Fiscal policy in a Real-Business-Cycle model with labor-intensive government services and endogenous public sector wages and hours

Aleksandar Vasilev*†

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Abstract

Motivated by the high public employment, and the public wage premia observed in Europe, a Real-Business-Cycle model, calibrated to German data (1970-2007), is set up with a richer government spending side, and an endogenous private-public sector labor choice. To illustrate the effects of fiscal policy, two regimes are compared and contrasted to one another - exogenous *vs.* optimal (Ramsey) policy case. The main findings from the computational experiments performed in this paper are: (i) The optimal steady-state capital tax rate is zero; (ii) A higher labor tax rate is needed in the Ramsey case to compensate for the loss in capital tax revenue; (iii) Under the optimal policy regime, public sector employment is lower, but government employees receive higher wages; (iv) The benevolent Ramsey planner provides the optimal amount of the public good, substitutes labor for capital in the input mix for public services production, and private output; (v) Government wage bill is smaller, while public investment is three times higher than in the exogenous policy case.

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*E-mail for correspondence: alvasilev@yahoo.com.

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1 Introduction

Since the early 1990s, many studies have investigated the effects of fiscal policy in general equilibrium setups, *e.g.* Christiano and Eichenbaum (1992), Baxter and King (1993), MacGrattan (1994), Mendoza and Tesar (1998), Chari, Christiano and Kehoe (1994, 1999), and Kocherlakota (2010).[1] The main focus of the computational experiments performed, however, has been on the effects of government purchases, public investment and taxes. With the exception of Ardagna (2007) and Fernandez-de-Cordoba *et al.* (2009, 2012), no previous studies have pursued a systematic study of government spending behavior in RBC models with endogenously-determined public employment and wages, as well as labor-intensive government services. After all, most of government consumption in national accounts consists of wage consumption, and only a small part actually corresponds to government purchases. (OECD 2012).[2]

This paper characterizes optimal fiscal policy and then evaluates it relative to the exogenous (observed) one. To this end, a Dynamic Stochastic General Equilibrium (DSGE) model will be st up with a richer government spending side, and an endogenous private-public sector labor choice in particular. The presence of a public employment decision margin, as a separate labor market choice made by the household, has not been sufficiently investigated in such setups properly. In addition, very few books on labor economics, with the exception of Bellante and Jackson (1979), adequately discuss public sector labor markets. This is regrettable, as governments, as the largest employers in Europe, usually have control over the wages, at least those in the public sector. In periods of fiscal contraction, wage- and employment cuts are often used as instruments to control wage bill spending. When incorporated in an RBC model, such wage-setting powers of the government could produce additional interactions between model variables. Thus, the paper argues that if public sector labor choice is ignored, then important effects on allocations and welfare, driven by government

[1] This paper abstracts away from nominal rigidities. Readers interested in studying the effects of fiscal policy in frameworks with price stickiness should consult Burnside *et al.* (2004), Pappa (2004), Gali *et al.* (2007), and the references therein.

[2] Rogoff (2010) also asserts that "it has to matter greatly what the government is spending money on." Reis (2010) adds to that that "the mechanism by which government policy stimulates the economy in standard models is a caricature of reality at best... Thinking harder about how is it that government consumption affects decisions to invest and work will require a transformation in these models, but the toolkits that macroeconomists use leaves much room for creativity."

wage-setting and household's hours decisions, will be missed. Therefore, it will be shown that aggregate labor market policies are also important for public finance management as well. The interplay between these two is the niche in the fiscal policy literature where this paper aims to position itself.

This paper goes on to propose optimal public/private employment- and wage ratios. These labor market dimensions are relevant for policy-makers because many economists since the 1970s have claimed that public administration is bloated, and have argued that government wages in continental Europe are too generous relative to measured productivity in the private sector. Since there are no other studies that address these labor aspects of fiscal policy, the work in this paper aims to contribute to the considerable debate on the effects of fiscal policy on aggregate variables. This is followed by a close study of the interactions generated by the presence of a public labor market in a general-equilibrium setup. In particular, the question of the real effects of exogenous and optimal fiscal policy is addressed using a representative-agent framework, where the household chooses hours worked in both sectors. To this end, a Real-Business-Cycle (RBC) model is augmented by a convex cost of working in the government sector. This new component is a useful device in the model, as it produces endogenously-determined public employment and wage rates. The quantitative importance of this transaction cost for the values of allocations in steady-state and their cyclical fluctuations is studied thoroughly. In addition, the paper investigates how fiscal policy instruments should optimally react to technology shocks, and how the responses differ between the exogenous and Ramsey (optimal) policy framework.

The novelty in the otherwise standard setup, the introduction of this government hours friction, is a plausible assumption. Mechanically, the cost representation helps the model to incorporate an important stylized fact: Ehrenberg and Schwarz (1986) and Gregory and Borland (1999) show that in the major EU countries, public sector wages feature a significant positive net mark-up above private sector wages. Furthermore, the convex cost function for varying public hours is broadly consistent with the view that the nature of work in the public sector is inherently different from supplying labor services in the private sector. As a result, the modeling approach generates different disutility of an hour of work across sectors.

The inclusion of such a cost in the otherwise standard RBC model with (exogenous) public employment, *e.g.* Finn (1998) has not been used in the literature before. Nevertheless, it can be justified on many different grounds. For practical purposes, the friction can be interpreted as a transaction (resource) cost that arises from working in the government sector. This real rigidity is due to the fact that for a worker in the public administration, the employment process and the general organization of work in the public sector are different from job seeking-strategy and the nature of working in the private sector. For an aspiring government bureaucrat, a strict pre-specified sequence of career steps should be taken in order to guarantee advance in the service by achieving regular promotions in the hierarchy. This may involve preparing for a civil service entry exam, which is costly in terms of both time and effort, as it may not be directly related to one's university degree. Another negative effect is that this process often leads to employing overqualified personnel. Additionally, in the public administration there is an over-emphasis on status and hierarchy (OECD 1993), as well as anonymity of the individual bureaucrat, who is merely an instrument of the public administration. Furthermore, once employed in the service, the newly-appointed civil servant starts at the lowest pay grade. Promotion is a wait-in-line process, it is automatically triggered with seniority and not directly based on performance.

Furthermore, employees in strategic sectors such as law and order, and security (police officers, fire fighters), do not enjoy certain civil liberties, *i.e.* soldiers and police officers are not allowed to be members of a political party and/or go on strike. In addition, those employees have to be always prepared to act on call in emergency situations outside normal working hours. Moreover, magistrates, such as prosecutors and judges could be exposed to threats and attacks from the criminal underworld. Those professionals are often exposed to higher levels of stress, and face life-threatening situations, especially in countries where the crime rate is high. Furthermore, Donahue (2008) points out that the private sector counterparts of the aforementioned professions, *e.g.* private detectives, security consultants, and notaries, generally enjoy a much more peaceful life, at the expense of a significantly lower wage rate (pp. 55-56).

In contrast, in the private sector, there is strong emphasis on personal contribution and performance; In terms of working patters, statutory working hours for government employees are usually a fixed full 8-hour shift as compared to flexible working time in the private sector. Additionally, private sector also offers more part-time employment opportunities, which can be the preferred option for female workers and single mothers, who want to spend more time with their children. Importantly, tasks in the civil service are to follow regulations and legislation in an impartial and strict manner. Furthermore, the span of power and job boundaries are clearly defined. This way of organizing work, however, is different from the approach adopted in the private sector, where involvement with a problem, customer-based individual approach, initiative-taking and originality in problem-solving are highly-valued.[3]

In effect, all the aforementioned aspects of government work could be viewed as generating hidden per-period resource- or transaction costs. However, the welfare effect of these costs is hard to measure directly. Thus, the paper will adopt Balestrino's (2007) modeling approach, which includes such costs as a term that directly decreases household's utility. In particular, in the model framework, utility costs of working in the public sector will decrease the amount of leisure enjoyed by the household, in a manner reminiscent to Kydland and Prescott (1982).[4,5] Given the case-study evidence in Box (2004) on the hidden costs of working in the public sector in the US, the difference in wages across sectors could be justified, at least partially, as a compensation for the additional transaction cost incurred from government work. However, in contrast to earlier literature, *e.g.* Alesina *et al.* (2002), Algan *et al.* (2002), Forni and Giordano (2003) and Ardagna (2007), in this paper the wage rate difference will persist despite the free mobility of labor between sectors.[6]

Additionally, the specific focus on government spending categories in this paper, and public hours choice in particular, provides a new explanation for the macroeconomic importance of government employment, as a rational supply decision made by the household in a clear-

[3] Independently from the arguments presented above, the time cost introduced in this paper is reminiscent of the psychic cost interpretation discussed in Tinbergen (1985). He argues that there is a strong negative non-monetary benefit of working in the public sector. In particular, job satisfaction experienced by government workers is usually low. Therefore, the modeling choice in this paper could be regarded as incorporating the psychic cost theory, *e.g.* as in Corbi *et al.* (2008), into the RBC framework, and applying it to the public sector labor choice context.

[4] In contrast to Kydland and Prescott (1982), transaction cost will be modeled as a quadratic function of current-period hours, and not as a polynomial function of lagged hours, as is the case in the original article.

[5] An alternative specification, *e.g.* modeling the transaction cost as a consumption/output cost is also possible. In that case, however, the disutility of an hour worked in the model would have been the same across sectors. Since this assumption is in contrast with the evidence provided above, this approach was not pursued further.

[6] There are other modeling approaches, *e.g.* setups that emphasize directed jobs search mechanisms, which are not going to be covered in this paper. Interested readers should consult Quadrini and Trigari (2007), Gomes (2009, 2012) and the references therein.

ing labor market. After all, the strong positive trend in public employment is a common stylized pattern observed in the post-WWII data series in the major European countries (OECD 2011). Earlier studies by Finn (1998), Cavallo (2005) and Linnemann (2009), however, circumvent the problem of optimal choice of hours between private and public sector by modeling public employment as a stochastic process that approximates data behavior.

The recent study by Fernandez-de-Cordoba *et al.* (2009, 2012) takes an alternative approach: Their model endogeneizes public hours using political-economy arguments as a major factor behind public employment dynamics. In their setup, employment in the government sector is an optimal decision made by a monopolistic public sector union, and not by the household, though. The union, being a special-interests group, is assumed to optimize over a weighted average of the public wage rate and public employment. In addition, the union's objective function is maximized subject to the government budget constraint, where government budget is balanced in every period. In the model, spending on government wages is productive, as public employment produces a positive externality on aggregate output. However, Fernandez-de-Cordoba *et al.* (2009, 2012) consider only exogenous income tax rate shocks, and leave outside the scope of their study interesting issues such as optimal tax-rate determination, public-wage- and employment-setting possibilities. Thus, previous literature has not adequately addressed either the household's choice at the margin between entering the public or the private sector, or the optimal policy framework, in which a benevolent government chooses all spending components in addition to the tax rates.[7] All these structural/labor-market aspects of public finance and public policy represent a gap in the literature, which the research in this paper aims to fill.

The second novelty in the framework, which adds value to earlier studies, is the more interesting and meaningful role attributed to government employees. In particular, the study models in greater detail the mechanism of public good provision. The setup models the government as an employer, needing labor hours to provide public goods. In contrast to Cavallo (2005) and Linnemann (2009), labor is combined with government capital (instead of government purchases) to produce valuable government services. Therefore, government

[7] The model in Fernandez-de-Cordoba *et al.* (2009, 2012) does not satisfy Hagedorn's (2010) conditions for the existence of a "sensible" Ramsey equilibrium, *i.e.* one that satisfies the non-negativity conditions. Strictly speaking, no direct mapping exists between the exogenous policy case and the optimal policy framework in their model.

investment is a productive government spending category in the setup, and public sector wage consumption is not entirely wasteful. Importantly, when hiring workers, the government will be able to set the public sector wage rate, an assumption which is consistent with data, *e.g.* Perez and Schucknecht (2003).

As pointed out in Ross (1985), many government programs are indeed labor-intensive, such as law and order, public administration, education, and health- and social care. Moreover, public administration, which is in charge of enforcing various rules and regulations, represents the biggest category of public employees in data that provides labor-intensive services. The other input in the public good production, government capital, will be an aggregate category that includes hospitals, schools, administrative buildings, infrastructure and equipment, etc. Thus, the paper expands on earlier research by endogeneizing public goods provision, as well as the determination of the wage rate and employment in the public sector. Furthermore, the presence of both public hours and government investment as inputs in the production of government services will have important external effects on the economy: In a competitive equilibrium, the household will ignore the utility-enhancing effect of more public hours supplied, which increases the level of public good provided. The same outcome occurs with public investment, which is taken by households as being set exogenously.

