The Inflation Bias under Calvo and Rotemberg Pricing*

Campbell Leith[†] Ding Liu[‡]

June 19, 2014

Abstract

New Keynesian models rely heavily on two workhorse models of nominal inertia - price contracts of random duration (Calvo, 1983) and price adjustment costs (Rotemberg, 1982) - to generate a meaningful role for monetary policy. These alternative descriptions of price stickiness are often used interchangeably since, to a first order of approximation they imply an isomorhpic Phillips curve and, if the steady-state is efficient, identical objectives for the policy maker and as a result in an LQ framework, the same policy conclusions.

In this paper we compute time-consistent optimal monetary policy in benchmark New Keynesian models containing each form of price stickiness. Using global solution techniques we find that the inflation bias problem under Calvo contracts is significantly greater than under Rotemberg pricing, despite the fact that the former typically exhibits far greater welfare costs of inflation. The rates of inflation observed under this policy are non-trivial and suggest that the model can comfortably generate the rates of inflation at which the problematic issues highlighted in the trend inflation literature emerge, as well as the movements in trend inflation emphasized in empirical studies of the evolution of inflation. Finally, we consider the response to cost push shocks across both models and find these can also be significantly different. The choice of which form of nominal inertia to adopt is not innocuous.

Key words: New Keynesian Model; Monetary Policy; Rotemberg Pricing; Calvo Pricing; Inflation Bias; Time-Consistent Policy.

JEL codes: E52, E63

*We are grateful for comments from Guido Ascari, Fabrice Collard, Richard Dennis, Charles Nolan and seminar participants at the University of Glasgow.

†Address: Economics, Adam Smith Business School, West Quadrangle, Gilbert Scott Building, University of Glasgow, Glasgow G12 8QQ. E-mail: campbell.leith@glasgow.ac.uk.

‡Address: Economics, Adam Smith Business School, 510, Robertson Building, University of Glasgow, Glasgow G11 6AQ. E-mail: dingliumath@gmail.com

1 Introduction

Mainstream macroeconomic analysis of both monetary and fiscal policy relies heavily on the New Keynesian model. The distinguishing feature of this model, relative to a more classical approach, is that it contains some form of nominal inertia. This allows monetary policy to have real effects, and widens the degree of interaction between monetary and fiscal policies, since monetary policy affects both the size of the tax base and real debt service costs in such models. Typically, one of two workhorse forms of nominal inertia are adopted in the literature - Calvo (1983) price contracts, and Rotemberg (1982) price adjustment costs. In the former, firms are only able to adjust their prices after random intervals of time, such that, outside of a zero inflation steady-state there will be a costly dispersion of prices across firms. While the latter implies that all firms behave symmetrically in setting the same price, but that they face quadratic adjustment costs in doing so. Despite this fundamental difference, researchers have typically treated the two approaches as being equivalent since the New Keynesian Phillips Curve (NKPC) they imply are, to a first order of approximation, isomorphic when linearized around a zero inflation steady state. Moreover, when that zero inflation steady-state is also efficient (that is, it matches the output level that would be chosen by a benevolent social planner) it can be shown that the second order approximation to welfare rewritten in terms of inflation and the output gap is also the same across the two approaches (see Nistico, 2007). Under these conditions, to a first order of approximation, the two approaches would yield the same policy implications. For these reasons the two approaches have largely been treated as synonymous within the New Keynesian literature.

However, despite this broad consensus, there are examples within the literature where the two approaches do differ. The first is where the steady-state around which we approximate the New Keynesian economy is not efficient. For example, Lombardo and Vestin (2008) relax the assumption of Nistico (2007) and consider the second order approximation to welfare when the steady state is not efficient. They find that the costs of such inefficiencies are typically larger in the Calvo economy. This mirrors the results in Damjanovic and Nolan (2011).

The second assumption underpinning the equivalence result, is that the economy is approximated around a zero inflation steady state (or that any steady-state inflation is perfectly indexed and therefore costless, see Yun (1996)). A literature considering the importance of trend inflation argues that this is not the case, and that the implications of failing to account for trend inflation can be dramatic, see Ascari and Sbordone (2013) for a survey. The presence of even a modest degree of (unindexed) steady-state inflation can radically overturn determinacy results, undermine the learnability of rational expectations equilibria, affect the monetary policy transmission mechanism and change the nature of optimal policy. Moreover, these effects can differ across the two forms of nominal inertia (Ascari and Rossi, 2012) with the larger impact of trend inflation being felt under Calvo. The large costs of trend inflation under Calvo is also reflected in the analysis of Damjanovic and Nolan (2010b) where the seigniorage maximizing rate of inflation is at double digit levels under Rotemberg pricing, but only single digits under Calvo. In short there appears to be significant non-linearities in the New Keynesian model which are affected by the size of the steady-state distortion, the degree of unindexed inflation and the type of nominal inertia adopted. However, this evidence largely comes from studies which linearize such economies, either to a first or second order approximation, after allowing for such factors.

In this paper we solve the benchmark New Keynesian model non-linearly using the two standard approaches to modelling price stickiness. Since we are not imposing any kind of approximation around a steady-state we can see clearly the extent to which the two approaches differ. Moreover, rather than consider the Ramsey problem or commitment to a simple monetary rule, we shall consider time-consistent optimal policy (commonly known as discretion). This in turn, given that we are not using any artificial devices to ensure the model's steady-state is efficient, implies that we can measure the extent of the inflationary bias problem under the two forms of nominal inertia. This identifies the extent to which a policy maker who is constrained to be time-consistent would be unable to prevent a costly rise in steady-state or trend inflation. This is an important measure of the non-linearities across the two descriptions of pricing behavior, but also serves as a plausibility check on the relevance of the effects highlighted in the literature on trend inflation. The inflation bias thus measures the maximum level of unindexed inflation that a policy maker would be forced to tolerate - the policy maker which allowed inflation to rise above this level is behaving sub-optimally even given the constraint that they cannot commit. Therefore if the level of inflation bias is significantly below that required to generate the perverse results found in the trend-inflation literature then we would need to find a reason why policy makers are not only failing to commit, but are generating inflation levels well beyond the maximum inflation bias before we need worry about these properties of the New Keynesian model. While if the model implies a sizeable inflation bias then the issues raised by the trend inflation literature and, more generally, the non-linearities inherent in the New Keynesian model need to be taken more seriously.

There is also an empirical literature which focusses on these two distortions in helping to explain inflation dynamics. Ireland (2007) allows for time variation in the Fed's inflation target to explain the evolution of US inflation. Cogley and Sargent (2002) argue that much of the movement in US inflation reflects movements in an underlying trend, rather than in fluctuations relative to that trend. While several authors have sought to identify the level of trend inflation using generalizations of the new Keynesian Phillips curve which allow for time varying (unindexed) trend inflation. As an example of the findings of this literature, Cogley and Sbordone (2008) argue that trend inflation rather than any kind of backward-looking indexation behavior is a major component of observed movements in inflation. Again we can ask - can the benchmark model, using either Rotemberg or Calvo pricing plausibly deliver the size of unindexed steady state or trend inflation these papers infer to explain the data?

Moving away from the Ramsey description of policy is important as such a policy implies that the optimal rate of inflation the policy maker would commit to would be zero in the benchmark model employing either Calvo or Rotemberg pricing (Woodford, 2003) and in the case of Calvo contracts very close to zero in models with other distortions due to, for example, fiscal policy (Schmitt-Grohe and Uribe, 2004) or a desire to generate seigniorage revenues (Damjanovic and Nolan, 2011). Under Rotemberg, the example of Damjanovic and Nolan suggests that since the welfare costs of nominal inertia do not rise as sharply as the rate of inflation rises under Rotemberg, that this may not be a general result across the two descriptions of nominal inertia. Nevertheless, the fact remains that Ramsey policy would typically imply that inflation was far lower and stable than appears to be found in the data.[1]

[1] Chen et al. (2014) assess the relative empirical performance of a New Keynesian model with habits and inflation inertia with policy described by not only by simple rules, but also optimal policy under discretion, commitment and quasi-commitment. They find that discretion fits the data far better than

There are some recent papers using global solution techniques which also consider optimal discretionary policy in the benchmark model under Calvo contracts - see Van Zandweghe and Wolman (2011) and Anderson et al. (2010), which is then extended in Ngo (2014) to allow for discount factor shocks which imply that policy must account for the zero lower bound (ZLB).[2] Other authors also consider issues relating to the ZLB in models which use Rotemberg pricing, but also introduce extensions such as capital (see Gavin et al. (2013), Braun and Korber (2011), Johannsen (2014)), consumption habits (Gust et al. (2012) and Aruoba and Schorfheide (2013)), labor market frictions (Roulleau-Pasdeloup (2013)) or fiscal policy (Nakata (2013), Niemann et al. (2013) and Johannsen (2014)).[3] Solving non-linear representations of an enriched New Keynesian model is typically far more computationally intensive than conventional perturbation methods, and these latter authors have all adopted the Rotemberg description of price stickiness since this reduces the number of state variables one must consider. Furthermore, in calibrating the Rotemberg price adjustment cost parameter almost all these authors use a conventional parameterization which matches the slope of the linearized NKPC across the Rotemberg and Calvo variants of the New Keynesian model after assuming a zero inflation steady-state. In other words the literature is typically implicitly assuming that the equivalence of the two forms of nominal inertia is retained in non-linear solutions of the New Keynesian model where the steady-state is distorted and the rate of inflation will typically not be zero. To our knowledge, the current paper is the first to formally compare and contrast time-consistent optimal policy under the two forms of price-setting using global solution algorithms and therefore to assess how innocuous the choice of one form of price-setting over the other actually is.

We find that the inflationary bias problem is non-trivial under both descriptions of nominal inertia, but is much greater under Calvo. This is despite earlier results implying that the costs of inflation are much higher under Calvo than Rotemberg. This essentially arises because of the different average mark-up behavior under the two models. Under Calvo higher inflation causes those firms who are able to adjust prices in a particular period to raise that price in anticipation of not being able to readjust that price for a prolonged period despite the general rise in the price level. This leads to an increase in the average mark-up as inflation rises. In contrast, under Rotemberg all firms set the same price, period by period, but face adjustment costs in doing so. In discounting future profits they also discount future price adjustment costs. As a result in the face of higher inflation the firms postpone some of the required price adjustment due to this discounting effect, which serves to reduce the average markup. Accordingly, for a given degree of monopolistic competition which induces an inflation bias, this further raises (lowers) the markup under Calvo (Rotemberg) and thereby worsens (improves) the inflationary bias problem. This effect also tends to imply that the inflationary impact of a given cost-push shock is greater under Calvo pricing, ceteris paribus. While the presence of an additional state variable under Calvo price-setting, namely price dispersion, can also result in a hump-shaped response in output to cost-push shocks which would not be the

any other description of policy, especially commitment which is simply too effective in stabilising the economy to be consistent with the data.

[2]Fernandez-Villaverde et al. (2012), Wieland (2013) and Richter et al. (2013) also explore equilibrium dynamics around the ZLB in variants of the New Keynesian model which adopt Calvo price contracts, but which adopt a rule-based description of policy.

[3]Within this group, Shibayama and Sunakawa (2012), Nakata (2013), and Niemann et al. (2013) explore optimal policy in various New Keynesian models using Rotemberg pricing. The others utilise a rule-based description of policy.

case under either Rotemberg pricing or the benchmark linearized model. The fact that steady-state inflation would ceteris paribus, and using standard calibration approaches, be significantly higher under Calvo also has implications for the probability of hitting the ZLB such that studies adopting Rotemberg pricing are more likely to experience such episodes.

The rest of the paper is organized as follows. In section 2, we describe the basic model under both Calvo and Rotemberg pricing. In section 3, we formulate the optimal discretionary policy problem with Rotemberg and Calvo pricing, respectively. In section 4, we present numerical results. In section 5, we extend the analysis to allow for a tax-driven cost-push shock to assess policy trade-offs. We conclude in section 6.

2 The Model

This section describes the basic economic structure in our model.

2.1 Households

There are a continuum of households of size one. We shall assume full asset markets, such that, through risk sharing, they will face the same budget constraint and make the same consumption plans. As a result, at period 0 the typical household will seek to maximize the following objective function,

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \tag{1}$$

where $0 < \beta < 1$ denotes the discount factor, C_t and N_t are a consumption aggregate, and labour supply at period t, respectively.

The household purchases differentiated goods in a retail market and combines them into composite goods using a CES aggregator:

$$C_t = \left(\int_0^1 C_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}, \, \epsilon > 1 \tag{2}$$

where $C_t(j)$ is the demand for differentiated goods of type j. The elasticity of substitution between varieties ϵ_t can be assumed to be time varying if we wish to allow for cost-push or mark-up shocks, but here we hold it fixed.

The budget constraint at time t is given by

$$\int_0^1 P_t(j)C_t(j)dj + E_t \left\{ Q_{t,t+1} D_{t+1} \right\} = \Xi_t + D_t + W_t N_t - T_t \tag{3}$$

where $P_t(j)$ is the nominal price of type j goods, D_{t+1} is the nominal payoff of the nominal bonds portfolio held at the end of period t, Ξ is the representative household's share of profits in the imperfectly competitive firms, W are wages, and T are lump-sum taxes. $Q_{t,t+1}$ is the stochastic discount factor for one period ahead payoffs. The labor market is perfectly competitive and wages are fully flexible.

Households must first decide how to allocate a given level of expenditure across the various goods that are available. They do so by adjusting the share of a particular good in their consumption bundle to exploit any relative price differences—this minimizes the costs of consumption. The demand curve for each good j is,

$$C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} C_t \tag{4}$$

where the aggregate price level P_t is defined to be

$$P_t = \left(\int_0^1 P_t(j)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}. \tag{5}$$

The dynamic budget constraint at period t can therefore be rewritten as

$$P_t C_t + E_t \{Q_{t,t+1} D_{t+1}\} = \Xi_t + D_t + W_t N_t - T_t. \tag{6}$$

2.1.1 Households' problem

The household's decision problem can be dealt with in two stages. First, regardless of the level of C_t the household purchases the combination of individual goods that minimizes the cost of achieving this level of the composite good. Second, given the cost of achieving any given level of C_t, the household chooses C_t, D_{t+1} and N_t optimally. We have solved the first stage problem above. For tractability, we assume that (1) takes the specific form

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}\right). \tag{7}$$

where $\sigma > 0$ is a risk aversion parameter and $\varphi > 0$ is the inverse of the Frisch elasticity of labor supply.

We can then maximize utility subject to the budget constraint (6) to obtain the optimal allocation of consumption across time,

$$\beta \left(\frac{C_t}{C_{t+1}}\right)^{\sigma} \left(\frac{P_t}{P_{t+1}}\right) = Q_{t,t+1}.$$

Taking conditional expectations on both sides and rearranging gives

$$\beta R_t E_t \left\{\left(\frac{C_t}{C_{t+1}}\right)^{\sigma} \left(\frac{P_t}{P_{t+1}}\right)\right\} = 1, \tag{8}$$

where $R_t \equiv \frac{1}{E_t(Q_{t,t+1})}$ is the gross nominal return on a riskless one period bond paying off a unit of currency in $t+1$. This is the familiar consumption Euler equation which implies that consumers are attempting to smooth consumption over time such that the marginal utility of consumption is equal across periods (after allowing for tilting due to interest rates differing from the households' rate of time preference).

The second first order condition concerning labour supply decision is given by

$$\left(\frac{W_t}{P_t}\right) = N_t^{\varphi} C_t^{\sigma}. \tag{9}$$

2.2 Firms

Each firm produces a differentiated good j using a constant returns to scale production function:

$$Y_t(j) = A_t N_t(j) \tag{10}$$

where $Y_t(j)$ is the output of firm j, and $N_t(j)$ denotes the hours hired by the firm, A_t is an exogenous aggregate productivity shock at period t, and $a_t = \log(A_t)$ is time varying and stochastic[4].

Similar to the household's problem, we first consider the cost minimization problem of firm j,

$$\min_{\{N_t(j)\}} \left(\frac{W_t}{P_t}\right) N_t(j) \text{ s.t. } Y_t(j) \leq A_t N_t(j).$$

which implies

$$mc_t = \frac{W_t}{P_t A_t}, \tag{11}$$

where mc_t is the Lagrange multiplier and also the real marginal cost of production. Note that the real marginal cost described in (11) does not depend on the output level of an individual firm, so long as its production function exhibits constant returns to scale and prices of inputs (here labor) are fully flexible.

The demand curve the firm j faces is given by

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t,$$

where $Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}$.

The intermediate-good sector is monopolistically competitive and the intermediate good producer therefore has market power. In the following, we consider two alternative forms of price stickiness - firstly that due to Rotemberg (1982) and then that of Calvo (1983).

2.2.1 Rotemberg Pricing

The Rotemberg model assumes that a monopolistic firm faces a quadratic cost of adjusting nominal prices, which can be measured in terms of the final good and given by

$$\frac{\phi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 Y_t \tag{12}$$

where $\phi \geq 0$ measures the degree of nominal price rigidity. The adjustment cost, which accounts for the negative effects of price changes on the customer–firm relationship, increases in magnitude with the size of the price change and with the overall scale of economic activity Y_t.

The problem for firm j is then to maximize the discounted value of nominal profits,

$$\max_{\{P_t(j)\}_{t=0}^{\infty}} E_t \sum_{s=0}^{\infty} Q_{t,t+s} \Xi_{t+s}$$

[4]Typically, the logarithm of A_t is assumed to follow an $AR(1)$ process: $a_t = \rho_a a_{t-1} + e_{at}$, $0 \leq \rho_a < 1$ where technology shock e_{at} is an $i.i.d.$ random variable, which has a zero mean and a finite standard deviation σ_a.

where nominal profits are defined as

$$\Xi_t = P_t(j)Y_t(j) - mc_t Y_t(j) P_t - \frac{\phi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 Y_t P_t \tag{13}$$
$$= P_t(j)^{1-\epsilon} P_t^{\epsilon} Y_t - mc_t P_t(j)^{-\epsilon} P_t^{1+\epsilon} Y_t - \frac{\phi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 Y_t P_t.$$

Firms can change their price in each period, subject to the payment of the adjustment cost. Hence, all the firms face the same problem, and thus will choose the same price, and produce the same quantity. In other words, $P_t(j) = P_t$ and $Y_t(j) = Y_t$ for any j. Hence, the first-order condition for a symmetric equilibrium is

$$(1-\epsilon) + \epsilon mc_t - \phi\Pi_t\left(\Pi_t - 1\right) + \phi\beta E_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{Y_{t+1}}{Y_t}\Pi_{t+1}\left(\Pi_{t+1} - 1\right)\right] = 0. \tag{14}$$

This is the Rotemberg version of the non-linear Phillips curve that relates current inflation to future expected inflation and to the level of output.

