocks. Short/medium term properties would then reflect variables’ behavior after impact and during the transition towards the new steady state. On impact, levels of production and consumption must remain constant. Thus any wealth effects generated by the shock must be offset by an instantaneous change in prices. In order to simplify the analysis, we assume that changes in wealth occurring on impact reflect only the shock’s direct effects. That is, changes in wealth due to subsequent changes in prices are accounted for in the long run. This assumption does not affect qualitatively the properties of the model.

We first assess the impact of permanent productivity and consumption shocks. We then introduce wage rigidities in the salaried sector. The results of all exercises are presented in Table 1.

Productivity Shock to the Tradables Sector

In the event of a productivity shock to the tradables sector, $\hat{A}_T > 0$, $\hat{A}_N = 0$ and $\gamma = 0$, both production of the sector as well as returns to capital and labor increase. This increases demands for both types of goods and causes the exchange rate to appreciate $p$.

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14 Reference equations for determining on-impact effects become: $\dot{Z} = \varphi_{\gamma} \hat{\gamma} + \varphi_{\hat{A}_N} \hat{A}_N$ and $\dot{C}_N = -\gamma + \dot{Z} - (\theta \gamma + (1-\gamma)) \dot{p} = 0$
rises) to clear the nontradables market. In addition, along the perfect foresight adjustment path, some self-employed find it more profitable to move to the salaried sector. The shadow value of their capital falls below 1 and, as it falls towards zero in the long run they disinvest. However, since capital adjusts with a lag, they cannot migrate until their capital has been completely dismantled. Tradable firms must also wait for the following period to adjust their capital. Therefore, on impact only prices adjust and average self-employed earnings follow the initial rise in $p$. As the economy adjusts, self-employed earnings fall relative to salaried sector wages as does the share of workers in self-employment. Hence, in both the short run and long run, $w_T / w_N$ increases, $L_T / L_N$ increases and, consistent with Balassa-Samuelson, $p$ rises relative to its initial level.

**Productivity Shock to the Nontradables Sector**

Consistent with standard models, if $\hat{A}_T = 0$, $\hat{A}_N > 0$, and $\gamma = 0$, in the steady state, the relative price of nontradables will decrease in proportion to the productivity shock in nontradables. Both capital intensity and earnings in the self-employed sector will be left unchanged. However, on impact $p$ rises due to increased demand for nontradables. It then falls along the transition path. This could be qualified as $p$ undershooting. Individuals who are already self-employed at the time of the shock and

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15 Workers whose sequence of returns from self-employment remains above that of the salaried wage face $q > 1$ and they accumulate more capital.

16 On impact, $p = \frac{\varphi_{LT}}{(\theta \gamma + (1-\gamma)) \hat{A}_T}$, which also corresponds to the initial rise in average earnings in the informal sector. The initial rise in formal wages is equal to $\hat{A}_T$ and remains larger than the rise in self-employed average earnings for reasonable values of $\gamma$ and $\varphi$. Total self-employed production and earnings, measured in tradables units, depends on the sign of $\frac{\hat{A}_T [1-\Omega_2 [1-\varphi_{LT} - \varphi_{se} -(1-\alpha_N) \gamma (1-\theta) - 1]]}{\Omega_{LT}}$. It is straightforward to check that the expression into brackets is always smaller than one. Since $\hat{w}_T = \frac{\hat{A}_T}{\Omega_{LT}}$, on average self-employed earnings fall in the long run relative to workers earnings in the salaried sector for any value of $\Omega_2$ and $\theta$.

17 In the steady state the direction of change of the employment share of self-employment depends on the sign of $[\varphi_{LT} + \varphi_{se} - 1 + (1-\alpha_N) \gamma (1-\theta) - 1]$. The expression is unambiguously negative implying that the share of self-employed workers falls.
who, with perfect foresight know that relative prices will continue to fall, do not modify their capital stock (their shadow value $q$ remains equal to unity), but the increase in productivity does, in the short run, increase their production and yield higher relative earnings. This induces migration from the tradables sector and will eventually drive returns back to the pre-shock level. However, to attract the marginal entrepreneur to self-employment, relative earnings in this sector will rise. Hence, in both the short and the long run, $w_T / w_N$, $L_T / L_N$ and $p$ decrease.

**Shift in Preferences toward Nontradables**

A shift in preferences, for instance, towards nontradables consumption $\hat{A}_T = 0$, $\hat{A}_N = 0$ and $\hat{\gamma} < 0$ increases self-employment as well as absolute and relative consumption of nontradables. On impact, the increased demand for nontradables causes the exchange rate to appreciate, and relative self-employed earnings and the shadow value of capital increase. This attracts new entrepreneurs to the sector, expanding nontradables supply and driving the relative price of nontradables, $p$, back to its initial, relative productivity-determined level. However, because marginal entrepreneurs are attracted to the sector, relative self-employment earnings must rise in the steady state. This represents an important case where both $w_T / w_N$ and $L_T / L_N$ fall with an initial appreciation and then continue to do so as the exchange rate depreciates again back to its initial level.

**Negative Salaried/Tradables Productivity Shock with Salaried Sector Wage Rigidities**

Unions or mandatory minimum wages may introduce downward nominal wage rigidities in the salaried sector that can reverse some of the above findings. As the derivation of the steady state is complex, detail is deferred to appendix A.1. In the case

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19 The sign of $(1 - \gamma (1 - \theta))$ determines the impact on self-employment. It is positive for any positive value of the intra-temporal elasticity of substitution.

20 $p = \frac{1}{(\theta \gamma + (1 - \gamma) \gamma)}$ and $\gamma < 0$. 

