Abstract: This paper examines the trade-off between strategic investment commitment and flexibility under oligopoly. The model consists of two periods, the first is characterised by demand uncertainty which is resolved in the second period. Firms decide whether to commit to capital in the first period or to postpone investment to the second period while choosing quantities in period two. We show that the lower cost firm will remain committed at higher levels of uncertainty. The Nash equilibrium of an endogenous investment timing game is determined for different levels of uncertainty and cost asymmetries. A welfare analysis is provided.

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1. Introduction

Strategically motivated investment commitment by rival firms implies the loss of the flexibility required for adjusting to unexpected demand shocks. A firm that undertakes a large capital investment so as to manipulate the future output or price choice of its rival may find itself overextended if the demand for the good turns out to be less than it anticipated. In contrast, a firm that delays its capital choice until it knows more about demand retains the flexibility to cope with demand fluctuations. Retention of flexibility however, may imply the surrender of a first-mover advantage. The purpose of this paper is to study the trade-off between investment commitment and flexibility in an oligopoly setting.

The trade-off between commitment and flexibility in a strategic environment has been discussed by Appelbaum and Lim (1985) and Daughety and Reinganum (1990). In the model we consider here, firms have a choice between investing early or later. If investment takes place in period one then the capital is chosen before the resolution of demand uncertainty, while if it is chosen in period two it is done so in the context of a known demand function. This choice of when to invest gives rise to endogenous timing in the investment game. Since the 1980s there has been considerable interest in the question of endogenous timing in the choice of strategic variables in oligopolistic markets. Hence another aim of this paper is to contribute to the understanding of the timing of investment decisions in an oligopolistic setting.

In this paper we derive equilibrium capital levels and investment move order under different levels of uncertainty and under different degrees of cost asymmetry. We find that the low cost firm will typically value commitment relative to flexibility more highly than its higher cost rival. We also examine the welfare rankings of different move orders.

In section two of the paper we set up the basic model in which two rival firms choose capital and output for a market characterised by demand uncertainty. In section three we calculate the firms payoffs for all possible investment timing combinations. We then compute the Nash equilibrium of the game for different levels of uncertainty and cost asymmetries. In section

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1 See for instance Gal-Or (1985), Dowrick (1986), Boyer and Moreaux (1987), Hamilton and Slutsky (1990), Brander and Spencer (1992) and Canoy and Van Cayseele (1996). There has also been an interest in timing in the strategic trade literature (see for instance Arvan (1991)).
four we calculate welfare rankings for the different commitment and flexibility combinations. Section five concludes with suggestions for future research.

2. The model: Capital investment with demand uncertainty

Two firms, producing an identical product and competing in a Cournot fashion, face uncertainty about demand in the market. The inverse demand function entails a stochastic component, denoted by \( u \), reflecting this uncertainty:

\[
p = a - Q + u
\]

with \( Q = q_1 + q_2 \) and \( u \) defined over the closed interval \( u \in [\underline{u}, \overline{u}] \) with zero mean and variance \( \sigma^2 \). Firms’ output are respectively denoted by \( q_1 \) and \( q_2 \). Firms know each other’s cost function, respectively given by expressions (2a) and (2b):

\[
C_1 = b - k_1 g + \frac{k_1^2}{2 \eta} \quad \text{with} \quad C_{1q_1} = -q_1 + \frac{k_1}{\eta} \quad \text{and} \quad C_{1q_2} = c_1 - k_1 \quad (2a)
\]

\[
C_2 = b - k_2 g + \frac{k_2^2}{2 \eta} \quad \text{with} \quad C_{2q_2} = -q_2 + \frac{k_2}{\eta} \quad \text{and} \quad C_{2q_2} = c_2 - k_2 \quad (2b)
\]

\( c_1 \) and \( c_2 \) are constants; \( \eta \) is a constant as well, affecting the marginal costs of both firms in the same way. Without loss of generality we assume that firm two’s marginal costs are at least as high as those of its rival \( (c_1 \leq c_2) \).

