

Adam Smith Business School

WORKING PAPER SERIES

Country default in a monetary union

Lovleen Kushwah

Paper no. 2020-19 September 2020

Country default in a monetary union

Lovleen Kushwah* University of Glasgow[†]

September 11, 2020

Abstract

We develop a simple model of borrowing and lending within the monetary union. We characterize the default decision of the borrowing country and explore the impact that the monetary union has on the amount of borrowing, the rate of interest and the default probability. The key assumptions of the modelling strategy are that in the monetary union, the lender is risk averse with monopoly power rather than risk neutral with perfect competition. We find that the borrowing member country of the monetary union borrows more at cheaper cost vis-à-vis a standalone borrowing country. Further, we find that forming a monetary union with high initial income disparity between the member countries leads to more and cheaper borrowing and higher default probabilities.

^{*}I am extremely grateful to Dr Salvador Ortigueira for his guidance and support. I would like to thank Professor Manuel Santos, Professor Andrés Erosa, Dr Matthias Kredler, Dr Abderrahim Tammouti, Dr Juan Pablo Rincon Zapatero, Dr Sandra Lizarazo, Dr Raphael Boleslavsky, Dr Hernán Seoane, Professor Charles Nolan, Professor Sayantan Ghosal, Professor José-María Da-Rocha, Dr Dania Thomas, Professor Tatiana Kirsanova, Dr Alessia Campolmi and all seminar/conference participants at the Universidad Carlos III de Madrid, University of Miami and University of Glasgow for their comments and suggestions; to Manuel Santos and University of A Miami for their hospitality.

[†]Email: Lovleen.Kushwah@glasgow.ac.uk

1 Introduction

The third stage of the European Economic Monetary Union (EMU) in 1999 led to the adoption of the Euro as a common currency among member countries and to their surrendering of national monetary policy in favour of the European Central Bank. The adoption of the Euro triggered a dramatic increase in external net borrowing among peripheral countries such as Greece, Italy, Portugal and Spain. The main lenders to these countries were France and Germany which together held around 45% of the Portuguese and the Spanish debt during the period 2001-2013. More specifically, around 77% of the French debt holdings were issued by countries of the European Union and 67% were issued by Eurozone countries. Likewise, 77% of the German debt holdings were issued by the European Union countries and 66% were issued by the Eurozone. The financial crisis of 2008, along with the above-mentioned high exposure of France and Germany to the debt of peripheral countries, increased the risk of default among the borrowing countries and led to the Greek debt crisis of 2010.

In this paper we present a model of borrowing and lending to study how a monetary union shapes the amount of borrowing, its price and the default probability. In particular, we investigate how the debt market responds to changes in both the costs and benefits associated with a monetary union. Our framework is also useful to shed light on how high initial income disparity between countries in a monetary union affects borrowing, lending and default. This is a question that has gained recent attention in light of the debt crisis of peripheral countries. Our model builds on existing literature on debt default (Eaton and Gersovitz, 1981, Arellano, 2008 and Lizarazo, 2013). We extend this literature by introducing a monetary union which we model as a technology that changes the income processes of the member countries. We describe a simple economy with two countries that receive exogenous income streams every period. Debt contracts are not enforceable and hence the borrowing country can choose to default on its debt. In case of default, the union breaks and both countries revert to autarky without any future interaction in the international credit market. A second main departure from the previous literature is that we model the lender as being risk averse with monopoly power, instead of a continuum of risk neutral lenders operating in a perfectly competitive market. The lender is also subjected to exogenous income shocks, which depend on whether it belongs to the monetary union or not. We model the benefit of a monetary union by assuming that the growth rate of income is higher relative to autarky. We model the cost of a monetary union by assuming higher income volatility than in autarky. Empirical evidence supporting this modelling strategy is provided by several papers.

We find that higher amount of borrowing at lower yield formed the equilibrium in the monetary union vis-à-vis standalone. Any two standalone countries engage in borrowing and lending only when the default cost is quite high. The costs and benefits associated with the monetary union add another level of trade-off with respect to the default decision. This not only increases the debt supply by the borrowing country in the monetary union but also increases the amount borrowed in equilibrium. The model yields additional result that higher income disparity between member countries leads to more and cheaper borrowing in the monetary union. The equilibrium outcomes, as were observed in the EMU debt market are the result of interplay between the market structure and agents' characteristics. The results when translated into policy recommendations call for either expanding the joining criterion to the real variables such as income disparity or developing tools to counter higher debt issuance when cross country income disparity is higher.¹

2 Some data on borrowing-lending within the EMU

In this section we present the empirical evidence on borrowing and lending within the EMU. We focus our attention on the debt securities issued by Greece, Italy, Portugal and Spain. Specifically, we look at the net lending by France and Germany to the above mentioned countries.

We use data from the International Monetary Fund's Coordinated Portfolio

¹This is line with the optimal currency area literature, seminal work by Robert Mundell (1961), where convergence of the real variables is crucial for the overall stability of a common currency area.

Investment Survey from 2001 till 2013 (CPIS, 2013).² This is an annual survey offering data on portfolio investments by the residents of a reporting country in the debt securities, short- and long-term instruments valued at market prices, of the issuing country. The CPIS collects data either on a security-by-security basis or on an aggregate basis.³ It uses a "from-whom-to-whom" approach and compiles information from either the end-investors or from custodians, or from a combination of the two. In an end-investor survey, the security owner reports directly, while in a custodian survey, financial institutions that hold securities report on behalf of the end-investors.⁴ The holders of the debt securities are either general government or financial institutions or non-financial corporations. General government consists of the central, state and local governments, social security funds, non-profit institutions and unincorporated enterprises that are controlled by the government units.⁵

We focus on short- and long-term debt securities. Short-term debt securities consist of money market instruments that yield the holder a fixed payment on a particular date, and that matures in less than a year. These include treasury bills and notes, commercial and financial paper, and bankers' acceptances, negotiable certificates of deposit, short-term notes issued under note issuance facilities or revolving underwriting facilities and promissory notes, debt securities that have been sold under repurchase agreements and debt securities that have been "lent"

²It is worth noting that both the CPIS and the QEDS (Quarterly External Debt Survey) are the databases on the private and public external debt. There is a discrepancy between the aggregate values of the total investment in the debt securities reported by each of them. While the CPIS reports not only the debt securities holding and the issuing countries but also the investors' profile in the reporting country, QEDS has only aggregate information with respect to the issuing country of the debt.

³A security is defined as a tradable instrument and is identified by the International securities identification number. We exclude equity securities for our analysis. Equity covers all instruments which are shares and stocks, participation documents, depository receipts, shares in mutual funds and investment trusts, securities that have been sold under repurchase agreements, "lent" under a securities lending arrangement etc. Financial derivatives and related non-resident enterprises (an enterprise group which has an equity interest of 10% or more or where a non-resident has more than 10% or more holdings in your group) are excluded from the survey.

⁴End-investors includes institutional investors, such as banks, security dealers, mutual funds, and pension and insurance funds. Custodians are also financial institutions but they manage securities on behalf of domestic residents.

⁵Governments are majorly the issuers of debt instruments rather than the buyers.

under a securities lending arrangement. Long term debt securities consist of bonds, debentures, and notes with maturity longer than a year. These include "straight" coupon bonds, non-participating preferred stocks or shares, convertible bonds and bonds with optional maturity dates, negotiable certificates of deposit, dual currency bonds, zero-coupon bonds and other deep discount bonds, floating rate bonds (FRNs), indexed bonds (IBs), asset-backed securities (ABSs), euro medium-term notes, Schuldscheine notes, debentures, bearer depository receipts denoting ownership of debt securities issued by non-residents, debt securities that have been sold under repurchase agreements and debt securities that have been "lent" under a securities lending arrangement.⁶

We use information on holdings and issuances of debt securities to calculate *ownership percentage* which is defined as the share of a lender in the total debt issuance of the borrower,

$$Ownership percentage = \frac{\text{debt securities issued by the borrower and owned by a lender}}{\text{total debt securities issued by the borrower}}$$

We also construct *exposure percentage*, which offers a measure of a lender's exposure to the debt securities of a given issuer in its portfolio,

$$Exposure percentage = \frac{\text{securities issued by a borrower and owned by the lender}}{\text{total debt securities owned by the lender}}$$

We now report net lending by France and Germany to Greece, Italy, Portugal and Spain from 2001 till 2013. We show net lending for two sub-periods: from 2001 until 2009 and from 2009 until 2013. The first sub-period corresponds to post-EMU formation and the second sub-period corresponds to the ongoing EMU debt crisis. Figures 1 to 4 illustrate an increase of more than 400% in net lending by France to Greece, Italy, Portugal and Spain during the first sub-period.⁷ It grew from USD 10 bn in 2001 to USD 80 bn in 2009 for Greece; from USD 36 bn in 2001 to USD 207 bn in 2009 for Italy; from USD 8 bn in 2001 to USD 60 bn in 2009 for Portugal and from USD 21 bn in 2001 to USD 175 bn in 2009 for

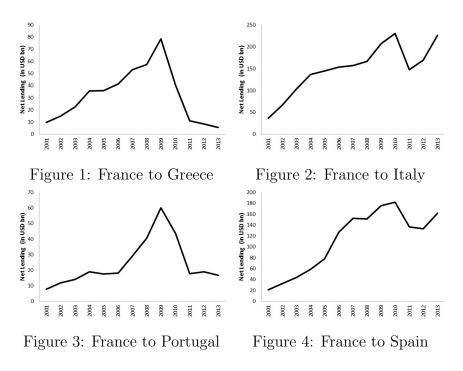
⁶FRNs such as perpetual-rate notes, variable-rate notes, structured FRN, reverse FRN, collared FRN, step-up recovery FRN, and range/corridor/accrual notes. IBs such as property index certificates. ABSs such as collateralized mortgage obligations and participation certificates.

⁷The net lending is valued at the market prices.

