GLASGOW SCIENCE FESTIVAL

Platonic Solids

Quick facts

- The Platonic solids are named after the philosopher Plato and have been known for thousands of years.
- A Platonic solid is an example of a *polyhedron* (plural: polyhedra). A polyhedron is a three-dimensional shape with flat faces, where each face is a polygon. For example a cuboid is a polyhedron, its faces are rectangles.
- A *regular polygon* is a polygon whose side lengths and interior angles are all equal, for example a square or an equilateral triangle.
- A *Platonic solid* is a *regular polyhedron*, which means that all the faces are regular polygons, all the faces are the same and the arrangement of the faces around each vertex is the same.
- There are exactly five Platonic solids: the tetrahedron (sometimes called a triangular pyramid), the cube, the octahedron, the dodecahedron and the icosahedron.

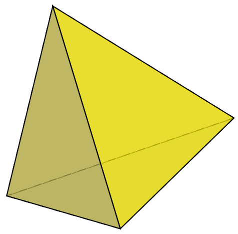
Activities

On the following pages you will find the nets for each of the five Platonic solids. A net is a two-dimensional diagram that can be cut, folded and glued to create a three-dimensional shape.

- Can you assemble each one of the nets into the corresponding Platonic solid?

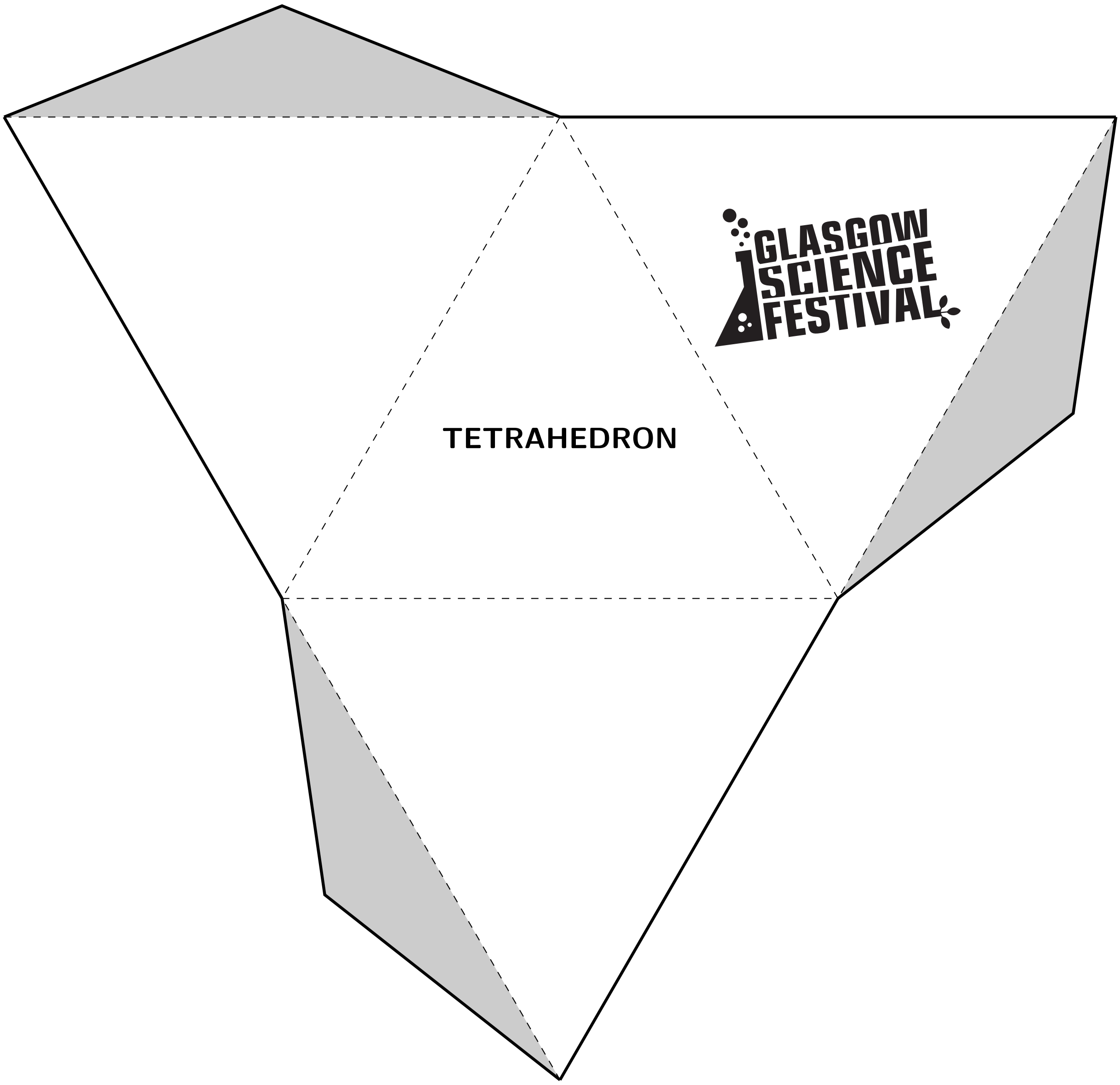
  You should print each net. Then cut along the solid lines and fold along the dashed lines. Then glue the ears (shaded in grey) to the faces to assemble the three-dimensional shape.