---
where $A_T < 0$, $A_N = 0$, and $\gamma = 0$, a negative shock to tradables' productivity puts downward pressure on nominal wages in the salaried sector. However, because of downward wage rigidities, consumption is not affected on impact and hence, there is no effect on relative prices, $p$. But the salaried sector will eventually adjust through quantities and released labor will flow into the non-traded sector increasing its production, driving down $p$, and reducing average self-employed earnings. For the already self-employed, the fall in $p$ observed along the transition path, leads to disinvestment in capital. However, since there is now rationing in the salaried sector, migration to the salaried sector is not possible and workers with relatively low entrepreneurial ability will earn less than what they would in the salaried sector. Hence, average earnings in the self-employed sector have fallen relative to the salaried sector while the size of self-employment has increased: we should see $w_T / w_N$ and $L_T / L_N$ moving against each other.\(^{21}\) This is the classic segmentation view: the informal sector absorbs released labor during downturns to which we also now add that $p$ falls as well. There are some parameter values which can lead to appreciation and a positive comovement of the labor market series. However, as detailed in appendix A.1, they are not very plausible and, while included for completeness they can be disregarded for most practical purposes.

3. Empirics

The previous section shows that very standard models anchored in the mainstream literature yield clear hypotheses of comovements among the three series. Two conclusions are important. First, independent of skill heterogeneity and adjustment costs, under no conditions can we generate a counter movement of relative sector sizes and earnings in the absence of a wage rigidity: observed counter movements imply

\(^{21}\) As long as wages in the salaried workers do not adjust, average earnings in self-employment unambiguously fall with respect to the former. Then, along the transition towards steady state, average earnings in the self-employed sector have fallen relative to the salaried sector while the size of self-employment has increased. As mentioned in the previous section, this is also true in the new steady state if appendix A.1, condition (1) is satisfied.
segmentation and if we detect them empirically, this is evidence of labor market distortions. Second, in all cases, the short run labor market dynamics move in the same direction as the steady state and only in the case of a shock to preferences for nontradables does the exchange rate overshoot in the short-run.

We explore the patterns of comovement between relative sector sizes, relative earnings and the real exchange rate for Argentina, Mexico, Brazil and Colombia using the multivariate Johansen (1988) approach. (see appendix A.3). Although cointegration is sometimes given the economic interpretation of capturing “long run” relations, as Granger (1991) and Hakkio and Rush argue (1991) at core it is a statistical relationship existing among non-stationary series that can occur at any frequency or span.\textsuperscript{22} In our case, relative sector sizes, earnings and the real exchange rate are plausibly I(1) and they always appear to be so in the analysis.\textsuperscript{23} Since overshooting or undershooting (as found in the case of a productivity shock or a demand shock respectively to the nontradables sector) can take a number of years to return to long run equilibrium, our short/medium runs can, in fact, represent quite persistent phenomena that will be identified by the cointegration relationship as well.

3.1 Data

We use quarterly data for Mexico, Brazil and Colombia and semi-annual data for Argentina (see Appendix A.4 for data definitions and details) to generate the earnings ratio of salaried over self-employed workers, $w_T/w_N$, and the ratio of the absolute size of the salaried over the self-employed sector, $L_T/L_N$. To the degree possible, we try to be consistent across surveys and in spirit be close to the ILO definitions: we treat the male population that reports being employed in firms of greater than 6 workers as salaried

\textsuperscript{22} See Hakkio and Rush (1991) \textit{Cointegration: How long is the long-run?}: "Clearly, the length of the 'long-run' may vary between problems, that is, for some issues the long-run may be a matter of decades while for others a matter of months."

\textsuperscript{23} Theoretically, however, it is legitimate to include an I(0) variable in the cointegrating relationship, although we would expect at least one cointegrating vector to emerge that captures simply the stationary series. In practice, these series were never stationary across our sample and the problem was moot.
(tradable) workers. Own-account workers or heads of firms employing fewer than 5 employees paying no social security contributions and excluding professionals and technicians, constitute the informal self-employed (nontradable) sector. Real exchange rates, \( p \), were taken from International Financial Statistics. The series are plotted in Figure 1 with the exchange rate inverted for greater graphical clarity (an upward movement here and here alone is a depreciation).

Three issues merit note. First, even if remuneration is equalized in both sectors, we do not observe non-monetary remuneration (independence, benefits foregone, taxes avoided, implicit returns to capital, etc.) and hence we may observe a wedge in observed returns even in equilibrium. We assume that these non-monetary components are a constant fraction of monetary earnings and hence that changes in relative monetary earnings are a good proxy for relative changes in total remuneration. Second, variations in definitions and the composition of payment can cause substantial differences in ratios of relative earnings across countries. As a final caveat, we do not model or study those salaried workers who are uncovered by labor legislation and hence are informal. While this group is substantially smaller than the informal self-employed, its particular cyclical behavior deserves independent study in another paper.

3.2 Results

We begin by estimating separate VAR models for Argentina, Mexico, Brazil and Colombia. We include a constant, lags for \( p \), \( w_T / w_N \) and \( L_T / L_N \) as well as time dummies in the cointegration space. These specifications prove sufficient to produce random errors. The model specifications for the three models are presented in Tables A.1-A.3 in the Appendix A.4 along with tests for long-run exclusion, stationarity and weak-exogeneity. All variables appear to be non-stationary and the diagnostics on the residuals of the system point towards the absence of autocorrelation, and normality. Sensitivity analysis for different lag lengths and with and without dummies further indicated robustness of the findings. Trace tests (\( \lambda_{trace} \)) indicate one significant cointegrating vector for all three models (Table A.2). Normalizing the cointegration vectors on the 1st
element, yields the estimates for the $\beta$s (Table 3) in a cointegration vector that can be read as:

$$L_T/L_N + \beta_w w_T/w_N + \beta_p p + \beta_C = 0$$

Thus, the signs on the first three parameters (beginning with the normalized coefficient on $L_T/L_N$) correspond directly to those in table 1. Hence, a finding of a positive coefficient on relative earnings implies an integrated labor market, while a negative coefficient indicates a segmented market.