Spain. The net lending by France stalled at the onset of the EMU debt crisis. It fell by 93% for Greece; by 72% for Portugal and by 8% for Spain from 2009 till 2013. Even though, Italy's debt market recovered immediately after the drop in 2009, the net lending grew at a meagre 9%.

Figures 5 to 8 report net lending by Germany to the aforementioned borrowing countries. Figure 5 plots net lending to Greece and it increased from USD 14 bn in 2001 to USD 38 bn in 2009, an increase of 156%. Post-debt crisis the net lending plummeted to USD 7 bn in 2013. While, we observe in figure 6 that net lending to Italy grew by 250% from 2001 until 2013, it dropped in 2003, 2005 and 2011. Figure 7 and 8 document the increase in net lending to Portugal and Spain by more than 450 % during the first sub-period. Up until 2002, the net lending to Spain was negligible; however, by 2007 it reached the peak at USD 178 bn. During the second sub-period net lending stalled for Portugal and Spain.

We present the evidence in support of two central assumptions of our theoretical model: risk averse lenders with monopoly power over the debt securities market of the borrower. Figures 9 and 10 show the ownership percentage of France and Germany in the debt issuance of Greece, Italy, Portugal and Spain. Germany remained one of the top holders of the Spanish debt securities with ownership percentage around 20% from 2001 till 2013. During the same period, Portuguese debt was held mostly by France with an ownership percentage around 30%.



Source: Coordinated Portfolio Investment Survey (IMF).

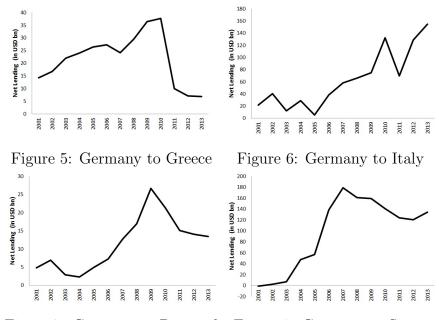
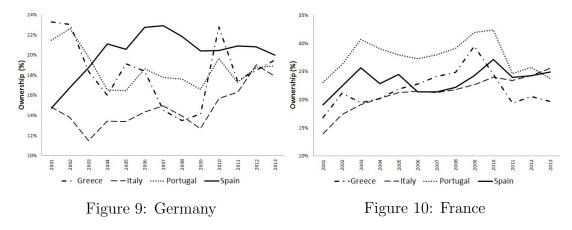


Figure 7: Germany to Portugal Figure 8: Germany to Spain

Source: Coordinated Portfolio Investment Survey (IMF).



Source: Coordinated Portfolio Investment Survey (IMF).

The right axis of figures 11 and 12 plots the Portuguese and Spanish total debt securities and the ownership percentage for major lenders in the debt securities markets on the left axis.⁸ We observe that France and Germany held 40-50%

⁸The figure plots the total debt securities values in logs.

of the Portuguese and Spanish debt securities consistently from 2001-2013. The holdings of Spanish debt by the Euro countries dwarf the holdings by non-Euro countries (US, UK, Japan, Norway and rest of the world). We measure the Portuguese and Spanish debt market concentration by calculating the Herfindahl-Hirschman index (HHI).⁹ Table 1 reports the HHI for all lenders and for the Euro zone lenders, which participated in the Portuguese and Spanish debt market. The HHI shows that debt markets have been moderately concentrated within the Eurozone countries.

Figures 13 and 14 plot the exposure percentages for France and Germany. On the left axis we document six of the debt issuing countries for which the exposure percentage was more than 5% from 2001 till 2013. The debt portfolio of France and Germany consist of securities issued by Italy, Netherlands, Spain, the United Kingdom and the United States. France and Germany also invested in each other's debt securities. On the right axis we plot the total debt held by France and Germany with exposure percentage greater than 1%.¹⁰ France and Germany had huge exposure to the debt securities of few countries. They held more than 75% of the debt securities issued by member countries of the European Union and more than 65% of the debt securities was issued by the European Union countries, Japan, United States and United Kingdom alone.

⁹HHI is a measure of the size of a lender in relation to the overall market. It is the sum of square of the market share of all the players. It ranges from close to zero to 10,000. A higher number indicates less competitive market.

¹⁰In the case of Germany, debt securities of Austria, Belgium, Denmark, Finland, France, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain, Sweden, United Kingdom, United States, Japan, Cayman Islands, Jersey and International Organizations.

France held securities of Austria, Belgium, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Netherlands Antilles, Portugal, Spain, Sweden, United Kingdom, United States, Japan, Cayman Islands, International Organizations.

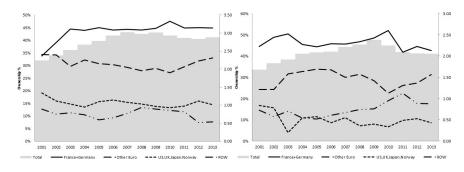


Figure 11: Spain Figure 12: Portugal

Source: Coordinated Portfolio Investment Survey (IMF).

Years	Spain		Portugal	
	All lenders	Euro Zone	All lenders	Euro Zone
2001	947	1643	1283	2017
2002	1114	1778	1472	2182
2003	1268	2003	1577	1931
2004	1274	1900	1429	1926
2005	1347	1993	1386	1865
2006	1280	1973	1372	1834
2007	1266	2026	1359	1943
2008	1266	2057	1434	1894
2009	1276	2022	1540	2041
2010	1382	2159	1674	2096
2011	1269	1961	1259	1726
2012	1290	1917	1388	1768
2013	1317	1918	1327	1613

Table 1: Herfindahl-Hirschman indexSource: Coordinated Portfolio Investment Survey (IMF).

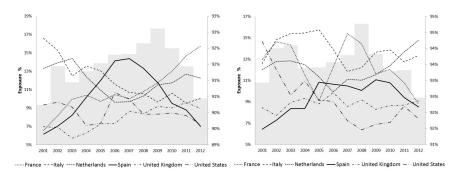


Figure 13: Germany Figure 14: France Source: Coordinated Portfolio Investment Survey (IMF).

3 The Environment

In the following section we will consider an economy with two countries, $i \in \{l, b\}$, which are inhabited by risk averse households of unit mass each. The households are identical within each country. Time t is discrete and infinite with lending and borrowing taking place in period, t = 1. There are two alternate regimes in our analysis, $j \in \{m, k\}$, where m represents the monetary union and k the autarky. The monetary union can be thought of as a technology that increases a country's potential income (higher trend) at the cost of giving up the ability to smooth income shocks (higher volatility).

Households The preferences of the representative household in country i are

$$E_1 \sum_{t=1}^{\infty} \beta_i^{t-1} u(c_{i,t}^j), \tag{1}$$

where $\beta_i \in (0, 1)$ is the household discount factor and $c_{i,t}^j$ is the consumption of the household in country *i* when it is in regime *j* in period *t*. The utility function is strictly increasing and concave, i.e. u'(.) > 0 and u''(.) < 0 and satisfies the standard Inada conditions. Households receive a stochastic endowment, $y_{i,t}^j$, every period and a lump sum transfer of goods from its government, a_t . The income process When country i is in regime j at any period t, income (endowment) follows the process,

$$log(y_{i,t+1}^{j}) = \alpha_{i}^{j} \cdot (t+1) + \rho_{i} log(y_{i,t}^{j}) + \epsilon_{i,t+1}^{j},$$
(2)

where, the trend parameter α_i^j is positive, the persistence parameter ρ_i is within the unit circle and the income shocks $\epsilon_{i,t+1}^j$ are i.i.d and normally distributed $N(0, \varsigma_i^{j2})$. ς_i^j is the standard deviation of the income shocks.

The income trend is higher for the member countries in the monetary union, which is attained through channels such as reduction in transaction costs, enhanced trade, financial deepening, seigniorage gains etc. (Papaioannou and Portes, 2008 and Frankel and Rose, 2002). At the same time, a higher income volatility is observed for the member countries on account of losing the monetary policy discretion (Luque et al., 2014 and OECD, 1999) and use of a single monetary policy when faced with asymmetric shocks across the member countries (Werning and Farhi, 2012). Thus,

$$\alpha_i^m > \alpha_i^k \quad \text{and} \quad \varsigma_i^m > \varsigma_i^k \quad \text{for} \quad i \in \{l, b\}$$
(3)

Governments The government of each country is benevolent and maximizes the utility of the representative household. In period t = 1 both countries are in the monetary union. They can smooth their consumption by trading a noncontingent bond which pays a time- and state-invariant return. We assume that $\beta_b < \beta_l$, so that country b is the net borrower and country l is the net lender. Also, we assume that the two countries have some initial asset/liability, a_1 , which has to be repaid by the borrowing country in period t = 1.

Borrowing and Lending In period t = 1, the borrowing country borrows an amount a_2 at a given discount q_2 , from the lending country.¹¹ The borrowing country lacks commitment to repay its debt obligation in period t=2. The default is an absorbing state, following which the monetary union breaks. The borrowing

¹¹Note here, q_2 represents the bond price or the discount at which the bond is sold and $\frac{1}{q_2}$ represents the interest rate for the same bond.

country is immediately penalized with γ fraction of its income realization in period t = 2. If however the debt is repaid, the monetary union continues to exist.

In Section 2 we provided the evidence that in the EMU only a few lending countries held most of the debt securities issued by the borrowing countries (CPIS, 2013). We follow this empirical finding and assume, for simplicity, that the single lending country is the sole source of borrowing and it has the monopoly power over the lending market instead of a continuum of lenders in a competitive market. In period t = 1, the lender chooses the bond price q_2 for a given debt. It remains in the monetary union if the borrowing country repays the debtin period t = 2.

3.1 Evidence on the "monetary union" technology

The "monetary union" is modelled as a technology, which allows member countries attain a higher long run growth in income at the expense of a higher income volatility. This reduced form definition of the monetary union captures parsimoniously the benefits and the costs of adopting a common currency.