Since investing in capital reduces the marginal cost of production, then in the absence of uncertainty the firms prefer committing to capital in the period before output is determined to delaying their investment decisions until outputs are chosen. Strategic commitment to capital allows firms to influence the outcome of the game in the product market to their advantage. However, when firms face demand uncertainty which is resolved only in the period when outputs are decided, such type of commitment loses part of its attraction as flexibility, i.e. delaying investment until uncertainty has cleared up, now becomes important. Throughout the paper, firms are assumed to be risk neutral which simplifies the analysis considerably. Risk aversion does not alter the qualitative nature of our results. We now examine the trade-off between commitment and flexibility generated by the demand uncertainty firms are confronted with.
3. The firm’s options: Capital commitment or flexibility

The game consists of two periods, the first of which is characterised by demand uncertainty which is resolved at the start of the second period. Firms decide whether to commit to capital in the first period or to postpone investing in capital to the second period while simultaneously determining outputs in period two.

Evidently, the firm’s choice between commitment or delay will depend on its conjecture of the competitor’s strategy. Throughout the game we assume that each strategy combination yields strictly positive output by both firms, thereby focusing on interior solutions only. Suppose the rival delays its capital investment, then the risk neutral firms simply compares expected profits from the two strategies to determine whether it should also delay its investment or commit instead.

We first look at the case where both firms delay investment. Nothing happens in the first period and both firms decide simultaneously on output and capital in the second period when uncertainty has evaporated. The first order conditions for outputs of the respective firms are given by

\[ A_1 - 2q_1 - q_2 + k_1 + u = 0 \]  \hspace{1cm} (3a)
\[ A_2 - q_1 - 2q_2 + k_2 + u = 0 \]  \hspace{1cm} (3b)

with \( A_i \equiv a - c_i \) and \( A_2 \equiv a - c_2 \). At the same time, differentiating expected profits with respect to capital leads to the following first order conditions

\[ q_1 - \frac{k_1}{\eta} = 0 \]  \hspace{1cm} (4a)
\[ q_2 - \frac{k_2}{\eta} = 0 \]  \hspace{1cm} (4b)

implying that capital levels are set equal to:

\[ k_1 = \eta q_1 \]  \hspace{1cm} (5a)
\[ k_2 = \eta q_2 \]  \hspace{1cm} (5b)

Substituting expressions (5a) and (5b) into (3a) and (3b) and solving for \( q_1 \) and \( q_2 \) yields the output levels for the respective firms:

\[ q_1 = \frac{b - \eta g - A_2 + b - \eta g}{3 - \eta b - \eta g} \]  \hspace{1cm} (6a)
Expected profits for the firm producing output $q_1$ are equal to $E\pi_1^{dd} = E \left( -\frac{di}{2\eta} \right)$, where the first superscript refers to delayed capital investment by firm one and the second to delayed capital investment by firm two. After substituting for expressions (5a) and (6a), expected profits can be written as

$$E\pi_1^{dd} = \frac{1 - \frac{b}{2\eta} \left[ b \eta g - A_2 \right]^2 + \sigma^2 \frac{b}{b - g}}{b - g \left[ b - \eta g \right]} \quad \text{(7)}$$

Note that profits are increasing in the variance (since $\eta < 1$), a result of the profit function being convex in $u$. This means that as uncertainty goes up, reflected in an increase in the mean-preserving spread, expected profits rise. The reason for this lies in the fact that firms can adjust their outputs, reducing them when negative demand shocks occur and expanding production when the opposite prevails.

Given that its rival delays investment, the firm’s alternative is to commit to investing in capital. In the second period, output decisions are made (expressions (3a) and (3b)) and the competing firm sets its capital (5b). Hence, outputs are given by

$$q_1 = \frac{b - \eta g - A_2 + b - \eta g + b - \eta g}{3 - 2\eta} \quad \text{(8a)}$$

$$q_2 = \frac{2A_2 - A_1 - k_i + u}{3 - 2\eta} \quad \text{(8b)}$$

Note $\frac{dq_1}{du} = \frac{1 - \eta}{3 - 2\eta} < \frac{dq_2}{du} = \frac{1}{3 - 2\eta}$. The loss in output flexibility for firm one is due to its commitment to capital in the period before the uncertainty has been resolved.