The theoretical model suggests that different shocks, or differing degrees of salaried sector rigidities, should lead to different regimes and hence different cointegration vectors across subsamples. To identify potential shifts in the degree of labor market segmentation, and for an indication of potential break dates we plot rolling correlation coefficient of the two labor variables (figures 4). We then test for specific cointegrating vectors across separate periods for our full model specification.

For both Argentina and Colombia, our correlation analysis suggests the only significant comovements between the two labor market variables to be negative with incipient but never statistically significant shifts toward the positive. This, combined with the relatively limited degrees of freedom in these cases led us to abandon search for regime change and we report only the full sample result in table 2. However, both Mexico and Brazil do suggest periods where the correlations flip signs and significantly so suggesting that in some moments the market is behaving in a more integrated fashion, and sometimes in a more segmented fashion. In Mexico, for example, the period around the crisis, roughly 1993 to 1997 shows a negative and significant correlation. However, on either side of the crisis, the boom prior to 1991 and after the recovery around 1999 and 2004, we identify periods of statistically significant positive correlation. As Brazil also exhibits both patterns with statistical significance, we proceed for these two countries to a subsample investigation and estimate cointegrating vectors for respective sub periods.
Our results provide strong evidence for two types of regimes. First, in Brazil during 1994-97 and in Mexico in the periods 1987-91 and 1998-04, we find a positive comovement of the two labor series. This suggests an integrated market and a positive demand shock to the nontradables sector. Traditionally, Mexican and Brazilian minimum wages are not especially binding, and in these samples, both economies were going through something of a boom. In the two Mexican cases, the appreciation of the exchange rate suggests that we are not yet in the long run. In the Brazilian case, the coefficient on the real exchange rate is statistically insignificant suggesting that we have reached the long run equilibrium where markets have adjusted to erode the short term overshooting.

In a second regime, Argentina, Brazil in the periods 1983-89 and 1998-02, Colombia, and Mexico during 1992-96, all correspond to the case of a negative shock to the formal/traded sector in the presence of wage rigidities. In this sense, we find the classic informal/non-tradable sector adjusting to take in labor no longer absorbed in the formal sector. This is historically plausible. Across these periods, all four countries experienced deep recessions where wages may not have been able to adjust sufficiently to prevent segmentation. In addition, Colombia’s minimum wage was the most binding in Latin America while Argentina, although not showing especially high minimum wages, nonetheless has been considered to have a quite rigid labor market.

4. Conclusion:

This paper has offered an integrated view of the developing country labor market and its behavior across macroeconomic fluctuations. We model a two sector labor market in a Rogoff-Obstfeld small economy model to include heterogeneous entrepreneurial ability and credit constraints to entering informal self-employment. This allows us to generate a set of hypotheses about the comovement of relative sector sizes and earnings and sectoral shocks as captured by the real exchange rate.

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24 See Maloney and Nunez (2004)
These patterns of comovement are then tested in a cointegration framework and offer provocative results. First, the informal self-employed and formal salaried sectors often appear as one integrated labor market, rather than segmented or dual labor markets as customarily envisaged: numerous periods show strong comovement between relativesector sizes and earnings. This suggests that a large component of the informal sector should not be viewed as somehow inferior or queuing for formal sector employment. However, it is also the case that rigidities in the formal salaried sector can become binding, as appears to be most dramatically the case in Colombia, and lead to patterns consistent with the traditional segmentation hypothesis of adjustment. These distinct patterns suggest that the pro or countercyclicality of the sectors may depend on the sectoral origin of the shocks, and the presence of binding wage rigidities.
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Table 1: Predicted Patterns of Comovement among Relative Earnings, Relative Sector Sizes, and the Real Exchange Rate

<table>
<thead>
<tr>
<th>Short / Medium Run</th>
<th>$\Delta (w_T/w_N)$</th>
<th>$\Delta (L_T/L_N)$</th>
<th>$\Delta p$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Flexible Wage</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta A_T &gt; 0$</td>
<td>$&gt; 0$</td>
<td>$&gt; 0$</td>
<td>$&gt; 0$</td>
</tr>
<tr>
<td>$\Delta A_N &gt; 0$</td>
<td>$&lt; 0$</td>
<td>$&lt; 0$</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td>$\Delta \gamma &lt; 0$</td>
<td>$&lt; 0$</td>
<td>$&lt; 0$</td>
<td>$0 &gt;$</td>
</tr>
<tr>
<td>(undersh.)</td>
<td></td>
<td></td>
<td>(oversh.)</td>
</tr>
<tr>
<td><strong>Wage Rigidities</strong></td>
<td>$\Delta A_T &lt; 0$</td>
<td>$&gt; 0$</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Long Run</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Flexible Wage</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta A_T &gt; 0$</td>
<td>$&gt; 0$</td>
<td>$&gt; 0$</td>
<td>$&gt; 0$</td>
</tr>
<tr>
<td>$\Delta A_N &gt; 0$</td>
<td>$&lt; 0$</td>
<td>$&lt; 0$</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td>$\Delta \gamma &lt; 0$</td>
<td>$&lt; 0$</td>
<td>$&lt; 0$</td>
<td>$0$</td>
</tr>
<tr>
<td><strong>Wage Rigidities</strong></td>
<td>$\Delta A_T &lt; 0$</td>
<td>$&gt; 0$</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 2: Cointegration Coefficients Among Relative Sector Sizes, Relative Earnings, and the Real Exchange Rate