In this section we assess the effects that the launch of the euro has on its member countries. Generally speaking, a monetary union benefits its members directly and indirectly through trade and financial market deepening. A common currency lowers the transaction costs, reduces price uncertainty and enhances the price transparency. At the same time adopting a common currency not only takes discretionary monetary policy away from each member country but also leave them facing asymmetric shocks with a single union-wide monetary policy.¹²

Frankel and Rose (2002) use a two-stage approach to quantify the effects of monetary unions on income through trade. They use cross-sectional data from over 180 countries for the period from 1970 until 1995. They provide econometric evidence that a monetary union triples trade. Further, an increase in total trade-to-GDP ratio by one percent raises income per capita by at least one-third of a percent.¹³

There are several papers estimating particularly the effect of the euro on trade.

 $^{^{12}}$ For a textbook treatment refer to De Grauwe (2012).

¹³The first estimate on the effect of currency unions on trade is larger if the currency union partnership has linguistic, historical, political and geographical links.

In a survey of the literature by Baldwin (2006), he concludes that the euro led to an increase in trade by five to ten percent within the euro area. Micco et al. (2003) use a panel data for 15 Euro countries from 1992 to 2002 and apply the difference-in-differences estimate approach. They find that the effect of the euro on trade between member countries is between 4-10%. The monetary union also enhances trade with non-Euro countries. Flam and Nordström (2006) estimate the increase in trade by 15% within the Euro area and by 8% with the other EU countries from 1989-2002 until 1998-2002.

Another channel which influences the output growth is through internationalization of the Euro. It reaps benefits to the monetary union in the form of seigniorage and expansion of the financial markets. Papaioannou and Portes (2008) discuss the positive effect of Euro adoption on deepening and integration of the financial markets. The financial market development affects the economic performance through channels such as risk sharing, lowering cost of capital, fast reallocation of investments etc.

We use data from the Organisation for Economic Co-operation and Development (OECD) database to calculate three-year averages of the real GDP's annual growth rates for France, Germany, Greece, Italy, Portugal and Spain, before and after joining the Euro.¹⁴ We observe that after joining the Euro the annual growth rate increased from 3.8% to 4.5% for Greece; from 1.5% to 2.3% for Italy; from 3.6% to 4.5% for Spain; from 2.4% to 3.1% for France and from 1.5% to 2.2% for Germany but it declined from 4.2% to 3.2% for Portugal.

Regarding the income volatility, Luque et al. (2014) report average volatilities of GDP per capita before and after the adoption of the euro for the periods, 1986-1998 and 1999-2011. They document that average volatilities increased after 1999 for almost all member countries. It increased from 6.67% to 14.94% for Greece; from 10.63% to 13.5% for Italy; from 19.38% to 23% for Portugal; from 9.54% to 18.91% for Spain and from 4.6% to 7.8% for France. However, it declined from 5.61% to 4.02% for Germany.

One of the explanations behind this increase in income volatility is that some

 $^{^{14}}$ France, Germany, Italy, Portugal and Spain joined the Eurozone in 1999 while Greece joined in 2001. For details on data refer to FRED or OECD (http://dx.doi.org/10.1787/data-00052-en).

member countries not only had high inflation variance historically but also were structurally different from other member countries. After adopting a common currency it is impossible to adjust the exchange rate in order to accommodate the inflation variation. Thus a union-wide single monetary policy would lead to higher output variability. Another mechanism leading to the increase in output volatility is that asymmetric shocks across member countries cannot be targeted by a single monetary policy (Lane, 2012; Shambaugh et al., 2012 and Werning and Farhi, 2012).¹⁵

3.2 Characterization of borrowing and lending

We solve the model by using the method of backward induction. We consider three distinct stages. In the first stage, the borrower takes the decision of default or repayment, given the amount of debt, a_2 , and the price, q_2 , paid for this amount. In the second stage, the borrower anticipates the default probability, $\delta(a_2, q_2)$, and chooses the amount of debt, $a_2(q_2)$, given the price, q_2 . Finally, in the last stage, the lender determines the price, q_2 , to offer, with the anticipation of both the default probability, $\delta(q_2)$, and the debt amount, $a_2(q_2)$.

Let us denote the value function of the borrowing country in period t = 1 as $V_{b,1}$. The maximization problem of the borrowing country is,

$$V_{b,1} = \underset{\{a_2\}}{Max} \left\{ u(y_{b,1}^m - a_1 + a_2q_2) + \beta_b E_1 \left[\underset{d_2 \in \{0,1\}}{Max} \mathcal{I}_{\{d_2=1\}} V_{b,2}^k + \mathcal{I}_{\{d_2=0\}} V_{b,2}^m \right] \right\},$$
(4)

where \mathcal{I} is the indicator function, d_2 represents borrowing country's decision to default $(d_2 = 1)$ or repay $(d_2 = 0)$ and it depends on the income realization in period t = 2. $V_{b,2}^k$ and $V_{b,2}^m$ are the value functions of the borrower in period t = 2associated with the default and the repayment decision.

$$V_{b,2}^m = u(y_{b,2}^m - a_2) + \beta_b E_2 \left[V_{b,3}^m \right] \text{ and } V_{b,2}^k = u((1-\gamma)y_{b,2}^m) + \beta_b E_2 \left[V_{b,3}^k \right],$$

 $^{^{15}{\}rm For}$ a discussion on the effect of the Euro through a political economy channel see Fratzscher and Stracca (2009).

where, $V_{b,3}^k$ and $V_{b,3}^m$ are the continuation values of the borrower in period t = 3 in the two monetary regimes and are discussed in detail later.

The value function of the lending country in period t = 1 is denoted by $V_{l,1}$. The maximization problem of the lending country is

$$V_{l,1} = \underset{\{q_2\}}{Max} \left\{ u(y_{l,1}^m + a_1 - a_2q_2) + \beta_l E_1 \left[\mathcal{I}_{\{d_2=1\}} V_{l,2}^k + \mathcal{I}_{\{d_2=0\}} V_{l,2}^m \right] \right\},$$
(5)

where $V_{l,2}^k$ and $V_{l,2}^m$ are the value functions of the lending country in period t = 2 corresponding to the default and the repayment decision of the borrowing country.

$$V_{l,2}^m = u(y_{l,2}^m + a_2) + \beta_l E_2 \left[V_{l,3}^m \right] \text{ and } V_{l,2}^k = u(y_{l,2}^m) + \beta_l E_2 \left[V_{l,3}^k \right],$$

where, $V_{l,3}^k$ and $V_{l,3}^m$ are the continuation values of the lender in period t = 3.

Continuation Value For all the periods t > 2, the lending and the borrowing country receive utility from consuming the income. The income realizations depend on the regimes that they are in, i.e. whether the borrowing country defaulted and remained forever as standalone or repaid the debt and remained forever in the monetary union. For a given i and j, let $E_2\left[V_{i,3}^j\right] = \widetilde{V}_i^j$, be the expectation of continuation value of period t = 3 when in period t = 2.

Under the case of CRRA preferences, for a given i and j, the continuation value is¹⁶

$$\widetilde{V} = \widetilde{V}(\beta, \sigma, \rho, \varsigma, \alpha, y_2).$$

Specifically, for the logarithmic preferences the continuation value is,

$$\widetilde{V} = \left[\frac{\rho log(y_2)}{1-\beta\rho} + \frac{\alpha}{1-\beta\rho}\frac{3-2\beta}{(1-\beta)^2}\right].$$

¹⁶The super and sub-scripts, *i* and *j* are suppressed for more clarity. The functional form of $\widetilde{V}(\beta, \sigma, \rho, \varsigma, \alpha, y_2)$ is described in the appendix.

Result 1 When the output trend is strictly positive, the continuation value \widetilde{V}_i^j exist iff $\sigma_i \geq \widetilde{\sigma}_i$, where $0 \leq \widetilde{\sigma}_i < 1$.

Proof in Appendix.

Decision of default/ repayment at period t = 2 We consider the default decision of the borrowing country for a given borrowed amount, a_2 and price, q_2 . The borrowing country chooses to repay the loan in period t = 2, depending on whether the value of repayment is greater than the value of default in period t = 2, i.e.

$$V_{b,2}^{m} \ge V_{b,2}^{k} \quad \Leftrightarrow \quad y_{b,2}^{m} \ge \widehat{y}, \tag{6}$$

where $\widetilde{\Omega}_{b} = \widetilde{V}_{b}^{k} - \widetilde{V}_{b}^{m}$ and $\widehat{y} = \widehat{y}(\beta_{b}, \gamma, \widetilde{\Omega}_{b}, a_{2}).$

The default decision of the borrowing country depends on the income realization in period t = 2. If the realized income is higher than the threshold, \hat{y} , the borrowing country will repay the loan. The income threshold is increasing in the level of borrowed amount. Alternatively, we can also use the above equation to define a threshold, $\hat{a}_2 = \hat{a}_2(y_{b,2}^m)$, at which the borrower is indifferent between its decision to either default or repay the debt in period t = 2. We define the repayment set as a set of debts where $a_2 \leq \hat{a}_2$ and the default set as a set of debts where $a_2 \geq \hat{a}_2$.

For the CRRA preferences, the debt level for which the borrower is indifferent between default and repayment is

$$\widehat{a}_2 = y_{b,2}^m - \left[((1-\gamma)y_{b,2}^m)^{1-\sigma_b} + \beta_b \widetilde{\Omega}_b \right]^{\frac{1}{1-\sigma_b}}$$

For the logarithmic preferences,

$$\widehat{a}_2 = y_{b,2}^m \left(1 - (1 - \gamma) \exp(\beta_b \widetilde{\Omega}_b) \right).$$

In the proposition below we will see the effect of changes in the parameters and the income realizations on the debt threshold \hat{a}_2 . We assume that in the benchmark economy the borrowing country's representative household has logarithmic preferences.