After presenting and discussing the benchmark exogenous policy case, this paper proceeds to study public employment, wage rate and government investment determination, as well as the level of public services provision, when all policy variables are optimally chosen by a benevolent government. In particular, under the optimal (Ramsey) policy regime, the benevolent government will choose the socially optimal levels of both private allocations and the public good. However, in a second-best world, the benevolent government will still finance its expenditure using proportional taxation, and remedy the existing distortions at the cost of introducing new ones. The comparison across the two regimes focuses on both the long-run behavior of government spending and the transitional dynamics of fiscal policy instruments and the other allocations in the economy. The model is calibrated to German data, which features both a significant public wage rate premium, and a large public employment share, and thus are a good testing ground for the theory.

Similar to earlier literature, *e.g.* Judd (1985), Chamley (1986), Zhu (1992), Ljungqvist and Sargent (2004), and Kocherlakota (2010), allowing for fiscal interventions in an RBC framework creates interesting trade-offs. On the one hand, public wage bill tends to increase welfare by providing higher consumption. In addition, higher public employment, government investment and public capital increase the level of utility-enhancing productive government services. On the other hand, more hours spent in the public sector, together with the transaction costs associated with government work, decrease household's utility from leisure. Furthermore, the proportional taxes on labor and capital are known to distort incentives to supply labor in the private and public sectors, and to accumulate physical capital. Therefore, higher taxes reduce consumption, which in turn lowers welfare. Lastly, despite the presence of public employment and government services in the model, the public finance problem is still to choose labor and capital tax rates to finance total government expenditure, while at the same time minimizing the allocative distortions created in the economy, as a result of the presence of proportional taxation.

The main findings from the computational experiments performed in this paper are: (i) as in Judd (1985), Chamley (1986) and Zhu (1992), the optimal steady-state capital tax rate is zero, as it is the most distortionary tax to use; (ii) A higher labor tax rate is needed to compensate for the loss in capital tax revenue; (iii) Under the optimal policy regime, public sector employment is lower. As a result, government employees are more valuable, and receive higher wages. (iv) The government wage bill is smaller, while public investment is three times higher than in the exogenous policy case. In other words, the model predicts that employment in Germany is too high, government employees are underpaid, and too little is invested in public capital; (v) The benevolent Ramsey planner substitutes labor for capital in the public services production, and private output.

The rest of this paper is organized as follows: Section 2 describes the model framework, Section 3 lays out the equilibrium system, Section 4 present the calibration, the steady-state model solution and some comparative statics. Sections 5 provides the model solution and discusses transitional dynamics of the model variables in response to technological innovations. Sections 6 proceeds with the optimal taxation (Ramsey) policy problem. Section 7 evaluates both the transitional dynamics and the long-run effects on the economy. Section 8 acknowledges the limitations of the study, and section 9 concludes the chapter.

2 Model setup

The model features a representative household, as well as a representative firm. The household owns the capital, which it supplies to the firm. Next, the household's unit endowment of time can be supplied to the private sector (firm), public sector, or enjoyed in the form of leisure. Due to the presence of additional transaction costs associated with government work, hours supplied in the public sector impose an additional utility cost on the household. The perfectly-competitive firm produces output using labor and capital, while the government hires labor and combines it with public capital to produce valuable government services. To finance the public wage bill, government investment and transfers, tax revenues from labor and capital income are collected, where the wage rate in the public sector is determined residually to balance the government budget in every period.

2.1 Households

There is an infinitely-lived representative household in the model economy, and no population growth. The household maximizes the following expected utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^h, L_t^h, S_t^g), \tag{2.1.1}$$

where E_0 is the expectation operator as of period 0; C_t^h, L_t^h and S_t^g are household's consumption, leisure and per household consumption of government services enjoyed at time t, respectively. The parameter β is the discount factor, $0 < \beta < 1$. The instantaneous utility function $U(.,.,.)$ is increasing in each argument and satisfies the Inada conditions. The particular form chosen for utility is:

$$U(C_t^h, L_t^h, S_t^g) = \psi_1 \ln C_t^h + \psi_2 \ln L_t^h + \psi_3 \ln S_t^g \tag{2.1.2}$$

where the parameters ψ_1, ψ_2 and $\psi_3 \equiv 1 - \psi_1 - \psi_2$ denote the weights attached to the utility of consumption, leisure and government services (public good consumption), respectively, $0 < \psi_1, \psi_2, \psi_3 < 1$. The level of government services is taken as given by the household.[8]

[8]The logarithmic specification for consumption is in line with risk-neutrality, *i.e.*, it can be interpreted as households pooling resources from public and private sector together, as in Merz (1995), and completely insuring themselves. The logarithmic form for leisure was chosen to avoid the issue of specifying Frisch elasticity of labor supply in the private *vs.* public sector.

The household has an endowment of one unit of time in each period t, which is split between work, N_t^h and leisure, L_t^h, so that

$$N_t^h + L_t^h = 1. \tag{2.1.3}$$

The household can supply hours of work in the public sector, N_t^{gh}, or in the private one, N_t^{ph}.[9] The wage rate per hour of work in public sector is w_t^g, and w_t^p in the private sector, which will be allowed to differ from one another. In particular, when the household chooses N_t^{gh}, it incurs an additional convex transaction cost, $\gamma(N^{gh})^2$, measured in terms of time, which will also depend on the level of public employment (where $\gamma > 0$).[10] Thus, the effective leisure for the household becomes

$$L_t = 1 - N_t^{ph} - N_t^{gh} - \gamma(N_t^{gh})^2. \tag{2.1.4}$$

Observe that in the polar case, when there is no transaction cost, or $\gamma = 0$, the two wage rates are equal $w_t^p = w_t^g$, because the disutility of an hour worked is equalized across sectors. Thus the model collapses to the representation used in Finn (1998), Cavallo (2005) and Linnemann (2009). Furthermore, Tinbergen (1985) argues that public sector employees suffer from lower job satisfaction, compared to their private sector counterparts, and incur a per-period psychic (non-monetary) cost of working for the government (p.36). Therefore, the usefulness of this particular modeling choice is that it generates a friction between the marginal disutility of an hour worked in the public and private sectors, which is generally consistent with the evidence and also helps the framework to accommodate the different wage rates in the two labor markets.[11,12]

In addition to the labor income generated from supplying hours in the two sectors, the representative household saves by investing in private capital I_t^h. As an owner of capital, the household receives interest income $r_t K_t^{ph}$ from renting the capital to the firms; r_t is the return to private capital, and K_t^{ph} denotes private capital stock in the beginning of period t.

[9] In this paper, due to the normalization of total population to unity, "hours" and "employment" will be used interchangeably.

[10] This is also consistent with the evidence that labor flows from the public to the private sector are much smaller than those from the private to the public sector.

[11] In addition, when both public and private hours are chosen by the households, the model will be consistent with the findings that public employment crowds out private employment, *e.g.* Malley and Moutos (1998) on Sweden, and Algan *et al.* (2002) on OECD countries.

[12] On a more abstract level, the convex cost could be interpreted as a short-cut, which substitutes for an explicit government optimization problem. Examples of the latter are Niskanen (1971) and Ardagna (2007).

Finally, the household owns all firms in the economy, and receives all profit (Π_t^h) in the form of dividends. Household's budget constraint is

$$C_t^h + I_t^h \leq (1-\tau_t^l)[w_t^p N_t^{ph} + w_t^g N_t^{gh}] + (1-\tau_t^k) r_t K_t^{ph} + G_t^t + \Pi_t^h, \tag{2.1.5}$$

where τ_t^l, τ_t^k are the proportional tax rates on labor and capital income, respectively, and G_t^t denotes the level of lump-sum government transfers per household.

Household's private physical capital evolves according to the following law of motion

$$K_{t+1}^{ph} = I_t^h + (1-\delta^p) K_t^{ph}, \tag{2.1.6}$$

where $0 < \delta^p < 1$ denotes the depreciation rate on private physical capital.

The representative household acts competitively by taking prices $\{w_t^p, w_t^g, r_t\}_{t=0}^{\infty}$, tax rates $\{\tau_t^l, \tau_t^k\}_{t=0}^{\infty}$, policy variables $\{K_{t+1}^g, S_t^g, G_t^t\}_{t=0}^{\infty}$ as given, and chooses allocations $\{C_t^h, N_t^{ph}, N_t^{gh}, I_t^h, K_{t+1}^{ph}\}_{t=0}^{\infty}$ to maximize Equation (2.1.1) subject to Equations (2.1.2)-(2.1.6), and initial conditions for private and public physical capital stocks, $\{K_0^{ph}, K_0^g\}$.

The optimality conditions from the household's problem, together with the transversality condition (TVC) for physical capital, are as follows: [13]

$$C_t\colon \ \frac{\psi_1}{C_t^h} = \Lambda_t \tag{2.1.7}$$

$$N_t^p\colon \ \frac{\psi_2}{1 - N_t^{ph} - N_t^{gh} - 2(N_t^{gh})^2} = \Lambda_t (1-\tau_t^l) w_t^p \tag{2.1.8}$$

$$N_t^g\colon \ \frac{\psi_2}{1 - N_t^{ph} - N_t^{gh} - 2(N_t^{gh})^2}[1 + 2\gamma N_t^{gh}] = \Lambda_t (1-\tau_t^l) w_t^g \tag{2.1.9}$$

$$K_{t+1}\colon \ \beta E_t \Lambda_{t+1}\Big[(1-\tau_{t+1}^k) r_{t+1} + (1-\delta^p)\Big] = \Lambda_t \tag{2.1.10}$$

$$\text{TVC:} \ \lim_{t\to\infty} \beta^t \Lambda_t K_{t+1}^p = 0, \tag{2.1.11}$$

[13] Detailed derivations are provided in Appendix 11.1.2.

where Λ_t is the Lagrange multiplier on the household's budget constraint. The household equates marginal utility from consumption with the marginal cost imposed on its budget. Private and public hours are chosen so that the disutility of an hour of work in each sector at the margin equals the after-tax return to labor in the corresponding sector. Next, the Euler equation describes the optimal private capital accumulation rule, and implicitly characterizes the optimal consumption allocations chosen in any two adjacent periods. The last expression is the TVC, imposed to ensure that the value of the physical capital that remains at the end of the optimization horizon is zero. This boundary condition ensures that the model equilibrium is well-defined by ruling out explosive solution paths.

2.2 Firms

There is a representative private firm in the model economy as well. It produces a homogeneous final product using a production function that requires private physical capital, K_t^p, and labor hours N_t^p. The production function is as follows

$$Y_t = A_t (N_t^p)^\theta (K_t^p)^{1-\theta}, \tag{2.2.1}$$

where A_t measures the level of total factor productivity at time t; $0 < \theta, (1-\theta) < 1$ are the productivity of labor and capital, respectively.

The representative firm acts competitively by taking prices $\{w_t^p, w_t^g, r_t\}_{t=0}^\infty$ and policy variables $\{\tau_t^k, \tau_t^l, K_{t+1}^g, S_t^g, G_t^t\}_{t=0}^\infty$ as given. Accordingly, K_t^p, and N_t^p are chosen every period to maximize the firm's static aggregate profit,

$$\Pi_t = A_t (N_t^p)^\theta (K_t^p)^{1-\theta} - r_t K_t^p - w_t^p N_t^p. \tag{2.2.2}$$

In equilibrium, profit is zero. In addition, labor and capital receive their marginal products, i.e [14]

$$w_t^p = \theta \frac{Y_t}{N_t^p} \tag{2.2.3}$$

$$r_t = (1-\theta) \frac{Y_t}{K_t^p}. \tag{2.2.4}$$

[14] Detailed derivations are provided in Appendix 11.1.1.

2.3 Government budget constraint

The government distributes transfers G_t^t to the household, invests G_t^i in public capital K_t^g, and hires labor N_t^g at the public sector wage w_t^g. Public employment and government capital are then combined to provide utility-enhancing government services, S_t^g, according to the following constant-returns-to-scale production function:

$$S_t^g = (N_t^g)^{\alpha}(K_t^g)^{(1-\alpha)}, \tag{2.3.1}$$

where α and $1-\alpha$ denote labor and capital share in government services, respectively, and $0 < \alpha, 1-\alpha < 1$. Since the household takes the level of government services as given, in competitive equilibrium there will be externalities arising from the presence of public employment and investment in the government services production function: More hours in the public sector generate more government services (a higher level of the public good available for public consumption), which increase utility directly. In addition, holding all else equal, an increase in public employment raises welfare indirectly by increasing the after-tax public sector labor income, and hence consumption. Lastly, more hours spent in the public sector decrease the amount of leisure the household can enjoy in a certain period, and thus lower welfare. Therefore, in the general case it is unclear *ex ante* whether public employment creates a positive or a negative externality in the economy.[15]

Next, total government expenditure, $G_t^t + G_t^i + w_t^g N_t^g$, is financed by levying proportional taxes on capital and labor income. Thus, the government budget constraint is[16]

$$G_t^t + G_t^i + w_t^g N_t^g = \tau_t^k r_t K_t^p + \tau_t^l \Big[w_t^p N_t^p + w_t^g N_t^g \Big]. \tag{2.3.2}$$

Next, public capital accumulates according to the following law of motion:

$$K_{t+1}^g = G_t^i + (1-\delta^g)K_t^g, \tag{2.3.3}$$

where $0 < \delta^g < 1$ denotes the depreciation rate on public capital.