2.2.2 Calvo Pricing

Each period, the firms that adjust their price are randomly selected, and a fraction $1-\theta$ of all firms adjust while the remaining θ fraction do not adjust. Those firms that do adjust their price at time t do so to maximize the expected discounted value of current and future profits. Profits at some future date $t+s$ are affected by the choice of price at time t only if the firm has not received another opportunity to adjust between t and $t+s$. The probability of this is θ^s.

The firm's pricing decision problem then involves picking $P_t(j)$ to maximize discounted nominal profits Using the demand curve for the firm's product, this objective function can be written as

$$E_t \sum_{s=0}^{\infty} \theta^s Q_{t,t+s}\left[P_t(j)\left(\frac{P_t(j)}{P_{t+s}}\right)^{-\epsilon} Y_{t+s} - mc_{t+s}\left(\frac{P_t(j)}{P_{t+s}}\right)^{-\epsilon} Y_{t+s}P_{t+s}\right].$$

where the discount factor $Q_{t,t+s}$ is given by $\beta^s\left(\frac{C_t}{C_{t+s}}\right)^{\sigma}\frac{P_t}{P_{t+s}}$, and mc_{t+s} is the marginal cost of production.

Let P_t^* be the optimal price chosen by all firms able to reset their price at time t. The first order condition for the optimal choice of P_t^* is,

$$\frac{P_t^*}{P_t} = \left(\frac{\epsilon}{\epsilon - 1}\right)\frac{K_t^p}{F_t^p} \tag{15}$$

where

$$K_t^p = C_t^{-\sigma} mc_t Y_t + \theta\beta E_t\left[\left(\frac{P_{t+1}}{P_t}\right)^{\epsilon} K_{t+1}^p\right]$$
$$F_t^p = C_t^{-\sigma} Y_t + \theta\beta E_t\left[\left(\frac{P_{t+1}}{P_t}\right)^{\epsilon-1} F_{t+1}^p\right].$$

The price index evolves according to

$$1 = (1-\theta)\left(\frac{P_t^*}{P_t}\right)^{1-\epsilon} + \theta(\Pi_t)^{\epsilon-1} \text{ with } \Pi_t \equiv \frac{P_t}{P_{t-1}} \,. \tag{16}$$

and price dispersion is described by

$$\Delta_t \equiv \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} dj = (1-\theta)\left(\frac{P_t^*}{P_t}\right)^{-\epsilon} + \theta\left(\frac{P_t}{P_{t-1}}\right)^{\epsilon}\Delta_{t-1}. \tag{17}$$

2.3 Aggregate Conditions

Under Rotemberg pricing, as all the firms will employ the same amount of labour, the aggregate production function is simply given by

$$Y_t = A_t N_t.$$

and the aggregate resource constraint is given by

$$Y_t = C_t + \frac{\phi}{2}\left(\Pi_t - 1\right)^2 Y_t.$$

Note that the Rotemberg adjustment cost creates an inefficiency wedge ψ_t^R between output and consumption

$$C_t = \left(1-\psi_t^R\right) Y_t = \left(1-\psi_t^R\right) A_t N_t \tag{18}$$

where $\psi_t^R = \frac{\phi}{2}\left(\Pi_t - 1\right)^2$.

In the case of Calvo pricing, firms changing prices in different periods will generally have different prices. Thus, the model features price dispersion. When firms have different relative prices, there are distortions that create a wedge between the aggregate output measured in terms of production factor inputs and aggregate demand measured in terms of the composite goods. Specifically,

$$N_t(j) = \frac{Y_t(j)}{A_t} = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon}\frac{Y_t}{A_t}$$

which yields,

$$N_t = \int_0^1 N_t(j)dj = \frac{Y_t}{A_t}\int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} dj = \frac{Y_t\Delta_t}{A_t}$$

after integrating across firms. $\Delta_t \geq 1$ implies that price dispersion is always costly in terms of aggregate output: the higher Δ_t, the more labour is needed to produce a given level of output. Moreover, under Calvo different firms with different prices will employ different amounts of labor. This explains why higher price dispersion acts as a negative productivity shift in the aggregate production function: $Y_t = (A_t/\Delta_t)N_t$. In addition, price dispersion is a backward-looking variable, and introduces an inertial component into the model.

Under Calvo, the aggregate resource constraint is simply given by

$$Y_t = C_t.$$

Hence, define $\psi_t^c = \Delta_t - 1$ as an inefficiency wedge under Calvo, then

$$C_t = Y_t = \frac{A_t N_t}{(1+\psi_t^c)} \tag{19}$$

Comparing (18) and (19), it is illuminating to note that the Rotemberg adjustment cost creates a wedge ψ_t^R between aggregate consumption and aggregate output, while the Calvo price dispersion creates a wedge ψ_t^c between aggregate hours and aggregate output. In addition, both wedges are non-linear functions of inflation, and they are minimized at one when steady-state inflation equals zero ($\Pi = 1$), and both wedges increase as trend inflation moves away from zero. See Ascari and Rossi (2012) for a discussion.

Appendix C.1 summarizes the models under Rotemberg and Calvo pricing.

3 Optimal Policy Problem Under Discretion

Under discretion, the monetary authority solves a sequential or period-by-period optimization problem, which maximizes the representative household's expected discounted utility subject to the optimality conditions from market participants, the aggregate conditions, and the law of motion for the state variables. Therefore, under optimal discretion, the policymaker cannot commit to a plan in the hope of influencing economic agents' expectations.

3.1 Rotemberg Pricing

Let $V(A_t)$ represents the value function at period t in the Bellman equation for the optimal policy problem. The optimal monetary policy then solves the following optimization problem:

$$V(A_t) = \max_{\{C_t, Y_t, \Pi_t\}} \left\{ \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{(Y_t/A_t)^{1+\varphi}}{1+\varphi} + \beta E_t \left[V(A_{t+1}) \right] \right\} \tag{20}$$

subject to,

$$C_t = \left[1 - \frac{\phi}{2} (\Pi_t - 1)^2 \right] Y_t \tag{21}$$

and,

$$(1-\epsilon) + \epsilon Y_t^{\varphi} C_t^{\sigma} A_t^{-\varphi-1} - \phi \Pi_t (\Pi_t - 1) + \phi \beta E_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \frac{Y_{t+1}}{Y_t} \Pi_{t+1} (\Pi_{t+1} - 1) \right] = 0 \tag{22}$$

Defining an auxilliary function,

$$M(A_{t+1}) \equiv C_{t+1}^{-\sigma} Y_{t+1} \Pi_{t+1} (\Pi_{t+1} - 1)$$

we can rewrite the Phillips curve (22) as,

$$(1-\epsilon) + \epsilon Y_t^{\varphi} C_t^{\sigma} A_t^{-\varphi-1} - \phi \Pi_t (\Pi_t - 1) + \phi \beta C_t^{\sigma} Y_t^{-1} E_t \left[M(A_{t+1}) \right] = 0$$

which captures the fact that the policy maker recognizes that any change in the state variable will affect expectations, but cannot promise to behave in a particular way tomorrow in order to influence expectations today. The optimal policy problem can then be formulated as the following Lagrangian,

$$\begin{aligned} \mathcal{L} = & \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{(Y_t/A_t)^{1+\varphi}}{1+\varphi} + \beta E_t \left[V(A_{t+1}) \right] + \lambda_{1t} \left\{ \left[1 - \frac{\phi}{2} (\Pi_t - 1)^2 \right] Y_t - C_t \right\} \\ & + \lambda_{2t} \left\{ (1-\epsilon) + \epsilon Y_t^{\varphi} C_t^{\sigma} A_t^{-\varphi-1} - \phi \Pi_t (\Pi_t - 1) + \phi \beta C_t^{\sigma} Y_t^{-1} E_t \left[M(A_{t+1}) \right] \right\} \end{aligned}$$

where λ_{1t} and λ_{2t} are the Lagrange multipliers. The first order conditions and complementary slackness conditions are given as follows,

$$C_t^{-\sigma} = \lambda_{1t} - \lambda_{2t}\left\{\sigma\epsilon Y_t^{\varphi} C_t^{\sigma-1} A_t^{-\varphi-1} + \sigma\phi\beta C_t^{\sigma-1} Y_t^{-1} E_t\left[M(A_{t+1})\right]\right\},$$

$$Y_t^{\varphi} A_t^{-1-\varphi} = \lambda_{1t}\left[1 - \frac{\phi}{2}\left(\Pi_t - 1\right)^2\right] + \lambda_{2t}\left\{\epsilon\varphi Y_t^{\varphi-1} C_t^{\sigma} A_t^{-\varphi-1} - \phi\beta C_t^{\sigma} Y_t^{-2} E_t\left[M(A_{t+1})\right]\right\},$$

$$\lambda_{1t}\phi\left(1 - \Pi_t\right) Y_t = \lambda_{2t}\phi\left(2\Pi_t - 1\right),$$

$$C_t = \left[1 - \frac{\phi}{2}\left(\Pi_t - 1\right)^2\right] Y_t,$$

$$0 = (1 - \epsilon) + \epsilon Y_t^{\varphi} C_t^{\sigma} A_t^{-\varphi-1} - \phi\Pi_t\left(\Pi_t - 1\right) + \phi\beta C_t^{\sigma} Y_t^{-1} E_t\left[M(A_{t+1})\right].$$

Note that consumption Euler equation is non-binding from the point of view of maximizing utility, because R_t (a variable of no direct interest in utility) can effectively be chosen to achieve the desired level of consumption.

The fully nonlinear problem is then to find five policy functions which relate the three choice variables $\{Y_t, C_t, \Pi_t\}$ and two Lagrange multipliers $\{\lambda_{1t}, \lambda_{2t}\}$ to the state variable A_t, that is, $Y_t = Y(A_t)$, $C_t = C(A_t)$, $\Pi_t = \Pi(A_t)$, $\lambda_{1t} = \lambda_1(A_t)$, and $\lambda_{2t} = \lambda_2(A_t)$. We will use the Chebyshev collocation method to approximate these five time invariant rules.

3.2 Calvo Pricing

Let $V(\Delta_{t-1}, A_t)$ denote the value function at period t in the Bellman equation for the optimal policy problem. The optimal monetary policy under discretion then can be described as a set of decision rules for $\{C_t, Y_t, \Pi_t, \frac{P_t^*}{P_t}, K_t^p, F_t^p, \Delta_t\}$ which maximize,

$$V(\Delta_{t-1}, A_t) = \max\left\{\frac{C_t^{1-\sigma} - 1}{1 - \sigma} - \frac{(\Delta_t Y_t / A_t)^{1+\varphi}}{1 + \varphi} + \beta E_t\left[V(\Delta_t, A_{t+1})\right]\right\}$$

subject to the following constraints,

Resource constraint:

$$Y_t = C_t$$

Phillips curve:

$$\frac{P_t^*}{P_t} = \left(\frac{\epsilon}{\epsilon - 1}\right)\frac{K_t^p}{F_t^p}$$

with

$$K_t^p = (\Delta_t Y_t)^{\varphi} A_t^{-\varphi-1} Y_t + \theta\beta E_t\left[M(\Delta_t, A_{t+1})\right]$$
$$F_t^p = Y_t C_t^{-\sigma} + \theta\beta E_t\left[L(\Delta_t, A_{t+1})\right],$$

where we have utilized two auxilliary functions,

$$M(\Delta_t, A_{t+1}) = \left(\Pi_{t+1}\right)^{\epsilon} K_{t+1}^p$$

and

$$L(\Delta_t, A_{t+1}) = \left(\Pi_{t+1}\right)^{\epsilon-1} F_{t+1}^p,$$

which highlights the fact that the policy maker recognizes that any change in the state variable will affect expections, but cannot promise to behave in a particular way tomorrow in order to influence expectations today. Inflation:

$$1 = (1 - \theta)\left(\frac{P_t^*}{P_t}\right)^{1-\epsilon} + \theta(\Pi_t)^{\epsilon-1}$$

Price dispersion:

$$\Delta_t = (1-\theta)\left(\frac{P_t^*}{P_t}\right)^{-\epsilon} + \theta\left(\Pi_t\right)^{\epsilon}\Delta_{t-1}.$$

As before, the policy problem can be written in Lagrangian form as follows:

$$\begin{aligned}
\mathcal{L} = & \frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{(\Delta_t Y_t/A_t)^{1+\varphi}}{1+\varphi} + \beta E_t\left[V(\Delta_t, A_{t+1})\right] \\
& + \lambda_{1t}[Y_t - C_t] \\
& + \lambda_{2t}\left[\frac{P_t^*}{P_t} - \left(\frac{\epsilon}{\epsilon-1}\right)\frac{K_t^p}{F_t^p}\right] \\
& + \lambda_{3t}\left\{K_t^p - (\Delta_t Y_t)^{\varphi} A_t{}^{-\varphi-1} Y_t - \theta\beta E_t\left[M(\Delta_t, A_{t+1})\right]\right\} \\
& + \lambda_{4t}\left\{F_t^p - Y_t C_t^{-\sigma} - \theta\beta E_t\left[L(\Delta_t, A_{t+1})\right]\right\} \\
& + \lambda_{5t}\left[1 - (1-\theta)\left(\frac{P_t^*}{P_t}\right)^{1-\epsilon} - \theta(\Pi_t)^{\epsilon-1}\right] \\
& + \lambda_{6t}\left[\Delta_t - (1-\theta)\left(\frac{P_t^*}{P_t}\right)^{-\epsilon} - \theta\left(\Pi_t\right)^{\epsilon}\Delta_{t-1}\right]
\end{aligned}$$

where λ_{jt} $(j = 1, .., 6)$ are the Lagrange multipliers. The first order conditions are given as follows: for consumption,

$$C_t^{-\sigma} - \lambda_{1t} + \sigma Y_t C_t^{-\sigma-1}\lambda_{4t} = 0$$

output,

$$-(\Delta_t/A_t)^{1+\varphi} Y_t^{\varphi} + \lambda_{1t} - (1+\varphi)(\Delta_t Y_t)^{\varphi} A_t{}^{-\varphi-1}\lambda_{3t} - C_t^{-\sigma}\lambda_{4t} = 0$$

optimal price,

$$\lambda_{2t} + (1-\theta)(\epsilon-1)\left(\frac{P_t^*}{P_t}\right)^{-\epsilon}\lambda_{5t} + \epsilon(1-\theta)\left(\frac{P_t^*}{P_t}\right)^{-\epsilon-1}\lambda_{6t} = 0$$

inflation,

$$-(\epsilon-1)\theta\lambda_{5t} - \epsilon\theta\Delta_{t-1}\Pi_t\lambda_{6t} = 0$$

numerator of optimal price K_t^p,

$$-\left(\frac{\epsilon}{\epsilon-1}\right)\frac{1}{F_t^p}\lambda_{2t} + \lambda_{3t} = 0$$

denominator of optimal price F_t^p,

$$\left(\frac{\epsilon}{\epsilon-1}\right)\frac{K_t^p}{(F_t^p)^2}\lambda_{2t} + \lambda_{4t} = 0$$

and price dispersion,

$$\begin{aligned}
0 = & -(Y_t/A_t)^{1+\varphi}\Delta_t^{\varphi} + \beta\frac{\partial E_t\left[V(\Delta_t, A_{t+1})\right]}{\partial\Delta_t} - \varphi(\Delta_t)^{\varphi-1}A_t{}^{-\varphi-1}Y_t^{\varphi+1}\lambda_{3t} \\
& - \theta\beta\frac{\partial E_t\left[M(\Delta_t, A_{t+1})\right]}{\partial\Delta_t}\lambda_{3t} - \theta\beta\frac{\partial E_t\left[L(\Delta_t, A_{t+1})\right]}{\partial\Delta_t}\lambda_{4t} + \lambda_{6t}
\end{aligned}$$

Note that the envelope theorem yields

$$\frac{\partial V(\Delta_{t-1}, A_t)}{\partial \Delta_{t-1}} = -\theta \left(\Pi_t\right)^{\epsilon} \lambda_{6t}$$

which allows us to rewrite the first order condition for price dispersion as,

$$\begin{aligned} 0 = &-(Y_t/A_t)^{1+\varphi}(\Delta_t)^{\varphi} - \theta\beta E_t\left[(\Pi_{t+1})^{\epsilon}\lambda_{6t+1}\right] - \varphi(\Delta_t)^{\varphi-1}A_t{}^{-\varphi-1}Y_t^{\varphi+1}\lambda_{3t} \\ &- \theta\beta\frac{\partial E_t\left[M(\Delta_t, A_{t+1})\right]}{\partial\Delta_t}\lambda_{3t} - \theta\beta\frac{\partial E_t\left[L(\Delta_t, A_{t+1})\right]}{\partial\Delta_t}\lambda_{4t} + \lambda_{6t} \end{aligned}$$

We can solve the nonlinear system consisting of these seven first order conditions and the six constraints to yield the time-consistent optimal policy under Calvo pricing. Specifically, without commitment, we need to find these thirteen time-invariant policy rules which are functions of the two state variables $\{\Delta_{t-1},\ A_t\}$. That is, we need to find policy functions such as $F_t^P = F^P(\Delta_{t-1}, A_t)$, $K_t^P = K^P(\Delta_{t-1}, A_t)$, and $\Pi_t = \Pi(\Delta_{t-1}, A_t)$. Similar to the Rotemberg case, the Chebyshev collocation method will be used to approximate these policy functions.

4 Numerical Analysis

4.1 Solution Method

We use the Chebyshev collocation method to globally approximate the policy functions.[5] In contrast to the linear-quadratic approximation method, this projection method can capture the extent to which the two approaches to modelling price stickiness differ, due to the non-linearities inherent in the New Keynesian models. First, we discretize the state space into a set of collocation nodes. In the Rotemberg model, there is one state variable (A_t), while in the Calvo model there are two state variables (Δ_{t-1}, A_t). Accordingly, the space of the approximating functions for the Rotemberg pricing consists of one-dimensional Chebyshev polynomials. In comparison, the space of approximating functions for the Calvo pricing is two-dimensional, and is, given by the tensor products of two sets of Chebyshev polynomials. Then we define the residual functions based on the equilibrium conditions. Gaussian-Hermite quadrature is used to approximate expectation terms. Under Calvo pricing, the partial derivatives with respect to price dispersion, are approximated by differentiating the Chebyshev polynomials. Finally, we solve the resultant system of nonlinear equations consisting of the residual functions evaluated over all the collocation nodes[6]. See appendix C.2 for details.