Proposition 1 For a borrowing country with logarithmic preferences, everything else remaining same,

i) a higher difference in the output growth, $\alpha_b^m - \alpha_b^k$, expands the repayment set (or contracts the default set) for all levels of income,

ii) a higher standalone income realization, $y_{b,2}^k$, leads to contraction of the repayment set (or expansion of the default set),

iii) a higher monetary union income realization, $y_{b,2}^m$, leads to expansion of the repayment set (or contraction of the default set) if $\rho_b > \frac{1}{2}$.

Proof in Appendix.

The above results remain robust if the borrowing country has CRRA preferences. We find that for a coefficient of relative risk aversion greater than one, a higher amount of debt will be repaid in the monetary union if and only if the income volatility difference, $\varsigma_b^m - \varsigma_b^k$, is lower. The results are intuitive as the relative benefits (or the relative costs) associated with the monetary union vis-à-vis standalone country goes up (or down), the value of repayment becomes higher relative to the value of default. Thus, the debt threshold, for which the two values are same, goes up as well and the repayment region expands. Further, we find that an increase in the coefficient of relative risk aversion decreases the debt threshold \hat{a}_2 . A higher risk aversion indicates a lower utility due to uncertainty, ceteris paribus. The value of default goes up for a higher risk aversion and thus the repayment region shrinks. The evidence in support of these assertions are provided in the appendix.¹⁷

We also explore the question whether the borrowing country has a larger repayment region when it is in the monetary union vis-à-vis standalone country. We assume that the default choice of the borrowing country, which is not a

¹⁷For any numerical analysis we consider the parameter space where, $\alpha_b^m \in [0.015, 0.02]$, $\alpha_b^k \in [0.0, 0.015]$, $\varsigma_b^m \in [0.0035, 0.007]$, $\varsigma_b^k \in [0, 0.0035]$, $\sigma \in \{1, 1.05, 1.15, 2\}$ and the support for incomes are around 0.85, i.e. $y_{b,2}^m \in [0.7, 1]$ and $y_{b,2}^k \in [0.7, 1]$.

member of the monetary union, results in immediate output cost. The income of the standalone country follows the same process irrespective of its default decision and thus the continuation values are same.

Proposition 2 In our benchmark economy the default set is smaller under the monetary union, ceteris paribus.

Proof in Appendix.

Corollory For a given debt level, a_2 , the 'indifference' income threshold is smaller in the monetary union when the output penalty is small.

Proof in Appendix.

A higher income trend, which is made possible by being in the monetary union, raises the value of repayment of the member borrowing country. This is not true in the case of a standalone country and the default region is bigger.

Default probability at period t = 1 The borrowing and the lending countries anticipate default in period t = 1. The borrowing country defaults if and only if its income realization in period t = 2 is lower than the income cutoff, i.e. $y_{b,2}^m \leq \hat{y}$. Thus, the default probability is $\delta_2(a_2) = \Pr[y_{b,2}^m \leq \hat{y}]$.

Result 2: The default probability,

$$\delta_2(a_2) = \frac{1}{2} \left[1 + erf\left(\frac{\log\left(\frac{\widehat{y}}{(y_{b,1}^m)^{\rho_b} \exp(2\alpha_b^m)}\right)}{\sqrt{2}\varsigma_b^m}\right) \right],\tag{7}$$

is non-negative and goes to zero (or one) for a very low (or high) income threshold \hat{y} . The default probability is increasing in the borrowed amount, a_2 .

Proof in Appendix.

In the above formula, erf(.) represents the error function. It is a special function, which is increasing in its argument and lies between zero and one in its domain of positive real numbers.

Proposition 3: In our benchmark economy, for a given level of borrowing,

 a_2 , the default probability is lower in the monetary union vis-à-vis standalone countries when the output penalty, γ , is small.

Proof: Using the corollary of Proposition 2, for any given level of debt, a_2 , it is immediate that the default probability in monetary union is lower whenever the output penalty is small. A detailed proof is available in the appendix.

Choice of the level of debt in period t = 1 The borrowing country chooses the amount of debt, given the bond price q_2 to maximize its lifetime utility given in Eq (4).

The debt supply by the borrowing country depends on two debt thresholds, \overline{a}_2 and \underline{a}_2 . The first threshold, \overline{a}_2 , corresponds to the case where even if the borrowing country receives the highest possible income shock, $\overline{y}_{b,2}^m$ it will default. Similarly, the second threshold, \underline{a}_2 , corresponds to the case where even after receiving the lowest possible income shock $\underline{y}_{b,2}^m$ it will not default. Formally the debt thresholds are expressed as,

$$\overline{a}_{2} = \left\{ a_{2} \left| V_{b,2}^{m}(\overline{y}_{b,2}^{m}) < V_{b,2}^{k}(\overline{y}_{b,2}^{m}) \right\} \text{ and } \underline{a}_{2} = \left\{ a_{2} \left| V_{b,2}^{m}(\underline{y}_{b,2}^{m}) \ge V_{b,2}^{k}(\underline{y}_{b,2}^{m}) \right\}.$$

The two thresholds, \overline{a}_2 and \underline{a}_2 , divide the debt space into three regions. The first region is the safe region in which the borrowing country never defaults. The second region is the risky region with a non-zero default probability. The borrowing country defaults depending on the income realization in period t = 2. The last region is the default region where the borrowing country always defaults. Hence,

$$\delta_2 = \begin{cases} 0 & \text{if} \quad a_2 \leq \underline{a}_2, \\ (0,1) & \text{if} \quad \overline{a}_2 > a_2 > \underline{a}_2, \\ 1 & \text{if} \quad a_2 \geq \overline{a}_2. \end{cases}$$

Given the bond price the borrowing country will borrow an amount in order to smooth its consumption across periods. We derive the first order condition of the borrowing country with logarithmic preferences.¹⁸ In this case, the debt supply function of the borrowing country is given by,

$$q_{2} = \begin{cases} \frac{\beta_{b}(y_{b,1}^{m}-a_{1})E_{1}\left[\frac{1}{y_{b,2}^{m}-a_{2}}\right]}{1-\beta_{b}a_{2}E_{1}\left[\frac{1}{y_{b,2}^{m}-a_{2}}\right]} & \text{if} \quad a_{2} \leq \underline{a}_{2}, \\ \frac{\beta_{b}(y_{b,1}^{m}-a_{1})\int_{\bar{y}}^{\overline{y}_{b,2}^{m}}\frac{1}{y_{b,2}^{m}-a_{2}}dF_{y_{b,2}^{m}}}{1-\beta_{b}a_{2}\int_{\bar{y}}^{\overline{y}_{b,2}^{m}}\frac{1}{y_{b,2}^{m}-a_{2}}dF_{y_{b,2}^{m}}} & \text{if} \quad \overline{a}_{2} > a_{2} > \underline{a}_{2}, \end{cases}$$
(8)
$$0 & \text{if} \quad a_{2} \geq \overline{a}_{2}. \end{cases}$$

The bond price function in period t = 1 For a given debt, the lending country with logarithmic preferences chooses a price which maximizes Eq (5) subject to q_2 being determined by Eq (8). The first order condition of the lender's maximization problem is,

3.3 Equilibrium

Definition: A monopolistic equilibrium with two countries–a lender and a borrower–is defined as a set of policy functions for debt supply, $a_2(q_2)$; default decision, $\hat{\delta}_2(q_2)$; price of debt, q_2 ; each agents' consumption in every period for a specific monetary regime, $\{c_{l,1}^m, c_{l,2}^m, c_{l,3}^m, c_{b,1}^m, c_{b,2}^m, c_{b,3}^m\}$, such that:

¹⁸We apply the Leibniz's rule for deriving the F.O.C. in the risky region. Note that $V_{b,2}^m(\hat{y}) = V_{b,2}^k(\hat{y})$.

- (i) Taking default decision and debt supply, $\{\widehat{\delta}_2(q_2), a_2(q_2)\}$ as given, price of debt, $\{q_2\}$ solves lender's maximization problem in period t = 1.
- (ii) Taking price of debt, $\{q_2\}$ as given, the default decision, $\{\widehat{\delta}_2(q_2)\}$ at time t = 2 and the choice of debt supply, $\{a_2(q_2)\}$, at time t = 1 maximizes borrower's utility in period t = 1.
- (iii) The resource constraints of the economy are satisfied every period.

3.4 An Example

The example in this section simplifies the full model and uses a stylized economy to illustrate the key results of the model. All the features of the full model are retained but two. First, the continuation values associated with the future are now restricted to only period t = 3. Second, the income shocks of the lender and the borrower are independently and identically distributed (i.i.d.) with two income states. We further assume that the lender and the borrower are risk averse with CRRA preferences and receive lower income state shock in period t = 1.

We set the values to parameters in our example as follows. The risk aversion parameters, σ_b and σ_l , of the borrower and the lender are assumed to be 3 and 2, respectively. The default output loss is set around 5%.¹⁹ The lender and the borrower are assumed to be equally patient with the discount factor, $\beta_b = \beta_l =$ 0.98. The initial debt level is $a_1 = 1.3$. The two state income shocks for the monetary union and the standalone countries are set at $\{\underline{y}^m, \overline{y}^m\} = \{4, 12\}$ and $\{\underline{y}^k, \overline{y}^k\} = \{4.5, 5\}$, respectively. We assume that the income structure is same for the lender and the borrower. In both of the monetary regimes, the probability of receiving a low income shock is 1%.

Figure 15 plots the debt supply as a function of the bond price and also depicts the corresponding equilibrium price and quantity (represented by the black dot). The three shaded regions are associated with repayment and default. The light grey region corresponds to the safe region and debt issued within this range is always repaid. The middle region is the risky region with default risk. The last and the dark grey region is the default region. The borrower's supply of debt

¹⁹In literature this may vary from 1-5%. Conesa and Kehoe (2012).

is increasing in the bond price within the safe and risky region. However, in the default region the borrower always supplies maximum amount of debt. The equilibrium debt and price for the given parameter space is $\{a_2^*, q_2^*\} = \{3.71, 0.0392\}$. As we increase the initial income of the lender from the low state to the high state, the new equilibrium shifts to $\{a_2^*, q_2^*\} = \{3.75, 0.0399\}$, depicted by the grey dot. The default probability remains same for both the equilibriums, which is the probability of receiving the low income shock, and does not depend on the debt supplied due to i.i.d. income structure. Further, we also evaluate the scenario when the two countries are standalone, where $\{\underline{y}^m, \overline{y}^m\} = \{\underline{y}^k, \overline{y}^k\}$, and assess the debt market as compared to when they are in the monetary union. In this example we find that under the assumption of standalone countries the two economies do not borrow or lend to each other.