The government takes market prices $\{w_t^p, r_t\}_{t=0}^{\infty}$ and allocations $\{N_t^p, N_t^g, K_t^p\}_{t=0}^{\infty}$ as given.

[15] *Ex post*, in the optimal policy framework, it will be shown that the public hours create a negative externality.

[16] Since public sector wage bill appears both as a revenue and expenditure category, the representation above is equivalent to a setup in which the government pays public wages net-of-taxes directly.

It will be assumed that the government chooses the public investment and transfers shares, where $G_t^{iy} = \frac{G_t^i}{Y_t}$ and $G_t^{ty} = \frac{G_t^t}{Y_t}$. Thus, the level of government investment $G_t^i = G_t^{iy} Y_t$, and government transfers $G_t^t = G_t^{ty} Y_t$ will both respond to output.

Only four of the following five policy instruments, $\{\tau_t^k, \tau_t^l, w_t^g, \frac{G_t^i}{Y_t}, \frac{G_t^t}{Y_t}\}_{t=0}^{\infty}$, will be exogenously set. First, government transfers share will be set to match the average public/private employment ratio in data. Next, government investment share, $\frac{G_t^i}{Y_t}$, as well as capital and labor income tax rates will be set equal to the average rates in data. Lastly, public sector wage rate will be determined as a residual to ensure that the government runs a balanced budget in every period. Thus, the government still acts to a certain degree as a regulator of the labor supplied in the public sector.[17]

2.4 Stochastic processes for the exogenous variables

The exogenous stochastic variable is total factor productivity A_t, which is assumed to follow AR(1) processes in logs, in particular

$$\ln A_{t+1} = (1-\rho_a)\ln A_0 + \rho_a \ln A_t + \epsilon_{t+1}^a, \tag{2.4.1}$$

where $A_0 > 0$ is steady-state level of the total factor productivity process, $0 < \rho_a < 1$ is the first-order autoregressive persistence parameter and $\epsilon_t^a \sim iidN(0, \sigma_a^2)$ are random shocks to the total factor productivity progress. Hence, the innovations ϵ_t^a represent unexpected changes in the total factor productivity process.

2.5 Decentralized competitive equilibrium

Given the fixed values of government transfers/output and government investment/output ratios $\{G^{ty}, G^{iy}\}$, the exogenous process followed by total factor productivity $\{A_t\}_{t=0}^{\infty}$, the initial conditions for the state variables $\{A_0, K_0^{ph}, K_0^g\}$, a decentralized competitive equilibrium (DCE) is defined to be a sequence of allocations $\{C_t^h, G_t^i, N_t^{ph}, N_t^{gh}, I_t^h, K_{t+1}^{ph}, K_{t+1}^g, S_t^g, G_t^t\}$ for all h, and prices $\{r_t, w_t^p, w_t^g\}_{t=0}^{\infty}$ such that (i) the representative household maximizes utility;

[17]Note that in general equilibrium, the two wage rates will be inter-related, which is in line with the empirical study in Lamo, Perez and Schucknecht (2007, 2008). Thus, the level of government investment will respond to output. (K_{t+1}^g will be exogenously determined as well, subject to the initial condition K_0^g and the law of motion for G^i.) Note also that the public sector wage rate schedule implicitly determines government's endogenous demand for public labor.

(ii) the stand-in firm maximizes profit every period; (iii) government budget constraint is satisfied in each time period; (iv) all markets clear.[18]

3 Data and model calibration

The model in this paper is calibrated for German data at annual frequency. The choice of this particular economy was made based on the large public employment share, as well as the significant public wage premium observed in this country. Since there is no EU-wide fiscal authority, an individual country was chosen, rather than calibrating the model for the EU Area as a whole. In addition, payment in the public sector in the model is determined not by marginal productivity of labor, but rather by budgetary considerations and thus most likely by non-market factors. Lastly, given the importance of government transfers in matching average employment ratio in data by the model, the calibration for a particular country is preferable as transfers are to a great extent driven by political consideration and are also determined at national-, and not at EU level.

The paper follows the methodology used in Kydland and Prescott (1982), as it is the standard approach in the literature. Both the data set and steady-state DCE relationships of the models will be used to set the parameter values, in order to replicate relevant long-run moments of the reference economy for the question investigated in this paper.

3.1 Model-consistent German data

Due to data limitations, the model calibrated for Germany will be for the period 1970-2007 only, while the sub-period 1970-91 covers West Germany only.[19,20,21] For Germany, data on real output per capita, household consumption per capita, government transfers and population was taken from the World Development Indicators (WDI) database. The

[18]The system of equations that characterizes the DCE is provided in Appendix 11.2.

[19]The time period is particularly suitable for the study of public employment, and government wage bill spending; Hughes (1994), for example, argues that "[i]n the 1970s, intellectual arguments were mounted by conservative economists that government was the economic problem restricting economic growth and freedom."(p. 11)

[20]The values of the model parameters used in the calibration do not differ significantly from those computed for periods 1970-91, and 1991-2007 separately.

[21]Despite the fact that Germany is an open economy, here the focus is on the closed-economy case, as fiscal policy is mainly serves domestic interests. Thus, the open-economy dimension is left for future research.

OECD statistical database was used to extract the long-term interest rate on 10-year generic bonds, CPI inflation, average annual earnings in the private and public sector, average hours, private, public and total employment in Germany. Investment and capital stock series were obtained from the EU Klems database (2009). German average annual real public compensation per employee was estimated by dividing the real government wage bill (OECD 2011) by the number of public employees.

3.2 Calibrating model parameters to German data

In German data, the average public/private employment ratio over the period 1970-2007 is $n^g/n^p = 0.17$, and the average public/private wage ratio is $w^g/w^p = 1.20$. Next, the average effective tax rates on labor and physical capital, obtained from McDaniel's (2009) dataset are $\tau^l = 0.409$ and $\tau^k = 0.16$, respectively. McDaniel's approach was preferred to that used by Mendoza et al. (1984) and the subsequent updates, *e.g.* Martinez-Mongay (2000), Carey and Tchilinguirian (2000) and Carey and Rabesona (2002), due to the more careful treatment of property and import taxes in the former. The labor share, $\theta = 0.71$, was computed as the average ratio of compensation of employees in total output. Alternatively, average capital share, $1 - \theta = 0.29$, can be obtained as the mean ratio of gross capital compensation in output from EU Klems Database (2009). Private capital depreciation rate was found to be $\delta^p = 0.082$, while public capital depreciation rate is $\delta^p = 0.037$ over the period.

The discount rate $\beta = 0.979$ was calibrated from the steady-state consumption Euler equation to match the average private capital-to-output ratio in data. Parameter $\alpha = 0.62$, which measures the weight on public sector hours in the public good production is obtained as the average ratio of public sector wage bill to total government expenditure less transfers and subsidies, as in Cavallo (2005) and Linnemann (2009). The value is consistent with OECD (1982) estimates for the period 1960-78 for Germany, which was obtained from a log-linear regression estimation. Additionally, the calibrated value of public capital elasticity, $1 - \alpha = 0.38$, is consistent with the government capital effect estimated in Aschauer (1989) and Hjerppe *et al.* (2006). In the exogenous policy setup, parameter α does not affect allocations, since the household ignores the externality. Thus, the level of government services will be residually determined. Nevertheless, there is a negative monotone relationship between public hours elasticity and welfare.

Next, using the estimate obtained in Finn (1994), the weight attached to productive government services in utility is set equal to $\psi_3 = 0.16$.[22] This value is consistent with the one used in Klein *et al.* (2008) for the weight attached by the household to utility derived from the consumption of the public good. The weight on private consumption in the household's utility function in this paper was then set equal to $\psi_1 = 0.31$. This produces a ratio $\psi_1/\psi_3 = 1.94$, which is also consistent with the ratios in Bouakez and Rebel (2007), Leeper *et al.* (2009) and Conesa *et al.* (2009), who argue that the private consumption good is on average twice more valuable for the household, compared to the public good. Next, the weight on utility is determined residually as $\psi_2 = 1 - \psi_1 - \psi_3$. The calibrated value for ψ_2 is also in line with earlier RBC studies, which usually attach a weight on leisure, which is twice as large as the weight attached to private consumption in the household's utility function. In this model, the ratio in question is $\psi_2/\psi_1 = 1.8$. Note that in contrast to Kydland (1995), ψ_1 was set slightly higher than the average steady-state total hours of work in data as a share of total hours available, $n = 0.296$, to account for the presence of transaction costs, which decrease the effective leisure.[23] Nevertheless, total employment is consistent with the estimates of the fraction of time spent working in Ghez and Becker (1975). Together with the employment ratio, this yields the model-consistent steady-state values for private and public hours, $n^p = 0.253$ and $n^g = 0.043$, respectively.

Next, the scale parameter $\gamma = 2.576$ in the public employment convex utility cost was set to match the average public/private wage ratio from data in steady-state. The wage ratio was chosen as a target, as the higher average wage in the public sector is viewed to a certain degree as a compensation for the transaction costs incurred from government work. In line with the RBC literature, the steady-state level of technology, A, is normalized at unity.

Total factor productivity moments, $\rho^a = 0.9427$ and $\sigma^a = 0.0131$, were obtained in several steps: First, using the model's aggregate production function specification and data series for physical capital and labor, Solow residuals (SR) were computed in the following

[22] Robustness checks with $\psi_3 = 0.1$ and $\psi_3 = 0.2$ to reflect the standard error of the estimation performed in Finn (1994) do not significantly affect either the steady-state results, or the transitional dynamics of the model.

[23] In this setup with two types of endogenously-determined hours and public hours transaction costs, it is not possible to derive labor supply functions explicitly.

way:

$$\ln SR_t = \ln y_t - (1-\theta)\ln k_t^p - \theta \ln n_t^p. \tag{3.2.1}$$

The logged series are then regressed on a linear trend ($b > 0$) to obtain

$$\ln SR_t = bt + \epsilon_t^{SR}. \tag{3.2.2}$$

Observe that the residuals from the regression above,

$$\epsilon_t^{SR} = \ln SR_t - bt \equiv \ln a_t, \tag{3.2.3}$$

represent the stationary, or detrended, component of the logged TFP series.

Next, the AR(1) regression

$$\ln a_t = \beta_0 + \beta_1 \ln a_{t-1} + \epsilon_t^a \tag{3.2.4}$$

was run using ordinary least squares (OLS) to produce the estimates (denoted by the "hat" symbol) for the persistence and standard deviation parameters of the total factor productivity process to be used in the calibration of the model. In particular,

$$\hat{\beta}_1 = \rho^a \tag{3.2.5}$$

$$\hat{\epsilon}_t^a \sim N(0, \sigma_a^2). \tag{3.2.6}$$

Table 1 on the next page summarizes all the model parameters used in the calibration.

Table 1: Model Parameters

Param.	Value	Definition	Source
β	0.979	Discount factor	Calibrated
θ	0.710	Labor income share	Data average
$1-\theta$	0.290	Capital income share	Calibrated
δ^p	0.082	Depreciation rate on private capital	Data average
δ^g	0.037	Depreciation rate on government capital	Data average
ψ_1	0.310	Weight on consumption in utility	Set
ψ_2	0.530	Weight on leisure in utility	Calibrated
ψ_3	0.160	Weight on government services in utility	Set
γ	2.576	Scale parameter for public hours transaction cost	Calibrated
α	0.620	Labor share in public services production	Data average
$1-\alpha$	0.380	Govt. capital share in public services production	Calibrated
τ^k	0.160	Effective tax rate on capital income	Data average
τ^l	0.409	Effective tax rate on labor income	Data average
A	1.000	Steady-state level of total factor productivity	Set
ρ^a	0.943	AR(1) parameter total factor productivity	Estimated
σ_a	0.013	SD of total factor productivity innovation	Estimated

4 Steady state results

Once model parameters were obtained, the unique steady-state of the system was computed numerically[24]for the Germany-calibrated model. Results are reported in Table 3 on the next page. The net returns to public and private labor,

$$\tilde{w}^p = (1 - \tau^l)w^p \quad \text{and} \tag{4.0.7}$$

$$\tilde{w}^g = (1 - \tau^l)w^g, \tag{4.0.8}$$

respectively, as well as the after-tax net of depreciation real return to capital,

$$\bar{r} = (1 - \tau^k)(r - \delta^p), \tag{4.0.9}$$

are also reported. These prices will be useful for comparisons across tax regimes in the analysis that follows. Finally, social welfare U is the discounted stream of instantaneous utilities evaluated at the steady-state allocations of consumption, hours and government services in every period.

