Uncertainty Averse Bank Runners*

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Abstract

Bank runs are relatively rare events characterized by highly pessimistic depositors' expectations. How would pessimistic depositors expect to be treated in a bank run? How will this affect their behavior? How can banks handle this kind of risk? In the framework of a Diamond-Dybvig-Peck-Shell banking model, in which a broad class of feasible contractual arrangements (including "suspension schemes") is allowed and which admits a run equilibrium, we analyze a scenario in which depositors are uncertain of their treatment should a run occur. We check whether bank runs are more likely or less likely to happen, in particular, if depositors are maxmin decision makers. We assess the utility of suspension schemes in the presence of pessimistic bank runners.

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1 Introduction

Starting from the seminal Diamond and Dybvig's (1983) paper (D-D henceforth), a stream of literature has developed which looks at bank runs as phenomena originating from a coordination failure driven by an extrinsic random variable, namely a *sunspot*. D-D have in fact constructed a simple banking model in which the optimal demand deposit contract gives rise to two equilibria: a "good" equilibrium, in which depositors truthfully reveal their type (impatient or patient) and act accordingly (run or wait); a "bad" equilibrium (*bank run*), in which all depositors, independently of their type, decide to run to the bank, driven by an "irrational shift of expectations" which makes them believe that everybody else is running. If there is no aggregate uncertainty (that is, if the bank knows how many patients and impatients populate the economy), then the *total suspension of convertibility* (TSC) eliminates the bank run equilibrium. Under aggregate uncertainty TSC is not implementable - simply because the bank does not know where to stop in returning deposits -, and the alternative solution proposed by D-D is the deposit insurance. Wallace (1988) has however criticized the - feasibility of the - optimal contract under aggregate uncertainty designed by D-D by arguing that, in it, the sequential service constraint (SSC) is indeed not "taken seriously"[1]. Wallace (1990) has then proven

[1] The solution under aggregate uncertainty proposed by D-D requires that the bank find the optimal contract as a function of the proportions of the two types (that it does not know yet) and that, after each depositor has contacted the bank and revealed her true type (which she is going to do, since the incentive compatibility constraint is satisfied), the bank eventually comes to know these proportions and is then able to implement the optimal contract. The Wallace's (1988) criticism is concerned with the assumption that the bank be able to observe each agent's type before starting the distribution of pay-offs to the depositors. This assumption is in fact not in line with the sequential service constraint, which is inspired by a "first come-first

that, under aggregate uncertainty and SSC taken seriously, *partial suspension of convertibility* (PSC) characterizes the optimal banking contract, because it enhances the risk-sharing among depositors.

In enlarging the set of feasible contractual arrangements from the "simple contracting" (the demand deposit contract) to a class of banking mechanisms that allow for suspension schemes[2], Wallace (1990) has brought about a significant departure from the original D-D framework, and has inspired a number of subsequent works in this field. An important contribution along these lines is Peck and Shell (JPE, 2003), which designs a banking model admitting a multiplicity of equilibria (one of which being a bank run), and further develops the issue of the selection among them. Peck and Shell (2003) can be interpreted as a response to the banking model developed by Green and Lin (2000), whose optimal mechanism only admits the good equilibrium. Peck and Shell (2003) in fact show that 1. the non-existence of the bank run equilibrium in Green and Lin (2000) crucially depends upon the - unrealistic - assumption that depositors know *exactly* their position in the queue to the bank; and that 2. bank run equilibria re-emerge as soon as depositors are assumed to have only a *probabilistic* knowledge of their place in line.

This paper enters the debate about the existence of sunspot-driven bank runs. It suggests that, once a class of banking mechanisms including suspension

served" principle, and which thus implies that the pay-off to each depositor is to be given as soon as she gets to the bank.

[2]Allowing for suspension schemes simply means that the bank, in finding the optimal contract, is allowed to assign different pay-offs across depositors as a function of their place in line.

schemes is taken into account, panic crises may disappear exactly because of depositors' *uncertain beliefs* about how they will be treated in the case of panic. In Peck and Shell (2003) depositors evaluate each place in line as equally likely independently of whether or not a bank run is expected. We further weaken the assumption about the depositors' prior beliefs by assuming that, not only do these depositors not know exactly their position in the queue, but they also have an "imprecise" probabilistic knowledge of their position *when expecting a run.*

Uncertainty here is to be intended in the sense, first given by Knight (1921), that the information of each depositor is too vague to be represented by a (single additive) probability distribution. We suppose that, when each depositor expects a bank run to occur, she feels no longer able to evaluate reliably the probability distribution of her position number in the queue. Bank runs are in fact relatively rare events, thereby preventing depositors from forming well informed expectations on their position should such an event occur. In particular, as we will clarify below, the patient depositor might fear to be unfavorably treated by the bank in the case she runs and, because of that, might be eventually discouraged from running.

