Instabilities in a Planar-Aligned Active Liquid Crystal

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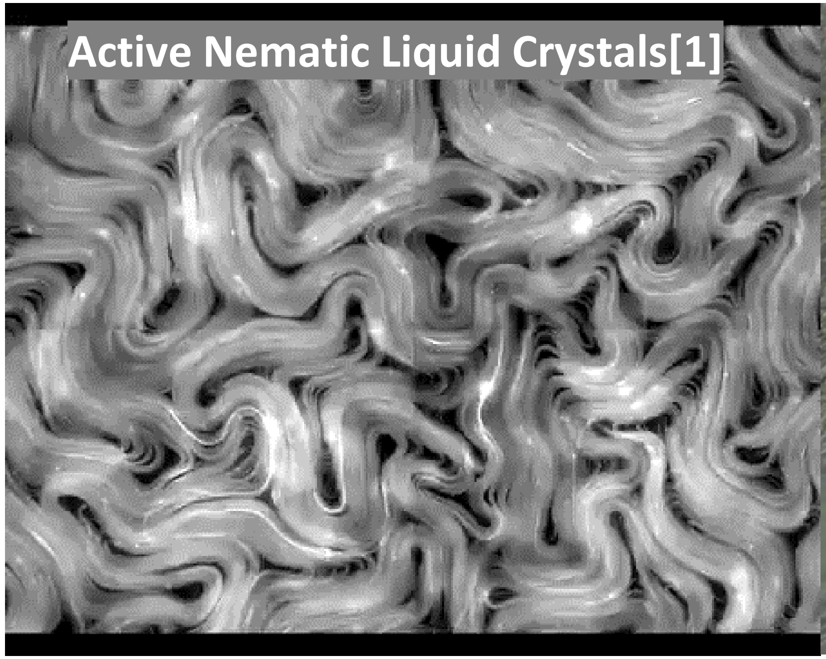
Introduction

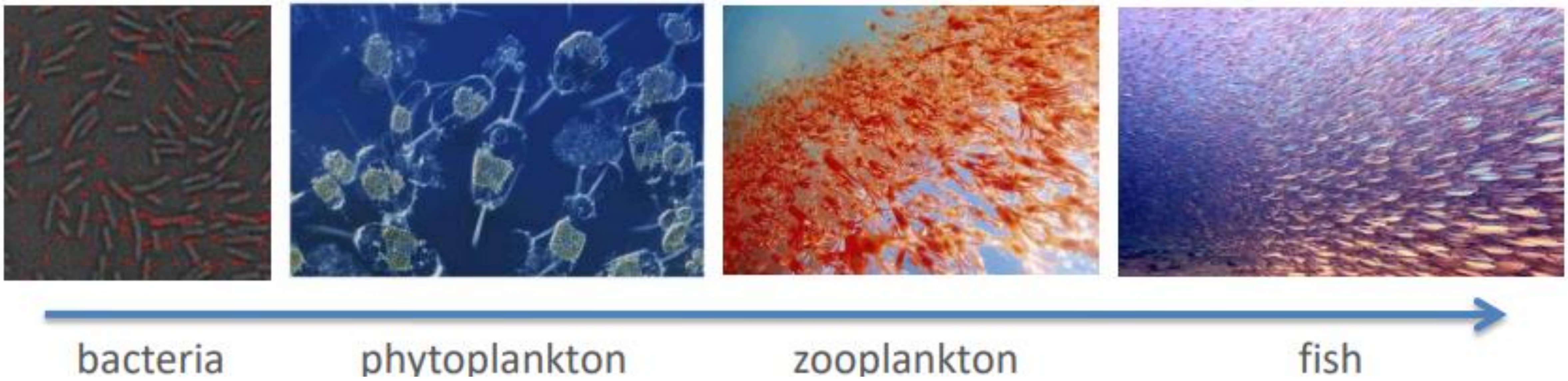
- ❖ Liquid crystals are intermediate phases, between a solid crystal and an isotropic liquid.
- ❖ Active liquid crystals are made of objects that can generate flow.
- ❖ These systems swim in patterns that suggest **long-range collective ordering**.
- ❖ The **activity** in these systems relies on **continuous energy production** by the individual particles.