The model performs relatively well *vis-a-vis* data. It slightly overestimates average consumption and underestimates the investment shares in output. This mismatch is due to the fact that the model treats government wage bill consumption as a transfer payment, and not as final public consumption, as is the case in the national accounts. This is not an issue here as the main objective of the model is to replicate the stylized facts in the labor markets. However, the model captures accurately the long-run after-tax capital return, where the latter is proxied by the average return on 10-year generic bonds net of CPI inflation. Moreover, the imputed government services is also predicted to make a significant share of output.[25]

Along the labor market dimension, the average time spent working is also close to its empirical counterpart over the period. By construction, the model was set to match the wage and employment ratios in data. Given the focus on the labor effects of fiscal policy, exogenous policy framework was calibrated to reproduce those stylized facts in the steady state, as this framework will provide an important benchmark for fiscal policy experiments in the later sections. Next, the ratio of the time cost of working in the public sector relative to public

[24]Appendix 11.3 summarizes the steady-state DCE system.

[25]In addition, this figure is close to the average government consumption-to-output ratio in German data (0.20).

Table 2: Data averages and long-run solution: exogenous policy

	GE Data	Exogenous
c/y	0.590	0.784
i/y	0.210	0.192
g^i/y	0.023	0.023
k^p/y	2.350	2.346
k^g/y	0.630	0.630
s^g/y	N/A	0.224
g^t/y	0.170	0.228
$w^p n^p/y$	0.710	0.710
$w^g n^g/y$	0.130	0.145
rk/y	0.290	0.290
w^g/w^p	1.200	1.200
n	0.296	0.266
n^p	0.253	0.227
n^g	0.043	0.039
n^g/n^p	0.17	0.170
$\gamma(n^g)^2/n^g$	N/A	0.199
$\bar{r}$	0.036	0.035
U	N/A	-47.91

labor supply is a non-trivial figure in steady-state, $\frac{\gamma(n^g)^2}{n^g} = 0.20$. Thus, transaction costs in the government sector are likely to be an important factor for a worker who is considering whether to enter the public sector labor market or the private sector one. In other words, these additional transaction costs should be incorporated in a(n) (equilibrium) wage offer accepted by a rational worker, who decides to supply labor services in the public sector.[26]

In the next section, the public/private wage ratio target is relaxed in order to perform an important comparative-statics exercise. The focus of the analysis will fall on the new parameter, γ, as its properties within the otherwise standard model framework have not been investigated so far. In particular, the computational experiments in the following section

[26]The average magnitude of the transaction cost is comparable to the size of another type of transaction costs in labor economics literature, namely the cost of commuting, as shown in Ehrenberg and Smith (2003).

will aim to uncover whether a systematic relationship exists between transaction cost scale parameter γ and the steady-state values of the other model variables.

4.1 Comparative Statics: changes in transaction cost parameter

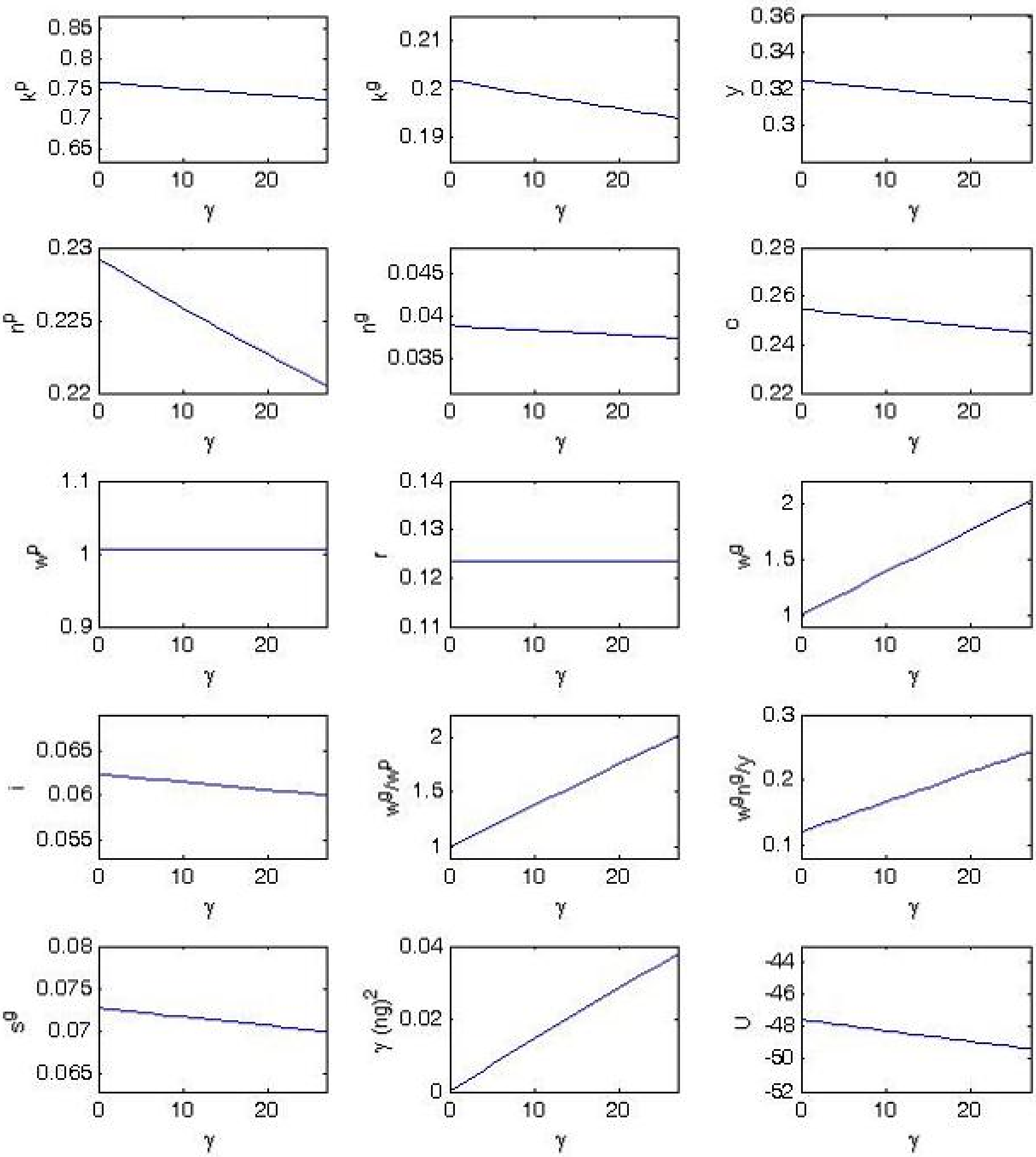
This section investigates the dependence of the steady-state results obtained in the previous section on the value of γ. In particular, the changes in this new parameter could be mapped to concrete institutional reforms in the government administration. For example, a decrease in the value of parameter γ can be interpreted as a reform in the civil service sector, which decreases transaction cost of working in the public sector, for any chosen amount of hours. In practice, reform measures could include, but are not limited to, new human resource practices in the public sector that try to follow as closely as possible the practices used in the private sector. For example, the measures could involve the introduction of flexible hours and performance pay for government employees, as well as a "fast track career path" in the public sector. Other possible measures include decentralization, job evaluation, the breakdown of old established work demarcations and job boundaries, and greater use of public relationships (PR) and information technology (IT) (OECD 1993).

In the literature, this move toward market-based labor relationships in the 1990s was called "New Public Management"(NPM) and discussed at length in Hughes (1991), Ferlie *et al.* (1996) and Nolan (2001). In particular, the major Western governments were trying to apply the best business practices used in the private sector into the public sector. The NPM approach proposed that public managers should be put in charge of the size, evaluation and qualification level of the workforce in their respective government agencies. The expected effect of the NPM was to empower heads of agencies to adjust employment size and skill level and implement best business practices, or the so-called "market-based public administration."

Therefore, in the model setup, all those measures listed above would effectively decrease the hidden costs of working in the government for any amount of public hours chosen by the household. The fall in the transaction cost is expected to affect consumption, private hours, capital, and output in a positive way. In addition, a fall in γ is expected to drive down the public wage premium, as well as the public wage bill share in output. As shown in Fig. 1 on the next page, the model is consistent with the economic intuition. The computational experiment performed shows a negative monotone relationship between the transaction cost scale parameter γ (or equivalently, total transaction cost $\gamma(n^g)^2$) and the model variables. Only public/private wage ratio and the public wage bill share in output positively co-move with the cost parameter. Clearly, a reduction in the waste, embodied in the government-work friction, is welfare-improving.

In the next section, the model's behavior outside of the steady-state is investigated. In particular, the transitional dynamics of the model economy and the responses of the variables in the face of a surprise technology innovation is presented and discussed. Moreover, a robustness check will be performed for the effect of γ on the cyclical fluctuations exhibited by the model variables as well.

Figure 1: Comparative Statics: exogenous policy case



5 Model solution and impulse responses

Since there is no closed-form general solution for the model in this paper, a typical approach followed in the RBC literature is to log-linearize the stationary DCE equations around the steady state, where $\hat{x}_t = \ln x_t - \ln x$, and then solve the linearized version of the model. The log-linearized system of model equations is derived and summarized in Appendix 13.4-13.5. The linearized DCE system can be represented in the form of first-order linear stochastic difference equations as in King, Plosser and Rebello (1988):

$$\mathbf{A}E_{t+1}\mathbf{\hat{x}_t} \;=\; \mathbf{B}\mathbf{\hat{x}_t} + \mathbf{C}\varepsilon_{\mathbf{t}}, \tag{5.0.1}$$

where $\mathbf{A}, \mathbf{B}$, and $\mathbf{C}$ are coefficient matrices, $\varepsilon_{\mathbf{t}}$ is a matrix of innovations, and $\mathbf{\hat{x}_t}$ is the stacked vector of state (also called 'predetermined') variables, $\mathbf{\hat{s}_t} = \begin{bmatrix} \hat{a}_t & \hat{k}_t^p & \hat{k}_t^g \end{bmatrix}'$, and control variables, $\mathbf{\hat{z}_t} = \begin{bmatrix} \hat{y}_t & \hat{c}_t & \hat{i}_t & \hat{n}_t & \hat{n}_t^p & \hat{n}_t^g & \hat{w}_t^p & \hat{w}_t^g & \hat{\lambda}_t & \hat{g}_t^t & \hat{g}_t^i & \hat{s}_t^g \end{bmatrix}'$. Klein's (2000) generalized eigenvalue decomposition algorithm was used to solve the model. Using the model solution, the impulse response functions (IRFs) were computed to analyze the transitional dynamics of model variables to a surprise innovation to productivity.[27]

5.1 The Effect of a positive productivity shock

Figure 1 shows the impact of a 1% surprise TFP innovation on the model economy. There are two main channels through which the TFP shock affects the model economy. A higher TFP increases output directly upon impact. This constitutes a positive wealth effect, as there is a higher availability of final goods, which could be used for private and public consumption, as well as for investment. From the rules for the government investment and transfers in levels, a higher output translates into higher level of expenditure in each of the two categories (not pictured). Next, the positive TFP shock increases both the marginal product of capital and labor, hence the real interest rate (not pictured) and the private wage rate increase. The household responds to the price signals and supplies more hours in the private sector, as well as increasing investment. This increase is also driven from both the intertemporal consumption smoothing and the intra-temporal substitution between private consumption and leisure. In terms of the labor-leisure trade-off, the income effect ("work

[27] Sensitivity of IRFs for different values of γ are presented in Appendix 11.6, the dependence of second momenets on the magnitude of the transaction cost parameter, and utility weights are presented in Appendix 11.7.1-11.7.2.