<table>
<thead>
<tr>
<th></th>
<th>Argentina</th>
<th>Brazil</th>
<th>Colombia</th>
<th>Mexico</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variables</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(L_T/L_N)</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>(W_T/W_N)</td>
<td>-2.336</td>
<td>1.135</td>
<td>-0.067</td>
<td>2.432</td>
</tr>
<tr>
<td></td>
<td>(-7.087)</td>
<td>(1.761)</td>
<td>(-2.297)</td>
<td>(16.7)</td>
</tr>
<tr>
<td>(p)</td>
<td>-0.376</td>
<td>-1.685</td>
<td>-0.224</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>(-4.16)</td>
<td>(-4.603)</td>
<td>(-4.07)</td>
<td>(0.045)</td>
</tr>
<tr>
<td>(const.)</td>
<td>3.300</td>
<td>-0.071</td>
<td>1.330</td>
<td>0.443</td>
</tr>
<tr>
<td></td>
<td>(-8.42)</td>
<td>(-0.418)</td>
<td>(41.7)</td>
<td>(17.1)</td>
</tr>
<tr>
<td>Regime</td>
<td>(\Delta A_T &lt; 0)</td>
<td>Mixed</td>
<td>(\Delta A_T &lt; 0)</td>
<td>(\Delta \gamma &lt; 0)</td>
</tr>
<tr>
<td>(From Table 1)</td>
<td>rigidities</td>
<td>rigidities</td>
<td>LR</td>
<td>rigidities</td>
</tr>
</tbody>
</table>

Note: Cointegration vectors between relative tradable/nontradables (Formal/Informal Self-employment) size measured in employment, relative tradable/nontradable earnings, and the real exchange rate. Vector presented as \(L_T L_N + \beta_W W_T / W_N + \beta_p p + \beta_C = 0\).
(t-statistics in parentheses).
Figure 1: Relative Sector Shares and Earnings, Real Exchange Rate
Argentina, Brazil, Colombia and Mexico

Source: Own estimates based on EPH (Encuesta Permanente de Hogares) and IFS.

Source: Own estimates based on PME (Pesquisa Mensal de Emprego) and IPEA.
Notes: Wage F/SE captures the relative earnings of the formal salaried vs informal self employed sector. Formal/SE captures the relative size of these sectors as a ratio of employed population.
**Figure 2:** Self-employment and gradual capital adjustment

![Diagram showing self-employment and capital adjustment](image)

**Figure 3:** Capital decumulation for migrating self-employed

![Diagram showing capital decumulation for migrating self-employed](image)
Figure 4: Rolling Correlation between relative sector size and relative wages

Note: 16 period rolling window, dashed lines represent 10% level of significance (0.426). Values above (below) 0.426 (-0.426) indicate significant positive (negative) comovement of relative sector size and relative wages.
Appendix A.1: Details on Negative Tradables Productivity Shock with Salaried Sector Wage Rigidities

Unions or mandatory minimum wages may introduce downward nominal wage rigidities in the salaried sector that can reverse many of the findings above.

A negative shock to productivity in the tradables sector translates into nominal wage downward pressures in the salaried sector. Nominal wage downward rigidities, if persistent, would lead to a non-optimal and thus unstable equilibrium. In order to obtain a possibly stable equilibrium, we assume that nominal wages are adjusted to satisfy the first order conditions of firms operating in the tradables sector. However, we assume that wage variations represent the last element of adjustment. That is, labor movements are precluded after wages have been adjusted. As a consequence, the pivotal individual could end up in a situation where belonging to either one or the other sector does make a difference. This is a case of segmentation where the nontradables sector behaves in part as a residual sector.

As both capital and labor are assumed not to move instantaneously, two adjustment scenarios are possible. In the first scenario, capital would move first, then labor and finally wages. In that scenario, capital adjustment is a two-step process. Capital first adjusts to meet (1) in a context of constant salaried labor force. It further adjusts to meet (1) considering labor variation obtained by solving (2) with constant wage. Wages adjust in a final stage to satisfy (2). In the second scenario, labor would move first, then capital and finally wages. In that scenario, capital adjustment is one-step process. Capital adjusts to meet (1) after salaried labor has changed to meet (2) with constant capital and wage. Wages adjust to meet (2) after labor and capital adjusted. We can expect results to be qualitatively similar, as we expect factors of production adjustments to be identically signed in both scenarios.

Taking for instance the case of a Cobb-Douglas production function, salaried labor outflow in the first scenario corresponds to

$$\hat{L}_r = \frac{\hat{A}_r}{\eta_{ILr}} (1 - \eta_{ILr}) \leq 0,$$

$$\hat{K}_r = \frac{(1 + \eta_{ILr})\hat{A}_r}{\eta_{ILr} (1 - \eta_{ILr})} < 0 \text{ and } \hat{w} = \frac{\hat{A}_r}{\eta_{ILr}} < 0.$$

Equilibrium of the demand and supply conditions in the nontradables sector

$$\hat{C}_N = ((2 - \eta_{ILr})\varphi_{ILr} - \varphi_{ILr} \Psi) \frac{\hat{A}_r}{\eta_{ILr} (1 - \eta_{ILr})} + \left[ \varphi_{ILr} \frac{1}{1 - \alpha_N} - (\theta \gamma + (1 - \gamma)) \right] \hat{p}$$

and

$$\hat{Y}_N = \frac{\alpha_N}{1 - \alpha_N} \hat{p} - \Psi \frac{\hat{A}_r}{\eta_{ILr} (1 - \eta_{ILr})}$$

give

$$L_{r,s} = \left[ \frac{(1 - \eta_{ILr})A_{r,s}}{w_{r,s}} \right]^{\eta_{ILr}} K_{r,s}.$$