In the first period differentiating expected profits with respect to capital then yields

$$E \frac{d\pi_1^{dd}}{dk_1} = Eq_1 - k_i \frac{1}{\eta} + E \frac{dq_2}{dk_1} \left|_{k_1=0} \right. \quad \text{(9)}$$

with $\pi_{1u_2} = -q_1$ and $\frac{dq_2}{dk_1} = -\frac{1}{3 - 2\eta}$.

The capital commitment implied by this condition is equal to
\[ k_1 = \frac{2\eta B - \eta G}{3 - 2\eta} \]  

(10)

Since \( 2B - \eta G > 3 - 2\eta \), it is obvious from expressions (5a) and (10), that, given that the rival delays investment, firm one’s expected capital when investing in period one is larger than in period two. This is logical as firm one strategically overinvests in capital when deciding to precommit. Expected profit is now

\[ E\pi_{1}^{sd} = 4 \left[ \frac{b - \eta g - A_2}{3 - 2\eta} + \frac{b - \eta g}{3 - 2\eta} \right] \]  

(11)

Now suppose the rival firm commits to capital in the first period. Again, firm one decides whether to delay or commit to capital by comparing expected profits associated with each strategy.

We first consider the delay option. Firm one’s capital choice is then given by expression (10) and, given that the opponent has committed to capital in the previous period, \( k_2 \) is taken as given and outputs are equal to:

\[ q_1 = \frac{2A_1 - A_2 - k_2 + u}{3 - 2\eta} \quad (12a) \]

\[ q_2 = \frac{b - \eta g - A_1 + \eta g + b - \eta g}{3 - 2\eta} \quad (12b) \]

The opponent differentiates expected profits with respect to capital in period one, which yields the optimal committed capital level:

\[ k_2 = \frac{2b - \eta g}{3 - 2\eta} \]  

(13)

After substituting (13) in (12a) and given (10), we calculate expected profits for delay given that the rival firm commits:

\[ E\pi_{1}^{dc} = 4 \left[ \frac{\eta}{2} \left[ \frac{b - \eta g - 1}{3 - 2\eta} + \frac{b - \eta g}{3 - 2\eta} \right] + \frac{\eta g}{\frac{3 - 2\eta}{b - \eta g}} \right] \]  

(14)

The final scenario involves capital commitment by both firms. Then, in the first period, capital levels are equal to

\[ k_1 = \frac{4}{3} \eta E q_1 \]  

(15a)
\[ k_2 = \frac{4}{3} \eta E q_2 \]  

(15b)

Since capital investments are strategic substitutes\(^2\), firms are now committed to a lower capital level than in the case where the opponent delays \( \frac{4}{3} < \frac{2 \eta C}{3 - 2 \eta} \) from comparing expression (10) and (15a)).

Expected profits under this last scenario are:

\[ E \pi^{ce}_1 = \frac{1}{9} \left[ \frac{b g h A_1 - A_2}{1 - \frac{3}{4} c} + \frac{\sigma^2}{\eta} \right] \]  

(16)

3.1. The symmetric cost case

After having calculated the payoffs of the all possible strategy combinations, we now solve the game. In this section, we focus on the symmetric case \( c_1 = c_2, \) hence \( A_1 = A_2). \) The actual level of uncertainty measured by the variance \( (\sigma^2) \) and the cost of capital (inversely related to \( \eta \)) are crucial in the timing of firms’ investments. We calculate two loci determining values of \( \sigma^2 \) and \( \eta \) for which each firm is indifferent between capital commitment and investment delay. One of the indifference loci is derived given rival commitment \( (E \pi^{cc}_1 = E \pi^{dc}_1), \) while the other is valid when the competitor delays investment \( (E \pi^{cd}_1 = E \pi^{dd}_1). \) As symmetry is imposed, the indifference loci are for both firms are identical. In figure 1 these loci are graphically represented with the normalised variance \( (\bar{\sigma}^2 = \sigma^2 / A_1^2) \) on the vertical axis.