This situation closely resembles the one depicted in the Ellsberg paradox (Ellsberg (1961)): two urns are given, each of which contains ten balls, whose color is either white or black. One of them is known to contain five white balls and five black balls, while no information is given on the distribution of the balls' colors in the other urn. If the decision maker is asked to bet on the color of the first ball drawn at random from either urn, which urn would she prefer for this bet? The paradox arises because most people show a clear preference for the "known" urn, that is, for the urn containing five white and five black balls: they are indifferent as to the color to bet on in both urns, but strictly prefer to bet on the known urn rather than on the unknown one. This choice behavior cannot be explained in the subjective expected utility (SEU) framework, since there is no subjective (additive) prior supporting these preferences. In particular, people showing this preference order do not act as if there were five white and five black balls in the unknown urn (otherwise they would show indifference), to the same extent as our depositors do not act as if their place in line were equally probable when fearing a bank run.

Our formalization of the depositor's attitude towards uncertainty is inspired by the multiple prior maxmin expected utility (MEU) theory axiomatized by Gilboa and Schmeidler (1989). This approach is a well-established generalization of the SEU theory, which can accomodate the choice behavior of Ellsberg-type situations, in which individuals are not able to estimate reliably probabilities. In representing subjective beliefs, the MEU decision rule replaces the "classic" single prior with a closed and convex set of priors (multi-prior beliefs). The agent evaluates every act by computing the *minimal* expected utility over this set of priors; she will then select the act which maximizes this minimal expected utility (hence the phrase "maxmin"). The agent is said to be uncertainty averse if this set is not a singleton. The application of this decision rule to our framework leads us to assume a depositor who maximizes her expected pay-off with respect to the binary choice - whether or not to withdraw -, while selecting the worst probability distribution (over her position in the queue) among all the admissible ones. As we will see in the next section, since in a mechanism design approach pay-offs generally vary as a function of the position number, uncertainty aversion may alter the agent's withdrawal strategy.

We will show that, *coeteris paribus*, "cautious" depositors (in the sense of Gilboa-Schmeidler) are "less willing" to run and, hence, that panic-driven bank runs may disappear once this conservative attitude towards uncertainty is taken into account and incorporated into the model. A remarkable implication is that, with purely maxmin depositors (in the sense of Wald (1950)), bank runs induced by sunspots disappear completely, because these agents do *never* run independently of what they think the others will do. Interestingly, the reason why panic-driven bank runs vanish is exactly opposite to the one pointed out by Green and Lin (2000) and criticized by Peck and Shell (2003): here depositors know "very little" about their place in line and, because of that, they may be dissuaded from running. Finally, a policy implication of our results is that banking mechanisms allowing for suspension schemes are worthy, not only because they improve risk-sharing (Wallace (1990)), but also because they may eliminate panic-driven bank runs in a potentially general class of banking models.

2 Aversion to Uncertainty and Propensity to Run

The banking model developed in Peck and Shell (2003) is characterized by aggregate uncertainty on the distribution of the agent's type and by the observance of the so-called sequential service constraint (which forces the bank to deal with customers sequentially). There are three periods, and N potential depositors, α being the number of impatients and $N-\alpha$ that of patients. Each of them is endowed with y units of consumption in period 0 regardless of type. Impatient agents evaluate utility of period 1 only, through a function $u(c^1)$, while patient agents, who are allowed to costlessly store consumption across periods, evaluate utility of both periods 1 and 2 through the function $v(c^1+c^2)$, where c^1 and c^2 represent respectively consumption received in periods 1 and 2. Both functions are assumed to be strictly increasing, concave, and twice continuously differentiable. The bank, whose target is to maximize the ex-ante expected utility of consumers[3], knows the probability distribution over the possible realizations of types [$f(\alpha)$ for $\alpha = 0, 1, ..., N$] and, as usual, is not able to recognize the agent's type. As to technology, 1 unit of consumption invested in period 0 yields R units in period 2 and 1 unit in period 1. As a consequence of the technology and preference assumptions, in autarchy patient depositors strictly prefer to consume in period 2.

In Peck and Shell (2003) an essential distinction is made between pre- and

[3] The implicit assumption here is the existence of a perfectly competitive banking sector.

post-deposit game. In the latter consumers are assumed to have already deposited their endowments and, after learning their type (at the beginning of period 1), must only decide whether to withdraw in period 1 or in period 2. The pre-deposit game also encompasses the agent's choice between deposit and autarchy: this choice is indeed not trivial since, for instance, the agent would decide not to deposit if she had the belief that a bank run would occur. Here we focus on the post-deposit game: all the findings about the pre-deposit game obtained in Peck and Shell (2003) apply, *mutatis mutandis*, to our framework as well.