1. S. J. DeCamp, G. S. Redner, A. Baskaran, M. F. Hagan, and Z. Dogic, *Nat Mater* 14, 1110, 2015.

Introduction

Examples of Active Liquid Crystals

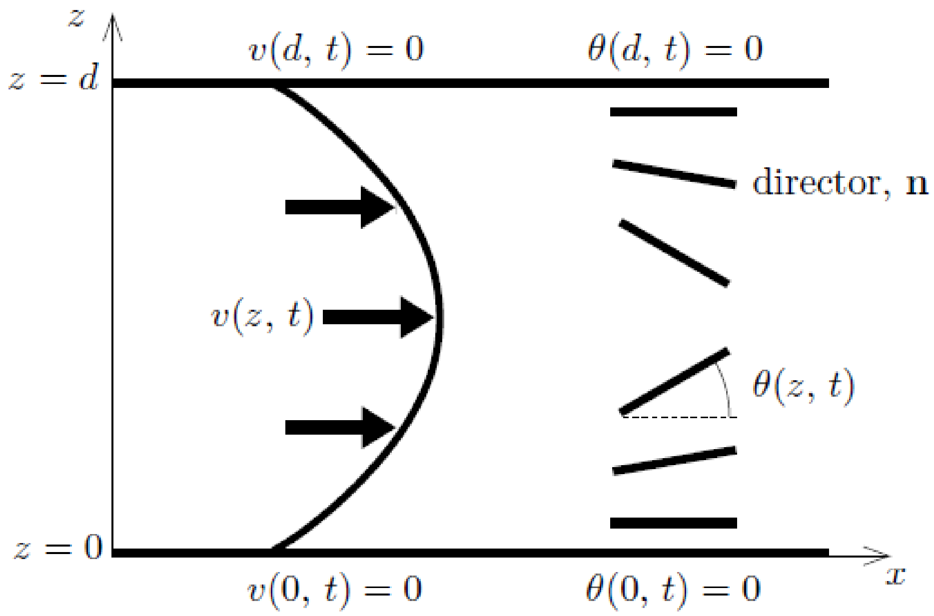


Motivation

- Will an orienting field increase or decrease the flow generated by active nematics?
- The use of active liquid crystals in designing sensors is an interesting possible application.
- We have used mathematical modelling of active nematics using an adapted version of the continuum theory of nematics.

Activity-Driven Channel Flow

Model geometry



An active nematic between two solid plates at z=0, d with flow in the *x*-direction [2].

Ericksen-Leslie Equations for an Active Nematic

❖ The simplest model uses the Ericksen-Leslie theory with just one extra term in the stress tensor [3]

$$\underbrace{\gamma_1\theta_t}_{\text{director rotation}} = \underbrace{(K_1\cos^2\theta + K_3\sin^2\theta)\theta_{zz} + (K_3-K_1)\sin\theta\cos\theta(\theta_z)^2}_{\text{elastic terms}} - \underbrace{m(\theta)v_z}_{\text{director-flow coupling}},$$

$$\underbrace{\rho v_t}_{\text{flow inertia}} = \underbrace{(g(\theta)v_z}_{\text{fluid viscosity}} + \underbrace{m(\theta)\theta_t}_{\text{director-flow coupling}} + \underbrace{\zeta\cos\theta\sin\theta)_z}_{\text{activity}}$$

❖ The stress tensor is written as: $\tau = \tau_{nem} + \underbrace{\zeta(\mathbf{n}\otimes\mathbf{n})}_{\text{activity}}$

❖ The configuration and organization of the active agents are deformed by flow, and the active agents also generate flow.

1. F. M. Leslie. Continuum theory for nematic liquid crystals. *Continuum Mechanics and Thermodynamics*, 4(3):167–175, 1992.
2. J. Walton, G. McKay, M. Grinfeld, and N. J. Mottram. *Mathematical modelling of active nematic liquid crystals in conned regions.* PhD thesis, University of Strathclyde, 2020

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Results

❖ Linearisation around a planar alignment

$$\gamma_1 \frac{\partial \theta}{\partial t} - K_1 \frac{\partial^2 \theta}{\partial z^2} - \alpha_3 \frac{\partial v}{\partial z} = 0, \quad \eta_1 \frac{\partial^2 v}{\partial z^2} + \alpha_3 \frac{\partial^2 \theta}{\partial z \partial t} + \zeta \frac{\partial \theta}{\partial z} = 0.$$

❖ The director angle and flow velocity

$$\theta(z,t) = \theta_0 \left[\cos\left(q - \frac{2q}{d}z\right) - \cos(q)\right] \exp\left(-\frac{t}{\tau}\right),$$

$$v(z,t) = v_0 \left[d \sin\left(q - \frac{2q}{d}z\right) + (2z - d)\sin(q)\right] \exp\left(-\frac{t}{\tau}\right).$$