---

25 In that case, labor demand in the salaried sector at period $s$ can be expressed as
The sign of both real exchange rate and average earnings depends upon the sign and magnitude of expression \( \alpha_N - \varphi_w + (1 - \alpha_N)(\gamma \theta + (1 - \gamma)) \). This expression is increasing with \( \theta \), the elasticity of substitution. For \( \theta = 1 \) (preferences are Cobb-Douglas) for instance, the expression is equal to \( 1 - \varphi_w \). In that case, both the real exchange rate and average self-employed earnings are decreasing unambiguously. A sufficient condition for the real exchange rate to depreciate is

\[
\frac{(1 - \varphi_w)}{(1 - \alpha_N) \gamma} > (1 - \theta).
\]

Unless \( \alpha_N \) and \( \theta \) are both very close to zero and \( \gamma \) very close to one the condition is likely to be always satisfied. However, the sufficient condition for both the relative price of nontradables and average earnings to be decreasing is more restrictive, namely \( \theta \leq 1 \). Average earnings in the self-employment sector could rise despite the fall in \( p \) because in the context of an expansion of the sector the contribution per unit of entrepreneurial ability is higher for less able workers. This is a feature of the model essentially due to the fact that ability enters in a linear manner in the production function of self-employed workers.

As far as relative earnings are concerned, self-employed workers would become on average worst off with respect to salaried workers if

\[
\left[ \frac{(2 - \eta_{LT}) \varphi_{LT} + (1 - \varphi_w) \Psi}{\alpha_N - \varphi_w + (1 - \alpha_N)(\gamma \theta + (1 - \gamma))} - \left[ \Psi - \frac{\phi^*}{1 - \phi^*} \right] \right] > (1 - \eta_{LT})
\]

for \( \theta = 1 \) we must have that

\[
\left[ \frac{(2 - \eta_{LT}) \varphi_{LT} + \phi^*}{(1 - \varphi_w) + \frac{\phi^*}{1 - \phi^*}} \right] > (1 - \eta_{LT})
\]

This condition is likely to be satisfied for any plausible set of parameters values.

In the second scenario, we obtain

\[
\hat{L}_r = \frac{\hat{A}_r}{(1 - \eta_{LT})} = \hat{\phi}^* < 0, \quad \hat{K}_r = \frac{\hat{A}_r}{\eta_{LT}(1 - \eta_{LT})} < 0 \quad \text{and} \quad \hat{\omega} = \frac{\hat{A}_r}{\eta_{LT}} < 0.
\]

The real exchange rate and average self-employed earnings vary according to
\[
\hat{p} = \frac{\hat{A}_t}{\eta_{st}} \left( 1 - \alpha_N \right) \left( \phi_{st} + \eta_{st} \left( 1 - \phi_{se} \right) \Psi \right) \left( \alpha_N - \phi_{se} + (1 - \alpha_N)\left(\theta + (1 - \gamma)\right) \right)
\]

and

\[
E(p_{Y_s}) = \frac{\hat{A}_t}{\eta_{st}} \left( \frac{\phi_{st} + \eta_{st} \left( 1 - \phi_{se} \right) \Psi}{\alpha_N - \phi_{se} + (1 - \alpha_N)\left(\theta + (1 - \gamma)\right)} \right) - \eta_{st} \left( \Psi - \frac{\phi^*}{1 - \phi^*} \right)
\]

Conditions presented in the case of the first scenario also apply to the second scenario.

When conditions presented above are satisfied, as labor migrates towards the self-employed sector, production rises, the real exchange rate depreciates, and average earnings in the self-employed sector fall. Moreover, as workers cannot migrate back to the salaried sector, those whose entrepreneurial ability is relatively low earn less than what they would get in the salaried sector. For those workers “trapped” in the self-employed sector earnings performance has worsened relative to those employed in the salaried sector as earnings in the salaried sector are preserved by institutional rigidities. The two labor force series move against each other. Critically, the same result would hold in the case where indexation of wages to past inflation forces salaried sector wages above equilibrium: we should see relative sector sizes and incomes move against each other.

Appendix A.2: Migration Timing

Because we assume that the self-employed individual, who is willing to move to the wage-work sector, has to disinstall the capital she borrowed before moving, migration occurs whenever,

\[
p_{s,t} + A_{n,t} + \phi_{st} k_{N,t} \leq \frac{\chi \left( t^2 \right)}{2 h(k_{j,t})} - rk_{n,t} + \sum_{s-t+1}^{\infty} \left( \frac{1}{1+r} \right)^{t-t} \left[ p_{s,t} + A_{n,s} + \phi_{st} k_{N,s} \right] - \frac{\chi \left( t^2 \right)}{2 h(k_{j,s})} - rk_{n,s}
\]

\[
\leq p_{s,t} + A_{n,t} + \phi_{st} k_{N,t} \leq \frac{\chi \left( t^2 \right)}{2 h(k_{j,t})} - rk_{n,t} + \sum_{s-t+1}^{\infty} \left( \frac{1}{1+r} \right)^{t-t} w_{r,s}
\]

Labor could adjust within the first period following the shock. However, because individuals are non homogenous when producing in the self-employed sector, the optimal time for leaving the latter may differ across workers.

The Left Hand Side of the above expression is increasing with entrepreneurial ability. Namely, more able individuals earn more than less able ones. Then the opportunity cost of migrating to the salaried sector at time \( t \), without considering the direct migration costs corresponding to capital disinstallation, is increasing in the level of entrepreneurial capability. The last term of the RHS, which represents the present value of labor earnings in the salaried sector is identical for all individuals at time \( t \). However, the first term of the RHS is likely to be different. The sign of the partial derivative of the latter with respect to \( \phi^* \) is given by
If the above expression appears to be positive, that would imply that the cost of migrating to the salaried sector at time is increasing with the level of entrepreneurial capability. If this is the case, then the total cost of migration is unambiguously increasing with \( \phi_j \). As a consequence we may expect more able entrepreneurs to postpone their migration towards the wage sector with respect to less able ones.