Our departure from the standard framework is concerned with the subjective prior of the depositor relative to her position in the queue. In Peck and Shell (2003) the agent evaluates each place in line as equally likely independently of whether or not a bank run is expected. Conversely, for the reasons stated in the introduction we allow probabilities to vary across position numbers whenever depositors believe that a run is about to occur. Following the Gilboa and Schmeidler's (1989) MEU principle, we further assume that, when a bank run is expected

1. the agent's subjective belief about her own position in the queue is modeled as a set of additive probability measures (*multiple prior belief);*

2. the agent's choice behavior is represented as a maxmin strategy, which leads her to evaluate the pay-off associated with withdrawing in period 1 according to the worst prior.

The depositor's strategy is taken into account by the bank when designing the optimal contract. Inside the set M of feasible banking mechanisms, the optimal mechanism m^*

$$m^* = (c^1(1), ..., c^1(z), ..., c^1(N), c^2(0), ..., c^2(N-1))$$

(where z refers to the depositor's position in the queue) is the set of pay-offs which maximizes total welfare - defined as the sum of the utilities of the two types weighted by the probabilities of all possible realizations - subject to the resource constraint and to an incentive compatibility constraint (ICC)[4]. ICC ensures that patient depositors, in comparing the expected pay-off associated with the "truth telling" strategy (withdrawing in period 2) to the one associated with the strategy of "lying" (withdrawing in period 1), prefer to tell the truth. The economy may be subject to a panic-driven run when ICC holds and the following condition, called no-bank run condition (NBC), is violated:

$$\sum_{z=1}^{N} q_z^*(m^*) v(c^1(z)) \leq v\left(\left[Ny - \sum_{z=1}^{N-1} c^1(z)\right] R\right) \qquad \text{(NBC)}$$

where q_z^* is the depositor's prior of her position in the queue which minimizes - over a set of given priors - her expected pay-off from running[5]. NBC states that, even though the patient depositor had the belief that any other agent would be running, she would be however interested in waiting until period 2. We can now state the following proposition.

[4] Since we do not need the formal representation of the banking problem to derive our results, we prefer to state it explicitely in the appendix.

[5] See the appendix for a more formal treatment.

Proposition 1 *For the post-deposit game there always exists a positive measure set of minimizing priors which makes the bank run equilibrium disappear.*

Proof. Assume the following set of priors:

$$q_z \in [0+\varepsilon, 1-\varepsilon] \text{ for } \varepsilon \in [0, \frac{1}{N}) \text{ and } \forall z = 1, ..., N \tag{1}$$

Also assume that the lowest pay-off in period 1 be strictly higher than 0, and -w.l.o.g., as it will be argued below - that weak PSC characterizes the optimal mechanism m^*:

$$c^1(1) \geq c^1(2) \geq ... \geq c^1(N-1) \geq Ny - \sum_{z=1}^{N-1} c_1(z)$$

The relation above identifies two possible cases.

1. The optimal solution is

$$c^1(1) = c^1(2) = ... = c^1(N-1) = Ny - \sum_{z=1}^{N-1} c_1(z) \tag{2}$$

In this case the minimizing distribution is anyone among all possible additive distributions belonging to the set defined in (1). Then the NBC becomes

$$v\left(Ny - \sum_{z=1}^{N-1} c_1(z)\right) < v\left[\left(Ny - \sum_{z=1}^{N-1} c_1(z)\right) R\right]$$

which is always satisfied $\forall R > 0$ and no bank run can occur. Notice that (2) corresponds to the "autarchic solution".

2. In the optimal solution, at least one pay-off is strictly greater than the others. Suppose (w. l. o. g.) that

$$c^1(1) \geq c^1(2) \geq ... \geq c^1(N-1) > Ny - \sum_{z=1}^{N-1} c_1(z)$$

In this case the minimizing prior with respect to (1) would be

$$[q_z^* = \varepsilon \ \forall z = 1, ..., N-1; q_N^* = 1-(N-1)\varepsilon]$$

and the NBC becomes

$$\sum_{z=1}^{N-1} \varepsilon v(c^1(z)) + [1-(N-1)\varepsilon]\, v\left(Ny - \sum_{z=1}^{N-1} c_1(z)\right) < v\left[\left(Ny - \sum_{z=1}^{N-1} c_1(z)\right) R\right]$$

We argue that, $\forall R > 0$, there is at least an $\varepsilon > 0$ that satisfies the condition stated above. The threshold value of ε *below which* the bank run disappears is

$$0 < \varepsilon = \frac{v\left[\left(Ny - \sum_{z=1}^{N-1} c_1(z)\right) R\right] - v\left(Ny - \sum_{z=1}^{N-1} c_1(z)\right)}{\sum_{z=1}^{N-1} v(c^1(z)) - (N-1)v\left(Ny - \sum_{z=1}^{N-1} c_1(z)\right)}$$