- On each page there is information about the number of faces, edges and vertices (corners) for each assembled shape. Try calculating $F - E + V$ for each one of the solids, where F is the number of faces, E is the number of edges and V is the number of vertices. What do you notice?

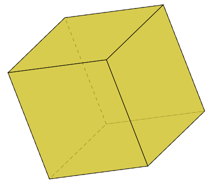


Tetrahedron

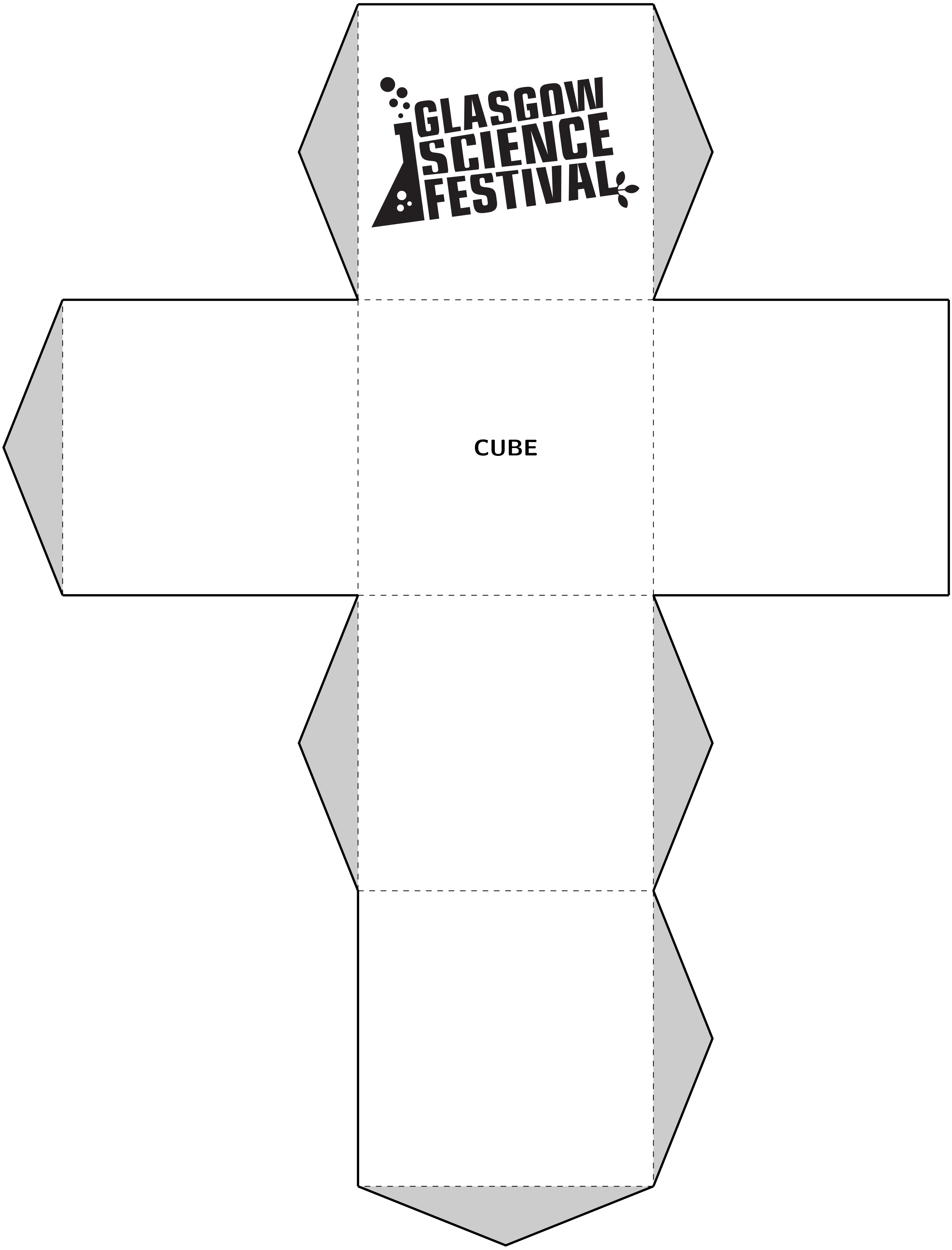
- Faces: 4, Edges: 6, Vertices: 4
- Each face is an equilateral triangle.
- The hydrocarbon methane CH_4 has four hydrogen atoms at the vertices of a tetrahedron.