In the case of a shock leading to an expansion of the self-employed sector, migration can occur within the first period following the shock, even though capital accumulation may take more than a period because of installation costs. Individuals migrate at the end of period \( s \) whenever

\[
w_{T,s} - \chi \left( \frac{k_{j,s+1}^2}{h(0)} \right) + \sum_{s' \neq s} \left( \frac{1}{1 + r} \right)^{t-s'} P_i A_{n,s} \Phi_j k_{n,s}^2 k_{s,s} - \frac{\chi}{2} \left( \frac{r_{j,s}^2}{h(k_{j,s})} \right) \geq w_{T,s} + \sum_{s' \neq s} \left( \frac{1}{1 + r} \right)^{t-s'} w_{T,s}
\]

Following arguments similar to those presented above, we can infer that more able entrepreneurs will leave the salaried sector first, in order to "cash in" the expected earnings differential the soonest.

Appendix A.3: Details of Johansen Cointegration Procedure

The Johansen procedure allows us to test for cointegration in a multivariate system. Starting from an unrestricted vector autoregressive model (VAR), the hypothesis of cointegration is formulated as a hypothesis of reduced rank of the long run impact matrix \( \Pi \) (Johansen, 1988; Johansen and Juselius, 1990). The VAR is generated by the vector \( z_t \), which defines the potential endogenous variables of the model, in our case, the three series. Taking first differences of the variables, the VAR can be transformed into an error correction model

\[
\Delta z_t = \Gamma_1 \Delta z_{t-1} + \ldots + \Gamma_{k-1} \Delta z_{t-k+1} + \Pi z_{t-k} + \psi D_t + \epsilon_t, \quad \epsilon_t \sim IN(0, \Sigma)
\]

where the estimates of \( \Gamma_i = -(I - A_i - \ldots - A_i), (i = 1, \ldots, k - 1) \) describe the short run dynamics to changes in \( z \) and \( \Pi = -(I - A_i - \ldots - A_i) \) captures the long run adjustments and \( D \) contains deterministic terms.

Cointegration occurs in the case of reduced rank of \( \Pi \). If the rank is reduced \( r<n \) it is possible to factorize \( \Pi \) into \( \Pi = \alpha \beta' \) where \( \alpha \) denotes the adjustment coefficients and \( \beta \) the cointegration vectors. The cointegration vectors \( \beta \) have the property that \( \beta' z_t \) is stationary even though \( z_t \) itself is non-stationary. If the rank is reduced it is also possible to interpret the VAR in first differences as a vector error correction model and to obtain estimates of \( \alpha \) and \( \beta \) via the reduced rank regression. Since the rank of \( \Pi \) is equal to the number of independent cointegration vectors and the rank of \( \Pi \) is also equal
to the number of non-zero eigenvalues, the test of cointegration thus amounts to a test for
the number of non-zero eigenvalues. The trace statistics, $\lambda_{\text{trace}}$, is a non-standard
distributed likelihood-ratio test, which is commonly used to determine the number of
cointegration vectors, (Johansen, 1988). The trace statistic tests the null hypothesis that
there are at most $r$ cointegration vectors:

$H_0: \lambda_i = 0, \text{ for } i = r+1,..n$

where only the first $r$ eigenvalues, $\lambda$, are non-zero against the unrestricted hypothesis
that $\lambda_n$.

26 The null hypothesis of at most $r$ cointegration vectors implies that there are $n-r$ unit roots and,
theoretically, $n-r$ zero eigenvalues. This is because the hypothesis of cointegration is formulated as the
reduced rank of $\Pi = \alpha \beta'$ and the full rank of $\alpha_{\perp}^\top \Gamma \beta_{\perp}$, where $\alpha$ and $\beta$ are $n \times r$ matrices and $\alpha_{\perp}$ and
$\beta_{\perp}$ are $n \times (n-r)$ matrices orthogonal to $\alpha$ and $\beta$. This allows us then to distinguish between $r$
cointegrating I(0) relations and $n-r$ non-cointegrating I(1) relations.
### Appendix A.3 Details on Data