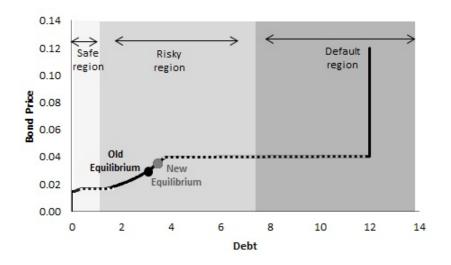


Figure 15: Debt supply and Equilibrium

3.5 Monetary union vis-à-vis non-Monetary union

We are interested in comparing the borrowing and lending behaviour in two monetary regimes: Monetary union and non-Monetary union. The first regime refers to the one described above where in period t = 1 the countries are in the monetary union. The income of the member countries incorporates both the benefits and the costs of being in the monetary union. In the second regime, countries are not the members of the monetary union in period t = 1 and follow the income processes of a standalone.

In order to analyse the borrowing and lending behaviour, we solve the model numerically under a specific parameter space as described in Table 2. We assume that the lending and the borrowing countries have logarithmic preferences. We follow the literature to assign values to the parameters in the model, β_b , β_l , ρ_b , ρ_l and γ . In this economy, we assume that the income trend and volatility are same for the borrowing and the lending countries when in a particular monetary regime. The borrowing country's initial income and debt are set such that it has positive initial wealth, $y_{b,1}^m - a_1 > 0$. We assume that the initial income of the lending country is higher than the initial income of the borrowing country.

When the default output cost is low, as in Table 2, a standalone borrowing country supplies either no or very high debt. As a result, we observe no borrowing and lending in equilibrium. For a very high default output cost, at 70%, the standalone borrowing country supplies a positive amount of debt (Figure 16). The former case illustrates that the monetary union by itself promote borrowing and lending amongst the member countries. The latter case, even though unrealistic, strengthens the above argument and emphasizes the key role that the monetary union plays, i.e. it facilitates borrowing and lending with lower debt yields even though the default probability is higher (refer to Result 2).

Parameter	Description	Value
α^m	Income trend (MU)	3%
α^k	Income trend (non MU)	2%
ς^m	Income volatility (MU)	8%
ς^k	Income volatility (non MU)	2%
$\mathbf{y}_{1,b}^m$	Borrower's initial income	5
$\mathbf{y}_{1,l}^m$	Lender's initial income	8
γ	Default output cost	1%
$ ho_b$	Borrower's income persistence	0.98
$ ho_l$	Lender's income persistence	0.98
β_b	Borrower's discount factor	0.7
β_l	Lender's discount factor	0.98
a_1	Initial debt	4.8

Table 2: Baseline economy

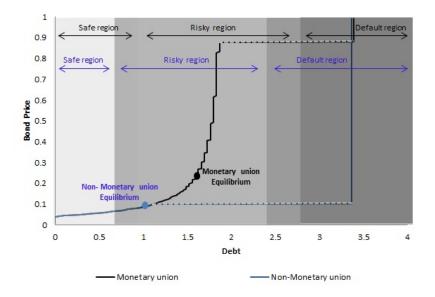


Figure 16: Debt supply with high default output cost

4 A cross country income disparity

We now turn our attention to another important question, which we address in this paper: Does forming the monetary union with high initial income disparity between the member countries influences the debt market equilibrium outcome?

We use the baseline economy as reported in Table 2. We change the income trend parameter in the monetary union to $\alpha^m = 6\%$ for illustrative purpose. In the first step, we calculate the debt supply of the borrowing country for a given price and then compute the equilibrium. Later, we increase the initial income disparity between the member countries of the monetary union. It is worth stating that the debt supply will remain unchanged if we change the parameters specific to the lending country. In the new equilibrium, the richer lending country buys higher amount of debt issued at lower yield from the borrowing country.

Further, we also find that the lending country with a higher discount factor, β_l , and a lower income persistence, ρ_l , has similar effects as with an increase in initial income. Figure 17 plots the debt supply of the borrowing country and

corresponding debt issuance and its price in the old (represented by the black dot) and the new (represented by the grey dot) equilibria.

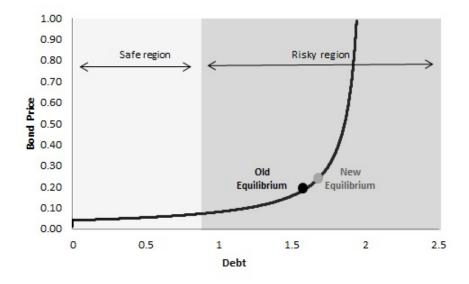


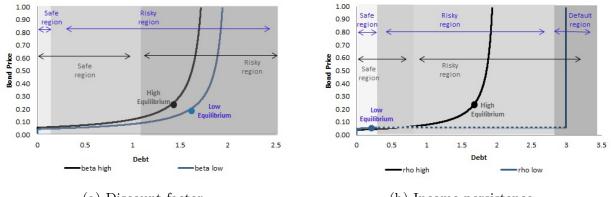
Figure 17: Comparative statics

5 Comparative statics

We perform additional comparative static exercises with respect to the borrowing country. We change the discount factor, β_b , and the income persistence, ρ_b , of the borrowing country. A more impatient borrowing country supplies a higher amount of debt at all levels of price. In equilibrium, higher amount of debt is issued at a lower price. Figure 18 plots the debt supply and equilibria associated with the change in the borrowing country's discount factor in panel (a).

An increase in borrowing country's income persistence leads to lower debt supply for higher price and higher debt supply for lower prices. In equilibrium, both debt issuance and its price increase. However, the new equilibrium has associated default risk. These changes are illustrated in panel (b) of figure 18.

We also look at the effect of change in initial indebtedness of the borrower. A higher initial debt increases the amount of debt supplied by the borrower. In equilibrium, the amount of debt issued increases but the bond price reduces. This result is in line with the theoretical literature, as higher initial debt is associated with higher borrowing in the future and has a 'debt roll-over' effect. The cost of borrowing may not necessarily go down as a highly indebted country poses higher risk of default. Figure 19 plots the debt supplies and respective equilibria.



(a) Discount factor

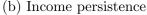


Figure 18: Comparative statics

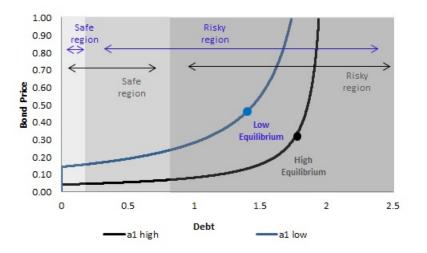


Figure 19: Initial Indebtedness

6 Concluding Remarks

A monetary union entails both costs and benefits for its member countries and has profound implications for the overall global economy. The recent debt crisis in the peripheral economies of the EMU has highlighted several vulnerabilities of the union. Our paper explores two main questions: Does being in the monetary union change the way its debt market evolves? Further, does higher initial income differential between member countries affect the amount of debt issuance, its price and incentives to default?

We develop a model of lending and borrowing in the monetary union, which is based on the endogenous sovereign debt default literature. We have two innovations in our framework. First, we introduce a reduced form representation of the monetary union and second, the lender is assumed to be risk averse with monopoly power. These key assumptions are based on the findings from several papers and on our own calculations with data from the IMF's CPIS and OECD database.

A monetary union provides another layer of trade-off associated with default in addition to the standard output default cost. In the event of default, the borrower has to forgo not only a fraction of the output as penalty but also future benefits associated with the union. However, it also leads to reduction in income volatility due to discretion over the monetary policy. Similarly, the default affects the lender in more than one way. The lender cannot collect the outstanding amount of debt and faces the consequences of a breakup of the monetary union.

We find that monetary union facilitates existence of the debt market and allows its member countries to borrow and lend more at lower cost of borrowing under circumstances when it is not possible do so if the countries were standalones. We also observe in the stylized economy that when the initial income difference grows between the lending and the borrowing member countries, debt issuance increases and debt yields drop. We illustrate that similar effects are observed when the lender has incentives to save for future and smooth consumption inter-temporally, for e.g. with higher discount factor and lower income persistence. A higher initial income not only allows lender to save more today but also increase the future benefits of staying in the union. Both higher discount factor and lower income persistence boost saving and mitigate credit rationing. The lender is able to receive higher monopoly rent today by buying more debt, which is issued by the borrower, at lower rate of interest and also reap benefits of staying in the union. The changes in borrowing country specific features not only shift the debt supply but also influence the equilibrium outcome. The borrowing country supplies higher debt when it is more impatient or indebted. Since these are conducive for default, higher debt is issued at higher cost of borrowing in equilibrium. For lower income persistence of the borrower, safe but costlier debt is issued in equilibrium.

Our model is simple but it has interesting implications for the monetary union. It confirms the growing interest in forming monetary union, as was the case of the EMU, from the lens of the debt market evolution. It also shows that interplay between the attributes of the member countries in the monetary union is crucial in determining the equilibrium of the debt market. We conclude that an empirical exercise along the lines of our model will help future research shed more light on the debt market in the monetary union. However, this will require more frequently reported and longer time series data.