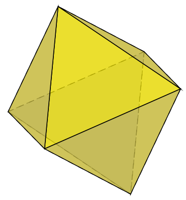
Cube



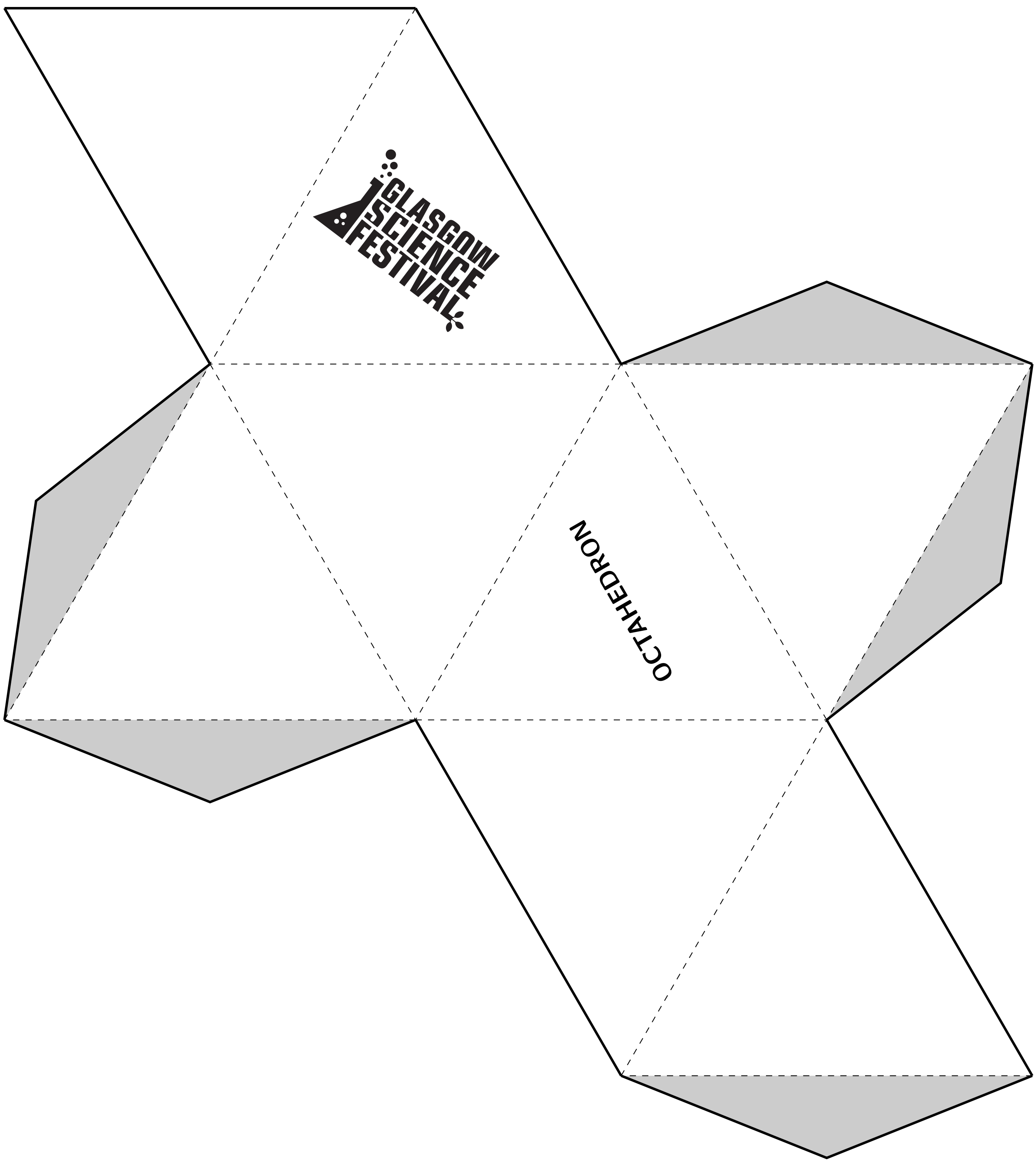
- Faces: 6, Edges: 12, Vertices: 8
- Each face is a square.