4.2 Numerical Results

4.2.1 Benchmark Parameters and Solution Accuracy

The benchmark parameters for Calvo pricing are taken from Anderson et al. (2010) and are standard. We conduct a sensitivity analysis below. To make the results from

[5] Judd (1992) and Judd (1998) are good references.

[6] We also tried the time iteration method. That is, a smaller system of nonlinear equations, composed of the residual functions evaluated at each collocation node, is solved repeatedly. For the benchmark case in this paper, both methods find identical solutions. However, the time iteration method will be used for other cases since it is generally faster and more robust.

Rotemberg pricing comparable, the value of price adjustment cost is assigned so that the linear quadratic approximation for both cases are equivalent[7]. This implies an equivalence between the two forms of pricing provided the steady-state is undistorted with a rate of inflation of zero. Such an approach is typically adopted in the literature even where authors are considering models where these conditions are not met. Table 1 summarizes the relevant parameter values.

With this benchmark parameterization, we solve the fully nonlinear models via Chebyshev collocation method. Following Anderson et al. (2010), the relative price dispersion Δ_t is bounded by $[1, 1.02]$, and the logged productivity a_t takes values from $[-2\sigma_a/(1-\rho_a), 2\sigma_a/(1-\rho_a)] = [-0.4, 0.4]$. For the Rotemberg case, the order of approximation n_a is chosen to be 6, and the number of nodes for Gauss-Hermite quadrature $q = 12$. This combination is quite accurate, since the maximum Euler equation error is on the order of 10^{-8}. For the Calvo case, the order of approximation n_a and n_Δ are both assigned to be 6, and $q = 12$ for Gauss-Hermite quadrature. The maximum Euler equation error over the full range is on the order of 10^{-7}. As suggested by Judd (1998), this order of accuracy is reasonable.

4.2.2 Steady State Inflation Bias

Figure 1 illustrates the solution of the discretionary equilibria for the Calvo case. Similar to the results in Anderson et al. (2010), the red dotted line plots the value of Δ_t as a function of Δ_{t-1} in a narrow interval of $[1, 1.02]$. The steady state relative price dispersion is about 1.0026 which is the intersection point between the red line and the 45-degree solid line. At this fixed point, the value of optimal gross inflation $\overline{\Pi}$ (the dashed line) is about 1.0054, implying an annualized inflation rate of 2.2%. In contrast, the discretionary inflation rate for the Rotemberg case is 1.0047 or 1.89% per year. It is well known that the optimal rate of inflation under commitment is zero, hence the inflation bias is equal to the optimal rate of inflation under discretion. Therefore, the inflation bias problem under Calvo pricing is more severe than that under Rotemberg pricing for the benchmark parameters. We now turn to discuss this result, as well as undertaking a sensitivity analysis.

To explore this difference further, we change the value of the monopolistic competition distortion defined by $\epsilon/(\epsilon-1)$ by varying ϵ and assessing its effect on the equilibrium inflation bias. We interchangeable describe this measure of the monopolistic competition distortion as the flexible-price markup since it measures the markup that would be observed under flexible prices. This approach is based on the fact that the size of the inflation bias depends on the degree of monopolistic distortion, which makes steady state (even flexible-price) output inefficient and hence higher inflation is attractive. Figures 2 and 3 present how the size of inflation bias changes as the markup is varied for the Calvo and Rotemberg pricing, respectively. The benchmark $\epsilon = 11$ yields a felxible-price markup of 1.1. When ϵ decreases, the corresponding monopolistic competition distortion and inflation bias increases. To illustrate the impact of the monopoly distortion on the non-linearity, the inflation bias for both cases under the linear-quadratic approximation (LQ) are also presented. The traditional linear-quadratic method becomes increasingly inaccurate for larger distortions.

[7] That is, $\phi = \frac{(\epsilon-1)\theta}{(1-\theta)(1-\beta\theta)}$.

Finally, we do some comparative statics with the model under both pricing approaches, in order to explore how other parameters affect the severity of the inflation bias problem and the sensitivity of the results obtained from the linear-quadratic approach. Table 2 and Table 3 summarize the robustness outcomes for the Calvo and Rotemberg pricing, respectively. In general, the inflation bias problem is much worse under Calvo pricing.

4.3 Discussion

4.3.1 The Average Markup and Inflation Bias

We find that the inflationary bias problem is significantly greater under Calvo, especially as the monopolistic competition distortion is increased. At the same time consumption falls by more, and hours worked by less under Calvo as we increase this distortion, and the average markup rises above the flexible price markup under Calvo, while decreases under Rotemberg as a result of the non-linear effects of the inflation bias. See Figures 2 and 3.

In understanding the results it is helpful to consider the effects of inflation on the two models. Ascari and Rossi (2012) discuss how inflation affects both models through a 'wedge' effect as well as an average markup effect. We shall consider the wedge effect first, before turning to the average markup effects, which will turn out to be key. Under both forms of nominal inertia the 'wedge' implies that the representative household's aggregate consumption will be lower for a given level of labour input as inflation rises. Under Calvo this is because the dispersion of prices means that they need to consume relatively more of the cheaper goods to compensate for the expensive goods given diminishing marginal utility in the consumption of each good. As Damjanovic and Nolan (2010a) note this is akin to a negative productivity shock, where we can combine the resource and aggregate production function to yield,

$$C_t = \frac{A_t}{(1+\psi_t^c)} N_t$$

where the inefficient wedge under Calvo, $\psi_t^c = \Delta_t - 1$, captures the extent to which price dispersion has been raised above one.

Under Rotemberg the micro-foundations of the wedge is different - adjusting prices uses up consumption goods directly. However, we can similarly combine the aggregate production function and resource constraint to obtain a similar expression under Rotemberg,

$$C_t = A_t(1 - \psi_t^R) N_t$$

where the Rotemberg wedge, $\psi_t^R = \frac{\phi}{2}(\Pi_t - 1)^2$ reflects the costs per unit of output of changing prices. Therefore in both cases the labour costs of attaining a particular level of aggregate consumption are higher, ceteris paribus, as inflation rises.

In order to assess how this affects the inflation bias problem facing the policy maker it is helpful to imagine how a social planner would respond to the existence of such wedges were he to imagine them to be exogenously given in the manner of a technology shock. Given the form of household utility, the social planner would choose an optimal level of labour input of

$$N_t^{\sigma+\varphi} = \left(\frac{A_t}{(1+\psi_t^c)}\right)^{1-\sigma}$$

under Calvo, and

$$N_t^{\sigma+\varphi} = \left(A_t(1-\psi_t^R)\right)^{1-\sigma}$$

under Rotemberg. Therefore, for our benchmark calibration of $\sigma = 1$ the social planner would not seek to adjust the labour input into the production process as a result of increases in either of the wedges, but would simply allow consumption to fall. In other words, for our benchmark calibration the efficiencies implied by these wedges do not give the policy maker a further desire to generate a surprise inflation, ceteris paribus. While if $\sigma > 1$ the social planner would seek to reduce the labour input as either of these inefficiency wedges increased. That is, in this case the wedges would reduce the desire to encourage firms to employ more workers ceteris paribus. We can see this from Tables 2 and 3 where raising the inverse of the intertemporal elasticity of substitution, σ, reduces the inflation bias under both pricing models. Therefore the different inefficiency wedges under Calvo and Rotemberg are not responsible for the observed inflation biases.

Instead the differences in inflation bias across the two models are generated by their average mark-up behavior, which is fundamentally different. Consider the steady-state of the average markup (equal to the inverse of real marginal cost) under Rotemberg which is obtained by rearranging the deterministic steady state of the new Keynesian Phillips curve (NKPC) under Rotemberg as,

$$mc^{-1} = \left[\frac{\epsilon - 1}{\epsilon} + \frac{(1-\beta)}{\epsilon} \phi (\Pi - 1)\Pi \right]^{-1}$$

The second term in square brackets exists as a combination of steady-state inflation and discounting on the part of firms (on behalf of their owners, the representative household). Essentially as the firm discounts future profits they also discount future price adjustment costs. As a result in the face of ongoing inflation, they will opt to partially delay the required price adjustment such that the average mark-up is decreasing in inflation.

The effect of inflation on the average mark-up under Calvo is,

$$mc^{-1} = \frac{\epsilon}{\epsilon - 1} \left(\frac{1 - \theta\beta\Pi^{\epsilon-1}}{1 - \theta\beta\Pi^{\epsilon}} \right) \left(\frac{1 - \theta\Pi^{\epsilon-1}}{1-\theta} \right)^{\frac{1}{\epsilon-1}}$$

In this case the effects of inflation on the average markup are ambiguous. However, following King and Wolman (1996) this can be decomposed into two elements - the marginal markup,

$$\frac{P^*}{MC} = \frac{\epsilon}{\epsilon - 1} \left(\frac{1 - \theta\beta\Pi^{\epsilon-1}}{1 - \theta\beta\Pi^{\epsilon}} \right)$$

and the price adjustment gap,

$$\frac{P}{P^*} = \left(\frac{1 - \theta\Pi^{\epsilon-1}}{1-\theta} \right)^{\frac{1}{\epsilon-1}}.$$

Here we can see that higher inflation raises the marginal markup. Firms facing the possibility of being stuck with the current price for a prolonged period will tend to raise their reset price when that price is likely to be eroded by inflation throughout the life of that contract. The effect of inflation on the price adjustment gap will tend to reduce this element of the average markup. However, except at very low rates of inflation, the effects of inflation on the average markup through the marginal mark-up effect are positive.

Therefore we would expect to see average markups rise with inflation under Calvo, but fall under Rotemberg. This, in turn, implies that the inflationary bias problem is worsened under Calvo as the rising markups increase the policy makers incentives to introduce a surprise inflation ceteris paribus, at the same time as it is mitigated under Rotemberg. As a result the inflation bias problem is significantly higher under Calvo where consumption falls by more and hours by less than it does under Rotemberg.

4.3.2 Sensitivity Analysis

Tables 2 and 3 consider the robustness of our results across various parameters for Calvo and Rotemberg pricing, respectively. The first three rows of each Table increase the degree of nominal inertia (where the Rotemberg price adjustment parameter is adjusted in line with the changes in the Calvo parameter such that the linearized NKPC is equivalent across both forms of nominal inertia). As we increase the degree of nominal inertia, we find that the inflation bias rises under Calvo, but falls under Rotemberg. This is for the reasons discussed above. Under Calvo greater price stickiness means that firms are likely to be stuck with their current price for longer, meaning that they aggressively raise prices when given the opportunity to do so. This will tend to raise average markups and worsen the inflationary bias problem. In contrast under Rotemberg, higher price adjustment costs result in firms wishing to delay price adjustment which reduces average markups and reduces the inflation bias problem.

The next piece of sensitivity analysis looks at various parameterization of the inverse of the intertemporal elasticity of substitution, σ. As noted above, at the benchmark value of $\sigma = 1$, the social planner would not wish to expand employment as either of the efficiency wedges due to the two forms of nominal inertia increase. While if $\sigma < (>) 1$ then they would wish to increase (decrease) the labour input as either efficient wedge increased. Therefore we see the inflationary bias falling as σ increases across both forms of nominal inertia. Finally, we consider an increase in the inverse of the Frisch elasticity of labour supply, φ, which serves to reduce the inflationary bias problem across both types of price stickiness. As labour supply becomes less elastic there is less desire to use costly inflation surprises to achieve only marginal increases in the level of output and the inflation bias falls.

4.3.3 Relevance of Results

In order to assess the implications of our calculated levels of inflation bias under the Rotemberg and Calvo forms of nominal inertia, we contrast our inflation rates with both the empirical estimates of trend inflation and the critical values of trend inflation at which the standard model develops non-standard properties.

Empirical Estimates of Trend Inflation Cogley and Sargent (2002)'s estimates of trend inflation in a Bayesian VAR with time varying coefficient suggests that a large part of the movements in inflation in the post-war period (its rise in the 1970s to its fall in the 1980s) was due to the evolution of trend inflation rather than fluctuations around that trend. Similarly, Cogley and Sbordone (2008) find that there is no inertia in price-setting behavior due to indexation-type behavior, but that the inertia in the data can be described by the evolution of trend-inflation in a generalized NKPC. While Ireland (2007) finds that changes in the Fed's inflation target can help explain inflation dynamics, where that target rose from 1.25% in 1959 to over 8% in the late 1970s, before falling back to 2.5% in 2004. Therefore, to the extent that observed inflation reflects movements in an underlying trend it would suggest that the empirical measures of trend inflation could easily be consistent with our measures of the inflationary bias without having to resort to implausibly high monopolistic competition distortions. Moreover, when we augment the model with an estimated process for mark-up shocks the magnitude of the resultant inflation volatility can easily account for the observed volatility of inflation around its time-varying trend.

Trend Inflation and Determinacy In order to further assess the implications of our calculated levels of inflation bias under the Rotemberg and Calvo forms of nominal inertia, we contrast our inflation rates with the key values of trend inflation at which the standard model develops issues with the determinacy of standard interest rate rules. We could, of course, have looked at other features highlighted in the trend inflation literature such as the learnability of the model as trend inflation rises or its impulse responses to monetary policy shocks and so on, but since the bifurcation in determinacy conditions reflects a common underlying non-linearity which drives all the phenomena in the trend inflation literature we choose to focus on this as a straight-forward way of assessing whether or not our calculations suggest the concerns raised by the trend inflation literature are significant or not.

Accordingly, we follow Ascari and Rossi (2012) and linearize our two economies around a deterministic steady-state with an arbitrary rate of steady-state inflation (details of the linearized models are provided in appendix C.4). We then assume a standard parameterization of a Taylor rule for monetary policy, $R_t = 1.5\pi_t + 0.5y_t$, and for a range of values of the monopolistic competition distortion/flexible price mark-up, $\epsilon/(\epsilon - 1)$, we compute the steady-state rate of inflation at which the standard Taylor rule flips from being determinate to being indeterminate. We then plot this determinacy frontier in distortion-inflation space along side our inflation bias estimates, see Figure 4. We find that at low levels of the monopolistic competition distortion the inflationary bias number lies below the determinacy frontier - in other words the standard Taylor rule would remain determinate at the rates of inflation implied by our inflationary bias calculations. However, as the markup is increased the inflation bias estimates cross the determinacy frontier implying that at the rates of inflation implied by the inflation bias estimates a standard Taylor rule would be indeterminate in a log-linearized representation of the model. This is particularly true in the case of Calvo where a flexible price markup of just over 11%, which is well within the range of standard parameterization in the literature. In contrast, under Rotemberg the flexible-price markup needs to be double that push us beyond the determinacy frontier.

5 The Effects of Cost-push Shocks

In the analysis above we have focussed on the stochastic steady state of the non-linear policy problem to reveal the extent of the inflation bias. However, the response to shocks can also be markedly different across the two forms of nominal inertia. In order to explore the effect of cost-push shocks[8] on policy trade-offs under discretion in our fully nonlinear model, we, adopt the estimated shock process from Chen et al. (2014) which is modelled as a revenue tax rate fluctuating around a steady state value of zero,

$$\ln(1 - \tau_{pt}) = (1 - \rho^{\tau_p}) \ln(1 - \tau_p) + \rho^{\tau_p} \ln(1 - \tau_{pt-1}) - e_{\tau t}$$

[8]The technology shocks already present in our model do not create meaningful policy trade-offs under our benchmark calibration largely resulting in offsetting interest rate movements regardless of the form of nominal inertia.

where $e_{\tau t} \sim N(0, 0.00486^2)$ and $\rho^{\tau_p} = 0.939$. In a log-linearized model this is equivalent to allowing for fluctuations in a desired mark-up through variations in ϵ. However, in our non-linear model allowing ϵ to be time varying has a direct impact on the measure of price dispersion in a way which would not normally be considered to be an inherent part of a cost push shock. Therefore we focus on variations in a revenue tax a means of generating an autocorrelated cost push which is consistent with the data. The complete model with the time-varying revenue tax rate is presented in appendix C.5.

We present two sets of results. In the first we consider the impact of an inflationary cost push shock with our benchmark parameterization, but with $\theta = 0.625$, and $\phi = 57.3684$. These respective measure of price stickiness imply an identical steady-state rate of inflation of 1.95%. Figure 5 reveals that even at this relatively modest degree of inflation bias, there are non trivial differences in the impulse responses to an identical cost push shock. These are driven by the same economic mechanisms observed in the steady state analysis above as average markups rise under Calvo exacerbating the effects of the cost push shock.

It should be noted that the conventional way of parameterizing the Rotemberg price adjustment cost parameter such that the slopes of the linearized Phillips curves are identical would have implied a far lower value of $\phi = 43.7$. In fact given the significant differences in the inflation bias across the two forms of price-stickiness it is generally not possible to calibrate the Rotemberg parameter by seeking to mimic the steady-state rate of inflation observed under Calvo, ceteris paribus. Therefore, in a second exercise we ensure a common steady-state rate of inflation of 2.54% by adopting the following set of parameters, $\epsilon = 11$, $\theta = 0.8$ under Calvo, and $\epsilon = 9.7076$, $\phi = 57.3684$ under Rotemberg. This calibration ensures that both forms of nominal inertia generate identical steady-state rates of inflation and levels of output. Despite sharing a steady state in these dimensions, the response to the identical shock is markedly different across Calvo and Rotemberg. In Figure 6 we can see that inflation is 0.2% higher on impact from an identical cost push shock under Calvo, while other variables, particularly output and consumption, exhibit a hump-shaped response to the shock due to the gradual evolution of price dispersion, which is not a feature of the response to the shock under Rotemberg pricing.

6 Conclusion

In this paper we have contrasted the properties of the Calvo and Rotemberg forms of nominal inertia which are commonly used in New Keynesian analyses of macroeconomic policy. They are often treated as being interchangeable, largely because they generate equivalent NKPC and policy implications when linearized around an efficient zero-inflation steady state. However, our non-linear solution of the discretionary policy problem reveals some striking differences across the two models of price stickiness.

Firstly, the inflation bias problem is far greater under Calvo pricing than Rotemberg pricing, despite the fact that the costs of inflation are significantly higher under the former. The reason for this is that inflation raises the average markup under Calvo pricing as firms seek to raise their prices more aggressively whenever they can to avoid the erosion of their relative price due to inflation. This increase in average markups worsens the inflationary bias problem. In contrast, under Rotemberg pricing firms can adjust prices in every period, and will moderate their average markups as inflation rises as they attempt to delay some of the costs of price adjustment due to the discounting inherent in their objective function.