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\(^2\) Substituting output levels into equations (15a) and (15b) immediately shows

\[ \frac{dk_2}{dk_1} = \frac{dk_1}{dk_2} = -\frac{b g h}{1 - \frac{3}{4} c} \quad < 0 \]
Indifference loci given rival commitment lie above the loci for rival delay ($\bar{\sigma}^2_{\eta_1}=\eta_1\bar{\sigma}^2_{\eta_2}>\bar{\sigma}^2_{\eta_1}=\eta_1\bar{\sigma}^2_{\eta_2}$ in Figure 1). Firms will prefer giving up flexibility at higher levels of uncertainty given rival commitment as opposed to delay, thereby avoiding being in a strategically disadvantageous position. If capital investment is relatively cheap ($\eta$ is high), commitment becomes relatively attractive and firms will commit at higher levels of uncertainty. This implies that the curves are sloping upwards.

The $\bar{\sigma}^2,\eta$-space is divided into three areas by the loci. Ultimately, the actual values of uncertainty and capital cost will pin down in which area firms find themselves.

Now we determine the outcome of the game in each area. In region I, commitment is a dominant strategy for both firms, implying that commitment by both firms is the unique equilibrium when uncertainty is sufficiently low (i.e., for $\bar{\sigma}^2<\bar{\sigma}^2_{\eta_1}=\eta_1\bar{\sigma}^2_{\eta_2}$). The horizontal axis in figure 1 reflects the certainty case, where firms always commit as there is no need for flexibility. For sufficiently low levels of uncertainty firms trade off flexibility for commitment as the strategic advantage from commitment dominates the gain from flexibility associated with delay.
The other extreme prevails in area III. Here, demand uncertainty is high enough (i.e., \( \bar{\sigma}^2 > \bar{\sigma}^2 \mid \bar{E} \bar{E} = 1 \)) for firms to prefer flexibility to commitment. Both firms delaying capital investment is the unique equilibrium in this area.

In the region between the two loci (area II), commitment nor delay is a dominant strategy for either firm. Given that the rival commits, each firm will commit too, while if the opponent delays, firms will opt for flexibility. Thus, there are two equilibria in area II, with the double delay equilibrium being the focal equilibrium, as from the firms’ viewpoint it Pareto dominates double commitment. With symmetry, this equilibrium always yields higher profits since in the alternative equilibrium firms strategically overinvest in capital thereby reducing profits. Note that this also implies that a prisoner’s dilemma prevails in area I where firms are forced into strategic overinvestment by the relatively low degree of demand uncertainty.

3.2. Commitment versus flexibility with cost asymmetries

When firms’ costs differ, their indifference loci no longer coincide, which obviously influences the outcome of the game. Especially for intermediate levels of uncertainty, the width of the cost wedge between firms crucially determines the equilibrium. We distinguish between ‘small’ and ‘large’ cost differences, where the latter imply that the two indifference loci of the low cost firm both lie entirely above those of its high cost counterpart. Without loss of generality, firm one is assumed to have the lowest cost of the two rivals.

When the cost asymmetry between firms is ‘small’, each indifference locus of the high-cost firm lies below the equivalent locus of its low-cost competitor. Figure 2 depicts this situation for \( A_2 = 0.97A_1 \). The high-cost firm’s locus given rival commitment and the low-cost firm’s locus given rival delay delineate four areas in the \( \bar{E} \bar{E} \)-diagram. For intermediate levels of uncertainty, there are now two areas. The multiple equilibria region (area IIb) survives for relatively low capital costs (\( \eta > 0.1 \)). But, for relatively high capital costs (\( \eta < 0.1 \)), commitment by the low-cost firm and delaying investment by the high-cost competitor is the unique equilibrium. This is explained by the fact that the ‘small’ cost difference is magnified by the huge capital costs.
Hence, the high cost firm’s dominant strategy involves delaying investment since it cannot afford to invest in strategic capital. Conversely, since area IIa is situated below its indifference locus given that the rival delays, the low-cost firm opts for capital commitment.