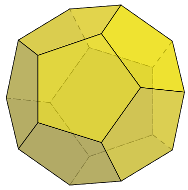
Octahedron



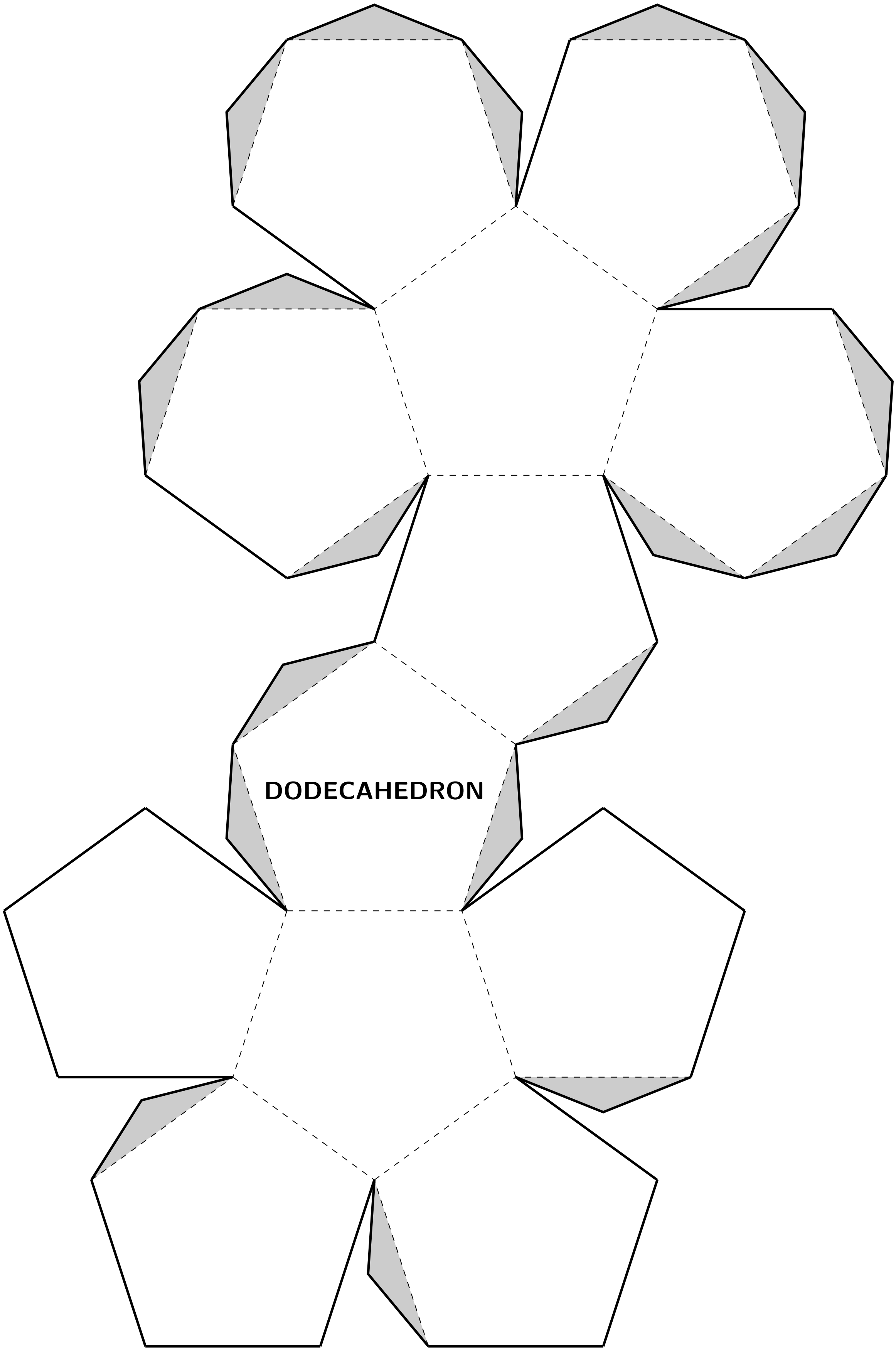
- Faces: 8, Edges: 12, Vertices: 6
- Each face is an equilateral triangle.