Secondly, for empirically reasonable levels of monopoly power the inflation bias that emerges from both models implies that the rates of inflation identified as being 'trend' inflation in empirical studies are reasonable. Moreover, the rates of inflation implied by the model are sufficient for the non-linearities inherent in the model to place the economy in the zone where the effects of trend inflation are found, in studies which approximate the economy around a non-zero rate of steady-state inflation, to have profound implications for the determinacy properties of rules, the learnability of the rational expectations equilibirum and the transmission of monetary policy. That is, the degree of inflation bias generated by the model implies that the non-linearities inherent in the model and the choice of form of nominal inertia matter.

Thirdly, we extended the model to consider the stabilization of the economy in the face of mark-up shocks. Here we find that the non-linearities that generate radically different degrees of inflation bias in the steady state also imply profound differences in the monetary policy response to the same shock both across models, with the inflation response to a cost-push shock being significantly greater under Calvo, while possibly also being associated with a hump-shaped output/consumption response as a result of the evolution of price dispersion which is absent from the Rotemberg model and typically ignored in the linearized New Keynesian model.

References

Anderson, G. S., J. Kim, and T. Yun (2010). Using a projection method to analyze inflation bias in a micro-founded model. *Journal of Economic Dynamics and Control 34*(9), 1572–1581.

Aruoba, S. B. and F. Schorfheide (2013). Macroeconomic dynamics near the ZLB: A tale of two equilibria. *NBER Working Paper No. 19248*.

Ascari, G. and L. Rossi (2012). Trend inflation and firms price-setting: Rotemberg versus Calvo. *The Economic Journal 122*(563), 1115–1141.

Ascari, G. and A. M. Sbordone (2013). The macroeconomics of trend inflation. *FRB of New York Staff Report No. 628*.

Braun, R. A. and L. M. Korber (2011). New Keynesian dynamics in a low interest rate environment. *Journal of Economic Dynamics and Control 35*(12), 2213–2227.

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of monetary Economics 12*(3), 383–398.

Chen, X., T. Kirsanova, and C. Leith (2014). How optimal is US monetary policy? *University of Glasgow Discussion Paper Series*.

Cogley, T. and T. J. Sargent (2002). Evolving post-World War II us inflation dynamics. *NBER Macroeconomics Annual 2001 16*, 331–388.

Cogley, T. and A. M. Sbordone (2008). Trend inflation, indexation, and inflation persistence in the New Keynesian Phillips Curve. *American Economic Review 98*(5), 2101–26.

Damjanovic, T. and C. Nolan (2010a). Relative price distortions and inflation persistence. *The Economic Journal 120*(547), 1080–1099.

Damjanovic, T. and C. Nolan (2010b). Seigniorage-maximizing inflation under sticky prices. *Journal of Money, Credit and Banking 42*(2-3), 503–519.

Damjanovic, T. and C. Nolan (2011). Second-order approximation to the Rotemberg model around a distorted steady state. *Economics Letters 110*(2), 132–135.

Fernandez-Villaverde, J., G. Gordon, P. A. Guerron-Quintana, and J. Rubio-Ramirez (2012). Nonlinear adventures at the zero lower bound. *NBER Working Paper No. 18058*.

Gavin, W. T., B. D. Keen, A. Richter, and N. Throckmorton (2013). Global dynamics at the zero lower bound. *Federal Reserve Bank of St. Louis Working Paper Series, 2013-007A*.

Gust, C., D. Lopez-Salido, and M. E. Smith (2012). The empirical implications of the interest-rate lower bound. *Federal Reserve Board Finance and Economics Discussion Series, 2012-83*.

Ireland, P. N. (2007). Changes in the federal reserve's inflation target: causes and consequences. *Journal of Money, Credit and Banking 39*(8), 1851–1882.

Johannsen, B. K. (2014). When are the effects of fiscal policy uncertainty large? *Federal Reserve Board Finance and Economics Discussion Series, 2014-40*.

Judd, K. L. (1992). Projection methods for solving aggregate growth models. *Journal of Economic Theory 58*(2), 410–452.

Judd, K. L. (1998). *Numerical methods in economics*. MIT press.

King, R. G. and A. L. Wolman (1996). Inflation targeting in a St. Louis model of the 21st century. *Federal Reserve Bank of St. Louis Review 78*(May/June 1996).

Lombardo, G. and D. Vestin (2008). Welfare implications of Calvo vs. Rotemberg-pricing assumptions. *Economics Letters 100*(2), 275–279.

Nakata, T. (2013). Optimal fiscal and monetary policy with occasionally binding zero bound constraints. *Federal Reserve Board Finance and Economics Discussion Series, 2013-40*.

Ngo, P. V. (2014). Optimal discretionary monetary policy in a micro-founded model with a zero lower bound on nominal interest rate. *Journal of Economic Dynamics and Control, forthcoming*.

Niemann, S., P. Pichler, and G. Sorger (2013). Public debt, discretionary policy, and inflation persistence. *Journal of Economic Dynamics and Control 37*(6), 1097–1109.

Nistico, S. (2007). The welfare loss from unstable inflation. *Economics Letters 96*(1), 51–57.

Rotemberg, J. J. (1982). Sticky prices in the United States. *Journal of Political Economy 90*(6), 1187–1211.

Roulleau-Pasdeloup, J. (2013). Why is the government spending multiplier larger at the zero lower bound? not (only) because of the zero lower bound. *Working paper*.

Schmitt-Grohe, S. and M. Uribe (2004). Optimal fiscal and monetary policy under sticky prices. *Journal of Economic Theory 114*(2), 198–230.

Shibayama, K. and T. Sunakawa (2012). Inflation bias and stabilization bias in Rotemberg pricing. *Mimeo, University of Kent*.

Van Zandweghe, W. and A. Wolman (2011). Discretionary monetary policy in the Calvo model. *Federal Reserve Bank of Richmond Working Paper Series, WP11-03*.

Walsh, C. E. (2003). *Monetary Theory and Policy*. The MIT Press.

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press.

Yun, T. (1996). Nominal price rigidity, money supply endogeneity, and business cycles. *Journal of monetary Economics 37*(2), 345–370.

A Tables

Table 1: Parameterization

Parameter	Value	Definition
β	0.99	Quarterly discount factor
σ	1	Relative risk aversion coefficient
φ	1	Inverse Frisch elasticity of labor supply
ϵ	11	Elasticity of substitution between varieties
θ	0.75	Probability of fixing prices in each quarter
ρ_a	0.95	AR-coefficient of technology shock
σ_a	0.01	Standard deviation of technology shock
ϕ	116	Rotemberg adjustment cost

Table 2: Sensitivity analysis for Calvo pricing

Parameter	Values				Numerical results		
	θ	σ	φ	ϵ	Price dispersion	Nonlinear solution	LQ solution
θ	0.05	1	1	11	1.0000	1.71	1.66
	0.3	1	1	11	1.0001	1.78	1.66
	0.5	1	1	11	1.0003	1.84	1.66
	0.75	1	1	11	1.0026	2.18	1.65
	0.85	1	1	11	1.0275	3.01	1.64
	0.90	1	1	11	1.0726	2.55	1.60
σ	0.75	0.3	1	11	1.0308	5.64	2.54
	0.75	1	1	11	1.0026	2.18	1.65
	0.75	5	1	11	1.0002	0.60	0.56
φ	0.75	1	0.36	11	1.0139	4.31	2.42
	0.75	1	1	11	1.0026	2.18	1.65
	0.75	1	4.75	11	1.0002	0.60	0.56

Note: the last two columns contain the annualized inflation rate in percentage solved by the projection method and the LQ method, respectively.

Table 3: Sensitivity analysis for Rotemberg pricing

Parameter	Values				Annualized Inflation rate (%)	
	θ	σ	φ	ϵ	Nonlinear solution	LQ solution
θ	0.05	1	1	1	1.94	1.83
	0.3	1	1	1	1.94	1.83
	0.5	1	1	11	1.93	1.83
	0.75	1	1	11	1.90	1.82
	0.85	1	1	11	1.83	1.80
	0.90	1	1	11	1.72	1.76
σ	0.75	0.3	1	11	2.95	2.54
	0.75	1	1	11	1.90	1.82
	0.75	5	1	11	0.64	0.56
φ	0.75	1	0.36	11	2.83	2.42
	0.75	1	1	11	1.90	1.82
	0.75	1	4.75	11	0.64	0.56

Note: for comparison, θ, which affects ϕ, is included.

B Figures

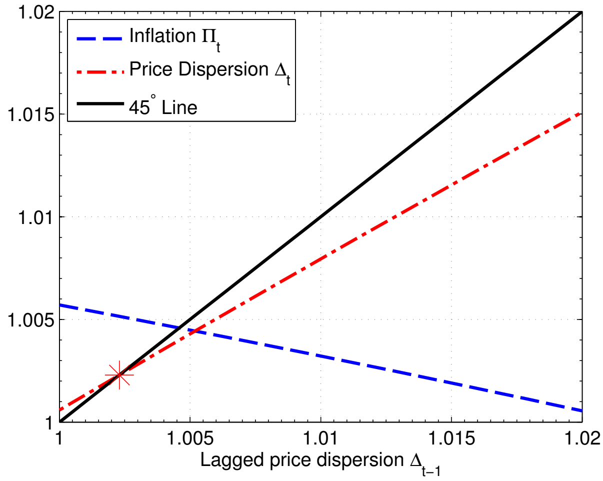

Figure 1: Relative price dispersion and inflation as functions of lagged relative price dispersion. This figure shows how the relative price dispersion converges to its steady state.

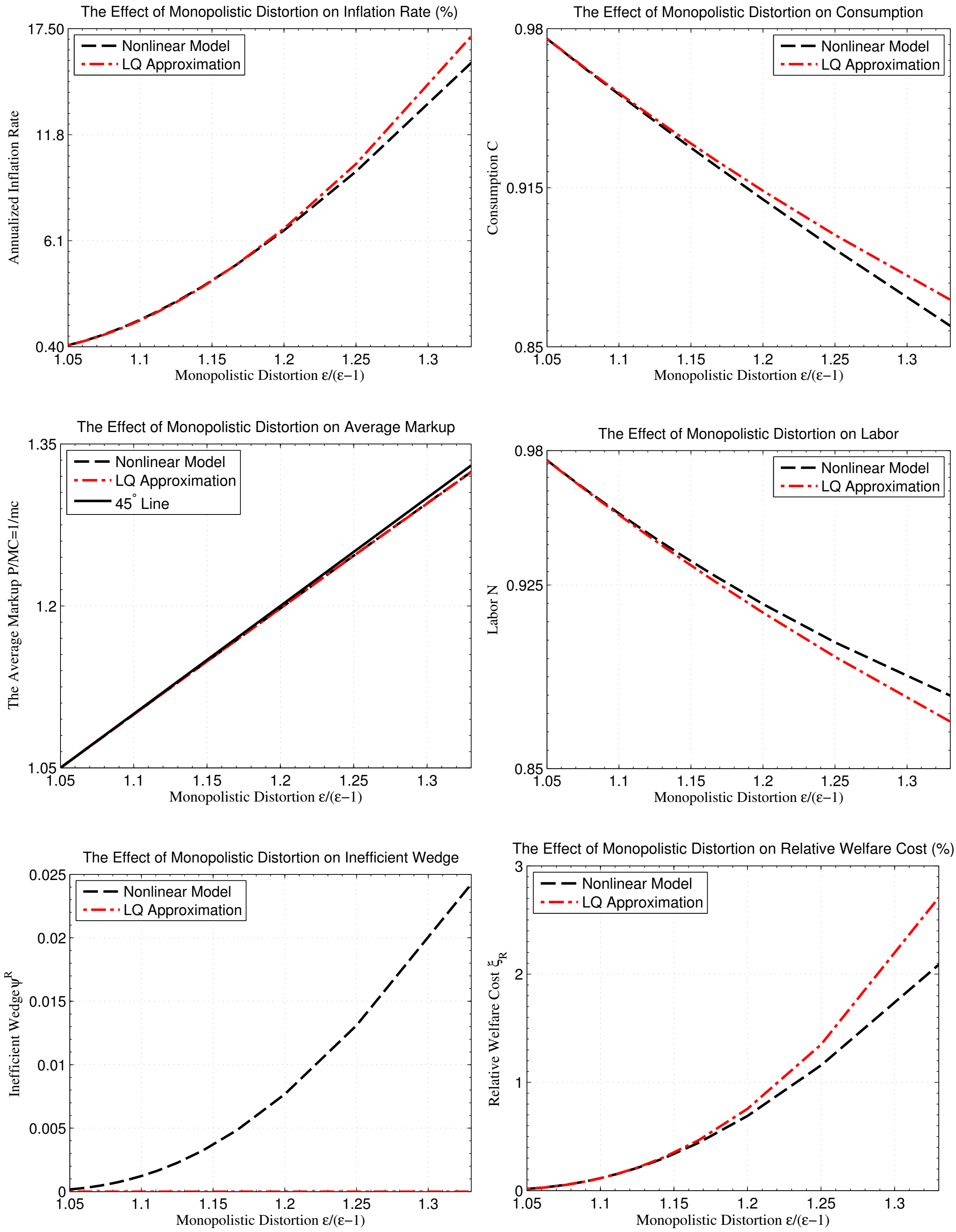

Figure 2: This figure shows the effect of monopolistic distortion under Rotemberg pricing. The monopolistic distortion is measured by markup at the deterministic steady state with zero inflation rate. The results from LQ and projection method are compared.

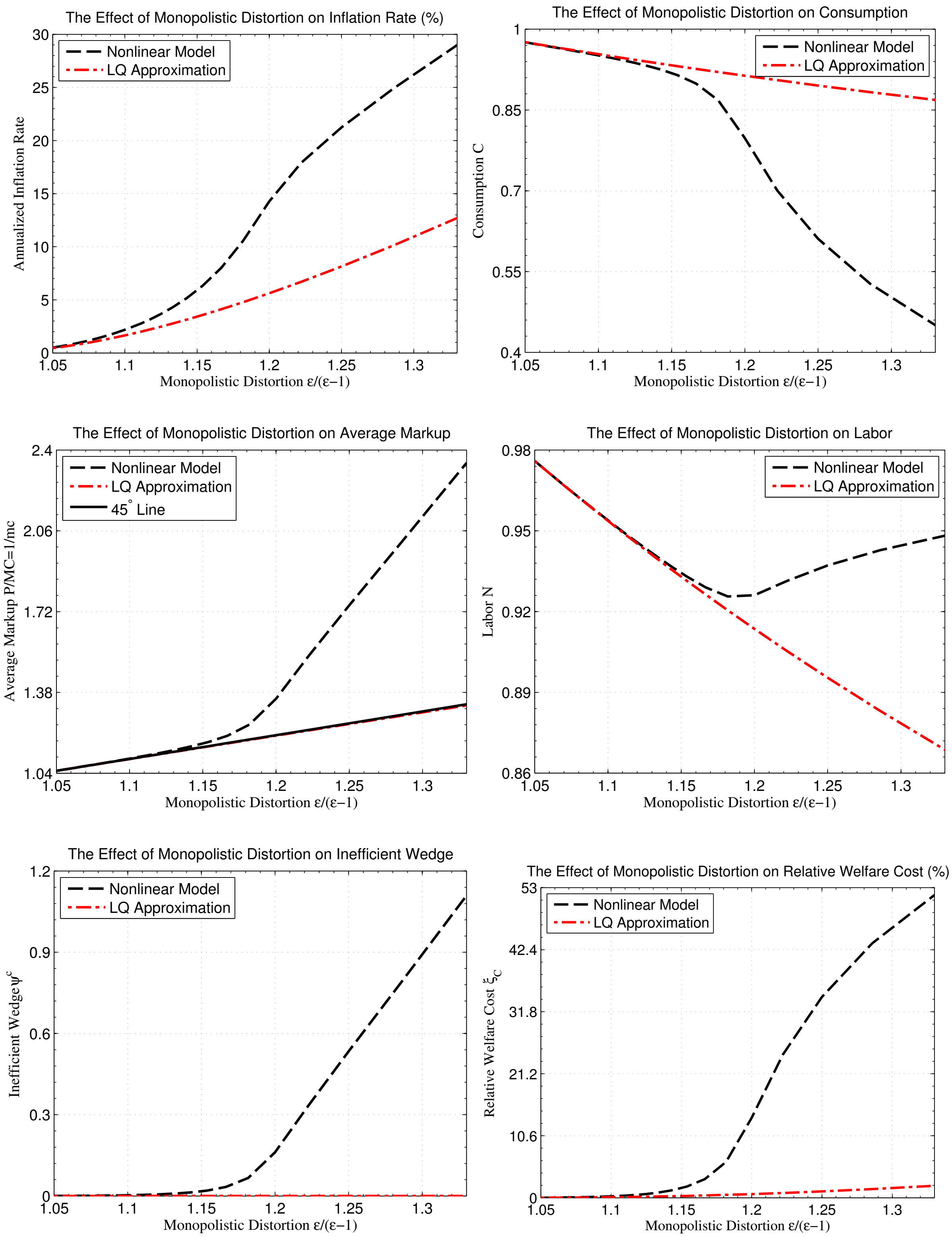

Figure 3: This figure shows the effect of monopolistic distortion under Calvo pricing. The monopolistic distortion is measured by markup at the deterministic steady state with zero inflation rate. The results from LQ and projection method are compared.