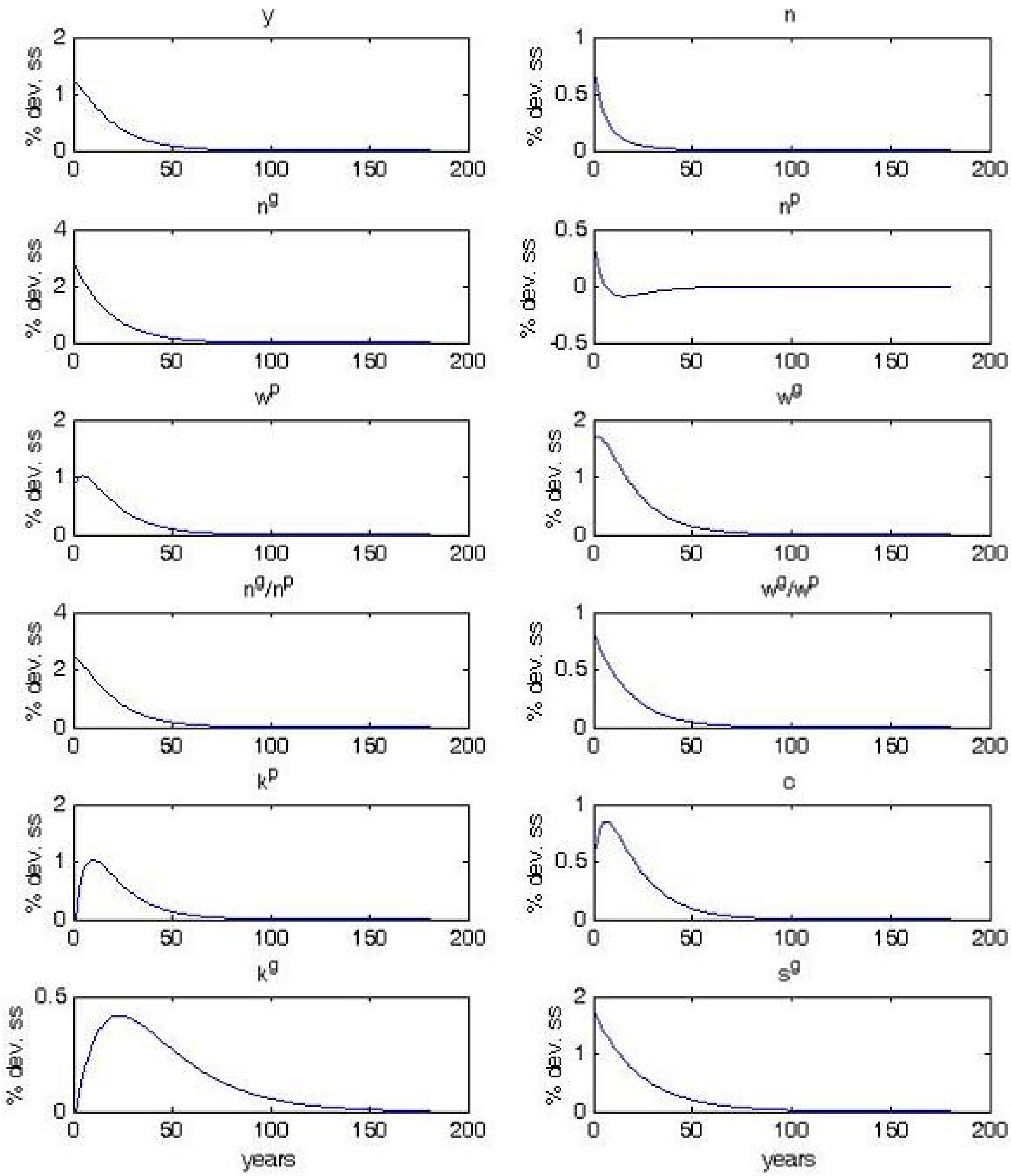
more") produced by the increase in the private wage dominates the substitution effect ("work less"). Furthermore, the increase of private hours expands output even further, thus both output and government spending categories increase more than the amount of the shock upon impact. Over time, as private physical capital stock accumulates, marginal product of capital falls, which decreases the incentive to invest. In the long-run, all variables return to their old steady-state values. Due to the highly-persistent TFP process, the effect of the shock is still present after 50 periods.

With regard to public sector labor dynamics, however, there is the additional effect of an increase in productivity leading to an increase in income and consumption. Higher income and consumption lead to larger tax revenue. In particular, the growth in government revenue exceeds the increase in the fiscal spending instruments. As a result, the additional funds available are spent on government investment, transfers and the wage bill. In turn, the increase in the latter leads to an expansion in both public sector wage and hours.[28] In addition, the model in this paper generates an interesting dynamics in the wage and hours ratio, which is not present in models with stochastic public employment, such as Finn (1998), Cavallo (2005) or Linnemann (2009). The two wage rates, as well as the two types of hours move together, making the model consistent with the empirical evidence presented in Lamo, Perez and Schuknecht (2007, 2008).

Overall, a positive innovation to total factor productivity has a positive effect on the allocations and prices in the economy. The novelty is that the endogenous public sector hours model generates an important difference in the composition of household's labor income with

[28]The effect on total hours in Germany is very small. Nonetheless, the increase in hours is much larger in magnitude than the responses reported in Fernandez-de-Cordoba *et al.* (2009, 2012) and the model with public sector union presented in Ch. 1.

Figure 2: Impulse Responses to a positive 1% productivity shock in Germany



the public sector share increasing at a much faster rate than the private sector labor income. Another important observation to make is that the TFP shocks, being the main driving force in the model, induce pro-cyclical behavior in public wage and hours. The shock effects are smaller and variables reach their peak very quickly. This means the impulse effect dies out relatively fast. Still, the transition period can still take up to 100 years. This illustrates the important long-run effects of TFP shocks on the wage- and hours ratios.

However, an important limitation of the exogenous policy analysis performed so far is that the effect of each tax rate was taken in isolation. In addition, government investment share was exogenously set, and public wage rate was a residually-determined instrument that always adjusted accordingly to balance the budget. In effect, by construction all interaction between the two tax rates was precluded, by fixing each to the corresponding average effective rate in data over the chosen period of study. These restrictions will be lifted in the next section, and the optimal fiscal policy framework will be considered in an environment, in which the two tax rates, government investment, public employment (and hence also government services) and public sector wage rate, will be chosen jointly by a benevolent government, whose preferences are perfectly aligned with the household's utility function.

6 The Ramsey problem (Optimal fiscal policy under full commitment)

In this section, the government will assume the role of a benevolent planner, who takes into account that the representative household and the firm behave in their own best interest, taking fiscal policy variables as given. The instruments under government's control in this section are labor and capital tax rates, next-period public capital (hence public investment), public employment and public sector wage rate. Government transfers will be held fixed at the level from the exogenous policy case. It is assumed that only linear taxes are allowed, and that the government can credibly commit to those. Thus, given the restriction to a set of linear distortionary tax rates, only a second-best outcome is feasible. However, the emphasis on the second-best theory makes the setup more realistic, and thus can be taken as a better approximation to the environment in which policymakers decide on a particular fiscal policy.

It is important to emphasize that each set of fiscal policy instruments implies a feasible allocation that fully reflects the optimal behavioral responses of the household and firm. Alternatively, each set of fiscal policy instruments can be thought of generating a different competitive equilibrium allocation, i.e. allocations and prices are contingent on the particular values chosen for the fiscal instruments. The difference from the analysis performed so far in the paper, is that in Ramsey framework, the government chooses all instruments, instead of taking them as being exogenous. At the same time, the government also chooses optimally the allocations of agents, as dictated by the dual approach to the Ramsey problem as in Chamley (1986).[29] It is also be assumed that the government discounts time at the same rate as the representative household. The constraints which the government takes into account when maximizing household's welfare include the government budget constraints, and the behavioral responses of both the household, and the firm. These are summarized in the DCE of the exogenous fiscal policy case.[30,31] In other words, in the dual approach of the Ramsey problem, which will be utilized in this section, the choice variables for the government are $\{C_t, N_t^p, N_t^g, K_{t+1}^p, K_{t+1}^g, w_t^p, w_t^g, r_t\}_{t=0}^{\infty}$ plus the two tax rates $\{\tau_t^l, \tau_t^k\}_{t=0}^{\infty}$.[32] The initial conditions for the state variable $\{A_0, K_0^p, K_0^g\}$, as well as the sequence of government transfers $\{G^t\}_{t=0}^{\infty}$ and the process followed by total factor productivity $\{A_t\}_{t=0}^{\infty}$ are taken as given.

Following the procedure in Chamley (1986) and Ljungqvist and Sargent (2004), the Ramsey problem will be transformed and simplified, so that the government chooses after-tax interest rate $\tilde{r}_t$ and wage rates $\tilde{w}_t^p$ and $\tilde{w}_t^p$ directly, instead of setting tax rates and prices separately, where

$$\tilde{r}_t \equiv (1-\tau_t^k) r_t \tag{6.0.1}$$

$$\tilde{w}_t^p \equiv (1-\tau_t^l) w_t^p \tag{6.0.2}$$

$$\tilde{w}_t^g \equiv (1-\tau_t^l) w_t^g. \tag{6.0.3}$$

Thus, the transformed government budget constraint becomes

$$A_t(N_t)^\theta K_t^{1-\theta} - \tilde{r}_t K_t - \tilde{w}_t^p N_t^p = \tilde{w}_t^g N_t^g + K_{t+1}^g - (1-\delta^g) K_t^g + G_t^t. \tag{6.0.4}$$

[29]In contrast, in the primal approach, all the policy variables and prices are solved as functions of the allocations, thus the government decides only on the optimal allocation.

[30]The DCE system is summarized in Appendix 11.5.

[31]Stockman (2001) shows that the absence of debt and thus the inability of the government to run surpluses and deficits has no dramatic effect on the optimal policies in the full commitment case.

[32]Note that by choosing next-period public capital, the planner is choosing public investment optimally.

Once the optimal after-tax returns are solved for, the expression for the before-tax real interest rate and private wage can be obtained from the DCE system. Solving for optimal capital and labor tax rates is then trivial.

The transformed Ramsey problem then becomes:[33]

$$\max_{C_t,N_t^p,N_t^g,K_{t+1}^p,K_{t+1}^g,\tilde{w}_t^p,\tilde{w}_t^g,\tilde{r}_t} E_0\sum_{t=0}^{\infty}\beta^t\Bigg\{\psi_1\ln C_t+\psi_2\ln\Big[1-N_t^p-N_t^g-\gamma(N_t^g)^2\Big] \\ +(1-\psi_1-\psi_2)\ln\Big[(N_t^g)^{\alpha}(K_t^g)^{1-\alpha}\Big]\Bigg\} \tag{6.0.5}$$

s.t

$$\frac{1}{C_t} = \beta E_t\frac{1}{C_{t+1}}\left[1-\delta^p+(1-\tau_{t+1}^k)(1-\theta)\frac{Y_{t+1}}{K_{t+1}^p}\right] \tag{6.0.6}$$

$$\psi_2 C_t=\psi_1\Big[1-N_t^p-N_t^g-\gamma(N_t^g)^2\Big](1-\tau_t^l)w_t^p \tag{6.0.7}$$

$$\psi_2 C_t[1+2\gamma N_t^g]=\psi_1\Big[1-N_t^p-N_t^g-\gamma(N_t^g)^2\Big](1-\tau_t^l)w_t^g \tag{6.0.8}$$

$$A_t(N_t^p)^{\theta}K_t^{(1-\theta)}=C_t+K_{t+1}^g-(1-\delta^g)K_t^g+K_{t+1}^p-(1-\delta^p)K_t^p \tag{6.0.9}$$

$$A_t(N_t)^{\theta}K_t^{1-\theta}-\tilde{r}_tK_t-\tilde{w}_t^pN_t^p=\tilde{w}_t^gN_t^g+K_{t+1}^g-(1-\delta^g)K_t^g+G_t^t \tag{6.0.10}$$

$$K_{t+1}^p = I_t+(1-\delta^p)K_t^p \tag{6.0.11}$$

$$r_t=(1-\theta)\frac{Y_t}{K_t^p} \tag{6.0.12}$$

$$w_t^p=\theta\frac{Y_t}{N_t^p} \tag{6.0.13}$$

$$S_t^g=(N_t^g)^{\alpha}(K_t^g)^{(1-\alpha)} \tag{6.0.14}$$

[33]Detailed derivations in Appendix 13.6.

$$K^g_{t+1} \quad = \quad G^i_t + (1-\delta^g)K^g_t \tag{6.0.15}$$

After numerically solving for the unique steady-state,[34] the full characterization of the long-run Ramsey equilibrium is summarized in Table 6 on the next page, where the same values for the parameters from the exogenous policy section (see Table 1) were used.[35]

As in Lucas (1990), Cooley and Hansen (1992) and Ohanian (1997), parameter ξ is introduced to measure the consumption-equivalent long-run welfare gain of moving from the steady-state allocations in the exogenous policy case to the equilibrium values obtained under Ramsey policy. In other words, the value of ξ measures the share of steady-state consumption under the exogenous policy that the household has to be compensated with, in order to achieve the same level of utility as the one under the Ramsey policy. A fraction $\xi > 0$, which is the case reported in Table 6 on the next page, demonstrates that the agent is better-off under Ramsey, while $\xi < 0$ would have implied that the agent is worse-off under Ramsey. The derivation of the analytic expression for ξ is presented in Appendix 11.10.

There are several additional important findings in the Ramsey equilibrium that can be seen in Table 6 on the next page. First, as expected, total discounted welfare is higher in the Ramsey regime. Next, private consumption share is lower, while private capital- and investment shares are higher, and thus the interest rate is lower. The model generates a zero steady-state optimal capital tax, and a higher labor tax rate. All these results are consistent with the findings in earlier studies, *e.g.* Judd (1985), Chamley (1986), Zhu (1992), Ljungqvist and Sargent (2004) and Kocherlakota (2010). In addition, earlier studies that use the representative-agent setup, *e.g.* Lucas (1990), Cooley and Hansen (1992), have shown that tax reforms which abolish capital taxation, even at the expense of a higher tax burden on labor, still produce significant welfare gains for the society.