Time constant

$$\tau = \alpha\gamma_1 \left(\frac{4K_1 q^2}{d^2} - \frac{\alpha_3 \zeta}{\eta_1}\right)^{-1}$$

Mode number

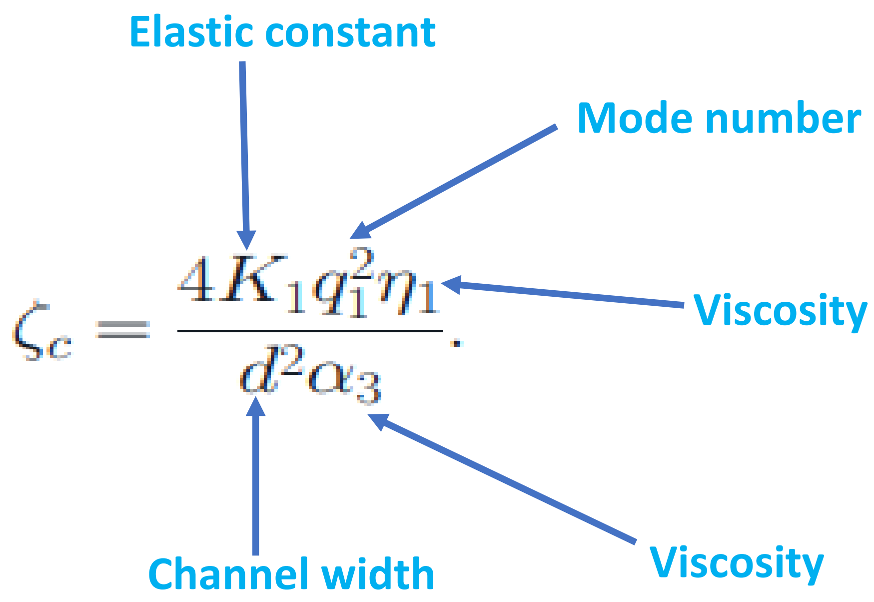
$$r(q) = q - (1-\alpha)\tan q + \frac{\alpha_2 \xi \alpha}{\eta_2}\left(\frac{4K_3 q^2}{d^2} + \frac{\alpha_2 \xi}{\eta_2}\right)^{-1} \tan q = 0.$$

❖ When the viscosity $\alpha_3 < 0$ we can see that $\tau > 0$ whenever $\zeta > 0$, so that the system is **stable.**

❖ When the viscosity $\alpha_3 > 0$ we can see that $\tau > 0$ whenever $\zeta < 0$, so that the system is **stable.**

Results

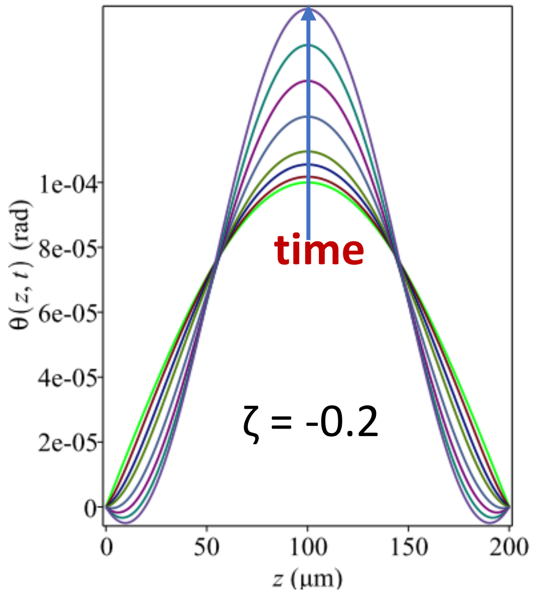
- **Instability**: The critical activity occurs at the **first mode**, n = 1, which leads to instability when $\zeta = \zeta_c$