Notice also that the assumption of PSC has been made w.l.o.g. Indeed suppose that the pay-off associated with the last position is not the minimum because there exists

$$v(c^1(i)) < v\left(Ny - \sum_{z=1}^{N-1} c_1(z)\right) \text{ for some } i \in [1, N-1]$$

then it will also be

$$v(c^1(i)) < v\left(Ny - \sum_{z=1}^{N-1} c_1(z)\right) < v\left[\left(Ny - \sum_{z=1}^{N-1} c_1(z)\right) R\right]$$

and the reasoning of the proof can be repeated identically. ■

Notice immediately that, when the risk of a bank run vanishes (due to the depositors' uncertain beliefs), the unique equilibrium for the post-deposit game is also the *unique* equilibrium for the *pre-deposit game*: the bank simply offers the optimal contract, and all the agents decide to deposit their endowments, and to withdraw them according to their true type.

An interesting corollary of this proposition is that, under the *maxmin return criterion* (Wald (1950)), panic-driven bank runs cease to exist. Whenever patient depositors fear that the worst case is going to happen, they will commit themselves to a truth telling strategy no matter what (they think) the others do. The proof is straightforward and can be obtained from the one above by setting $\varepsilon = 0$. This condition would in fact give rise to the following *unrestricted* set of priors (which always characterizes these types of decision makers): $\hat{q}_z \in [0, 1]$ $\forall z = 1, ..., N$. The minimization over this set will lead to assign probability 1 to the worst position in the queue and 0 to all the others.

As a result, banking mechanisms would be again immune to sunspot-induced runs as in Green and Lin (2000) but, somewhat paradoxically, for the opposite reason: while there the assumption that each depositor knows her place in line lies behind the backward induction argument which makes bank run equilibria disappear, here it is the strong uncertainty and a highly conservative attitude towards it which discourage depositors from running.

We have provided a theoretical argument in favor of the implementation of suspension schemes in banking contracts, which adds to the “classic” argument provided by Wallace (1990). Wallace proved that, in conditions of aggregate uncertainty on the distribution of the agent’s type, the possibility of designing more sophisticated contracts - including suspension schemes - increases the depositors’ welfare by improving risk-sharing among them. We claim that, if depositors - in a situation in all respects similar to the one giving rise to the Ellsberg paradox - hold pessimistic beliefs upon their position in the queue when expecting a run, these suspension schemes can also make these contracts immune to panic-driven bank runs.

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A Appendix

In this Section we formulate the optimal problem solved by the bank. We first need the following definition: conditional on an agent's being patient, the probability that the number of impatient agents is α can be expressed as

$$f_p(\alpha) = \frac{\left[1 - \frac{\alpha}{N}\right] f(\alpha)}{\sum\limits_{\alpha'=0}^{N-1} \left[1 - \frac{\alpha'}{N}\right] f(\alpha')}$$

for $\alpha = 0, 1, ..., N$. The optimal mechanism m^* is found by maximizing the expected welfare of depositors:

$$\max_{[c^1(1),....,c^1(N-1)]} W = \sum_{\alpha=0}^{N-1} f(\alpha) \left[\sum_{z=1}^{\alpha} u(c^1(z)) + (N-\alpha) v \left(\frac{\left[Ny - \sum\limits_{z=1}^{\alpha} c^1(z) \right] R}{N - \alpha} \right) \right] +$$

$$+ f(N) \left[\sum_{z=1}^{N-1} u(c^1(z)) + u \left(Ny - \sum_{z=1}^{N-1} c^1(z) \right) \right]$$

subject to the following incentive compatibility constraint (the resource constraint is already incorporated into the objective function):

$$s.t. \ \sum_{\alpha=0}^{N-2} f_p(\alpha) \left[\frac{1}{\alpha+1} \sum_{z=1}^{\alpha+1} v(c^1(z)) \right] + f_p(N-1) \left[\sum_{z=1}^{N} q_z^*(m^*) v(c^1(z)) \right] \leq$$

$$\leq \sum_{\alpha=0}^{N-1} f_p(\alpha) v \left(\frac{\left[Ny - \sum_{z=1}^{\alpha} c^1(z) \right] R}{N - \alpha} \right)$$

where

$$q_z^*(m^*) = \arg\min \left\{ \sum_{z=1}^{N} q_z v(c^1(z)) \right\}$$

$$s.t. \sum_{z=1}^{N} q_z = 1$$

$$s.t. q_z \in \left[q_z^l; q_z^h \right] \forall z = 1, ..., n \text{ and } q_z^l < q_z^h$$

The superscripts l and h stand respectively for "low" and "high" and delimit the extension of priors. q_z^* is the minimizing prior which the bank takes into account when choosing the optimal contract.