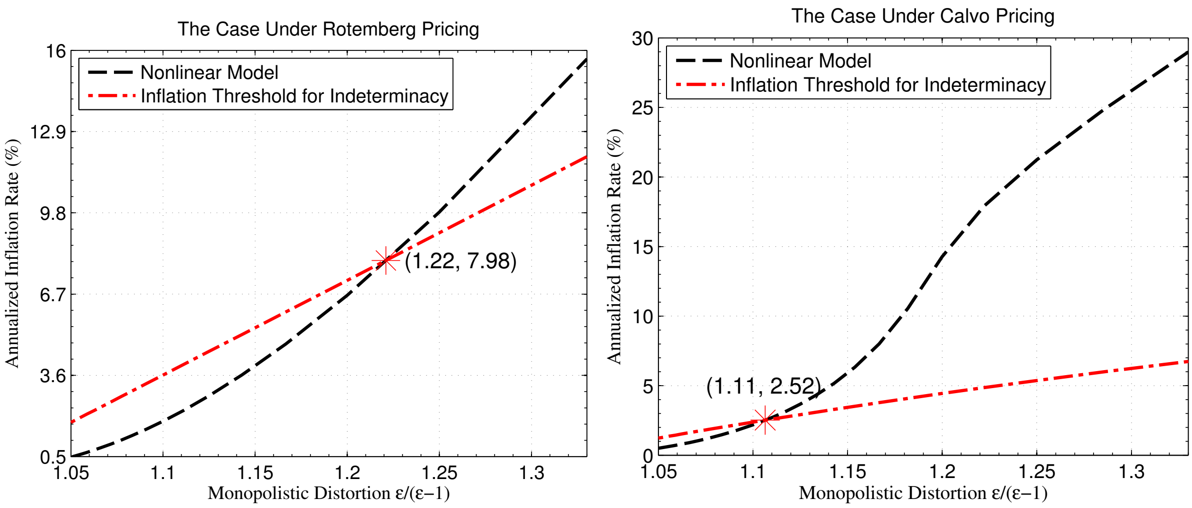

Figure 4: The threshold of inflation rate for indeterminacy versus the inflation bias from the nonlinear method

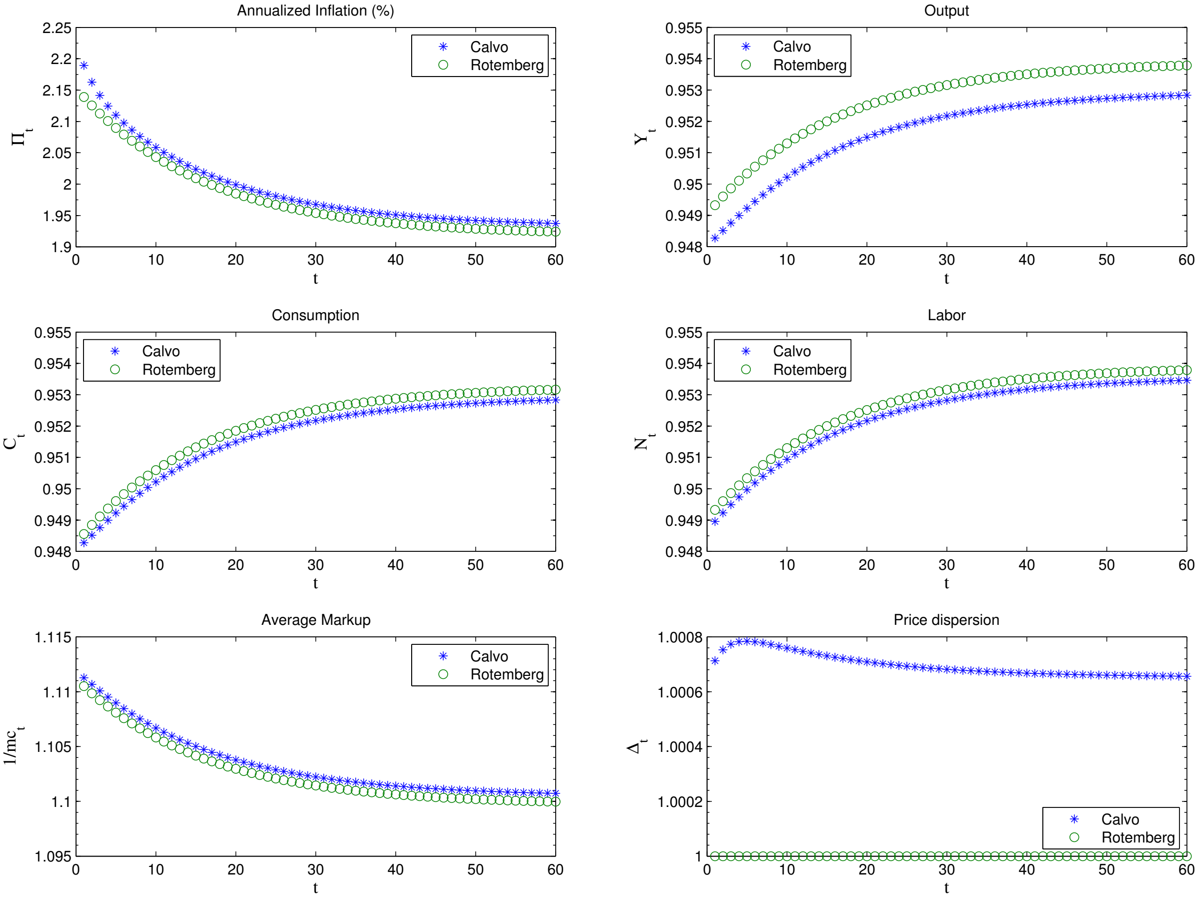

Figure 5: The impulse response functions to one percent positive tax-driven cost-push shock under Calvo and Rotemberg pricing. Note that the two cases are calibrated so that the steady state inflation rate is equal.

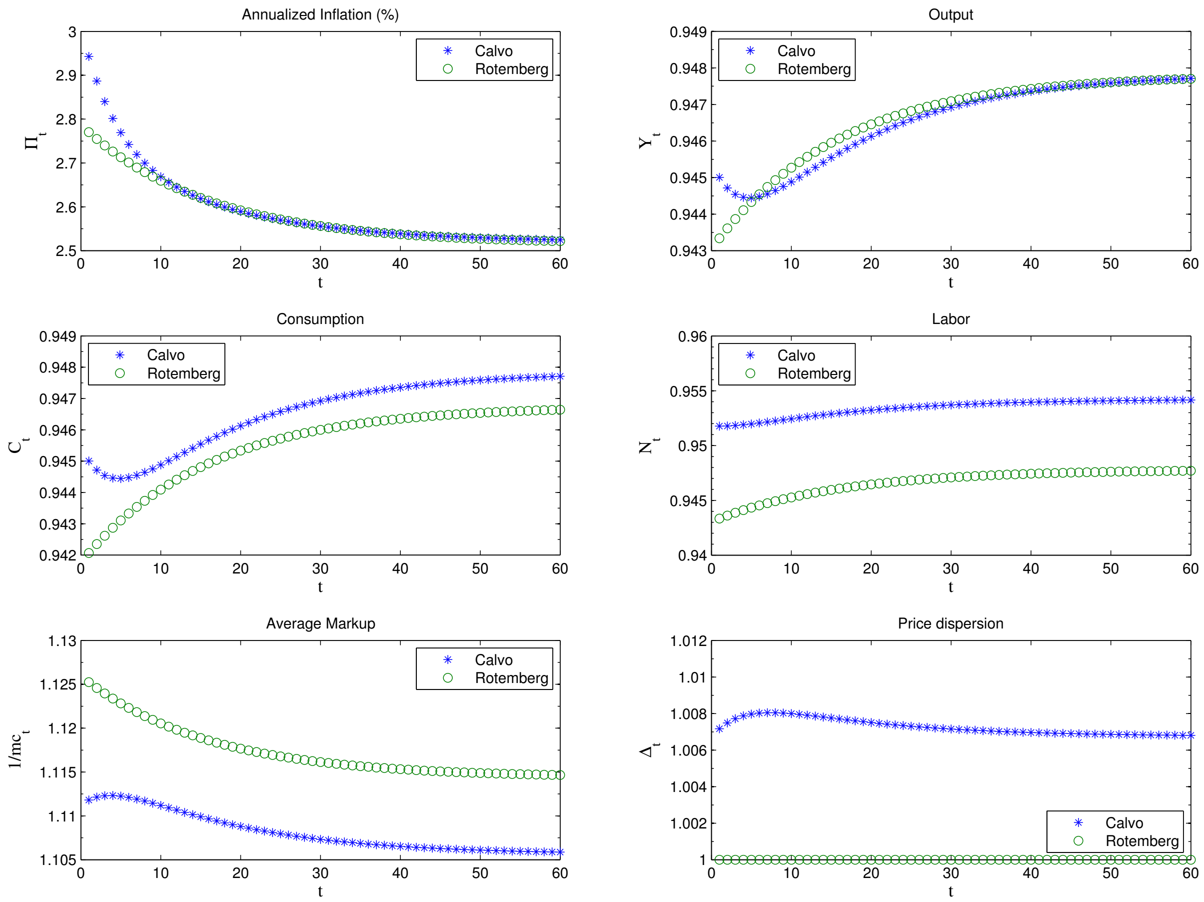

Figure 6: The impulse response functions to one percent positive tax-driven cost-push shock under Calvo and Rotemberg pricing. Note that each model is calibrated so that the steady state output and inflation rates are equal across both cases.

C Technical Appendix (Not for Publication)

C.1 Summary of Models

C.1.1 Rotemberg Pricing

The equilibrium conditions are given as follows:

Consumption Euler equation:

$$\beta R_t E_t \left\{ \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right\} = 1$$

Labor supply:

$$\left(\frac{W_t}{P_t} \right) = N_t^{\varphi} C_t^{\sigma}$$

Resource constraint:

$$\left[1 - \frac{\phi}{2} \left(\Pi_t - 1 \right)^2 \right] Y_t = C_t$$

Phillips curve:

$$(1 - \epsilon) + \epsilon mc_t - \phi \Pi_t \left(\Pi_t - 1 \right) + \phi \beta E_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \frac{Y_{t+1}}{Y_t} \Pi_{t+1} \left(\Pi_{t+1} - 1 \right) \right] = 0$$

Technology:

$$Y_t = A_t N_t$$

Marginal costs:

$$mc_t = \frac{W_t}{P_t A_t} = \frac{N_t^{\varphi} C_t^{\sigma}}{A_t} = \frac{(Y_t / A_t)^{\varphi} C_t^{\sigma}}{A_t} = Y_t^{\varphi} C_t^{\sigma} A_t^{-\varphi - 1}$$

We can simplify these equilibrium conditions by eliminating the interest rate and labour supply from the constraints, so that consumption can be considered as the monetary policy instrument. Specifically,

Resource constraint:

$$\left[1 - \frac{\phi}{2} \left(\Pi_t - 1 \right)^2 \right] Y_t = C_t$$

Phillips curve:

$$(1 - \epsilon) + \epsilon Y_t^{\varphi} C_t^{\sigma} A_t^{-\varphi - 1} - \phi \Pi_t \left(\Pi_t - 1 \right) + \phi \beta E_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \frac{Y_{t+1}}{Y_t} \Pi_{t+1} \left(\Pi_{t+1} - 1 \right) \right] = 0$$

while the objective function is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1 - \sigma} - \frac{(Y_t / A_t)^{1+\varphi}}{1 + \varphi} \right)$$

Note that the state variables are productivity (and any other exogenous shock processes we choose to add).

C.1.2 Calvo Pricing

The equilibrium conditions are given below:

Consumption Euler equation:

$$\beta R_t E_t \left\{ \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right\} = 1$$

Labor supply:

$$\left(\frac{W_t}{P_t} \right) = N_t^{\varphi} C_t^{\sigma}$$

Resource constraint:

$$Y_t = C_t = \frac{A_t N_t}{\Delta_t}$$

Phillips curve:

$$\frac{P_t^*}{P_t} = \left(\frac{\epsilon}{\epsilon - 1} \right) \frac{K_t^p}{F_t^p}$$

Inflation:

$$1 = (1-\theta) \left(\frac{P_t^*}{P_t} \right)^{1-\epsilon} + \theta (\Pi_t)^{\epsilon - 1}$$

Price dispersion:

$$\begin{aligned} \Delta_t &= (1-\theta) \left(\frac{P_t^*}{P_t} \right)^{-\epsilon} + \theta \left(\frac{P_t}{P_{t-1}} \right)^{\epsilon} \Delta_{t-1} \\ &= (1-\theta) \left(\frac{P_t^*}{P_t} \right)^{-\epsilon} + \theta \left(\Pi_t \right)^{\epsilon} \Delta_{t-1} \end{aligned}$$

Marginal costs:

$$mc_t = \frac{W_t}{P_t A_t} = \frac{N_t^{\varphi} C_t^{\sigma}}{A_t} = (Y_t \Delta_t)^{\varphi} C_t^{\sigma} A_t{}^{-\varphi - 1}$$

Note that the state variables are not just productivity, but also price dispersion.

C.2 The Chebyshev Collocation Method

C.2.1 Algorithm for Rotemberg Pricing

In the following, let s_t denote the state of the economy at time t. There are five functional equations associated with five endogenous variables $\{C_t, Y_t, \Pi_t, \lambda_{1t}, \lambda_{1t}\}$.

The state is $s_t = a_t \equiv \ln A_t$, which evolves according to the following motion equation:

$$a_{t+1} = \rho_a a_t + e_{at}$$

where $0 \leq \rho_a < 1$ and technology innovation e_{at} is an *i.i.d.* normal random variable, which has a zero mean and a finite standard deviation σ_a.

Let's define a new function $X : \mathbb{R} \to \mathbb{R}^5$, in order to collect the policy functions of endogenous variables as follows:

$$X(s_t) = (C_t(s_t), Y_t(s_t), \Pi_t(s_t), \lambda_{1t}(s_t), \lambda_{2t}(s_t))$$

Given the specification of the function X, the equilibrium conditions can be written more compactly as,

$$\Gamma(s_t, X(s_t), E_t[Z(X(s_{t+1}))]) = 0$$

where $\Gamma : \mathbb{R}^{1+5+1} \to \mathbb{R}^5$ summarizes the full set of dynamic equilibrium relationship, and $Z(X(s_{t+1})) = M(A_{t+1})$. Then the problem is to find a vector-valued function X that Γ maps to the zero function. Projection methods, hence, can be used.

The Chebyshev collocation method which we use to approximate policy functions under Rotemberg pricing can be described as follows:

1. Choose an order of approximation n_a, compute the $n_a + 1$ roots of the Chebychev polynomial of order $n_a + 1$ as

$$z_a^i = \cos\left(\frac{(2i-1)\pi}{2(n_a+1)}\right)$$

   for $i = 1, 2, ..., n_a + 1$, and formulate initial guesses for θ_y and θ_Π.

2. Compute collocation points

$$a_i = \frac{\overline{a} + \underline{a}}{2} + \frac{\overline{a} - \underline{a}}{2} z_a^i = \frac{\overline{a} - \underline{a}}{2}\left(z_a^i + 1\right) + \underline{a}$$

   for $i = 1, 2, ..., n_a + 1$, where $a = \log(A)$ is logged technology shock. Note that the number of collocation nodes is $n_a + 1$.

3. Formulate the approximating policy functions. Let $T_i(z) = \cos(i \cos^{-1}(z))$ denote the Chebyshev polynomial of order i, $z \in [-1, 1]$, and let ξ denote a linear function mapping the domain of $x \in [\underline{x}, \overline{x}]$ into $[-1, 1]$. In this way, $T_i(\xi(x))$ are Chebyshev polynomials adapted to $x \in [\underline{x}, \overline{x}]$ for $i = 0, 1, ...$ Apparently, $\xi(x) = 2(x - \underline{x}) / (\overline{x} - \underline{x}) - 1$. Then, a degree n_a Chebyshev polynomial approximation for the five decision rules at each nodes a_i can be written as in vector form

$$X(a_i) = T(\xi(a_i))\Theta_X$$

   where $\Theta_X = [\theta_y, \theta_c, \theta_\pi, \theta_{\lambda_1}, \theta_{\lambda_2}]$ is a $(1 + n_a) \times 5$ matrix comprised of the Chebyshev collocation coefficients, and $T(\xi(a_i))$ is a $1 \times (1 + n_a)$ matrix of the Chebyshev polynomials evaluated at node a_i.

4. At each collocation point a_i, calculate the values of the five residual functions defined by the five equilibrium conditions as follows: assuming a Gaussian distribution for the shock $e_{at} \sim N(0, \sigma_a^2)$, To compute the integral part, we make the following change of variables: $z = e_a/\sqrt{2\sigma_a^2} \sim N(0, 1/2)$, then

$$\begin{aligned}
& E_t[M(s_{t+1})] \\
&= \frac{1}{\sigma_a\sqrt{2\pi}} \int_{-\infty}^{+\infty} C_{t+1}^{-\sigma} Y_{t+1} \Pi_{t+1}(\Pi_{t+1} - 1) \exp\left(-\frac{e_{at+1}^2}{2\sigma_a^2}\right) de_{at+1} \\
&= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} C_{t+1}^{-\sigma} Y_{t+1} \Pi_{t+1}(\Pi_{t+1} - 1) \exp\left(-z^2\right) dz
\end{aligned}$$

where we employ a Gauss-Hermite quadrature to approximate the integral. We compute the nodes z_j and weights ω_j for the quadrature such that

$$\begin{aligned}
&\frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty} C_{t+1}^{-\sigma} Y_{t+1}\Pi_{t+1}\left(\Pi_{t+1}-1\right)\exp\left(-z^2\right)dz \\
&\approx \frac{1}{\sqrt{\pi}}\sum_{j=1}^{q}\left[\begin{array}{c} \omega_j\widehat{C}\left(\rho_a a_i + z_j\sqrt{2\sigma_a^2};\theta_y,\theta_\pi\right)^{\sigma}\widehat{Y}\left(\rho_a a_i + z_j\sqrt{2\sigma_a^2};\theta_y\right)\times \\ \widehat{\Pi}\left(\rho_a a_i + z_j\sqrt{2\sigma_a^2};\theta_\pi\right)\left(\widehat{\Pi}\left(\rho_a a_i + z_j\sqrt{2\sigma_a^2};\theta_\pi\right)-1\right)\end{array}\right] \\
&\equiv \Psi\left(a_i;\theta_y,\theta_\pi,q\right)
\end{aligned}$$

for $i = 1, 2, ..., n_a + 1$.

Then, the residual functions can be written as

$$R_1 = \widehat{C}(a_i;\theta_c)^{-\sigma} - \widehat{\lambda}_1(a_i;\theta_{\lambda_1}) + \widehat{\lambda}_2(a_i;\theta_{\lambda_2})\left[\begin{array}{c}\sigma\epsilon\widehat{Y}(a_i;\theta_y)^{\varphi}\widehat{C}(a_i;\theta_c)^{\sigma-1}\exp\left(a_i\right)^{-\varphi-1} \\ +\sigma\phi\beta\widehat{C}(a_i;\theta_c)^{\sigma-1}\widehat{Y}(a_i;\theta_y)^{-1}\Psi\left(a_i;\theta_y,\theta_\pi,q\right)\end{array}\right]$$

$$\begin{aligned}
R_2 &= \widehat{Y}(a_i;\theta_y)^{\varphi}\exp\left(a_i\right)^{-\varphi-1} - \left[1-\frac{\phi}{2}\left(\widehat{\Pi}(a_i;\theta_\pi)-1\right)^2\right]\widehat{\lambda}_1(a_i;\theta_{\lambda_1}) \\
&\quad - \widehat{\lambda}_2(a_i;\theta_{\lambda_2})\left[\begin{array}{c}\epsilon\varphi\widehat{Y}(a_i;\theta_y)^{\varphi-1}\widehat{C}(a_i;\theta_c)^{\sigma}\exp\left(a_i\right)^{-\varphi-1} \\ -\phi\beta\widehat{C}(a_i;\theta_c)^{\sigma}\widehat{Y}(a_i;\theta_y)^{-2}\Psi\left(a_i;\theta_y,\theta_\pi,q\right)\end{array}\right] \\
R_3 &= \widehat{\lambda}_1(a_i;\theta_{\lambda_1})\left(1-\widehat{\Pi}(a_i;\theta_\pi)\right)\widehat{Y}(a_i;\theta_y) - \widehat{\lambda}_2(a_i;\theta_{\lambda_2})\left(2\widehat{\Pi}(a_i;\theta_\pi)-1\right) \\
R_4 &= \widehat{C}(a_i;\theta_c) - \left[1-\frac{\phi}{2}\left(\widehat{\Pi}(a_i;\theta_\pi)-1\right)^2\right]\widehat{Y}(a_i;\theta_y)
\end{aligned}$$

$$\begin{aligned}
R_5 &= (1-\epsilon) + \epsilon\widehat{Y}(a_i;\theta_y)^{\varphi}\widehat{C}(a_i;\theta_c)^{\sigma}\exp\left(a_i\right)^{-\varphi-1} \\
&\quad - \phi\widehat{\Pi}(a_i;\theta_\pi)\left(\widehat{\Pi}(a_i)-1\right) + \phi\beta\widehat{C}(a_i;\theta_c)^{\sigma}\widehat{Y}(a_i;\theta_y)^{-1}\Psi\left(a_i;\theta_y,\theta_\pi,q\right)
\end{aligned}$$

where the hat symbol indicates the corresponding approximate policy functions.