Next, due to the presence of a second labor market, as well as an endogenous public sector hours, sophisticated labor market interactions are generated. In the framework presented in this paper, the labor market structure allows for labor flows between sectors. Further-

[34] Appendix 11.8-11.9 contains the detailed derivation of the Ramsey problem and steady-state system representation.

[35] The dynamic version of the model was also solved to check that the model possesses saddle-path stability, *i.e.* that for $t \geq 1$, given initial conditions for K^p, K^g the model has a unique set of sequences for $\{C_t, N^p_t, N^g_t, K^p_{t+1}, K^g_{t+1}, w^p_t, w^g_t, r_t, \tau^l_t, \tau^k_t\}^{\infty}_{t=0}$ that converges to the steady-state.

Table 3: Data averages and long-run solution: exogenous vs. optimal policy

	GE Data	Exogenous	Ramsey
c/y	0.590	0.784	0.709
i/y	0.210	0.192	0.229
g^i/y	0.023	0.023	0.062
k^p/y	2.350	2.346	2.793
k^g/y	0.630	0.630	1.669
s^g/y	0.193	0.224	0.289
g^t/y	0.170	0.228	0.221
$w^p n^p/y$	0.710	0.710	0.710
$w^g n^g/y$	0.130	0.145	0.143
rk/y	0.290	0.290	0.290
w^g/w^p	1.200	1.200	1.339
w^p	N/A	1.006	1.079
w^g	N/A	1.207	1.445
$\tilde{w}^p$	N/A	0.595	0.541
$\tilde{w}^g$	N/A	0.714	0.724
n	0.296	0.266	0.251
n^p	0.253	0.227	0.218
n^g	0.043	0.039	0.033
n^g/n^p	0.170	0.170	0.150
$\gamma(n^g)^2/n^g$	N/A	0.199	0.171
$\bar{r}$	0.036	0.035	0.022
τ^k	0.160	0.160	0.000
τ^l	0.409	0.409	0.499
U	N/A	-47.91	-46.22
ξ	N/A	0	0.123

more, the government internalizes the public services in its choice. Thus, it picks the socially optimal levels of public hours and capital stock to provide the optimal level of the public consumption good. In addition, the planner chooses a different mix between the inputs used in the provision of government services: a higher level of government investment is undertaken, while fewer public hours are employed than the in the DCE solution. As a result, public investment (and thus public capital) share almost triples, which increases the amount of the public good produced relative to output. The same substitution of labor for capital is observed in the production of the private consumption good. Furthermore, the Ramsey planner finds it optimal to produce a higher level of the private output using more capital and fewer private hours. As a result, total employment also decreases.

In terms of the relative price of labor in the two labor markets, the after-tax private wage decreases after the higher labor tax is levied, while the after-tax public wage increases slightly. The higher public/private wage ratio, and thus the higher public wage premium in the optimal policy case overcompensate for the increase in the labor tax. Furthermore, the public/private hours ratio is lower, due to the substitution away from labor in the government sector. In other words, the increase in the public wage premium is driven by budgetary considerations, as the public wage is the residually-determined fiscal instrument that balances the per-period government budget constraint. In addition, the result is consistent with economic logic and the scarcity argument: relatively fewer hours are employed in the public sector, thus the steady-state public wage rate is higher. Furthermore, the optimal government wage consumption is a little lower.[36] In addition, given the substantial increase in public investment, the fixed level of government transfers, and the loss of capital income tax revenue, steady-state labor tax is 10% higher relative to the exogenous policy case. Overall, these changes in the distribution of spending are new results in the optimal policy literature. As seen from Table 6, if these aspects are ignored, important adjustment mechanisms are missed.

The value-added of the model with endogenous hours and wages is that it generates new predictions about the long-run effects of fiscal policy on the labor markets, such as the wage- and employment ratios, the optimal composition of the government wage bill consumption, and the distribution of spending across government expenditure categories. These results, generated from the incorporation of a richer government spending side, are new and interesting for policy makers, as previous research had ignored those important dimensions.

In the next section, the analysis is extended to the behavior of the Ramsey economy outside of the steady-state. The transitional dynamics of model variables under optimal policy setup is analyzed as well. In particular, the optimal responses of the fiscal instrument and the other prices and allocations to positive shocks to TFP is presented and discussed.

[36]The result that cuts in the wage bill have an expansionary effect on the economy is not new to the empirical macroeconomic studies, *e.g.* Algan *et al.* (2002), Alesina (1997), Alesina *et al.* (2001), Alesina *et al.* (2002), and Giavazzi and Pagano (1990). However, the optimal public wage and employment aspects in the analysis are novel in the modern macroeconomic literature, given the predominance of setups with single wage rates, and exogenously-determined public employment. Lastly, the results are robust: changes in the relative utility weights do not significantly affect the results obtained here.

7 Optimal reaction of fiscal policy instruments to productivity shocks

The optimal policy model is now solved using the first-order linearization procedure from Schmitt-Grohe and Uribe (2004) to study the dynamics of prices and allocations outside the steady-state.[37] The model solution is then used to study transitional behavior in response to a surprise innovation in total factor productivity. Under Ramsey, endogenous variables would generally behave differently, as compared to the responses to a positive technology shock under the exogenous fiscal policy case. Fig. 3 summarizes all responses to a 1 % surprise innovation to total factor productivity. To highlight differences across regimes, Fig. 4 plots on the same graph both the IRFs from the exogenous policy case and the optimal ones. The new variables in the system are the five fiscal policy instruments - capital and labor taxes, as well as public investment (hence public capital), public wage rate and public employment. By intervening in the public sector labor market, the benevolent government can influence the private sector labor market, and thus affect the course of the economy.

In period 0, after the realization of the unexpected technological innovation, capital tax remains unchanged.[38] This result is in line with previous findings in the literature, *e.g.* Chari and Kehoe (1994, 1999) who show that in a standard RBC model capital tax rate does not respond to productivity shocks. In other words, the benevolent government would not deviate from the optimal zero steady-state capital tax rate even in the face of uncertain productivity shocks.[39] Next, labor income tax rate increases upon the impact of the positive surprise innovation in TFP and then slowly returns to its old steady-state; the substantial persistence observed is in line with earlier studies (Chari and Kehoe, 1994, 1999). However, due to the richer structure of the and endogenously-determined government spending, the magnitude of the response in the labor tax is greater.

Furthermore, given that public spending categories are optimally chosen in this framework,

[37] Given the absence of curvature in the model, the second-order approximation to the equilibrium system of equations did not change results significantly.

[38] At first glance the huge percentage deviation from the steady-state for capital tax can be misleading. However, noting that the steady-state capital tax rate under Ramsey is $\tau^k = 2.58 \times 10^{-12} \approx 0$, it follows that the log-deviation from the steady-state is an extremely large number, as the denominator is close to zero, even though the absolute value of change is tiny.

[39] This is also a result of the logarithmic specification of the household's utility of consumption.

the setup generates much more interaction among the variables than does the standard RBC model. For example, public investment increases substantially, as the government under the Ramsey regime also chooses public capital and government services optimally. Next, as in the exogenous policy case, public sector wages will increase more than the private sector wage. The higher volatility in public wages, as discussed in earlier sections, is an artifact of the presence of transaction costs from government work. The change in the public/private wage ratio in turn triggers a reallocation of labor resources from the private to the public sector. The net effect on labor supply, however, is negative, as the transaction cost friction depresses the response of public hours. Next, the outflow of hours from the firm leads to an increase in the marginal product of capital, and hence the real interest rate increases. However, from the complementarity between private labor and capital in the Cobb-Douglas production function, private capital decreases. Therefore, due to the fall in the levels of the two private inputs, output increases by less than the size of the technology shock.

In addition, given the jump in government investment, private consumption and investment fall upon the impact of the shock. Overall, the difference in the dynamics in the main model variables under the Ramsey regime is due to the fact that the government chooses the optimal levels of public hours and capital (and hence also public investment).

Figure 3: Impulse Responses to a positive 1% productivity shock under Ramsey policy

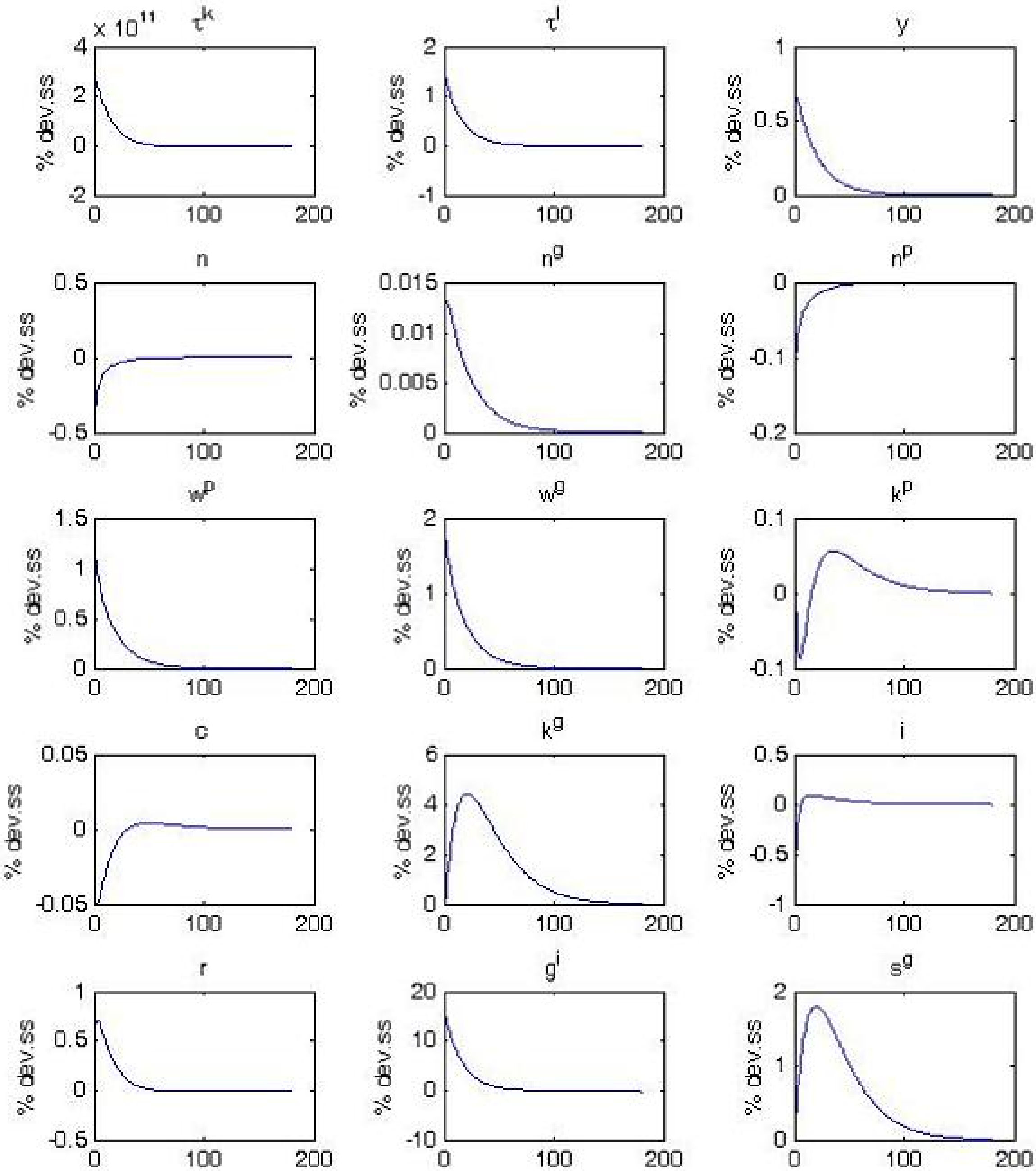
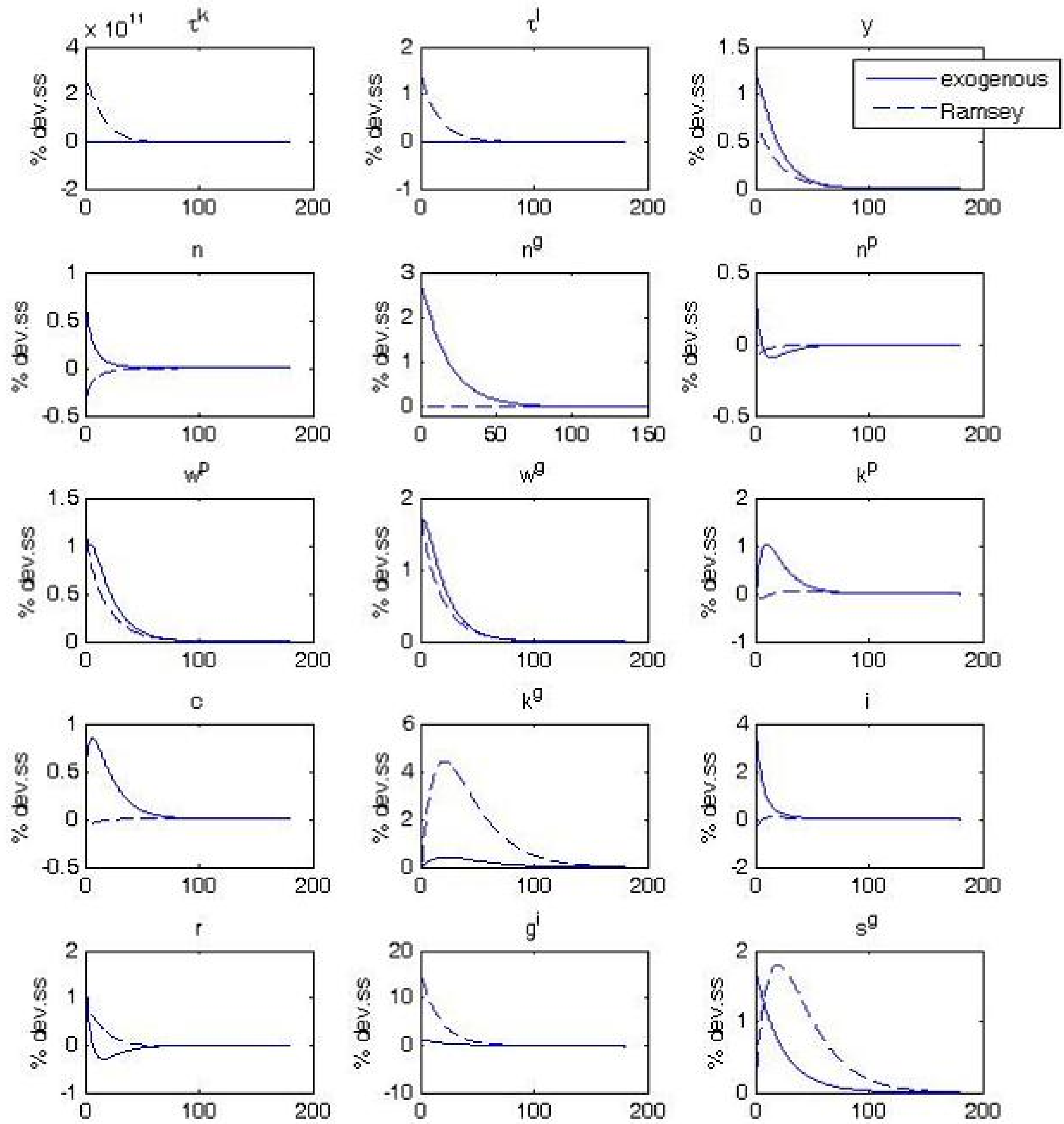