<table>
<thead>
<tr>
<th>Country</th>
<th>Survey</th>
<th>Time Coverage and Frequency</th>
<th>Spatial Coverage</th>
<th>Sample</th>
<th>Definition of Formal Sector All who declared:</th>
<th>Definition of S.E. Sector All who declared:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>Pesquisa Mensal de Emprego - PME (Monthly Employment Survey)</td>
<td>From first quarter of 1983 to fourth quarter of 2002&lt;br&gt;Each quarter is represented by the last month of that quarter.</td>
<td>6 major metropolitan regions (covering 25% of the national labor market): Paulo, Rio de Janeiro, Belo Horizonte, Porto Alegre, Recife and Salvador.</td>
<td>Males above 15 years old</td>
<td>- to be working or to have a work during the survey's week&lt;br&gt;- to be employees in their work&lt;br&gt;- to have a work-card (carteira de trabalho)&lt;br&gt;- to have NOT a work-card and to be working in some activity related to the public sector</td>
<td>- to be working or to have a work during the survey's week&lt;br&gt;- to be employers&lt;br&gt;- to be self employed</td>
</tr>
<tr>
<td>Mexico</td>
<td>Encuesta Nacional de Empleo Urbano – ENEU (National Survey of Urban Employment)</td>
<td>From first quarter of 1987 to fourth quarter of 2004</td>
<td>16 major urban areas, covering 60% of urban population</td>
<td>Males between 11 and 99 years</td>
<td>- to be employees of firms with more than 5 employees(*) with social benefits&lt;br&gt;- to be owners of firms with more than 5 employees&lt;br&gt;- to be commission workers with social benefits</td>
<td>- to be owners of firms with 5 or less employees&lt;br&gt;- to be self employed&lt;br&gt;- to be commission workers without benefits</td>
</tr>
<tr>
<td>Argentina</td>
<td>Encuesta Permanente de Hogares – EPH (Permanent Employment Survey)</td>
<td>From second wave of 1985 to first wave of 2003 (two waves per year, one in May, one in October)</td>
<td>Gran Buenos Aires</td>
<td>Males between 12 and 75 years</td>
<td>- to be working during the survey's period&lt;br&gt;- to be employees in their work&lt;br&gt;- to have a pension plan in their current employment</td>
<td>- to be working or to have a work during the survey's week&lt;br&gt;- to be self employed&lt;br&gt;- to be employers in firms with 5 or less workers (**)</td>
</tr>
<tr>
<td>Colombia</td>
<td>Encuesta Nacional de Hogares – ENH (National Household Survey)</td>
<td>From first quarter of 1985 to second quarter of 2004</td>
<td>7 major metropolitan areas (Barranquilla, Bucaramanga, Bogota, Manizales, Medellin, Cali, and Pasto)</td>
<td>Males between 11 and 99 years with less than 12 years of education(****)</td>
<td>Not possible to identify Formal Salaried, just Salaried:&lt;br&gt;- those who declared to be working for a private firm or for the Government</td>
<td>- to be employers&lt;br&gt;- to be self employed</td>
</tr>
</tbody>
</table>

(*) Due to a modification in the questionnaire (1994), a firm is considered to be small if it has 6 or less workers for all periods before to third quarter of 1994.

(**) Employers in big firms were dropped to avoid unnecessary pro-cyclicality in formal wages. These individuals account for a reduced number so the sector sizes are not affected after dropping them (e.g. in 2003 I, 2.7% of the formal workers were employers in big firms).

(****) All observations with incomplete monetary income declarations are dropped from the sample.
Appendix A.4: Model Specification Tests for the VAR models: Table A.1: Tests for Long-Run Exclusion, Stationarity and Weak Exogeneity

<table>
<thead>
<tr>
<th>Model Specification:</th>
<th>Test for Long-Run Exclusion: $\chi^2(r)$</th>
<th>Test for Stationarity: $\chi^2(p - r)$</th>
<th>Test for Weak-Exogeneity: $\chi^2(r)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LR-Test</td>
<td>LR-Test</td>
<td>LR-Test</td>
</tr>
<tr>
<td>$r$</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$dgf$</td>
<td>1</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>$\chi^2(5)$</td>
<td>3.84</td>
<td>7.81</td>
<td>3.84</td>
</tr>
<tr>
<td>$\chi^2(5)$</td>
<td>5.99</td>
<td>5.99</td>
<td>5.99</td>
</tr>
<tr>
<td><strong>Mexico</strong></td>
<td><strong>16.97</strong></td>
<td><strong>37.96</strong></td>
<td><strong>0.57</strong></td>
</tr>
<tr>
<td>Lag length: 4</td>
<td><strong>10.54</strong></td>
<td><strong>38.52</strong></td>
<td><strong>12.06</strong></td>
</tr>
<tr>
<td>Dummies: P</td>
<td><strong>26.06</strong></td>
<td><strong>35.22</strong></td>
<td><strong>21.22</strong></td>
</tr>
<tr>
<td>1995 Q1</td>
<td>Constant</td>
<td>12.6</td>
<td>14.4</td>
</tr>
<tr>
<td>(Peso Crisis)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Brazil</strong></td>
<td><strong>10.27</strong></td>
<td><strong>15.05</strong></td>
<td><strong>17.34</strong></td>
</tr>
<tr>
<td>Lag Length: 3</td>
<td><strong>4.23</strong></td>
<td><strong>16.3</strong></td>
<td><strong>1.69</strong></td>
</tr>
<tr>
<td>Dummies: P</td>
<td><strong>13</strong></td>
<td><strong>14.8</strong></td>
<td><strong>0.13</strong></td>
</tr>
<tr>
<td>1994 Q2, 1991 Q1</td>
<td>Constant</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>(Currency conversion from Cruzerio Real to Real; Real Devaluation)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Colombia</strong></td>
<td><strong>21.58</strong></td>
<td><strong>28.18</strong></td>
<td><strong>2.83</strong></td>
</tr>
<tr>
<td>Lag Length: 2</td>
<td><strong>11.25</strong></td>
<td><strong>27.76</strong></td>
<td><strong>3.33</strong></td>
</tr>
<tr>
<td>No dummies</td>
<td><strong>12.81</strong></td>
<td><strong>28.96</strong></td>
<td><strong>24.83</strong></td>
</tr>
<tr>
<td>Constant</td>
<td><strong>20.6</strong></td>
<td><strong>23.1</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Argentina</strong></td>
<td><strong>8.78</strong></td>
<td><strong>31.54</strong></td>
<td><strong>18.33</strong></td>
</tr>
<tr>
<td>Lag Length: 4</td>
<td><strong>27.01</strong></td>
<td><strong>23.8</strong></td>
<td><strong>0.34</strong></td>
</tr>
<tr>
<td>Dummies</td>
<td><strong>10.23</strong></td>
<td><strong>32.07</strong></td>
<td><strong>1.34</strong></td>
</tr>
<tr>
<td>1991:1; 2002:1</td>
<td>Constant</td>
<td><strong>9.45</strong></td>
<td><strong>12.73</strong></td>
</tr>
</tbody>
</table>