5. If all residuals are close enough to zero then stop, else update $\{\theta_y, \theta_c, \theta_\pi, \theta_{\lambda_1}, \theta_{\lambda_2}\}$, and go back to step 3.

The last step uses Christopher A. Sims' *csolve*[9] to solve the system of nonlinear equations, $R_j = 0$ for $j = 1, ..., 5$. When implementing the above algorithm, we first use lower order Chebyshev polynomials where steady states can be good initial guesses. Then, we increase the order of approximation and take as starting value the solution from the previous lower order approximation. This informal homotopy continuation method follows the advice from Anderson et al. (2010).

[9] The solver can be found at `http://dge.repec.org/codes/sims/optimize/csolve.m`.

C.2.2 Algorithm for Calvo Pricing

Now the state space is $s_t = (\Delta_{t-1}, A_t)$, where price dispersion Δ_{t-1} is endogenous and technology A_t is exogenous and respectively, with the following law of motion:

$$\Delta_t = (1-\theta)\left(\frac{P_t^*}{P_t}\right)^{-\epsilon} + \theta\left(\Pi_t\right)^{\epsilon}\Delta_{t-1}$$
$$a_t = \rho_a a_{t-1} + e_{at}$$

There are 7 endogenous variables and 6 Lagrangian multipliers, hence 13 functional equations. Similar to Rotemberg pricing, we can rewrite this nonlinear system a more compact form,

$$\Gamma\left(s_t, X(s_t), E_t\left[Z\left(X(s_{t+1})\right)\right], E_t\left[Z_\Delta\left(X(s_{t+1})\right)\right]\right) = 0$$

where $\Gamma : \mathcal{R}^{2+13+3+3} \to \mathcal{R}^{13}$ summarizing the equilibrium relationship,

$$X(s_t) = \left(C_t(s_t), Y_t(s_t), \Pi_t(s_t), \frac{P_t^*(s_t)}{P_t(s_t)}, K_t^p(s_t), F_t^p(s_t), \Delta_t(s_t), \lambda_{1t}(s_t), \lambda_{2t}(s_t), \lambda_{3t}(s_t), \lambda_{4t}(s_t), \lambda_{5t}(s_t), \lambda_{6t}(s_t)\right)$$

collecting the policy functions we need to solve, with $X : \mathcal{R}^2 \to \mathcal{R}^{13}$, and

$$Z\left(X(s_{t+1})\right) = \begin{bmatrix} Z_1\left(X(s_{t+1})\right) \\ Z_2\left(X(s_{t+1})\right) \\ Z_3\left(X(s_{t+1})\right) \end{bmatrix} = \begin{bmatrix} M(\Delta_t, a_{t+1}) \\ L(\Delta_t, a_{t+1}) \\ \left(\Pi_{t+1}\right)^{\epsilon}\lambda_{6t+1} \end{bmatrix}$$

and

$$Z_\Delta\left(X(s_{t+1})\right) = \begin{bmatrix} \frac{\partial Z_1(X(s_{t+1}))}{\partial \Delta_t} \\ \frac{\partial Z_2(X(s_{t+1}))}{\partial \Delta_t} \\ \frac{\partial Z_3(X(s_{t+1}))}{\partial \Delta_t} \end{bmatrix} = \begin{bmatrix} \frac{\partial M(\Delta_t, a_{t+1})}{\partial \Delta_t} \\ \frac{\partial L(\Delta_t, a_{t+1})}{\partial \Delta_t} \\ \frac{\partial[(\Pi_{t+1})^{\epsilon}\lambda_{6t+1}]}{\partial \Delta_t} \end{bmatrix}$$
$$= \begin{bmatrix} \epsilon\left(\Pi_{t+1}\right)^{\epsilon-1} K_{t+1}^p \frac{\partial \Pi_{t+1}}{\partial \Delta_t} + \left(\Pi_{t+1}\right)^{\epsilon}\frac{\partial K_{t+1}^p}{\partial \Delta_t} \\ (\epsilon-1)\left(\Pi_{t+1}\right)^{\epsilon-2} F_{t+1}^p \frac{\partial \Pi_{t+1}}{\partial \Delta_t} + \left(\Pi_{t+1}\right)^{\epsilon-1}\frac{\partial F_{t+1}^p}{\partial \Delta_t} \\ \epsilon\left(\Pi_{t+1}\right)^{\epsilon-1}\lambda_{6t+1}\frac{\partial \Pi_{t+1}}{\partial \Delta_t} + \left(\Pi_{t+1}\right)^{\epsilon}\frac{\partial \lambda_{6t+1}}{\partial \Delta_t} \end{bmatrix}$$

Note we are assuming $E_t\left[Z_\Delta\left(X(s_{t+1})\right)\right] = \partial E_t\left[Z\left(X(s_{t+1})\right)\right]/\Delta_t$, which is normally valid using the Interchange of Integration and Differentiation Theorem.

The Chebyshev collocation method which we use to solve this nonlinear system can be described as follows:

1. Choose an order of approximation n_Δ and n_a for each dimension of the state space $s_t = (\Delta_{t-1}, a_t)$, then there are $N_s = (n_\Delta + 1) \times (n_a + 1)$ nodes in the state space.

2. Compute the $n_\Delta + 1$ and $n_a + 1$ roots of the Chebychev polynomial of order $n_\Delta + 1$ and $n_a + 1$ as

$$z_\Delta^i = \cos\left(\frac{(2i-1)\pi}{2(n_\Delta+1)}\right), \text{ for } i = 1, 2, ..., n_\Delta + 1.$$
$$z_a^i = \cos\left(\frac{(2i-1)\pi}{2(n_a+1)}\right), \text{ for } i = 1, 2, ..., n_a + 1.$$

and formulate initial guesses for the approximating coefficients.

3. Compute collocation points a_i as

$$a_i = \frac{\overline{a}+\underline{a}}{2} + \frac{\overline{a}-\underline{a}}{2} z_a^i = \frac{\overline{a}-\underline{a}}{2}\left(z_a^i + 1\right) + \underline{a}$$

for $i = 1, 2, ..., n_a + 1$, where $a = \log(A)$ is logged technology shock. Note that the number of collocation nodes is $n_a + 1$. Similarly, compute collocation points Δ_i as

$$\Delta_i = \frac{\overline{\Delta}+\underline{\Delta}}{2} + \frac{\overline{\Delta}-\underline{\Delta}}{2} z_\Delta^i = \frac{\overline{\Delta}-\underline{\Delta}}{2}\left(z_\Delta^i + 1\right) + \underline{\Delta}$$

for $i = 1, 2, ..., n_\Delta + 1$, which map $[-1, 1]$ into $[\underline{\Delta}, \overline{\Delta}]$.

4. At each node (Δ_i, a_j), for $i = 1, 2, ..., n_\Delta + 1$ and $j = 1, 2, ..., n_a + 1$, compute $X(s_t)$, that is,

$$X(\Delta_i, a_j) = \Omega(\Delta_i, a_j)\Theta_X$$

where $\Omega(\Delta_i, a_j) \equiv [T_{j_\Delta}(\xi(\Delta_i)T_{j_a}(\xi(a_j))]$, $j_\Delta = 0, ..., n_\Delta$, and $j_a = 0, ..., n_a$, is a $1\times N_s$ matrix of two-dimensional Chebyshev polynomials evaluated at node (Δ_i, a_j), and

$$\Theta_X = \left[\theta^c, \theta^y, \theta^\pi, \theta^p, \theta^k, \theta^f, \theta^\Delta, \theta^{\lambda_1}, \theta^{\lambda_2}, \theta^{\lambda_3}, \theta^{\lambda_4}, \theta^{\lambda_5}, \theta^{\lambda_6}\right]$$

is a $N_s \times 13$ matrix of the collocation coefficients.

5. At each node (Δ_i, a_j), for $i = 1, 2, ..., n_\Delta + 1$ and $j = 1, 2, ..., n_a + 1$, compute the possible values of future policy functions $X(s_{t+1})$ for $k = 1, ..., q$. That is,

$$X_{t+1}(\Delta_i, a_j) = \Omega_{t+1}(\Delta_i, a_j)\Theta_X$$

where q is the number of quadrature nodes, and the subscript $t + 1$ indicates next period values. Note that

$$\Omega_{t+1}(\Delta_i, a_j) \equiv \left[T_{j_\Delta}(\xi(\widehat{\Delta}(\Delta_i, a_j; \theta^\Delta)))T_{j_a}(\xi(\rho_a a_j + z_k\sqrt{2\sigma_a^2}))\right]$$

with $j_\Delta = 0, ..., n_\Delta$, and $j_a = 0, ..., n_a$, is a $q \times N_s$ matrix of Chebyshev polynomials evaluated at $t+1$ nodes (Δ_t, a_{t+1}), and the hat symbol indicates the corresponding approximate policy functions.

The two auxilliary functions can be calculated as follows:

$$M(s_{t+1}) \approx \left(\widehat{\Pi}(s_{t+1}; \theta^\pi)\right)^\epsilon \widehat{K}(s_{t+1}; \theta^k)$$

and,

$$L(s_{t+1}) \approx \left(\widehat{\Pi}(s_{t+1}; \theta^\pi)\right)^{\epsilon-1} \widehat{F}(s_{t+1}; \theta^f).$$

6. Calculate the expectation terms at each node (Δ_i, a_j). Let $z = e_a/\sqrt{2\sigma_a^2}$, and we have

$$\begin{aligned}
E_t\left[M(s_{t+1})\right] &= \frac{1}{\sigma_a\sqrt{2\pi}}\int_{-\infty}^{+\infty} \left(\Pi_{t+1}\right)^\epsilon K_{t+1}^p \exp\left(-\frac{e_{at+1}^2}{2\sigma_a^2}\right) de_{at+1} \\
&= \frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty} \left(\Pi_{t+1}\right)^\epsilon K_{t+1}^p \exp\left(-z^2\right) dz \\
&\approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q} \omega_k \left(\widehat{\Pi}(s_{t+1}; \theta^\pi)\right)^\epsilon \widehat{K}(s_{t+1}; \theta^k) \\
&\equiv \Psi\left(\Delta_i, a_j, q\right),
\end{aligned}$$

$$
\begin{aligned}
E_t\left[L(s_{t+1})\right] &= \frac{1}{\sigma_a\sqrt{2\pi}}\int_{-\infty}^{+\infty}\left(\Pi_{t+1}\right)^{\epsilon-1}F_{t+1}^{p}\exp\left(-\frac{e_{at+1}^2}{2\sigma_a^2}\right)de_{at+1}\\
&\approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q}\omega_k\left(\widehat{\Pi}(s_{t+1};\theta^{\pi})\right)^{\epsilon-1}\widehat{F}(s_{t+1};\theta^{f})\\
&\equiv \Theta\left(\Delta_i, a_j, q\right),
\end{aligned}
$$

and

$$
\begin{aligned}
E_t\left[\left(\Pi_{t+1}\right)^{\epsilon}\lambda_{6t+1}\right] &= \frac{1}{\sigma_a\sqrt{2\Pi}}\int_{-\infty}^{+\infty}\left(\Pi_{t+1}\right)^{\epsilon}\lambda_{6t+1}\exp\left(-\frac{e_{at+1}^2}{2\sigma_a^2}\right)de_{at+1}\\
&\approx \frac{1}{\sqrt{\Pi}}\sum_{k=1}^{q}\omega_k\left(\widehat{\Pi}(s_{t+1};\theta^{\Pi})\right)^{\epsilon-1}\widehat{\lambda}_6(s_{t+1};\theta^{\lambda_6})\\
&\equiv \Lambda\left(\Delta_i, a_j, q\right).
\end{aligned}
$$

7. Calculate the two partial derivatives under expectation, that is,

$$
\begin{aligned}
\frac{\partial E_t\left[M(s_{t+1})\right]}{\partial\Delta_t} &= E_t\left[\frac{\partial M(s_{t+1})}{\partial\Delta_t}\right]\\
&= E_t\left[\epsilon\left(\Pi_{t+1}\right)^{\epsilon-1}K_{t+1}^{p}\frac{\partial\Pi_{t+1}}{\partial\Delta_t}+\left(\Pi_{t+1}\right)^{\epsilon}\frac{\partial K_{t+1}^{p}}{\partial\Delta_t}\right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E_t\left[L(s_{t+1})\right]}{\partial\Delta_t} &= E_t\left[\frac{\partial L(s_{t+1})}{\partial\Delta_t}\right]\\
&= E_t\left[\left(\epsilon-1\right)\left(\Pi_{t+1}\right)^{\epsilon-2}F_{t+1}^{p}\frac{\partial\Pi_{t+1}}{\partial\Delta_t}+\left(\Pi_{t+1}\right)^{\epsilon-1}\frac{\partial F_{t+1}^{p}}{\partial\Delta_t}\right].
\end{aligned}
$$

Hence, we only need to compute $\partial\Pi_{t+1}/\partial\Delta_t$, $\partial K_{t+1}^{p}/\partial\Delta_t$ and $\partial F_{t+1}^{p}/\partial\Delta_t$. Note that

$$
\frac{\partial\Pi_{t+1}}{\partial\Delta_t}\approx\sum_{j_\Delta=0}^{n_\Delta}\sum_{j_a=0}^{n_a}\frac{2\theta_{j_\Delta j_a}^{\pi}}{\overline{\Delta}-\underline{\Delta}}T_{j_\Delta}'(\xi(\Delta_i))T_{j_a}(\xi(a_j))\equiv\widehat{\Pi}_\Delta
$$

$$
\frac{\partial K_{t+1}^{p}}{\partial\Delta_t}\approx\sum_{j_\Delta=0}^{n_\Delta}\sum_{j_a=0}^{n_a}\frac{2\theta_{j_\Delta j_a}^{k}}{\overline{\Delta}-\underline{\Delta}}T_{j_\Delta}'(\xi(\Delta_t))T_{j_a}(\xi(a_{t+1}))\equiv\widehat{K}_\Delta
$$

$$
\frac{\partial F_{t+1}^{p}}{\partial\Delta_t}\approx\sum_{j_\Delta=0}^{n_\Delta}\sum_{j_a=0}^{n_a}\frac{2\theta_{j_\Delta j_a}^{f}}{\overline{\Delta}-\underline{\Delta}}T_{j_\Delta}'(\xi(\Delta_t))T_{j_a}(\xi(a_{t+1}))\equiv\widehat{F}_\Delta
$$

Now, we can calculate

$$
\frac{\partial E_t\left[M(s_{t+1})\right]}{\partial\Delta_t}=\frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty}\left[\begin{array}{c}\epsilon\left(\Pi_{t+1}\right)^{\epsilon-1}K_{t+1}^{p}\frac{\partial\Pi_{t+1}}{\partial\Delta_t}\\+\left(\Pi_{t+1}\right)^{\epsilon}\frac{\partial K_{t+1}^{p}}{\partial\Delta_t}\end{array}\right]\exp\left(-z^2\right)dz
$$

$$
\begin{aligned}
&\approx\frac{1}{\sqrt{\pi}}\sum_{k=1}^{q}\omega_k\left[\epsilon\left(\widehat{\Pi}(s_{t+1};\theta^{\pi})\right)^{\epsilon-1}\widehat{K}(s_{t+1};\theta^{k})\widehat{\Pi}_\Delta+\left(\widehat{\Pi}(s_{t+1};\theta^{\pi})\right)^{\epsilon}\widehat{K}_\Delta\right]\\
&\equiv\widehat{M}\left(\Delta_i, a_j, q\right),
\end{aligned}
$$

$$\frac{\partial E_t\left[L(s_{t+1})\right]}{\partial \Delta_t} = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty}\left[\begin{array}{c}(\epsilon-1)\left(\Pi_{t+1}\right)^{\epsilon-2}F^p_{t+1}\frac{\partial \Pi_{t+1}}{\partial \Delta_t} \\ +\left(\Pi_{t+1}\right)^{\epsilon-1}\frac{\partial F^p_{t+1}}{\partial \Delta_t}\end{array}\right]\exp\left(-z^2\right)dz$$

$$\approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q}\omega_k\left[(\epsilon-1)\left(\widehat{\Pi}(s_{t+1};\theta^\pi)\right)^{\epsilon-2}\widehat{F}(s_{t+1};\theta^f)\widehat{\Pi}_\Delta + \left(\widehat{\Pi}(s_{t+1};\theta^\pi)\right)^{\epsilon-1}\widehat{F}_\Delta\right]$$

$$\equiv \widehat{L}\left(\Delta_i, a_j, q\right).$$

8. At each collocation point (Δ_i, a_j), calcuate the values of the thirteen residual functions defined by the equilibrium conditions as follows:

$$R_1 = \widehat{Y}(\Delta_i, a_j; \theta^y) - \widehat{C}(\Delta_i, a_j; \theta^c)$$

$$R_2 = \widehat{p}(\Delta_i, a_j; \theta^p) - \left(\frac{\epsilon}{\epsilon-1}\right)\frac{\widehat{K}(\Delta_i, a_j; \theta^k)}{\widehat{F}(\Delta_i, a_j; \theta^f)}$$