7 Appendix

7.1 Net lending to Spain by Japan, Norway, the USA and the UK

Japan, Norway, the UK and the USA formed the third biggest group of lenders owning Spanish debt from 2001-2013. Japan and Norway were net lenders throughout 2001-2013, while Spain was the net lender to the USA until 2010. The UK remained a net lender of Spain post-2004 and increased lending after a decline during the financial crises of 2007-2009. The net lending increased in the first sub-period from USD 10 bn in 2001 to USD 29 bn for Japan; from USD 3 bn in 2001 to USD 20 bn for Norway and from USD 7 bn in 2001 to USD 20 bn for the UK. It dropped by 22% for Japan; by 100% for Norway and increased by 136% for the UK in the second sub-period. The growth patterns in net lending had similarities between the UK and Norway, which are members of the European Union, with those of the lending Euro countries.

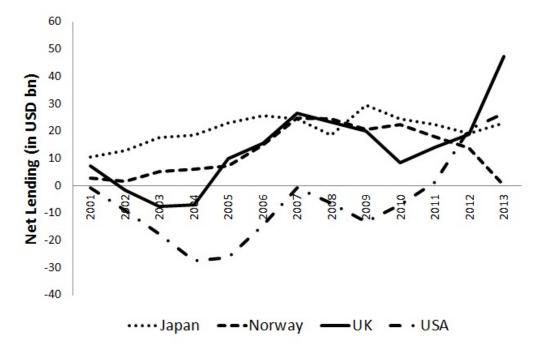


Figure 20: Net Lending

Source: Quarterly External Debt Statistics, Reinhart and Rogoff.

7.1.1	Spanish	dept	securities	holder's	profile	

Year	Financial Sector	General government	Non-Financial Sector
2013	90.3%	6.6%	3.1%
2012	90.0%	7.3%	2.8%
2011	92.8%	5.3%	1.9%
2010	90.5%	8.1%	1.4%
2009	98.9%	0.1%	1.0%
2008	99.1%	0.0%	0.8%
2007	99.0%	0.0%	0.8%

Table 3: Germany

Source: Coordinated Portfolio Investment Survey (IMF).

Financial Sector: Central Bank, Deposit-taking Corporations, Other Financial Corporations; Non-Financial Sector: Nonfinancial Corporations, Households and Non-profit institutions serving households.

Year	Financial Sector	General government	Non-Financial Sector
2013	96.0%	0.8%	3.2%
2012	95.3%	0.6%	4.1%
2011	98.3%	0.8%	0.9%
2010	97.7%	1.3%	0.9%
2009	97.7%	0.8%	1.5%
2008	96.8%	0.9%	1.5%
2007	96.6%	0.8%	2.6%

Table 4: France

Source: Coordinated Portfolio Investment Survey (IMF).

Financial Sector: Central Bank, Deposit-taking Corporations, Other Financial Corporations; Non-Financial Sector: Nonfinancial Corporations, Households and Non-profit institutions serving households.

7.2 Proofs

Continuation Value for period 3 In this section, we define the functional form of the continuation value with CRRA preferences. If the borrowing country chooses to repay (or default) its debt in period t = 2, it recieves income shocks associated with a monetary union (or standalone country) in period $t \ge 3$. So the continuation values when at period t = 2, for a given i and j, can be evaluated as follows,

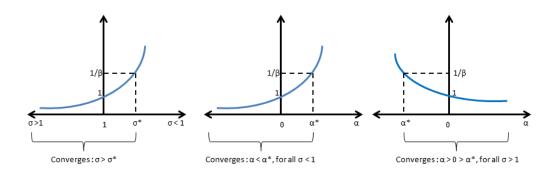
$$\widetilde{V} = E_2 \left[u(c_3) + \beta u(c_4) + \beta^2 u(c_5) + \dots \right] = \frac{S - \frac{1}{1 - \beta}}{1 - \sigma}$$

Proof of Result 1:

In the above equation, S represents the infinite sum of the sequence, s_n ,

$$S = \sum_{n=1}^{\infty} s_n, \text{ where } s_n = \beta^{n-1} y_2^{(1-\sigma)\rho^n} e^{\frac{(1-\rho^{2n})(1-\sigma)^2\varsigma^2}{2(1-\rho^2)}} e^{\frac{(-3\rho^{n-1} + \frac{n+2}{\rho} - \frac{1-\rho^{n-1}}{1-\rho})(1-\sigma)\alpha\rho}{1-\rho}}$$

For the convergence of the above infinite series, as per the 'Ratio test' and using the property that $\rho_b < 1$, $\beta_b < 1$ and $\rho^n \to 0$ as $n \to \infty$, we need that $\frac{s_{n+1}}{s_n} \to \beta e^{\frac{\alpha(1-\sigma)}{1-\rho}} < 1$ as $n \to \infty$. This shall be true for a positive growth, $\alpha > 0$ (since persistence, $\rho < 1$) iff $\sigma \ge \tilde{\sigma}$, where, $\tilde{\sigma} < 1$. Conversely, we may think that for all $\sigma < 1$ we need $\alpha < \alpha * (> 0)$ and for $\sigma > 1$ we need $\alpha > \alpha * (< 0)$



The continuation value for the benchmark case with logarithmic preferences is given by,

$$\widetilde{V} = E_2 \left[log(y_3) + \beta log(y_4) + \beta^2 log(y_5) + \dots \right]$$
$$= \left[\frac{\rho log(y_2)}{1 - \beta \rho} + \frac{3\alpha}{1 - \beta \rho} + \frac{4\beta\alpha}{1 - \beta \rho} + \frac{5\beta^2 \alpha}{1 - \beta \rho} + \dots \right]$$
$$= \left[\frac{\rho log(y_2)}{1 - \beta \rho} + \frac{\alpha}{1 - \beta \rho} \frac{3 - 2\beta}{(1 - \beta)^2} \right]$$

Decision to Default/Repayment at Period t = 2 For CRRA preferences and for a given debt, a_2 , the borrowing country decides to repay the loan in period t = 2 depending on the following condition:

$$(y_{b,2}^m - a_2)^{1-\sigma_b} - ((1-\gamma)y_{b,2}^m)^{1-\sigma_b} \ge \beta_b (1-\sigma_b)\widetilde{\Omega}$$
where $\widetilde{\Omega}_b = \frac{S^d - S^r}{1-\sigma}$
(10)

When the borrower has logarithmic utility,

$$\widetilde{\Omega}^m = \frac{\rho_b(\log(y_2^k) - \log(y_2^m))}{1 - \beta_b \rho_b} + \frac{\alpha_b^k - \alpha_b^m}{1 - \beta_b \rho} \frac{3 - 2\beta_b}{(1 - \beta_b)^2}$$

We look at the properties of the function $\mathcal{H}(y_{b,2}^m, a_2) = V_{b,2}^m(a_2) - V_{b,2}^k$. This is an increasing function of $y_{b,2}^m$ and we show it below.

$$\begin{aligned} \mathcal{H}(y_{b,2}^{m}, a_{2}) &= \log(y_{b,2}^{m} - a_{2}) + \beta_{b}\widetilde{V}_{b}^{m} - \log((1 - \gamma)y_{b,2}^{m}) - \beta_{b}\widetilde{V}_{b}^{k} \\ &= \log(\frac{y_{b,2}^{m} - a_{2}}{(1 - \gamma)y_{b,2}^{m}}) - \beta_{b}(\widetilde{V}_{b}^{k} - \widetilde{V}_{b}^{m}) \\ \text{Let } \mathcal{D} &= \frac{\beta_{b}\rho_{b}}{1 - \beta_{b}\rho_{b}} > 1 \text{ and } \mathcal{C} = \frac{(3 - 2\beta)(\alpha^{k} - \alpha^{m})}{(1 - \beta\rho)(1 - \beta)^{2}}. \\ &= -\log((1 - \gamma)) + \log(y_{b,2}^{m} - a_{2}) - \log(y_{b,2}^{m}) + \log((y_{b,2}^{m})^{\mathcal{D}}) - \log((y_{b,2}^{k})^{\mathcal{D}}exp(\mathcal{C})) \\ \text{Notice that } \frac{\partial \mathcal{H}}{\partial y_{b,2}^{m}} = \frac{a_{2}}{y_{b,2}^{m} - a_{2}} + \frac{\mathcal{D}}{y_{b,2}^{m}} > 0. \end{aligned}$$

Proof of Proposition 1:

We provide evidence in support of the above theorem by considering specific cases when $\sigma \in \{0.5, 1, 2\}$. In doing so we separate the default and repayment sets based on the risk appetite of the borrower.

For logarithmic preferences, when $\sigma = 1$,

$$\hat{a}_{2} = y_{2}^{m} \left(1 - (1 - \gamma) \exp\left(\beta_{b} \left[\frac{\rho_{b}(\log(y_{2}^{k}) - \log(y_{2}^{m}))}{1 - \beta_{b}\rho_{b}} + \frac{\alpha_{b}^{k} - \alpha_{b}^{m}}{1 - \beta_{b}\rho} \frac{3 - 2\beta_{b}}{(1 - \beta_{b})^{2}} \right] \right) \right)$$

First order derivative of \hat{a}_2 w.r.t to the parameters under consideration (ceteris paribus) are

$$\frac{\partial \widehat{a}_2}{\partial \alpha_b^m} = \frac{\beta_b (1-\gamma) y_2^m (3-2\beta_b) \exp\left(\beta_b \widetilde{\Omega}^m\right)}{(1-\beta_b \rho_b)^3 (1-\beta_b)^2} \ge 0$$
$$\frac{\partial \widehat{a}_2}{\partial \alpha_b^k} = -\frac{\beta_b (1-\gamma) y_2^m (3-2\beta_b) \exp\left(\beta_b \widetilde{\Omega}^m\right)}{(1-\beta_b \rho_b)^3 (1-\beta_b)^2} \le 0$$
$$\frac{\partial \widehat{a}_2}{\partial y_2^k} = -\frac{\beta_b (1-\gamma) \rho_b y_2^m \exp\left(\beta_b \widetilde{\Omega}^m\right)}{y_2^k (1-\beta_b \rho_b)} \le 0,$$