For ‘large’ cost differences between competitors (e.g., $A_2 = 0.8A_1$ in figure 3), leadership by the low cost firm is the only surviving equilibrium for intermediate levels of uncertainty, driving out the multiple equilibrium region. Compared to the case with a small cost asymmetry, the high cost firm is now relatively less efficient than its low cost rival. For all relevant costs of capital investment, delay is the dominant strategy for firm two, implying that it can never afford strategic investment given its rival’s cost advantage.
4. Welfare implications

In this section, the social ranking of the four possible commitment-delay permutations of our model is determined for various ranges of uncertainty. In other words, we derive which commitment-delay outcome is preferred from a social perspective, given non-intervention by the government. Since investment makes firms ‘tough’ \( \frac{dE\pi_i}{dk_i} < 0 \) and firms compete in a Cournot fashion, they typically produce more under capital commitment than under investment delay. Thus, although capital commitment by both firms raises the social cost of investment and possibly emerges as the unique equilibrium of a prisoner’s dilemma, it is likely to benefit consumers in the form of a lower market price. Still, since commitment leads to a loss of flexibility and profits are convex in \( u \), it induces social costs (in the form of lower profits) which, at high levels of uncertainty, are likely to outweigh the benefits for consumers.

Expected welfare is defined as the sum of expected profits and expected consumer surplus (the latter being equal to \( E\left(Q^2 / 2\right) \)):
\[ EW = \sum_{i=1,2} E\pi_i + E \frac{Q^2}{2} \]  

(17)

Cost asymmetries dramatically change the social ranking of the commitment-delay market outcomes. We first discuss the symmetric case.

4.1. Social ranking of market outcomes in the symmetric case

As discussed in section 3.1, if firms have identical costs, the indifference loci for each firm given the rival’s action coincide. Figure 4 gives the actual commitment-delay outcomes versus the socially preferred outcomes for different combinations of \( \sigma^2 \) and \( \eta \). The figure is divided in four zones (I-IV). For each of these zones we give the socially preferred commitment-delay outcome and ask how this compares with the actual outcome.

For sufficiently low levels of uncertainty (areas I and II), commitment by both firms prevails. Yet, this double commitment outcome yields only the socially preferred outcome for very low uncertainty (area I), whereas investment delay by both is socially preferred in area II. On the social indifference locus (\( EW^{cc} = EW^{dd} \)), these investment regimes generate precisely the same welfare, while anywhere above this locus the social ranking indicates a social preference for investment delay by both firms. The multiplicity of equilibria at intermediate levels of uncertainty (area III) implies that the socially preferred outcome (i.e., double delay) may or may not occur, while it always occurs for sufficiently high uncertainty (area IV).

\[ \text{footnote}{Unlike the optimal policy question, this question has a positive rather than a normative connotation.} \]
4.2. Social ranking of market outcomes with cost asymmetries

Since market outcomes depend crucially on the extent of the cost asymmetries, we distinguish again between ‘large’ and ‘small’ cost differences.

First, we look at the case where the cost difference between competitors is relatively ‘large’ (see figure 5 with $A_2 / A_1 = 0.8$). For relatively low levels of uncertainty (i.e., $\bar{\sigma}^2$ below the $EW^{cd} = EW^{dd}$ locus), the social ranking favours the relatively cost efficient firm emerging as the industry leader (areas I, II and III). In area I both firms commit as we see earlier. This implies that, for very low uncertainty, the market leads to “too much commitment” from a social point of view. The problem is that part of the additional output due to strategic commitment is here supplied by the relatively high-cost firm. Consumers would benefit more if the ‘natural leader’ (the lower cost firm) produced all the extra output, leading to a lower price. Conversely, both firms delay at relatively high levels of uncertainty (area III). Hence, the market entails “too little commitment” over that range.
Here, from a social point of view, the negative effects of the lack of flexibility on the part of the low-cost firm would be compensated by consumer benefits arising from such a commitment. Only for intermediate uncertainty (area II) where a leader-follower outcome ensues, and for very high levels of uncertainty (area IV) where both firms delay, the actual market outcomes are also the socially preferred ones.

The most complex case is that characterised by “small” cost differences between the firms. To explain this case it proves convenient to present the subcases of high $\eta$ and low $\eta$ on separate diagrams. This is because the area of $\sigma^{-2}, \eta$-space at low $\eta$ is fairly intricate (see figure 6b).