$$R_3 = \widehat{K}(\Delta_i, a_j; \theta^k) - (\widehat{\Delta}(\Delta_i, a_j; \theta^\Delta)\widehat{Y}(\Delta_i, a_j; \theta^y))^\varphi \exp(a_j)^{-\varphi-1}\widehat{Y}(\Delta_i, a_j; \theta^y) - \theta\beta\Psi\left(\Delta_i, a_j, q\right)$$

$$R_4 = \widehat{F}(\Delta_i, a_j; \theta^f) - \widehat{Y}(\Delta_i, a_j; \theta^y)\widehat{C}(\Delta_i, a_j; \theta^c)^{-\sigma} + \theta\beta\Theta\left(\Delta_i, a_j, q\right)$$

$$R_5 = 1 - (1-\theta)\widehat{p}(\Delta_i, a_j; \theta^p)^{1-\epsilon} - \theta\widehat{\Pi}(\Delta_i, a_j; \theta^\pi)^{\epsilon-1}$$

$$R_6 = \widehat{\Delta}(\Delta_i, a_j; \theta^\Delta) - (1-\theta)\widehat{p}(\Delta_i, a_j; \theta^p)^{-\epsilon} - \theta\widehat{\Pi}(\Delta_i, a_j; \theta^\pi)^\epsilon\Delta_i$$

$$R_7 = \widehat{C}(\Delta_i, a_j; \theta^c)^{-\sigma} - \widehat{\lambda}_1(\Delta_i, a_j; \theta^{\lambda_1}) + \sigma\widehat{Y}(\Delta_i, a_j; \theta^y)\widehat{C}(\Delta_i, a_j; \theta^c)^{-\sigma-1}\widehat{\lambda}_4(\Delta_i, a_j; \theta^{\lambda_4})$$

$$\begin{aligned} R_8 = &-\widehat{\Delta}(\Delta_i, a_j; \theta^\Delta)^{1+\varphi}\widehat{Y}(\Delta_i, a_j; \theta^y)^\varphi \exp(a_j)^{-1-\varphi} + \widehat{\lambda}_1(\Delta_i, a_j; \theta^{\lambda_1}) \\ &- (1+\varphi)(\widehat{\Delta}(\Delta_i, a_j; \theta^\Delta)\widehat{Y}(\Delta_i, a_j; \theta^y))^\varphi \exp(a_j)^{-1-\varphi}\widehat{\lambda}_3(\Delta_i, a_j; \theta^{\lambda_3}) \\ &- \widehat{C}(\Delta_i, a_j; \theta^c)^{-\sigma}\widehat{\lambda}_4(\Delta_i, a_j; \theta^{\lambda_4}) \end{aligned}$$

$$\begin{aligned} R_9 = &\widehat{\lambda}_2(\Delta_i, a_j; \theta^{\lambda_2}) + (1-\theta)(\epsilon-1)\widehat{p}(\Delta_i, a_j; \theta^p)^{-\epsilon}\widehat{\lambda}_5(\Delta_i, a_j; \theta^{\lambda_5}) \\ &+ \epsilon(1-\theta)\widehat{p}(\Delta_i, a_j; \theta^p)^{-\epsilon-1}\widehat{\lambda}_6(\Delta_i, a_j; \theta^{\lambda_6}) \end{aligned}$$

$$R_{10} = -(\epsilon-1)\theta\widehat{\lambda}_5(\Delta_i, a_j; \theta^{\lambda_5}) - \epsilon\theta\widehat{\Pi}(\Delta_i, a_j; \theta^\pi)\widehat{\lambda}_6(\Delta_i, a_j; \theta^{\lambda_6})\Delta_i$$

$$R_{11} = -\left(\frac{\epsilon}{\epsilon-1}\right)\widehat{F}(\Delta_i, a_j; \theta^f)^{-1}\widehat{\lambda}_2(\Delta_i, a_j; \theta^{\lambda_2}) + \widehat{\lambda}_3(\Delta_i, a_j; \theta^{\lambda_3})$$

$$R_{12} = \left(\frac{\epsilon}{\epsilon-1}\right)\frac{\widehat{K}(\Delta_i, a_j; \theta^k)}{\widehat{F}(\Delta_i, a_j; \theta^f)^2}\widehat{\lambda}_2(\Delta_i, a_j; \theta^{\lambda_2}) + \widehat{\lambda}_4(\Delta_i, a_j; \theta^{\lambda_4})$$

$$\begin{aligned} R_{13} = &-\widehat{Y}(\Delta_i, a_j; \theta^y)^{1+\varphi}\widehat{\Delta}(\Delta_i, a_j; \theta^\Delta)^\varphi \exp(a_j)^{-1-\varphi} - \theta\beta\Lambda\left(\Delta_i, a_j, q\right) \\ &- \varphi\widehat{\Delta}(\Delta_i, a_j; \theta^\Delta)^{\varphi-1}\widehat{Y}(\Delta_i, a_j; \theta^y)^{1+\varphi}\widehat{\lambda}_3(\Delta_i, a_j; \theta^{\lambda_3})\exp(a_j)^{-1-\varphi} \\ &- \theta\beta\widehat{M}\left(\Delta_i, a_j, q\right)\widehat{\lambda}_3(\Delta_i, a_j; \theta^{\lambda_3}) - \theta\beta\widehat{L}\left(\Delta_i, a_j, q\right)\widehat{\lambda}_4(\Delta_i, a_j; \theta^{\lambda_4}) + \widehat{\lambda}_6(\Delta_i, a_j; \theta^{\lambda_6}) \end{aligned}$$

9. Finally, check the stopping rules. If all residuals are close enough to zero then stop, else update the approximation coefficients and go back to step 4.

The equation solver *csolve* written by Christopher A. Sims is employed to solve the resulted system of nonlinear equations. When implementing the above algorithm, we start from lower order Chebyshev polynomials and formulate the initial guesses based on the results in Anderson et al. (2010). Then, we increase the order of approximation and take as starting value the solution from the previous lower order approximation. This informal homotopy continuation idea ensures us to find the solution.

C.3 Welfare Comparison

In order to compare the social welfare under Calvo and Rotemberg pricing in a fully nonlinear model, we first describe the second-order approximation to welfare. Then we transform the welfare as the fraction of the consumption path under the Ramsey allocation that must be given up in order to equalize welfare under the Ramsey policy and discretionary policy.

C.3.1 The Quadratic Approximation to Welfare

Individual utility in period t is

$$U_t \equiv U(C_t, N_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

Let $\widehat{X}_t \equiv \log\left(X_t/\overline{X}\right)$ denote the log-deviation of variable X_t from its steady state $\overline{X}$. In addition, let $\widetilde{X}_t = X_t - \overline{X}$ denote the linear deviation of X_t around its steady state value. Then using a second-order Taylor approximation,

$$\frac{X_t - \overline{X}}{\overline{X}} = \frac{\widetilde{X}_t}{\overline{X}} = \widehat{X}_t + \frac{1}{2}\widehat{X}_t^2 + o(2) \tag{23}$$

where $o(2)$ represents terms that are of order higher than 2 in the bound on the amplitude of the relevant shocks. We will repeatedly use (23) to derive a second-order approximation to the social welfare.

Now consider the second-order approximation to per period utility,

$$U_t = \overline{U} + \overline{C}^{1-\sigma}\left[\widehat{C}_t + \frac{1-\sigma}{2}\widehat{C}_t^2\right] - \overline{N}^{1+\varphi}\left[\widehat{N}_t + \frac{1+\varphi}{2}\widehat{N}_t^2\right] + o(2)$$

where

$$\overline{U} = \frac{\overline{C}^{1-\sigma} - 1}{1-\sigma} - \frac{\overline{N}^{1+\varphi}}{1+\varphi}$$

Rotemberg Pricing The second-order approximation to market clearing condition, $C_t = \left[1 - \frac{\phi}{2}\left(\Pi_t - 1\right)^2\right] Y_t$, is

$$\widehat{C}_t + \frac{1}{2}\widehat{C}_t^2 = \widehat{Y}_t + \frac{1}{2}\widehat{Y}_t^2 - \frac{\phi}{2}\widehat{\Pi}_t^2 + o(2)$$

such that,

$$\begin{aligned} U_t = \overline{U} - \frac{(\sigma+\varphi)\overline{C}^{1-\sigma}}{2}&\left[(x_t - x^*)^2 + \frac{\phi}{\varphi+\sigma}\widehat{\Pi}_t^2\right] \\ + \overline{C}^{1-\sigma}&\left[\begin{array}{c} \frac{\Phi^2}{2(\varphi+\sigma)} - \frac{(1-\sigma)(1-\Phi)-(1+\varphi)}{1+\varphi}\widehat{Y}_t^f \\ + \frac{(1-\sigma)(\sigma+\varphi)}{2(1+\varphi)}\left(\widehat{Y}_t^f\right)^2 \end{array}\right] + o(2) \end{aligned} \tag{24}$$

where $\widehat{Y}_t^f = \log\left(Y_t^f/\overline{Y}^f\right)$ denote the log-deviation of output from its steady state under flexible price, $x_t \equiv \widehat{Y}_t - \widehat{Y}_t^f$ is the output gap, $x^* \equiv \ln \overline{Y} - \ln \overline{Y}^f = -\ln(1-\Phi)/(\sigma+\varphi) \approx$

$\Phi / (\sigma + \varphi)$ is a measure of the distortion created by the presence of monopolistic competition alone, *t.i.p.* are terms independent of policy, and terms like $\Phi \left(\widehat{Y}_t^f \right)^2$ and $\Phi \widehat{Y}_t \widehat{Y}_t^f$ are omitted[10]. In addition, the fact that $\overline{N}^{1+\varphi} = (1 - \epsilon^{-1}) \overline{C}^{1-\sigma} \equiv (1 - \Phi) \overline{C}^{1-\sigma}$, and $\widehat{A}_t = (\varphi + \sigma) / (1 + \varphi) \widehat{Y}_t^f$ is used in deriving (24).

Hence,

$$W_R \equiv E_0 \sum_{t=0}^{\infty} \beta^t U_t = \frac{\overline{U}}{1-\beta} - \frac{(\sigma + \varphi) \overline{C}^{1-\sigma}}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[(x_t - x^*)^2 + \frac{\phi}{\sigma + \varphi} \widehat{\Pi}_t^2 \right]$$

$$+ \left[\frac{\Phi^2 \overline{C}^{1-\sigma}}{2 (\varphi + \sigma) (1 - \beta)} - \frac{(1-\sigma)(1+\varphi) \overline{C}^{1-\sigma} \sigma_a^2}{2 (\varphi + \sigma)(1-\beta)(1-\rho_a)} \right] + o(2)$$

$$= \frac{\overline{U}}{1-\beta} - \Omega_R E_0 \sum_{t=0}^{\infty} \beta^t \left[\lambda_R (x_t - x^*)^2 + \widehat{\Pi}_t^2 \right] \tag{25}$$

$$+ \left[\frac{\Phi^2 \overline{C}^{1-\sigma}}{2 (\varphi + \sigma) (1 - \beta)} - \frac{(1-\sigma)(1+\varphi) \overline{C}^{1-\sigma} \sigma_a^2}{2 (\varphi + \sigma)(1-\beta)(1-\rho_a)} \right] + o(2)$$

where

$$\Omega_R \equiv \frac{\phi \overline{C}^{1-\sigma}}{2}$$

$$\lambda_R \equiv \frac{\sigma + \varphi}{\phi}$$

Note that we derive the LQ welfare function explicitly retaining the relevant *t.i.p* in order to make a legitimate comparison with the social welfare obtained from the fully nonlinear model.

In order to calculate the inflation bias under LQ method, we write down the log-linearized IS equation and NKPC below. The IS curve is,

$$x_t = E_t x_{t+1} - \frac{1}{\sigma} \left(\widehat{R}_t - E_t \widehat{\Pi}_{t+1} \right) + \frac{1+\varphi}{\varphi + \sigma} (\rho_a - 1) \widehat{A}_t$$

and the NKPC is,

$$\widehat{\Pi}_t = \beta E_t \widehat{\Pi}_{t+1} + \frac{(\epsilon - 1)(\varphi + \sigma)}{\phi} x_t$$

Calvo Pricing The second-order approximation to market clearing condition is

$$\widehat{C}_t + \frac{1}{2} \widehat{C}_t^2 = \widehat{Y}_t + \frac{1}{2} \widehat{Y}_t^2 + o(2)$$

and it can be shown (see Woodford, 2003, chap 6) that,

$$\widehat{N}_t = \left(\widehat{Y}_t - \widehat{A}_t \right) + \frac{\epsilon}{2} var_j \left(\widehat{P}_t(j) \right) + o(2)$$

[10]When $\Phi = 1/\epsilon$ is so small that the product of Φ with a second-order term can be ignored as negligible.

Hence, similar to the Rotemberg case,

$$U_t = \overline{U} - \frac{(\varphi+\sigma)\,\overline{C}^{1-\sigma}}{2}\left[(x_t - x^*)^2 + \frac{\epsilon}{\varphi+\sigma} var_j\left(\widehat{P}_t(j)\right)\right]$$

$$+\overline{C}^{1-\sigma}\left[\frac{\Phi^2}{2(\varphi+\sigma)} - \frac{(1-\sigma)(1-\Phi)-(1+\varphi)}{(1+\varphi)}\widehat{Y}_t^f + \frac{(1-\sigma)(\sigma+\varphi)}{2(1+\varphi)}\left(\widehat{Y}_t^f\right)^2\right] + o(2)$$

The next step is to relate price dispersion $\Delta_t \equiv var_j\left(\widehat{P}_t(j)\right)$ to the average inflation rate across all firms. Walsh (2003, p.554) shows that

$$\Delta_t \approx \theta\Delta_{t-1} + \left(\frac{\theta}{1-\theta}\right)\pi_t^2$$

which implies

$$\sum_{t=0}^{\infty}\beta^t\Delta_t = \frac{\theta}{(1-\theta)(1-\theta\beta)}\sum_{t=0}^{\infty}\beta_t\pi_t^2$$

Therefore,

$$W_C = \frac{\overline{U}}{1-\beta} - \Omega_C E_0\sum_{t=0}^{\infty}\beta^t\left[\lambda_C(x_t-x^*)^2 + \pi_t^2\right]$$

$$+\left[\frac{\Phi^2\overline{C}^{1-\sigma}}{2(\varphi+\sigma)(1-\beta)} - \frac{(1-\sigma)(1+\varphi)\,\overline{C}^{1-\sigma}\sigma_a^2}{2(\varphi+\sigma)(1-\beta)(1-\rho_a)}\backslash\right] + o(2)$$

where

$$\Omega_C \equiv \frac{(\sigma+\varphi)\,\overline{C}^{1-\sigma}}{2}\frac{\epsilon}{\kappa}$$

$$\lambda_C \equiv \kappa/\epsilon$$

$$\kappa \equiv \frac{(1-\theta)(1-\theta\beta)(\sigma+\varphi)}{\theta}$$

The log-linearized IS equation and NKPC are given, respectively, as follows:

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}\left(\widehat{R}_t - E_t\widehat{\Pi}_{t+1}\right) + \frac{1+\varphi}{\varphi+\sigma}(\rho_a - 1)\widehat{A}_t$$

$$\widehat{\Pi}_t = \beta E_t\widehat{\Pi}_{t+1} + \kappa x_t$$

Note that when

$$\phi = \frac{(\epsilon-1)\,\theta}{(1-\theta)(1-\theta\beta)}$$

the NKPC under both Rotemberg pricing and Calvo pricing are the same. Also note that $\lambda_R = \left(\frac{\epsilon}{\epsilon-1}\right)\lambda_C$, and $\Omega_R = \left(\frac{\epsilon-1}{\epsilon}\right)\Omega_C$. The inflation weights λ_R and λ_C differ only marginally, since ϵ usually takes values between 7 and 10 in the applied literature.

C.3.2 Inflation Bias Under LQ Method

We can rewrite the above LQ model as follows, using $\pi_t = \Pi_t - 1 \approx \ln(\Pi_t) - \ln\left(\overline{\Pi}\right) = \widehat{\Pi}_t$ and $i_t = R_t - 1 \approx \ln(R_t) - \ln\left(\overline{R}\right) = \widehat{R}_t$:

$$\max_{\{x_t,\pi_t\}} -\Omega_j E_0 \sum_{t=0}^{\infty} \beta^t \left[\lambda_j (x_t - x^*)^2 + \pi_t^2\right]$$

subject to

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \tag{26}$$

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1}) + \frac{1+\varphi}{\varphi+\sigma}(\rho_a - 1)\widehat{A}_t$$

where $j = R, C$. Woodford (2003, p.471) shows that the equilibrium inflation under optimal discretion is

$$\pi_t = \frac{\lambda_j}{\lambda_j + \kappa^2}(\beta E_t \pi_{t+1} + \kappa x^*)$$

hence the steady state $\overline{\pi}$ under rational expectation satisfies

$$\overline{\pi} = \frac{\lambda_j}{\lambda_j + \kappa^2}(\beta \overline{\pi} + \kappa x^*)$$

that is,

$$\overline{\pi} = \frac{\lambda_j \kappa}{(1-\beta)\lambda_j + \kappa^2} x^* = \frac{\lambda_j \kappa}{(1-\beta)\lambda_j + \kappa^2} \frac{\Phi}{(\sigma+\varphi)}$$

with $j = R, C$. $\overline{\pi}$ is the so-called inflation bias, relative to the targeted zero rate of inflation which is optimal under perfect commitment.

C.3.3 Relative Welfare Cost

The welfare under discretion from the LQ method is calculated as follows. Unless stated otherwise, the superscript d denotes the discretion case, and subscripts R and C represent the Rotemberg and Calvo pricing, respectively. From (26), $\overline{x} = (1-\beta)\overline{\pi}/\kappa$, then using the log-linearized model we can solve for steady state values for deviations $\widehat{C}_t$ and $\widehat{N}_t$, denoted as $\widehat{C}$ and $\widehat{N}$, respectively. It is straightforward to show that $\widehat{C} = \widehat{N} = \widehat{Y} = \overline{x}$. Finally, the steady state values for levels C_t and N_t, are

$$\overline{C}_j^d = \overline{C}^r e^{\widehat{C}} \approx \overline{C}^r \left(1 + \widehat{C}\right) = \overline{C}^r (1 + \overline{x})$$

$$\overline{N}_j^d = \overline{N}^r e^{\widehat{N}} \approx \overline{N}^r \left(1 + \widehat{N}\right) = \overline{N}^r (1 + \overline{x})$$

where $j = R, C$, and

$$\overline{C}^r = \overline{N}^r = \left(\frac{\epsilon - 1}{\epsilon}\right)^{1/(\sigma+\varphi)}$$

are the Ramsey steady states around which we log-linearize the model. Therefore,

$$W_j = \frac{1}{1-\beta}\left[\frac{\left(\overline{C}_j^d\right)^{1-\sigma} - 1}{1-\sigma} - \frac{\left(\overline{N}_j^d\right)^{1+\varphi}}{1+\varphi}\right] - \frac{\Omega_j}{1-\beta}\left[\lambda_j\left(\frac{(1-\beta)\overline{\pi}}{\kappa} - \frac{\Phi}{(\varphi+\sigma)}\right)^2 + \overline{\pi}^2\right]$$

$$+\frac{\left(\overline{C}_j^d\right)^{1-\sigma}}{2\left(\varphi+\sigma\right)\left(1-\beta\right)}\left[\Phi^2-\frac{\left(1-\sigma\right)\left(1+\varphi\right)\sigma_a^2}{\left(1-\rho_a\right)}\right]$$

where $j = R, C$.