 $\frac{\partial \widehat{a}_2}{\partial y_2^m} = 1 + (1 - \gamma) \exp\left(\beta_b \widetilde{\Omega}^m\right) \frac{2\beta_b \rho_b - 1}{1 - \beta_b \rho_b} \ge 0 \text{ if } \beta_b \rho_b \ge \frac{1}{2} \text{ else it is ambigous.}$ We know $\beta_b < 1$ and applying this to our condition we get $\rho_b \ge \frac{1}{2\beta_b} > \frac{1}{2}$. This remains true under the parameter space that we consider.

$$\frac{\partial \widehat{a}_2}{\partial \gamma} = y_2^m \exp\left(\beta_b \widetilde{\Omega}^m\right) \ge 0,$$

Next we consider the case for CRRA preferences where $\sigma > 1$. The borrowers' indifference between default and repayment arises when

$$(y_2^m - a_2)^{1-\sigma} - ((1-\gamma)y_2^m)^{1-\sigma} = \beta(1-\sigma)\widetilde{\Omega}_b$$
(11)
here $(1-\sigma)\widetilde{\Omega}_b = S^D - S^R$ and $S = \sum_{n=1}^{\infty} \beta^{n-1} y_2^{(1-\sigma)\rho^n} e^{\frac{(1-\rho^{2n})(1-\sigma)^2\varsigma^2}{2(1-\rho^2)}} e^{\frac{(-3\rho^{n-1} + \frac{n+2}{\rho} - \frac{1-\rho^{n-1}}{1-\rho})(1-\sigma)\alpha\rho}{1-\rho}$

Thus, the debt level which shall make the borrower indifferent between repayment or default is given by:

$$\Rightarrow \widehat{a}_{2} = y_{2}^{m} - \left[((1-\gamma)y_{2}^{m})^{1-\sigma} + \beta_{b}(1-\sigma)\widetilde{\Omega}_{b} \right]^{\frac{1}{1-\sigma}}$$

Let, $C_{b} = \frac{(1-\sigma)^{2}(1-\rho_{b}^{2n})}{2(1-\rho_{b}^{2})} > 0$ and $Z_{b} = \frac{(-3\rho_{b}^{n-1} + \frac{n+2}{\rho_{b}} - \frac{1-\rho_{b}^{n-1}}{1-\rho_{b}})(1-\sigma)\rho_{b}}{1-\rho_{b}} > 0$ iff $\sigma < 1$.

An analysis of the derivatives is presented below for the case when $\sigma = 2$. However, this may be extended to the general case where $\sigma_b > 1$ and the results hold true. In the special case with $\sigma_b = 2$

$$\widehat{a} = y_2^m - \frac{1}{\frac{1}{(1-\gamma)y_2^m} + \sum_{n=1}^{\infty} \beta^n \left(\frac{\exp\left(\varsigma_b^{2k}C_b + Z_b\alpha_b^k\right)}{(y_2^k)^{\rho_b^n}} - \frac{\exp\left(\varsigma_b^{2m}C_b + Z_b\alpha_b^m\right)}{(y_2^m)^{\rho_b^n}}\right)}$$

The first order derivative of a_2 are given below. Of course, the expressions below may not be determined or are discontinuous if the denominator goes to zero. However, it is worth stating that the denominator is always non zero in the parameter and state space we conider.

$$\frac{\partial \hat{a}_2}{\partial \alpha_b^m} = -\frac{\sum_{n=1}^{\infty} \beta^n Z_b \frac{\exp\left(\varsigma_b^{2m} C_b + Z_b \alpha_b^m\right)}{(y_2^m)^{\rho_b^n}}}{\left[\frac{1}{(1-\gamma)y_2^m} + \sum_{n=1}^{\infty} \beta^n \left(\frac{\exp\left(\varsigma_b^{2k} C_b + Z_b \alpha_b^k\right)}{(y_2^k)^{\rho_b^n}} - \frac{\exp\left(\varsigma_b^{2m} C_b + Z_b \alpha_b^m\right)}{(y_2^m)^{\rho_b^n}}\right)\right]^2} \ge 0, \text{ unambigously.}$$

$$\frac{\partial \widehat{a}_2}{\partial \alpha_b^k} = \frac{\sum_{n=1}^{\infty} \beta^n Z_b \frac{\exp\left(\varsigma_b^{2k} C_b + Z_b \alpha_b^k\right)}{(y_2^k)^{\rho_b^n}}}{\left[\frac{1}{(1-\gamma)y_2^m} + \sum_{n=1}^{\infty} \beta^n \left(\frac{\exp\left(\varsigma_b^{2k} C_b + Z_b \alpha_b^k\right)}{(y_2^k)^{\rho_b^n}} - \frac{\exp\left(\varsigma_b^{2m} C_b + Z_b \alpha_b^m\right)}{(y_2^m)^{\rho_b^n}}\right)\right]^2} \le 0, \text{ unambigously.}$$

$$\frac{\partial \hat{a}_{2}}{\partial y_{2}^{m}} = 1 + \frac{-\frac{1}{(1-\gamma)y_{2}^{m2}} + \sum_{n=1}^{\infty} \beta^{n} \rho_{b}^{n} \frac{\exp\left(\varsigma_{b}^{2m} C_{b} + Z_{b} \alpha_{b}^{m}\right)}{(y_{2}^{m})^{1+\rho_{b}^{n}}}}{\left[\frac{1}{(1-\gamma)y_{2}^{m}} + \sum_{n=1}^{\infty} \beta^{n} \left(\frac{\exp\left(\varsigma_{b}^{2k} C_{b} + Z_{b} \alpha_{b}^{k}\right)}{(y_{2}^{k})^{\rho_{b}^{n}}} - \frac{\exp\left(\varsigma_{b}^{2m} C_{b} + Z_{b} \alpha_{b}^{m}\right)}{(y_{2}^{m})^{\rho_{b}^{n}}}\right)\right]^{2}} \ge 0 \text{ in the paper potential production}$$

rameter space under consideration.

$$\frac{\partial \hat{a}_2}{\partial y_2^k} = -\frac{\sum_{n=1}^{\infty} \beta^n \rho_b^n \frac{\exp\left(\varsigma_b^{2k} C_b + Z_b \alpha_b^k\right)}{(y_2^k)^{1+\rho_b^n}}}{\left[\frac{1}{(1-\gamma)y_2^m} + \sum_{n=1}^{\infty} \beta^n \left(\frac{\exp\left(\varsigma_b^{2k} C_b + Z_b \alpha_b^k\right)}{(y_2^k)^{\rho_b^n}} - \frac{\exp\left(\varsigma_b^{2m} C_b + Z_b \alpha_b^m\right)}{(y_2^m)^{\rho_b^n}}\right)\right]^2} \le 0, \text{ unambigously.}$$

$$\frac{\partial \widehat{a}_2}{\partial \varsigma_b^{2k}} = \frac{\sum_{n=1}^\infty \beta^n C_b \frac{\exp\left(\varsigma_b^{2k} C_b + Z_b \alpha_b^k\right)}{(y_2^k)^{\rho_b^n}}}{\left[\frac{1}{(1-\gamma)y_2^m} + \sum_{n=1}^\infty \beta^n \left(\frac{\exp\left(\varsigma_b^{2k} C_b + Z_b \alpha_b^k\right)}{(y_2^k)^{\rho_b^n}} - \frac{\exp\left(\varsigma_b^{2m} C_b + Z_b \alpha_b^m\right)}{(y_2^m)^{\rho_b^n}}\right)\right]^2} \ge 0, \text{ unambigously.}$$

$$\frac{\partial \widehat{a}_2}{\partial \varsigma_b^{2m}} = -\frac{\sum_{n=1}^{\infty} \beta^n C_b \frac{\exp\left(\varsigma_b^{2m} C_b + Z_b \alpha_b^m\right)}{(y_2^m)^{\rho_b^n}}}{\left[\frac{1}{(1-\gamma)y_2^m} + \sum_{n=1}^{\infty} \beta^n \left(\frac{\exp\left(\varsigma_b^{2k} C_b + Z_b \alpha_b^n\right)}{(y_2^k)^{\rho_b^n}} - \frac{\exp\left(\varsigma_b^{2m} C_b + Z_b \alpha_b^m\right)}{(y_2^m)^{\rho_b^n}}\right)\right]^2} \le 0, \text{ unambigously.}$$

$$\frac{\partial \widehat{a}_2}{\partial \gamma} = \frac{1}{\left[\frac{1}{(1-\gamma)y_2^m} + \sum_{n=1}^{\infty} \beta^n \left(\frac{\exp\left(\varsigma_b^{2k} C_b + Z_b \alpha_b^n\right)}{(y_2^k)^{\rho_b^n}} - \frac{\exp\left(\varsigma_b^{2m} C_b + Z_b \alpha_b^m\right)}{(y_2^m)^{\rho_b^n}}\right)\right]^2} \ge 0, \text{ unambigously.}$$

$$\frac{\partial \widehat{a}_2}{\partial \gamma} = \frac{1}{(1-\gamma)^2 y_2^m \left[\frac{1}{\gamma y_2^m} + \sum_{n=1}^\infty \beta^n \left(\frac{\exp\left(\varsigma_b^{2k} C_b + Z_b \alpha_b^k\right)}{(y_2^k)^{\rho_b^n}} - \frac{\exp\left(\varsigma_b^{2m} C_b + Z_b \alpha_b^m\right)}{(y_2^m)^{\rho_b^n}}\right)\right]^2} \ge 0, \text{ unambigously.}$$

(Q.E.D)

Proof of Proposition 2:

Let,
$$y_{b,2}^m = y_{b,2}^k = y_{b,2}$$
.