Consider first the subcase of small cost differences and high $\eta$. This is represented by $\eta$-levels above 0.25 in figure 6a. The social ranking again favours investment delay by both firms at high levels of uncertainty (area IV) and double commitment at low uncertainty (area Ia). When uncertainty is fairly low, leadership by the low cost firm is the socially preferred outcome if the cost of capital investment is relatively low (area Ib). High $\eta$, that is, low capital costs increase the social value of the leadership outcome since this means that the consumer surplus can be increased significantly at a relatively low cost of overinvestment.
In region II and III the social ranking favours double delay. However, in region II both firms commit (while in region III, which is a region of multiple equilibria, both double delay and double commitment may arise). The negative effects of this “strategic commitment war” which is a prisoner’s dilemma from the firms’ point of view exceeds the benefits to consumers in lower prices.

For intermediate ranges of the cost of capital (i.e., levels of η below 0.25 in figure 6a and above 0.09 in figure 6b), the comparison between actual market outcomes and the socially preferred market outcome is identical to the discussion with small cost differences and high η, except that area Ib does no longer exists. That is to say there the leader-follower outcome is never the socially preferred one.

Now consider the case where capital costs are high (η less than 0.09 in figure 6b). For very high capital costs, yet another situation emerges. In regions Ia and IIIa the capital cost and the uncertainty is at levels that induce the higher cost firm to remain flexible, while the lower cost competitor chooses commitment.

\[
\begin{align*}
E\pi_1^{cd} &= E\pi_1^{dd}, & \emptyset: E\pi_2^{ce} &= E\pi_2^{cd}, & x: EW^{rc} &= EW^{rd}, & \ast: EW^{rd} &= EW^{dd}, & \circ: EW^{rc} &= EW^{rd}
\end{align*}
\]
Hence we get a leader-follower outcome. However, this will never be socially optimal in these regions. The social indifference locus between both firms committing and double delay (i.e., $EW^{cc} = EW^{dd}$) separates these two regions. Above the locus (area IIIa), there is too much commitment from a social perspective and below the locus (area Ia) there is not enough.

Summarising, with very high or extremely low uncertainty the actual market outcome generally is also the socially preferred one, entailing both firms delaying investment and committing to capital, respectively. For a range of intermediate levels of uncertainty, the market outcome typically no longer coincides with the socially preferred one.

5. Concluding remarks

This paper has analysed the trade-off between strategic investment commitment and flexibility in an oligopoly model consisting of two periods. The first period is characterised by demand uncertainty which is resolved in the second period. In our model firms decide whether to commit to capital in the first period or to postpone investment to the second period while

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4 This area lies to the left of the intersection between the firms’ indifference loci.
choosing quantities in period two. We have shown that the degree of cost asymmetry between
the two firms plays a crucial role, together with the level of uncertainty in determining the
nature of the Nash equilibrium outcome of the game. In particular we find that the low cost
firm typically values strategic commitment relative to the retention of capital flexibility more
highly than its higher cost rival. We have shown that this implies that the lower cost firm will
remain committed at higher levels of uncertainty. For intermediate values of uncertainty, a
natural leader-follower outcome often emerges in which the lower cost firm is the investment
leader.

Our welfare results demonstrate that, with very high or low uncertainty, the actual market
outcome is also usually the socially preferred one, entailing both firms delaying investment and
committing to capital, respectively. However, at intermediate levels of uncertainty and high
capital costs, investment leadership by the low-cost firm tends to arise as an equilibrium while
double delay or double commitment is socially preferred for high and low levels of uncertainty
respectively. By contrast, capital leadership by the low-cost is the higher welfare outcome, but
not the market outcome when capital is relatively inexpensive.

In this paper our analysis has been confined to interior solutions in which both firms produce
positive quantities. However, in the presence of fluctuating demand some firms may not enter
when demand turns out to be low. Firms will anticipate this at the investment stage and this
implies a richer set of outcomes. Allowing for asymmetric information in the analysis would
lead to yet another interesting line of research. We could for instance consider a case in which
one firm has better information about the market demand. These issues seem worth exploring
in future research.
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