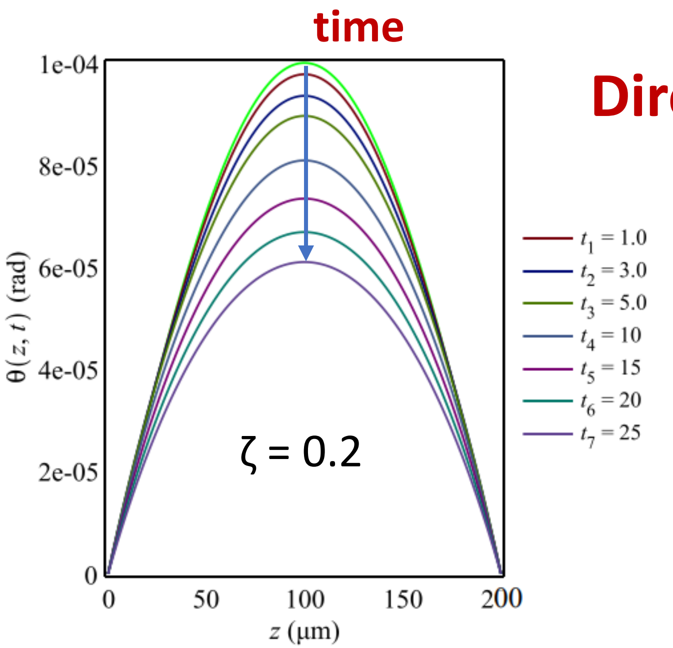


$$\zeta_c = \frac{4K_1 q_1^2 \eta_1}{d^2 \alpha_3}.$$

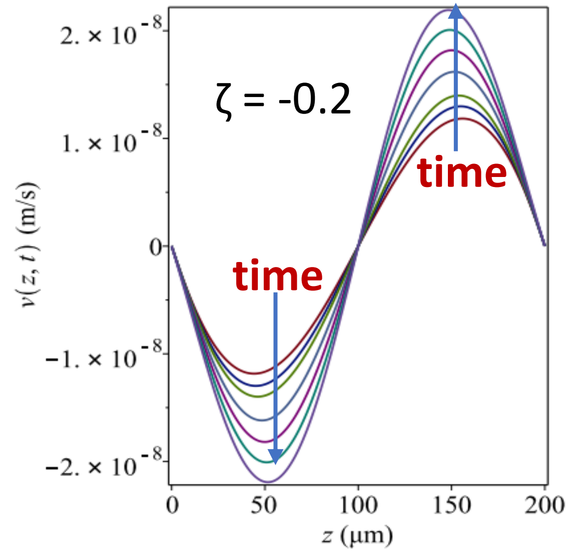
- When the active agent is rod-like then $\alpha_3 < 0$ and $\zeta_c < 0$.
- When the active agent is disc-like then $\alpha_3 > 0$ and $\zeta_c > 0$.