Figure 4: Impulse Responses to a positive 1% productivity shock under exogenous and Ramsey policy



Over time, attracted by the above-steady-state real interest rate, more private investment is undertaken by the government. In turn, private capital accumulation increases, and the usual hump-shape dynamics appears. Higher capital input increases the marginal productivity of labor, and private labor starts slowly to recover to its old steady-state. As time passes, private consumption response also turns positive, and the shape if its response follows the dynamic path of private capital. In general, positive innovations to TFP have a positive effect on the economy. Additionally, there is a long-lasting internal propagation effect in the economy. This is due to the fact that there are two labor markets featuring different wage rates, and labor can flow between sectors in response to changes in the relative wage. Moreover, there is complementarity between public hours and the public capital, which reinforces the complementarity between private and public consumption in the household's utility. The quantitative effect of public sector labor market, however, completely dominates capital response in terms of the initial dynamics. Nevertheless, in the long-run, private capital accumulation effects becomes dominant, as dictated by the standard neoclassical RBC model.

An interesting result is that a significant portion of the private gains are channeled to the public sector in the form of higher public wage bill spending and public investment. Indeed, in this model both categories are productive expenditures, as labor and capital are combined in the provision of the public good. Even though the household suffers a little from the lower private consumption, this negative effect is overcompensated for through the increase in leisure (as private hours fall by much more relative to the increase in private hours), and a higher level of public good consumption. Overall, it takes more than 100 years for all the model variables to return to their old steady states.

8 Limitations of the study

The analysis performed in this section was based on the strong assumption that the government budget is balanced every period and that the household can work in both sectors. However, labor supply decision is done sequentially in the real world. A worker usually decides on a sector first, and only then on the number of hours worked in the selected sector. Furthermore, it is a stylized fact in labor data that most of the variations in hours worked in data are driven by changes in employment rates rather than by changes in hours worked per person. Thus, the model is too simple and cannot distinguish between employment and hours per person in the two sectors. A possible extension, left for future research, is to setup a model with heterogeneous-agents, who search for work according to a directed search process,[40] similar to that used in Gomes (2009).

Next, the model setup presented in this paper abstracted away from debt issues, which are important for modern economies. In models with full commitment, however, Stockman (2001) has shown the presence of debt not to be relevant. Nevertheless, as a possible extension of the model, government bonds can be introduced, together with a long-run target debt/GDP ratio that has to be met in the long-run. Studying the transitional dynamics of the economic variables and the adjustment in the different of public finance categories, in the presence of endogenous public hours and wages, is another promising avenue for future work.

Additionally, since the analysis focused on the long-run full commitment case, the setup abstracted away from electoral uncertainty and thus ignores possible departures from the full-commitment case. Across the political spectrum in most democratic societies, there are different parties with diverse objectives that compete for the popularity vote at parliamentary/presidential elections. In the model setup in this paper, the benevolent government's utility function was assumed to coincide with that of the household. In reality, however, a party's utility function can be quite different when the party is in office, as compared to the case when the party is in opposition, as suggested in Philoppopoulos, Economides and Malley (2004) and Malley, Philippopoulos and Woitek (2007). Different parties might have different preferences for the level of public employment. In other words, jobs can be created in the public sector to generate political support and increase the chances of re-election. However, such considerations, as well as possible departures from the full commitment case, and a focus on "loose commitment" as in Debortoli and Nunes (2010), or time-consistent policies as in Klein and Rios-Rull (2003), Ortigueira (2006), Klein *et al.* (2008) and Martin (2010), will be put on the agenda for future research.

Lastly, aside from political considerations, the observed premium at macroeconomic level is also likely to be a by-product of aggregation of microeconomic data. In the German Socio-Economics Panel (SOEP), as well as in the US Panel Study of Income Dynamics (PSID) database, for example, the age-skill profile of public employees is skewed to the left:

[40] In particular, every agent searches for work in only one of the two sectors, and the sector is determined from the realization of a stochastic process.

the average public employee is older, more skilled, more experienced, and tend to occupy managerial positions as compared to his/her private sector counterpart. However, such distributional and occupational dimensions are outside the scope of a simple RBC model with a representative household. Therefore, further work to endogeneize public wage premium, perhaps within a heterogeneous-agents framework, needs to be undertaken.

9 Summary and Conclusions

This paper characterized optimal fiscal policy and evaluated it relative to the exogenous (observed) one. The focus was on the labor market effects of fiscal policy in a model with endogenously-determined public wages and hours, as well as labor-intensive government services. To this end, a Real-Business-Cycle model, calibrated to German data (1970-2007), was set up with a richer government spending side, and an endogenous private-public sector labor choice. The latter was achieved by the inclusion of a transaction cost of government work, generating a wedge between the marginal disutility of an hour worked in the public and private sectors, which is generally consistent with the evidence. This friction also helped the framework to accommodate the different wage rates in the two labor markets. To illustrate the effects of fiscal policy, two regimes were compared and contrasted to one another - exogenous *vs.* optimal (Ramsey) policy case, in terms of their long-run effects, as well as in terms of the dynamics of the economy in response to technology shocks. The interaction between the two labor markets was shown to be quantitatively important for both the short-run model dynamics, as well as for the steady-state values of the allocations.

The main findings from the computational experiments performed in this paper were: (i) as in Judd (1985), Chamley (1986) and Zhu (1992), the optimal steady-state capital tax rate is zero, as it is the most distortionary tax to use; (ii) Given a fixed level of government transfers, a higher labor tax rate is needed to compensate for the loss in capital tax revenue, despite the fact that the Ramsey planner chooses both components of the government wage bill, as well as the level of investment optimally; (iii) Under the optimal policy regime, public/private employment ratio is lower, while public/private wage rate is higher. In other words, employment in the public sector should be diminished, but government employees should receive better renumeration. (iv) The government wage bill is smaller, while public investment is three times higher relative to the exogenous policy case; (v) The government chooses the optimal level of government services provided. Additionally, it substitutes labor for capital in the input mix of the public good production, and private output. In turn, this substitution makes government hours more valuable. All these result are in line with Alesina (1997) and Alesina *et al.* (2002) who found that for OECD countries, fiscal adjustments that rely on cuts in government wage bill produce a stimulus to the economy through higher investment, and thus a leaner public sector creates an expansionary effect for the economy.

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10 Technical Appendix

10.1 Optimality conditions

10.1.1 Firm's problem

The profit function is maximized when the derivatives of that function are set to zero. Therefore, the optimal amount of capital - holding the level of technology A_t and labor input N_t^p constant - is determined by setting the derivative of the profit function with respect to K_t^p equal to zero. This derivative is

$$(1-\theta)A_t(K_t^p)^{-\theta}(N_t^p)^{\theta} - r_t \;=\; 0 \tag{10.1.1}$$

where $(1-\theta)A_t(K_t^p)^{-\theta}(N_t^p)^{\theta}$ is the marginal product of capital because it expresses how much output will increase if capital increases by one unit. The economic interpretation of this First-Order Condition (FOC) is that in equilibrium, firms will rent capital up to the point where the benefit of renting an additional unit of capital, which is the marginal product of capital, equals the rental cost, i.e the interest rate.

$$r_t \;=\; (1-\theta)A_t(K_t^p)^{-\theta}(N_t^p)^{\theta} \tag{10.1.2}$$

Now, multiply by K_t^p and rearrange terms. This gives the following relationship:

$$K_t^p(1-\theta)A_t(K_t^p)^{-\theta}(N_t^p)^{\theta} = r_tK_t^p \quad \text{or} \quad (1-\theta)Y_t = r_tK_t^p \tag{10.1.3}$$

because

$$K_t^p(1-\theta)A_t(K_t^p)^{-\theta}(N_t^p)^{\theta} = A_t(K_t^p)^{1-\theta}(N_t^p)^{\theta} \;=\; (1-\theta)Y_t$$

To derive firms' optimal labor demand, set the derivative of the profit function with respect to the labor input equal to zero, holding technology and capital constant:

$$\theta A_t(K_t^p)^{1-\theta}(N_t^p)^{\theta-1} - w_t^p = 0 \text{ or } w_t^p = \theta A_t(K_t^p)^{1-\theta}(N_t^p)^{\theta-1} \tag{10.1.4}$$

In equilibrium, firms will hire labor up to the point where the benefit of hiring an additional hour of labor services, which is the marginal product of labor, equals the cost, i.e the hourly wage rate.

Now multiply both sides of the equation by N_t^p and rearrange terms to yield

$$N_t^p\theta A_t(K_t^p)^{1-\theta}(N_t^p)^{\theta-1} = w_t^pN_t^p \quad \text{or} \quad \theta Y_t = w_t^pN_t^p \tag{10.1.5}$$

Next, it will be shown that in equilibrium, economic profits are zero. Using the results above one can obtain

$$\Pi_t \;=\; Y_t - r_t K_t^p - w_t^p N_t^p = Y_t - (1-\theta)Y_t - \theta Y_t = 0 \qquad (10.1.6)$$

Indeed, in equilibrium, economic profits are zero.

10.1.2 Consumer problem

Set up the Lagrangian

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \Bigg\{ \psi_1 \ln C_t + \psi_2 \ln \Big[1 - N_t^p - N_t^g - \gamma (N_t^g)^2 \Big] + \psi_3 \ln S_t^g$$
$$+\Lambda_t \Bigg[(1-\tau_t^l)(w_t^p N_t^p + w_t^g N_t^g) + (1-\tau_t^k) r_t K_t^p + G_t^t - C_t - K_{t+1}^p + (1-\delta) K_t^p \Bigg] \Bigg\} \qquad (10.1.7)$$

This is a concave programming problem, so the FOCs, together with the additional, boundary ("transversality") conditions for private physical capital and government bonds are both necessary and sufficient for an optimum.