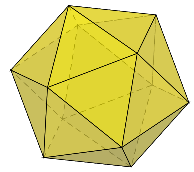


Dodecahedron



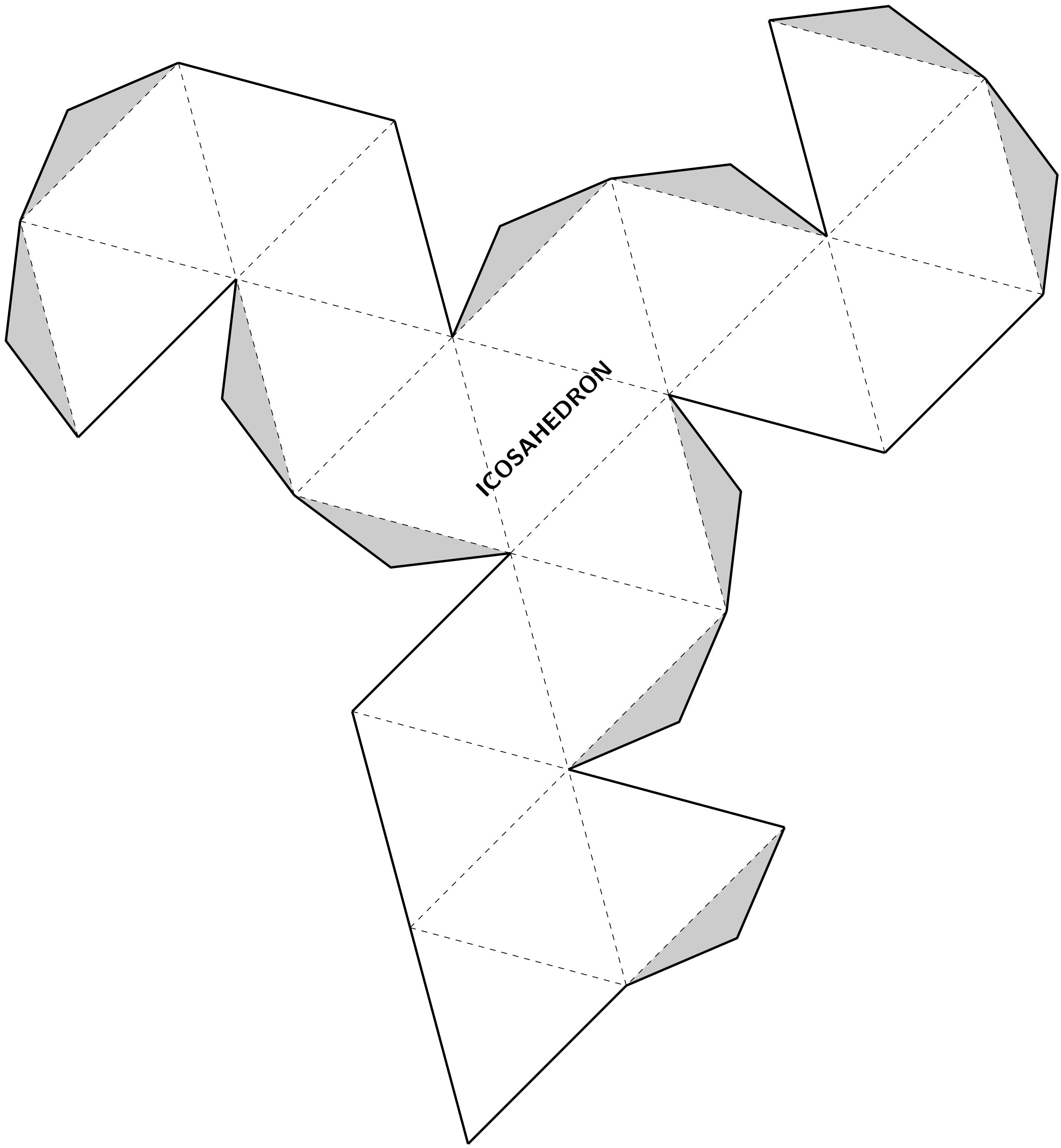
- Faces: 12, Edges: 30, Vertices: 20
- Each face is a regular pentagon.





Icosahedron

- Faces: 20, Edges: 30, Vertices: 12
- Each face is an equilateral triangle.
- The structures of many common viruses are based around icosahedral capsids.



Challenge

We can prove that there can be at most five Platonic solids by considering the arrangements of angles around a vertex. It helps to first calculate the internal angle for a regular n-gon. A regular n-gon is a convex polygon whose sides have equal length and whose internal angles are equal. Examples of regular polygons include the square, equilateral triangle and regular hexagon. Note that the *external angles* of any polygon add up to $360°$. For a regular polygon each external angle is therefore equal to

$$\frac{360°}{n}$$

and since angles on a straight line add up to to $180°$ then each *internal angle* is

$$180° - \frac{360°}{n} = 180° \times \left(\frac{n-2}{n}\right).$$

Now consider k regular n-gons meeting at a vertex in a Platonic solid (for example $k = 3$ and $n = 4$ would be three squares meeting at a vertex). We can imagine unfolding the faces around the vertex into a plane, but cutting along one of the edges. In this way we can make a *net*, which can be folded into the three-dimensional polyhedron. If k regular n-gons are arranged around a vertex then the sum of the internal angles at that vertex must be less than $360°$, since otherwise we would not be able to unfold it without overlaps. Therefore

$$k \times 180° \times \left(\frac{n-2}{n}\right) < 360°$$

or equivalently

$$\frac{1}{k} + \frac{1}{n} > \frac{1}{2}$$

where k and n are whole numbers and each bigger than or equal to 3.

- What pairs of values (k, n) satisfy the inequality above? Do the solutions correspond to any of the shapes you have constructed with the nets?

  For example $(k, n) = (3, 3)$ works because

  $$\frac{1}{3} + \frac{1}{3} = \frac{2}{3} > \frac{1}{2}$$

  and this corresponds to the tetrahedron, which has three equilateral triangles meeting at each vertex.