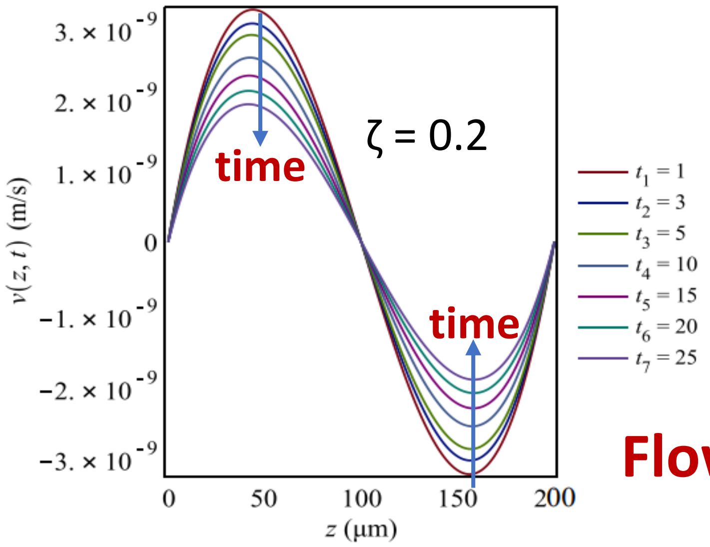
Results





Director angle



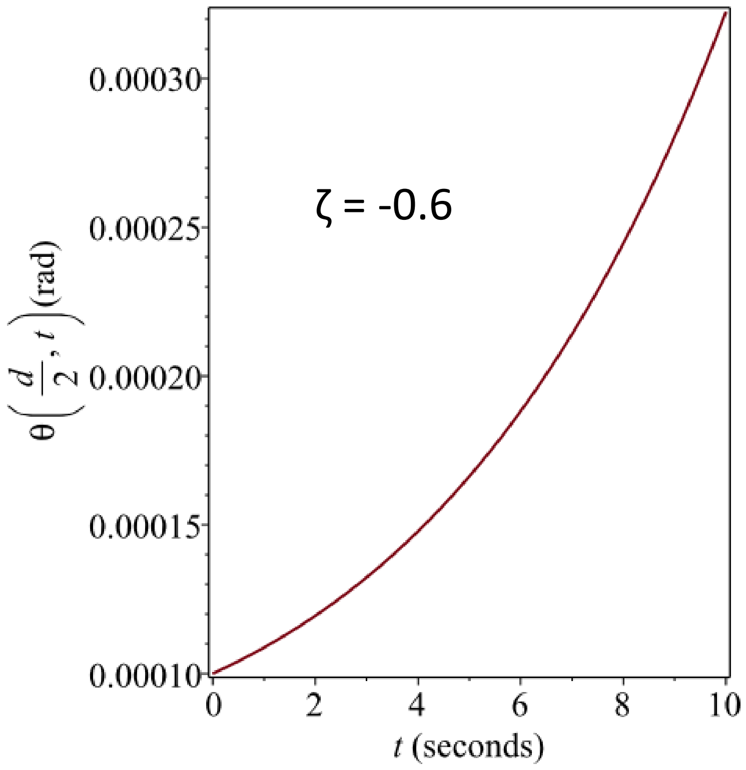


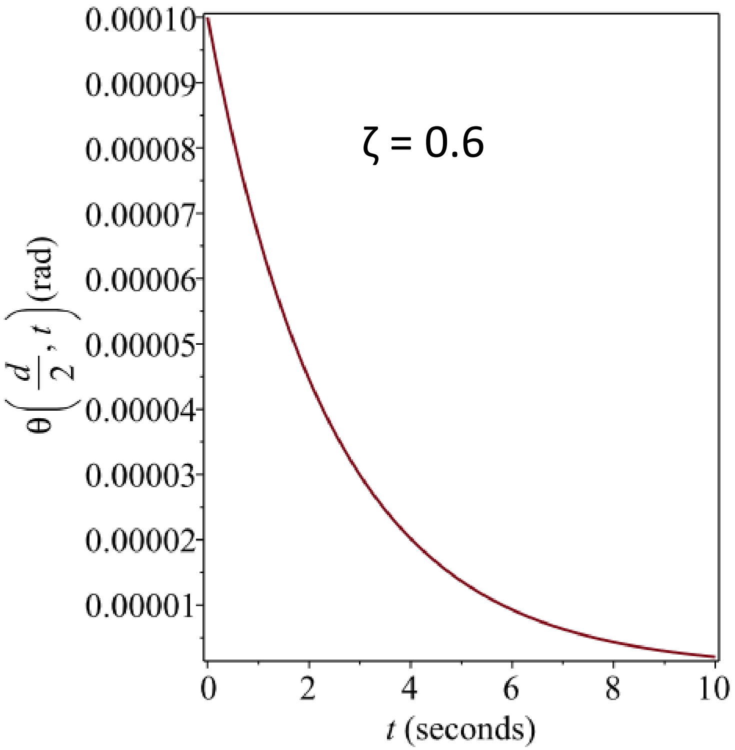
Flow velocity

- For the **negative activity** parameter the **flow grows** with time, for the **positive activity** parameter, the **flow decays**.
- As the **magnitude of the activity increases**, the active system tends to **generate flow**.
- The **fastest-growing mode** for each value of the activity is the mode with the **most negative real part** of the time constant.