For the fully nonlinear method, the welfare under discretion is calculated by adding corresponding policy functions into optimal policy problem and then approximated by the Chebyshev collocation method. That is,

$$W_{R,t}^d = W_R^d\left(A_t\right) = \frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{\left(Y_t/A_t\right)^{1+\varphi}}{1+\varphi} + \beta E_t\left[W_R^d(A_{t+1})\right]$$

$$W_{C,t}^d = W_C^d\left(\Delta_{t-1}, A_t\right) = \frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{\left(\Delta_t Y_t/A_t\right)^{1+\varphi}}{1+\varphi} + \beta E_t\left[W_C^d(\Delta_t, A_{t+1})\right]$$

and the steady state welfare, denoted as W_R^d and W_C^d for ease of notation, can be correspondingly found.

Note that W_R, W_R^d and W_C, W_C^d which represent the conditional expectation of lifetime utility, are absolute welfare measures under Rotemberg pricing and Calvo pricing, respectively. However, the utility function is ordinal, so a welfare measure based on the value function is not very revealing. Hence, we calculate the relative welfare cost in terms of the consumption equivalent units under the Ramsey allocation. Specifically, we want to find ξ such that

$$E_0\sum_{t=0}^{\infty}\beta^t U(C_t^d, N_t^d) = E_0\sum_{t=0}^{\infty}\beta^t U((1-\xi)C_t^r, N_t^r)$$

where the r superscript denotes the Ramsey allocation (under full commitment), and the d superscript stands for the allocation under discretion. Given the utility function adopted, the expression for ξ in percentage terms is

$$\xi = \left\{1-\exp\left[\left(1-\beta\right)\left(W^d-W^r\right)\right]\right\}\times 100 \tag{27}$$

where

$$W^d \equiv E_0\sum_{t=0}^{\infty}\beta^t\left(\ln C_t^d - \frac{\left(N_t^d\right)^{1+\varphi}}{1+\varphi}\right)$$

represents the unconditional expectation of lifetime utility in the economy under discretion, and

$$W^r \equiv E_0\sum_{t=0}^{\infty}\beta^t\left(\ln C_t^r - \frac{\left(N_t^r\right)^{1+\varphi}}{1+\varphi}\right) = \frac{1}{1-\beta}\left[\ln\overline{C}^r - \frac{\left(\overline{N}^r\right)^{1+\varphi}}{1+\varphi}\right]$$

is the unconditional expectation of lifetime utility associated with the economy under full commitment. Recall that $\sigma = 1$ is the benchmark case in our paper.

Hence, under the Rotemberg case,

$$\xi_R = \begin{cases} \left\{1-\exp\left[\left(1-\beta\right)\left(W_R-W^r\right)\right]\right\}\times 100 & \text{, using LQ method} \\ \left\{1-\exp\left[\left(1-\beta\right)\left(W_R^d-W^r\right)\right]\right\}\times 100 & \text{, using projection method} \end{cases}$$

and under the Calvo case,

$$\xi_C = \begin{cases} \left\{1-\exp\left[\left(1-\beta\right)\left(W_C-W^r\right)\right]\right\}\times 100 & \text{, using LQ method} \\ \left\{1-\exp\left[\left(1-\beta\right)\left(W_C^d-W^r\right)\right]\right\}\times 100 & \text{, using projection method} \end{cases}$$

C.4 Trend Inflation

In this section we explore the determinacy properties of our simple New Keynesian models at the levels of steady-state inflation implied by our non-linear optimal policy exercise.

C.4.1 The Rotemberg Case

Following Ascari and Rossi (2012) the linearized version of our New Keyensian model under Rotemberg pricing can be shown to be,

$$\pi_t = \gamma_f \beta E_t \pi_{t+1} + \gamma_y \beta(1-\sigma)\Delta E_t y_{t+1} + \gamma_{mc} mc_t$$

$$mc_t = (\sigma + \varphi) y_t - \zeta_c \sigma \pi_t - (1+\varphi) a_t$$
$$y_t = E_t y_{t+1} - \zeta_c \Delta E_t \pi_{t+1} - \frac{1}{\sigma} E_t (R_t - \pi_{t+1})$$

where

$$\zeta_c = \frac{\phi(\pi - 1)\pi}{1 - \frac{\phi}{2}(\pi-1)^2}$$
$$\frac{C}{Y} = 1 - \frac{\phi}{2}(\pi - 1)^2$$
$$\rho = (2\pi^2 - \pi)C/Y + \beta[(\pi-1)\pi]^2 \sigma\phi$$
$$\gamma_f = \frac{(2\pi^2 - \pi)C/Y + [(\pi-1)\pi]^2\sigma\phi}{\rho}$$
$$\gamma_y = \frac{(\pi^2 - \pi)C/Y}{\rho}$$
$$\gamma_{mc} = \frac{(\epsilon - 1 + \phi(\pi^2 - \pi)(1-\beta))C/Y}{\phi\rho}$$

This can be written in matrix form as,

$$A0 \begin{bmatrix} \pi_t \\ y_t \\ i_t \\ E_t\pi_{t+1} \\ E_t y_{t+1} \end{bmatrix} = A1 \begin{bmatrix} \pi_{t-1} \\ y_{t-1} \\ i_{t-1} \\ \pi_t \\ y_t \end{bmatrix}$$

where

$$A0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -\alpha_\pi & -\alpha_y & 1 & 0 & 0 \\ -\gamma_{mc}\zeta_c\sigma & -\gamma_y\beta(1-\sigma) + \gamma_{mc}(\sigma+\varphi) & 0 & \gamma_f\beta & \gamma_y\beta(1-\sigma) \\ \zeta_c & 0 & -\frac{1}{\sigma} & -\zeta_c + \frac{1}{\sigma} & 1 \end{bmatrix}$$
$$A1 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

C.4.2 The Calvo Case

$$\pi_t = [\beta\pi + \eta(\theta-1)]E_t\pi_{t+1} + \kappa y_t + \lambda\varphi s_t + \eta E_t\psi_{t+1}$$
$$\psi_t = (1-\sigma)(1-\theta\beta\pi^{\epsilon-1})y_t + \theta\beta\pi^{\epsilon-1}[(\epsilon-1)E_t\pi_{t+1} + E_t\psi_{t+1}]$$

$$s_t = \xi\pi_t + \theta\pi^{\epsilon}s_{t-1}$$
$$y_t = E_t y_{t+1} - \frac{1}{\sigma}E_t(R_t - \pi_{t+1})$$

where

$$\lambda = \frac{(1-\theta\pi^{\epsilon-1})(1-\theta\beta\pi^{\epsilon})}{\theta\pi^{\epsilon-1}}$$
$$\eta = \beta(\pi-1)(1-\theta\pi^{\epsilon-1})$$
$$\kappa = \lambda(\sigma+\varphi) + \eta(1-\sigma)$$
$$\xi = \frac{\epsilon\theta\pi^{\epsilon-1}(\pi-1)}{1-\theta\pi^{\epsilon-1}}$$

$$B0\begin{bmatrix} \pi_t \\ y_t \\ i_t \\ s_t \\ E_t\pi_{t+1} \\ E_t y_{t+1} \\ E_t\psi_{t+1} \end{bmatrix} = B1\begin{bmatrix} \pi_{t-1} \\ y_{t-1} \\ i_{t-1} \\ s_{t-1} \\ \pi_t \\ y_t \\ \psi_t \end{bmatrix}$$

where

$$B0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -\alpha_\pi & -\alpha_y & 1 & 0 & 0 & 0 & 0 \\ -\xi & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \kappa & 0 & \lambda\varphi & \beta\pi+\eta(\theta-1) & 0 & \eta \\ 0 & 0 & -\frac{1}{\sigma} & 0 & \frac{1}{\sigma} & 1 & 0 \\ 0 & (1-\sigma)(1-\theta\beta\pi^{\epsilon-1}) & 0 & 0 & \theta\beta\pi^{\epsilon-1}(\epsilon-1) & 0 & \theta\beta\pi^{\epsilon-1} \end{bmatrix}$$

$$B1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \theta\pi^{\epsilon} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

This enables us to assess the determinacy properties of the underlying dynamic systems by considering the eigenvalues of the transition matrices, $A0^{-1}A1$ and $B0^{-1}B1$, in the cases of Rotemberg and Calvo, respectively. We require two roots with modulus in excess of one to ensure determinacy in the case of Rotemberg, and three under Calvo.

Notice that when $\pi = 1$, the linearized systems reduce to.

$$\pi_t = \beta E_t\pi_{t+1} + \frac{\epsilon-1}{\phi}[(\sigma+\varphi)y_t - (1+\varphi)a_t]$$

under Rotemberg, and,

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta}[(\sigma+\varphi)y_t - (1+\varphi)a_t]$$

under Calvo, with both representations sharing the same Euler equation,

$$y_t = E_t y_{t+1} - \frac{1}{\sigma} E_t (R_t - \pi_{t+1})$$

Therefore, linearized around a zero inflation steady-state the two systems are identical provided,

$$\frac{\epsilon - 1}{\phi} = \frac{(1-\theta)(1-\beta\theta)}{\theta}$$

which is the standard means of calibrating the adjustment cost parameter, ϕ, with reference to evidence on the probability of price change under Calvo, $1-\theta$.

C.5 The Model With Time-Varying Tax Rate

To indirectly introduce cost push shock, we consider the revenue tax τ_{pt} which is assumed to follow the following autoregressive process,

$$\ln(1-\tau_{pt}) = (1-\rho^{\tau_p})\ln(1-\tau_p) + \rho^{\tau_p}\ln(1-\tau_{pt-1}) - e_{\tau t}$$

$$e_{\tau t} \overset{i.i.d}{\sim} N\left(0, \sigma_\tau^2\right)$$

With revenue tax τ_{pt}, the expected discounted sum of nominal profits under Rotemberg pricing is given by

$$E_t \sum_{s=0}^{\infty} Q_{t,t+s}\left[(1-\tau_{pt})P_t(j)Y_t(j) - mc_t Y_t(j) P_t - \frac{\phi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 Y_t P_t\right]$$

and under Calvo it can be written as

$$E_t \sum_{s=0}^{\infty} \theta^s Q_{t,t+s}\left[(1-\tau_{pt})P_t(j)Y_{t+s}(j) - mc_{t+s}Y_{t+s}(j)P_{t+s}\right]$$

Based on the derivation of the benchmark model, it is quite straightforward to write down the complete system of non-linear equations describing the discretionary equilibrium. We will use Chebyshev collocation with time iteration method to solve the models with time-varying tax for optimal policy functions.

C.5.1 The Rotemberg Pricing

Since we want to focus on the effect of tax rate, then the technology shock can be shut down by setting $A_t \equiv 1$. This, in fact, can simplify numerical computation.

The Lagrangian is

$$\begin{aligned}\mathcal{L} = &\frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} + \beta E_t\left[V(\tau_{pt+1})\right] + \lambda_{1t}\left\{\left[1 - \frac{\phi}{2}(\Pi_t - 1)^2\right]N_t - C_t\right\} \\ &+ \lambda_{2t}\left\{(1-\epsilon)(1-\tau_{pt}) + \epsilon C_t^{\sigma} N_t^{\varphi} - \phi\Pi_t(\Pi_t - 1) + \phi\beta C_t^{\sigma} Y_t^{-1} E_t\left[M(\tau_{pt+1})\right]\right\}\end{aligned}$$

where λ_{jt} $(j = 1, 2)$ are the Lagrange multipliers, and

$$M(\tau_{pt+1}) \equiv C_{t+1}^{-\sigma} N_{t+1} \Pi_{t+1} \left(\Pi_{t+1} - 1 \right)$$

The equilibrium conditions for time-consistent policy are,

$$\begin{aligned}
C_t^{-\sigma} &= \lambda_{1t} - \lambda_{2t} \left\{ \epsilon \sigma C_t^{\sigma-1} N_t^{\varphi} + \sigma \phi \beta C_t^{\sigma-1} N_t^{-1} E_t \left[M(\tau_{pt+1}) \right] \right\} \\
N_t{}^{\varphi} &= \lambda_{1t} \left[1 - \frac{\phi}{2} \left(\Pi_t - 1 \right)^2 \right] + \lambda_{2t} \left\{ \begin{array}{c} \epsilon \varphi N_t{}^{\varphi-1} C_t^{\sigma} \\ - \phi \beta C_t^{\sigma} N_t^{-2} E_t \left[M(\tau_{pt+1}) \right] \end{array} \right\} \\
\lambda_{1t} \phi \left(1 - \Pi_t \right) N_t &= \lambda_{2t} \phi \left(2\Pi_t - 1 \right) \\
C_t &= \left[1 - \frac{\phi}{2} \left(\Pi_t - 1 \right)^2 \right] N_t \\
0 &= (1-\epsilon)(1-\tau_{pt}) + \epsilon C_t^{\sigma} N_t^{\varphi} - \phi \Pi_t \left(\Pi_t - 1 \right) + \phi \beta \frac{C_t^{\sigma}}{N_t} E_t \left[M(\tau_{pt+1}) \right].
\end{aligned}$$

C.5.2 The Calvo Pricing

Similar to the Rotemberg case, we solve a simpler question by shutting down the technology shock. Then, there are two state variables, τ_{pt} and Δ_{t-1}. The Lagrangian is given as follow:

$$\begin{aligned}
\mathcal{L} = & \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t{}^{1+\varphi}}{1+\varphi} + \beta E_t \left[V(\Delta_t, \tau_{pt+1}) \right] \\
& + \lambda_{1t} [N_t / \Delta_t - C_t] \\
& + \lambda_{2t} \left[(1-\tau_{pt}) \frac{N_t}{\Delta_t C_t^{\sigma}} + \theta \beta E_t \left[L(\Delta_t, \tau_{pt+1}) \right] - F_t \right] \\
& + \lambda_{3t} \left[\frac{N_t^{\varphi+1}}{(1-\epsilon^{-1})\Delta_t} + \theta \beta E_t \left[M(\Delta_t, \tau_{pt+1}) \right] - S_t \right] \\
& + \lambda_{4t} \left[(1-\theta) \left(\frac{1 - \theta \Pi_t{}^{\epsilon-1}}{1-\theta} \right)^{\frac{\epsilon}{\epsilon-1}} + \theta \Pi_t^{\epsilon} \Delta_{t-1} - \Delta_t \right] \\
& + \lambda_{5t} \left[F_t \left(\frac{1 - \theta \Pi_t{}^{\epsilon-1}}{1-\theta} \right)^{\frac{1}{1-\epsilon}} - S_t \right]
\end{aligned}$$

where λ_{jt} $(j = 1, 2, 3, 4, 5)$ are the Lagrange multipliers, and

$$L(\Delta_t, \tau_{pt+1}) \equiv \Pi_{t+1}^{\epsilon-1} F_{t+1}$$

$$M(\Delta_t, \tau_{pt+1}) \equiv \Pi_{t+1}^{\epsilon} S_{t+1}$$

The equilibrium conditions for time-consistent policy are,

$$\begin{aligned}
C_t &= N_t / \Delta_t \\
F_t &= (1-\tau_{pt}) C_t^{1-\sigma} + \theta \beta E_t \left[\Pi_{t+1}^{\epsilon-1} F_{t+1} \right] \\
S_t &= \frac{N_t^{\varphi+1}}{(1-\epsilon^{-1})\Delta_t} + \theta \beta E_t \left[\Pi_{t+1}^{\epsilon} S_{t+1} \right]
\end{aligned}$$

$$\Delta_t = (1-\theta)\left(\frac{1-\theta{\Pi_t}^{\epsilon-1}}{1-\theta}\right)^{\frac{\epsilon}{\epsilon-1}} + \theta\Pi_t^{\epsilon}\Delta_{t-1}$$

$$S_t = F_t\left(\frac{1-\theta{\Pi_t}^{\epsilon-1}}{1-\theta}\right)^{\frac{1}{1-\epsilon}}$$

$$0 = 1 - \lambda_{1t}C_t^{\sigma} - \sigma(1-\tau_{pt})\lambda_{2t}$$

$$0 = \Delta_t C_t^{\sigma}{N_t}^{\varphi} - C_t^{\sigma}\lambda_{1t} - (1-\tau_{pt})\lambda_{2t} - \frac{(\varphi+1)\,C_t^{\sigma}N_t^{\varphi}\lambda_{3t}}{(1-\epsilon^{-1})}$$

$$0 = \lambda_{2t} - \lambda_{5t}\left(\frac{1-\theta{\Pi_t}^{\epsilon-1}}{1-\theta}\right)^{\frac{1}{1-\epsilon}}$$

$$0 = \lambda_{3t} + \lambda_{5t}$$

$$\begin{aligned} 0 = \epsilon&\left(\left(\frac{1-\theta{\Pi_t}^{\epsilon-1}}{1-\theta}\right)^{\frac{1}{\epsilon-1}} - \Delta_{t-1}\Pi_t\right)\lambda_{4t} \\ &- \frac{1}{1-\theta}\left(\frac{1-\theta{\Pi_t}^{\epsilon-1}}{1-\theta}\right)^{\frac{\epsilon}{1-\epsilon}}\lambda_{5t}F_t \end{aligned}$$

$$\begin{aligned} 0 = &\frac{C_t}{\Delta_t}\lambda_{1t} + (1-\tau_{pt})\frac{C_t^{1-\sigma}}{\Delta_t}\lambda_{2t} + \frac{N_t^{\varphi}C_t}{(1-\epsilon^{-1})\Delta_t}\lambda_{3t} \\ &+ \lambda_{4t} - \theta\beta\lambda_{2t}E_t\left[L_1(\Delta_t,\tau_{pt+1})\right] - \theta\beta\lambda_{3t}E_t\left[M_1(\Delta_t,\tau_{pt+1})\right] - \theta\beta E_t\left[\Pi_{t+1}^{\epsilon}\lambda_{4t+1}\right] \end{aligned}$$