$$\Rightarrow \beta_b \widetilde{\Omega}_b = \frac{\beta_b \rho_b (\log(y_{b,2}) - \log(y_{b,2}))}{1 - \beta_b \rho_b} + \frac{\beta_b \rho_b (3 - 2\beta_b) (\alpha_b^k - \alpha_b^m)}{(1 - \beta_b \rho_b) (1 - \beta_b)^2} = \frac{\beta_b \rho_b (3 - 2\beta_b) (\alpha_b^k - \alpha_b^m)}{(1 - \beta_b \rho_b) (1 - \beta_b)^2} \le 0$$

$$\Rightarrow \exp(\beta_b \widetilde{\Omega}_b) \le 1$$

$$\Rightarrow y_{b,2} \left(1 - (1 - \gamma) \exp(\beta_b \widetilde{\Omega}_b) \right) \ge y_{b,2} \left(1 - (1 - \gamma) \right)$$

$$\Rightarrow \widehat{a}_2^m \ge \widehat{a}_2^k$$

(Q.E.D)

Proof of Corollory:

We can write that $\hat{a}_2^m = \hat{a}_2^k = a_2$ iff

(i) $y_b^{*m} \ge y_b^{*k}$ and default penalty is big and trend growth difference is high for different regimes or when (ii) $y_b^{*m} \le y_b^{*k}$.

$$\begin{split} \widehat{a}_{2}^{m} &: V_{b,2}^{m}(a_{2}) = V_{b,2}^{k} \text{ and } \widehat{a}_{2}^{k} : V_{b,2}^{s}(a_{2}) = V_{b,2}^{k} \\ \Rightarrow \gamma y_{b,2}^{k} &= y_{b,2}^{m} \left(1 - (1 - \gamma) exp\left[\beta_{b}(\widetilde{V}^{k} - \widetilde{V}^{m}) \right] \right) \\ \Rightarrow \gamma \frac{y_{b}^{*k}}{y_{b}^{*m}} &= 1 - (1 - \gamma) \left(\frac{y_{b,2}^{k}}{y_{b}^{*m}} \right)^{\mathcal{D}} exp(\mathcal{C}) \\ \Rightarrow \gamma \frac{y_{b}^{*k}}{y_{b}^{*m}} + (1 - \gamma) exp(\mathcal{C}) \left(\frac{y_{b,2}^{k}}{y_{b}^{*m}} \right)^{\mathcal{D}} &= 1 \\ \text{where, } \mathcal{D} &= \frac{\beta \rho}{1 - \beta \rho} > 1 \text{ and } \mathcal{C} &= \frac{\beta(3 - 2\beta)(\alpha^{k} - \alpha^{m})}{(1 - \beta \rho)(1 - \beta)^{2}} < 0 \end{split}$$

Note that the above equality will not be satisfied for the cases when, $y_{b,2}^k < y_b^{*k} < y_b^{*m}$ and $y_b^{*k} < y_{b,2}^k < y_b^{*m}$. The above will be satisfied for a higher γ or a higher difference between α^m and α^m when $y_b^{*k} < y_b^{*m} < y_{b,2}^k$. Thus, if we have a small default penalty, we should have $y_b^{*k} > y_b^{*m}$.

Further, if we compare the case when the borrower recieves same shocks for the autarky when in either of the regimes we get,

$$\gamma \frac{y_b^k}{y_b^m} + (1 - \gamma) exp(\mathcal{C}) \left(\frac{y_b^k}{y_b^m}\right)^{\mathcal{D}} = 1$$

The above equation will never be satisfied if $\frac{y_b^k}{y_b^m} < 1$ or $y_b^k < y_b^m$, since γ , $1 - \gamma$ and $exp(\mathcal{C}) < 1$. However, it is possible for the equality to hold for $\frac{y_b^k}{y_b^m} > 1$ or $y_b^k > y_b^m$.

(Q.E.D)

Default probability in period 1

In order to prove this result we make use of the properties of the error function.

Since,
$$-1 \le \operatorname{erf}(.) \le 1$$

 $\Rightarrow 0 \le 1 - \operatorname{erf}(.) \le 2 \Rightarrow 0 \le \frac{1 - \operatorname{erf}(.)}{2} \le 1$
 $\Rightarrow 0 \le \delta_2 \le 1.$

Again, given $a_2 = y_b^* \left(1 - (1 - \gamma) \exp\left(\beta_b \left[\frac{\rho_b (\log(y_{b,2}^k) - \log(y_b^*))}{1 - \beta_b \rho_b} + \frac{\alpha_b^k - \alpha_b^m}{1 - \beta_b \rho} \frac{3 - 2\beta_b}{(1 - \beta_b)^2} \right] \right) \right)$ we take the derivative with respect to a_2 to obtain,

$$1 = \frac{\partial y_b^*}{\partial a_2} \left[1 - (1 - \gamma) \exp(\beta_b \widetilde{\Omega}_b) + \frac{\beta_b \rho_b}{1 - \beta_b \rho_b} (1 - \gamma) \exp(\beta_b \widetilde{\Omega}_b) \right]$$

$$\Rightarrow \frac{\partial y_b^*}{\partial a_2} = \frac{1}{\left[(1 - (1 - \gamma) \exp(\beta_b \widetilde{\Omega}_b)) + \frac{\beta_b \rho_b}{1 - \beta_b \rho_b} (1 - \gamma) \exp(\beta_b \widetilde{\Omega}_b) \right]} = \frac{1}{1 + (1 - \gamma) \exp(\beta_b \widetilde{\Omega}_b) \frac{2\beta_b \rho_b - 1}{1 - \beta_b \rho_b}} \ge 0 \text{ if } \rho_b \beta_b \ge \frac{1}{2}, \text{ which shall be the case in our setup.}$$

$$-\delta_{2}'(a_{2}) = \frac{-1}{2y_{b,2}^{*}\varsigma_{b}^{m}}\sqrt{\frac{2}{\pi}} \left[exp\left(-\frac{\log^{2}\left(\frac{y_{b,2}^{*}}{(y_{b,1}^{m})^{\rho_{b}}\exp(2\alpha_{b}^{m})}\right)}{2(\varsigma_{b}^{m})^{2}}\right) \right] \frac{\partial y_{b,2}^{*}}{\partial a_{2}} < 0.$$
(12)

where, $\frac{\partial y_{b,2}^*}{\partial a_2} \ge 0$ for all the values of σ . (Q.E.D)

Debt threshold changes as the risk aversion of the borrower changes (refer to

fig 21).

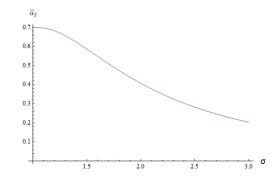


Figure 21: 'Indifference' debt v.s. risk aversion

Proof of Proposition 3:

Let us say, $y_{b,1}^m = y_{b,1}^k$. We already know that $y_b^{*k} = \frac{a_2}{\gamma}$ and $(y_b^{*m})^{\mathcal{B}-1}(y_b^{*m} - a_2) = (1-\gamma)(y_b^k)^{\mathcal{B}}e^{\mathcal{C}(\alpha^k - \alpha^m)} \ge 0 \Rightarrow y_b^{*m} \ge a_2$.

Thus, we are in one of the two cases, (i) $a_2 \leq y_b^{*m} \leq \frac{a_2}{\gamma} = y_b^{*k}$ or (ii) $a_2 \leq \frac{a_2}{\gamma} = y_b^{*k} \leq y_b^{*m}$. For an debt level a_2 , $y_{b,2}^{m*}(a_2) \leq y_{b,2}^{k*}(a_2)$ whenever $\gamma \mapsto \underline{\gamma}$ (where $\underline{\gamma}$ is close to zero). In this scenario we can say, since the error and logarithmic functions are monotonically increasing, that

$$\frac{y_{b,2}^{m*}}{(y_{b,1}^m)^{\rho_b} \exp(2\alpha_b^m)} \le \frac{y_{b,2}^{k*}}{(y_{b,1}^k)^{\rho_b} \exp(2\alpha_b^k)} \Rightarrow \delta_2^m \le \delta_2^k \ .$$

(Q.E.D)

References

Arellano, C. (2008). Default risk and income fluctuations in emerging economies. The American Economic Review, 690–712.

Baldwin, R. E. (2006). The euro's trade effects.

- Conesa, J. C. and T. J. Kehoe (2012). Gambling for redemption and self-fulfilling debt crises. Technical report, Federal Reserve Bank of Minneapolis.
- CPIS, I. T. F. (2013). Coordinated portfolio investment survey, **IMF** Task Force. International Monetary Fund.
- De Grauwe, P. (2012). Economics of monetary union. Oxford University Press.
- Eaton, J. and M. Gersovitz (1981). Debt with potential repudiation: Theoretical and empirical analysis. *The Review of Economic Studies* 48(2), 289–309.
- Flam, H. and H. Nordström (2006). Trade volume effects of the euro: Aggregate and sector estimates.
- Frankel, J. and A. Rose (2002). An estimate of the effect of common currencies on trade and income. *The Quarterly Journal of Economics* 117(2), 437–466.
- Fratzscher, M. and L. Stracca (2009). The political economy under monetary union: Has the euro made a difference? *Economic Policy* 24(58), 307–348.
- Lane, P. R. (2012). The european sovereign debt crisis. The Journal of Economic Perspectives 26(3), 49–67.
- Lizarazo, S. V. (2013). Default risk and risk averse international investors. Journal of International Economics 89(2), 317–330.
- Luque, J., M. Morelli, and J. Tavares (2014). A volatility-based theory of fiscal union desirability. *Journal of Public Economics* 112, 1–11.
- Micco, A., E. Stein, and G. Ordoñez (2003). The currency union effect on trade: early evidence from emu. *Economic policy* 18(37), 315–356.
- OECD (1999). European Monetary Union facts, challenges and policies.
- Papaioannou, E. and R. Portes (2008). Costs and benefits of running an international currency. Technical report, Directorate General Economic and Monetary Affairs (DG ECFIN), European Commission.

- Shambaugh, J. C., R. REIS, and H. REY (2012). The euro's three crises [with comments and discussion]. *Brookings Papers on Economic Activity*, 157–231.
- Werning, I. and E. Farhi (2012). Fiscal unions. Technical report, National Bureau of Economic Research.