Results

The director angle in the middle of the layer as a function of t

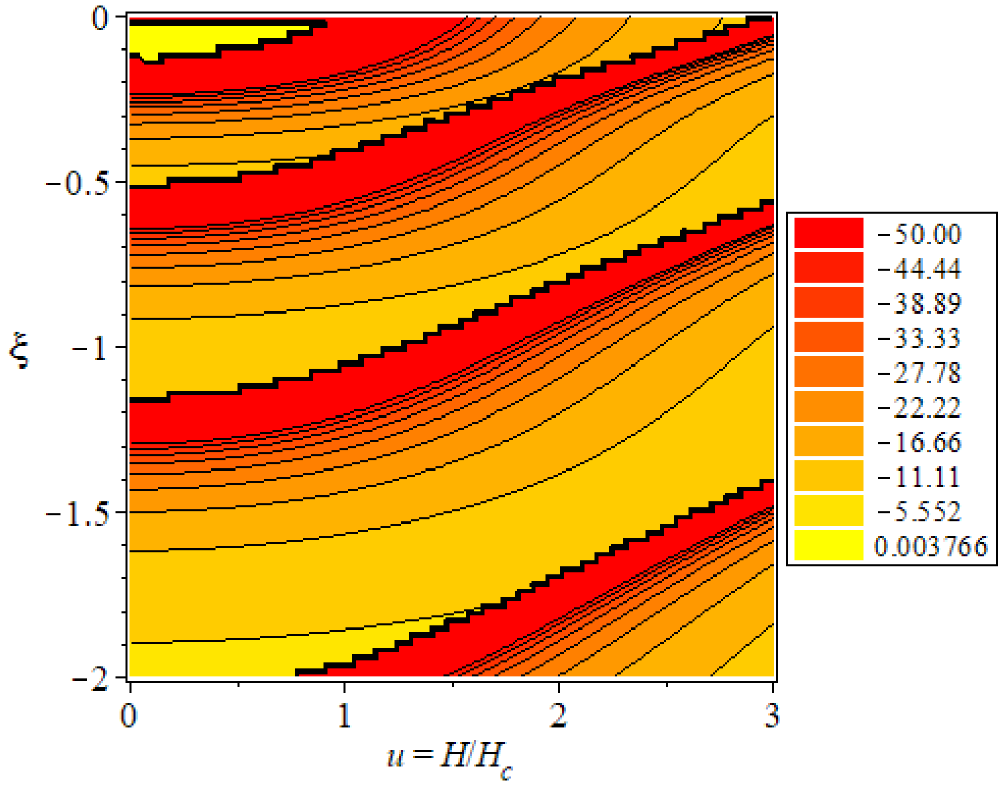


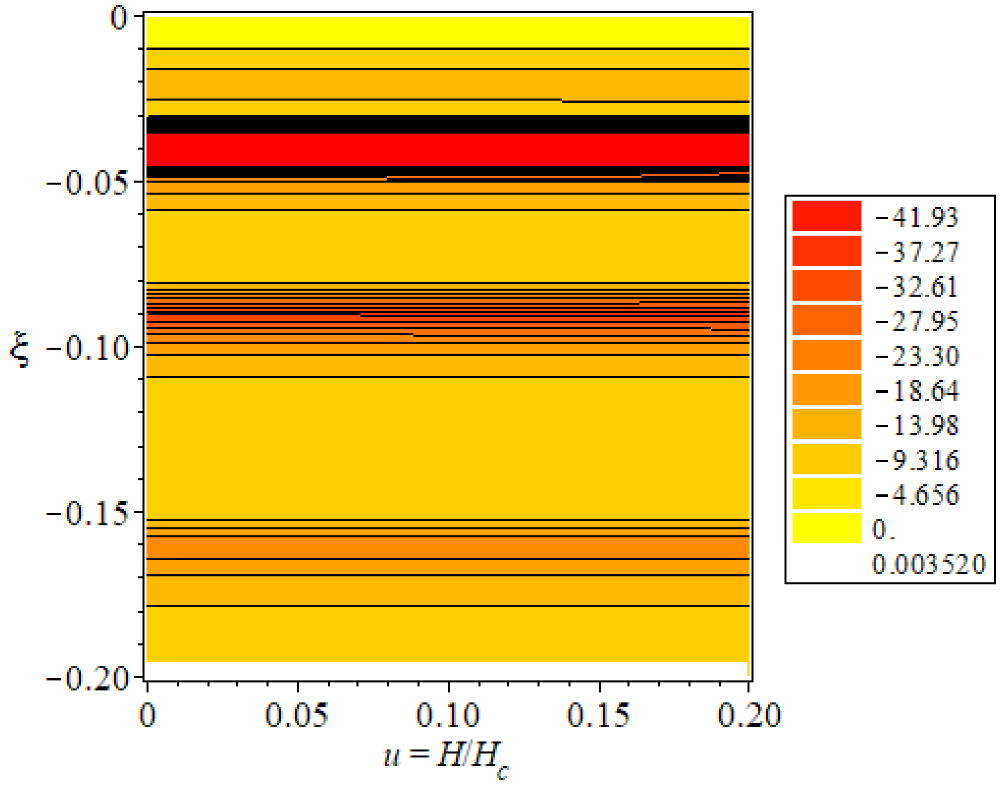


❖ The activity coefficient controls the generation and growth/decay of flow in the system.

Future Work

❖ Contour plots of the minimum values of the **time constant** τ and the corresponding imaginary values.





❖ In future research, we will solve the full coupled nonlinear PDEs using an appropriate numerical methods.