Note: The table above provides the results of model specification tests for the VAR models, including tests for long-run exclusion, stationarity, and weak exogeneity. The table includes model specifications, test statistics, and dummy variables for countries such as Mexico, Brazil, Colombia, and Argentina, along with their respective lag lengths and dummy variables.
### Table A.2: Multivariate Statistics (Residual Analysis)

<table>
<thead>
<tr>
<th></th>
<th>México:</th>
<th>Brazil:</th>
<th>Colombia:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Information Criteria:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SC</td>
<td>-21.09</td>
<td>-15.81</td>
<td>-13.37</td>
</tr>
<tr>
<td>HQ</td>
<td>-21.81</td>
<td>-16.52</td>
<td>-13.85</td>
</tr>
<tr>
<td><strong>Autocorrelation</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ljung-Box:</td>
<td>$\chi^2$ (114) = 126.3, p-val. = 0.05</td>
<td>$\chi^2$ (150) = 159, p-val. = 0.18</td>
<td>$\chi^2$ (99) = 129.5, p-value = 0.02</td>
</tr>
<tr>
<td>LM(1)</td>
<td>$\chi^2$ (9) = 8.1, p-value = 0.53</td>
<td>$\chi^2$ (9) = 4.52, p-value = 0.87</td>
<td>$\chi^2$ (9) = 7.6, p-value = 0.58</td>
</tr>
<tr>
<td>LM(4)</td>
<td>$\chi^2$ (9) = 17.2, p-value = 0.05</td>
<td>$\chi^2$ (9) = 13.1, p-value = 0.16</td>
<td>$\chi^2$ (9) = 7.9, p-value = 0.54</td>
</tr>
<tr>
<td><strong>Normality</strong></td>
<td>$\chi^2$ (6) = 9.94, p-value = 0.13</td>
<td>$\chi^2$ (6) = 16.4, p-value = 0.01</td>
<td>$\chi^2$ (6) = 31.5, p-value = 0.00</td>
</tr>
</tbody>
</table>

### Table A.3: Univariate Test Statistics (Residual Analysis)

<table>
<thead>
<tr>
<th></th>
<th>Mexico</th>
<th>Brazil</th>
<th>Colombia</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Skewness</strong></td>
<td>$\pi_T/\pi_N$</td>
<td>$W_T/W_N$</td>
<td>$p$</td>
</tr>
<tr>
<td></td>
<td>-0.066</td>
<td>-0.446</td>
<td>0.718</td>
</tr>
<tr>
<td><strong>Kurtosis</strong></td>
<td>2.345</td>
<td>3.785</td>
<td>3.940</td>
</tr>
<tr>
<td><strong>ARCH</strong></td>
<td>0.941</td>
<td>5.077</td>
<td>2.307</td>
</tr>
<tr>
<td><strong>Normality</strong></td>
<td>0.830</td>
<td>4.061</td>
<td>5.884</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.290</td>
<td>0.470</td>
<td>0.765</td>
</tr>
</tbody>
</table>
### Table A.4

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Alternative Hypothesis</th>
<th>Lag: 4 With Constant</th>
<th>Lag: 3 With Constant</th>
<th>Lag: 3 With Constant</th>
<th>Lag: 2 With Constant</th>
<th>95% Critical Value</th>
<th>90% Critical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{\text{trace test}}$</td>
<td>$r = 0$</td>
<td>$r &gt; 0$</td>
<td>36.65*</td>
<td>59.85*</td>
<td>51.06*</td>
<td>36.45*</td>
<td>35.10</td>
</tr>
<tr>
<td>$r \leq 1$</td>
<td>$r &gt; 1$</td>
<td>11.38</td>
<td>13.32</td>
<td>19.54</td>
<td>6.43</td>
<td>20.17</td>
<td>17.79</td>
</tr>
<tr>
<td>$r \leq 2$</td>
<td>$r &gt; 2$</td>
<td>0.53</td>
<td>5.71</td>
<td>5.36</td>
<td>1.69</td>
<td>9.10</td>
<td>7.50</td>
</tr>
</tbody>
</table>

*Rejection at the 5% level of significance.*

### Table A.5: Adjustment Coefficients

<table>
<thead>
<tr>
<th>Variables</th>
<th>Argentina</th>
<th>Brazil</th>
<th>Colombia</th>
<th>Mexico</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta L_t/L_N$</td>
<td>-0.356</td>
<td>-0.048</td>
<td>-0.609</td>
<td>-0.265</td>
</tr>
<tr>
<td>(-4.211)</td>
<td>(-5.715)</td>
<td>(-2.743)</td>
<td>(-1.710)</td>
<td>(-6.795)</td>
</tr>
<tr>
<td>$\Delta W_t/W_N$</td>
<td>-0.238</td>
<td>-0.010</td>
<td>-0.131</td>
<td>0.190</td>
</tr>
<tr>
<td>(-2.407)</td>
<td>(-0.457)</td>
<td>(-0.223)</td>
<td>(0.779)</td>
<td>(-0.520)</td>
</tr>
<tr>
<td>$\Delta p$</td>
<td>0.040</td>
<td>-0.014</td>
<td>-1.269</td>
<td>-0.332</td>
</tr>
<tr>
<td>(0.369)</td>
<td>(-0.509)</td>
<td>(-3.055)</td>
<td>(-6.233)</td>
<td>(-0.873)</td>
</tr>
</tbody>
</table>

*Note: $\Delta$ indicates a variable in first differences. t statistics in parentheses.*

---

27 The Reinsel-Ahn small sample corrected critical value at the 10% level is 43.13 for Mexico, 39.53 for Brazil and 36.32 for Colombia.