To derive the FOCs, first take the derivative of the Lagrangian w.r.t C_t (holding all other variables unchanged) and set it to 0, *i.e.* $\mathcal{L}_{C_t} = 0$. That will result in the following expression

$$\beta^t \Bigg\{ \frac{\psi_1}{C_t} - \Lambda_t \Bigg\} = 0 \quad \text{or} \quad \frac{\psi_1}{C_t} = \Lambda_t \qquad (10.1.8)$$

This optimality condition equates marginal utility of consumption to the marginal utility of wealth.

Now take the derivative of the Lagrangian w.r.t K_{t+1}^p (holding all other variables unchanged) and set it to 0, *i.e.* $L_{K_{t+1}^p} = 0$. That will result in the following expression

$$\beta^t \Bigg\{ -\Lambda_t + E_t \Lambda_{t+1} \Big[(1-\tau_{t+1}^k) r_{t+1} + (1-\delta^p) \Big] \Bigg\} \;=\; 0 \qquad (10.1.9)$$

Cancel the β^t term to obtain

$$-\Lambda_t + \beta E_t \Lambda_{t+1} \Big[(1-\tau_{t+1}^k) r_{t+1} + \tau^k \delta^p + (1-\delta^p) \Big] = 0 \qquad (10.1.10)$$

Move Λ_t to the right so that

$$\beta E_t \Lambda_{t+1}\Bigg[(1-\tau^k_{t+1})r_{t+1} + (1-\delta^p)\Bigg] = \Lambda_t \tag{10.1.11}$$

Using the expression for the real interest rate shifted one period forward one can obtain

$$r_{t+1} = (1-\theta)\frac{Y_{t+1}}{K^p_{t+1}}$$

$$\beta E_t \Lambda_{t+1}\Bigg[(1-\tau^k_{t+1})(1-\theta)\frac{Y_{t+1}}{K^p_{t+1}} + (1-\delta^p)\Bigg] = \Lambda_t \tag{10.1.12}$$

This is the Euler equation, which determines how consumption is allocated across periods.

Take now the derivative of the Lagrangian w.r.t N^p_t (holding all other variables unchanged) and set it to 0, *i.e.* $\mathcal{L}_{N^p_t} = 0$. That will result in the following expression

$$\beta^t\Bigg\{ -\frac{\psi_2}{1-N^p_t-N^g_t-\gamma(N^g_t)^2} + \Lambda_t(1-\tau^l_t)w^p_t\Bigg\} \;=\; 0 \tag{10.1.13}$$

Cancel the β^t term to obtain

$$-\frac{\psi_2}{1-N^p_t-N^g_t-\gamma(N^g_t)^2} + \Lambda_t(1-\tau^l_t)w^p_t = 0 \tag{10.1.14}$$

Rearranging, one can obtain

$$\frac{\psi_2}{1-N^p_t-N^g_t-\gamma(N^g_t)^2} = \Lambda_t(1-\tau^l_t)w^p_t \tag{10.1.15}$$

Plug in the expression for w^h_t, that is,

$$w^p_t = \theta\frac{Y_t}{N^p_t} \tag{10.1.16}$$

into the equation above. Rearranging, one can obtain

$$\frac{\psi_2}{1-N^p_t-N^g_t-\gamma(N^g_t)^2} + \Lambda_t(1-\tau^l_t)\theta\frac{Y_t}{N^p_t} \tag{10.1.17}$$

Take now the derivative of the Lagrangian w.r.t N^g_t (holding all other variables unchanged) and set it to 0, *i.e.* $\mathcal{L}_{N^g_t} = 0$. That will result in the following expression

$$\beta^t\Bigg\{ -\frac{\psi_2(1+2\gamma N^g_t)}{1-N^p_t-N^g_t-\gamma(N^g_t)^2} + \Lambda_t(1-\tau^l_t)w^g_t\Bigg\} \;=\; 0 \tag{10.1.18}$$

Cancel the β^t term to obtain

$$-\frac{\psi_2(1+2\gamma N_t^g)}{1-N_t^p-N_t^g-\gamma(N_t^g)^2}+\Lambda_t(1-\tau_t^l)w_t^g=0 \tag{10.1.19}$$

Rearranging, one can obtain

$$\frac{\psi_2(1+2\gamma N_t^g)}{1-N_t^p-N_t^g-\gamma(N_t^g)^2}=\Lambda_t(1-\tau_t^l)w_t^g \tag{10.1.20}$$

Transversality conditions need to be imposed to prevent Ponzi schemes, i.e borrowing bigger and bigger amounts every subsequent period and never paying it off.

$$\lim_{t\to\infty}\beta^t\Lambda_t K_{t+1}^p \quad = \quad 0 \tag{10.1.21}$$

10.2 Per capita stationary DCE

Since the model in stationary and per capita terms by definition, there is no need to transform the optimality conditions, i.e $Z_t^h = Z_t = z_t$. The system of equations that describes the DCE is as follows:

$$y_t = a_t(k_t^p)^{1-\theta}(n_t^p)^\theta \tag{10.2.1}$$

$$y_t = c_t + k_{t+1}^p - (1-\delta^p)k_t^p + g_t^i \tag{10.2.2}$$

$$\frac{\psi_1}{c_t} = \lambda_t \tag{10.2.3}$$

$$\lambda_t = \beta E_t \lambda_{t+1}\left[1 - \delta^p + (1-\tau^k)(1-\theta)\frac{y_{t+1}}{k_{t+1}^p}\right] \tag{10.2.4}$$

$$\frac{\psi_2}{1 - n_t^p - n_t^g - \gamma(n_t^g)^2} = \frac{\psi_1}{c_t}(1-\tau^l)\theta\frac{y_t}{n_t^p} \tag{10.2.5}$$

$$\frac{\psi_2}{1 - n_t^p - n_t^g - \gamma(n_t^g)^2}[1 + 2\gamma n_t^g] = \frac{\psi_1}{c_t}(1-\tau^l)w_t^g \tag{10.2.6}$$

$$k_{t+1}^p = i_t + (1-\delta^p)k_t^p \tag{10.2.7}$$

$$r_t = (1-\theta)\frac{y_t}{k_t^p} \tag{10.2.8}$$

$$w_t^p = \theta\frac{y_t}{n_t^p} \tag{10.2.9}$$

$$g_t^t + g_t^i + w_t^g n_t^g = \tau^k r_t k_t^p + \tau^l\left[w_t^p n_t^p + w_t^g n_t^g\right]. \tag{10.2.10}$$

$$k_{t+1}^g = g_t^i + (1-\delta^g)k_t^g \tag{10.2.11}$$

$$g_t^i = g_t^{iy} y_t \tag{10.2.12}$$

$$g_t^t = g_t^{ty} y_t \tag{10.2.13}$$

$$s_t^g = (n_t^g)^\alpha(k_t^g)^{1-\alpha} \tag{10.2.14}$$

Therefore, the DCE is summarized by Equations (13.1.1)-(13.1.14) in the paths of the following 14 variables $\{y_t, c_t, i_t, k_t^p, k_t^g, g_t^i, g_t^t, n_t^p, n_t^g, s_t^g, w_t^p, w_t^g, r_t, \lambda_t\}_{t=0}^\infty$ given the process followed by total factor productivity $\{a_t\}_{t=0}^\infty$, the values of government investment and government transfers shares g^{iy}, g^{ty}, and the fixed capital and labor tax rates $\{\tau^k, \tau^l\}$.

10.3 Steady-state

In steady-state, there is no uncertainty and variables do not change. Thus, eliminate all stochasticity and time subscripts to obtain

$$y = a(k^p)^{1-\theta}(n^p)^\theta \tag{10.3.1}$$

$$y = c + \delta^p k^p + g^i \tag{10.3.2}$$

$$\frac{\psi_1}{c} = \lambda \tag{10.3.3}$$

$$1 = \beta\left[1 - \delta^p + (1-\tau^k)(1-\theta)\frac{y}{k^p}\right] \tag{10.3.4}$$

$$\frac{\psi_2}{1 - n^p - n^g - \gamma(n^g)^2} = \frac{\psi_1}{c}(1-\tau^l)\theta\frac{y}{n^p} \tag{10.3.5}$$

$$\frac{\psi_2}{1 - n^p - n^g - \gamma(n^g)^2}[1 + 2\gamma n^g] = \frac{\psi_1}{c}(1-\tau^l)w^g \tag{10.3.6}$$

$$i^p = \delta^p k^p \tag{10.3.7}$$

$$r = (1-\theta)\frac{y}{k^p} \tag{10.3.8}$$

$$w^p = \theta\frac{y}{n^p} \tag{10.3.9}$$

$$g^t + g^i + w^g n^g = \tau^k r k^p + \tau^l\left[w^p n^p + w^g n^g\right]. \tag{10.3.10}$$

$$g^i = \delta^g k^g \tag{10.3.11}$$

$$g^i = g^{iy} y \tag{10.3.12}$$

$$g^t = g^{ty} y \tag{10.3.13}$$

$$s^g = (n^g)^\alpha (k^g)^{1-\alpha} \tag{10.3.14}$$

10.4 Log-linearization

10.4.1 Log-linearized production function

$$y_t = a_t(k_t^p)^{1-\theta}(n_t^p)^\theta \tag{10.4.1}$$

Take natural logs from both sides to obtain

$$\ln y_t = \ln a_t + (1-\theta)\ln k_t^p + \theta \ln n_t^p \tag{10.4.2}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln y_t}{dt} = \frac{d\ln a_t}{dt} + (1-\theta)\frac{d\ln k_t^p}{dt} + \theta\frac{d\ln n_t^p}{dt} \tag{10.4.3}$$

$$\frac{1}{y}\frac{dy_t}{dt} = \frac{1}{a}\frac{da_t}{dt} + \frac{1-\theta}{k^p}\frac{dk_t}{dt} + \frac{\theta}{n^p}\frac{dn_t^p}{dt} \tag{10.4.4}$$

Pass to log-deviations to obtain

$$0 = -\hat{y}_t + (1-\theta)\hat{k}_t^p + \hat{a}_t + \theta\hat{n}_t^p \tag{10.4.5}$$

10.4.2 Linearized market clearing

$$c_t + k_{t+1}^p - (1-\delta)k_t^p + g_t^i = y_t \tag{10.4.6}$$

Take logs from both sides to obtain

$$\ln[c_t + k_{t+1}^p - (1-\delta)k_t^p + g_t^i] = \ln(y_t) \tag{10.4.7}$$

Totally differentiate with respect to time

$$\frac{d\ln[c_t + k_{t+1}^p - (1-\delta)k_t^p + g_t^i]}{dt} = d\ln(y_t) \tag{10.4.8}$$

$$\frac{1}{c+\delta k^p + g^c}\left[\frac{dc_t}{dt}\frac{c}{c} + \frac{dk_{t+1}^p}{dt}\frac{k^p}{k^p} - (1-\delta^p)\frac{dk_t^p}{dt}\frac{k^p}{k^p} + \frac{dg_t^i}{dt}\frac{g^i}{g^i}\right] = \frac{dy_t}{dt}\frac{1}{y} \tag{10.4.9}$$

Define $\hat{z} = \frac{dz_t}{dt}\frac{1}{z}$. Thus passing to log-deviations

$$\frac{1}{y}\left[\hat{c}_t c + \hat{k}_{t+1}^p k^p - (1-\delta^p)\hat{k}_t^p k^p + g^i\hat{g}_t^i\right] = \hat{y}_t \tag{10.4.10}$$

$$\hat{c}_t c + \hat{k}_{t+1}^p k^p - (1-\delta^p)\hat{k}_t^p k^p + g^i\hat{g}_t^i = y\hat{y}_t \tag{10.4.11}$$

$$k^p\hat{k}_{t+1}^p = y\hat{y}_t - c\hat{c}_t + (1-\delta)k^p\hat{k}_t^p - g^i\hat{g}_t^i \tag{10.4.12}$$

10.4.3 Linearized FOC consumption

$$\frac{\psi_1}{c_t} = \lambda_t \tag{10.4.13}$$

Take natural logarithms from both sides to obtain

$$\ln \psi_1 - \ln(c_t) = \ln \lambda_t \tag{10.4.14}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln \psi_1}{dt} - \frac{d \ln c_t}{dt} = \frac{d \ln \lambda_t}{dt} \tag{10.4.15}$$

or

$$-\frac{d \ln c_t}{dt} = \frac{d \ln \lambda_t}{dt} \tag{10.4.16}$$

$$-\frac{dc_t}{dt}\frac{1}{c} = \frac{d\lambda_t}{dt}\frac{1}{\lambda} \tag{10.4.17}$$

Pass to log-deviations to obtain

$$-\hat{c}_t = \hat{\lambda}_t \tag{10.4.18}$$

10.4.4 Linearized no-arbitrage condition for capital

$$\lambda_t = \beta E_t \lambda_{t+1}[(1 - \tau^k_{t+1}) r_{t+1} + (1 - \delta^p)] \tag{10.4.19}$$

Substitute out r_{t+1} on the right hand side of the equation to obtain

$$\lambda_t = \beta E_t[\lambda_{t+1}((1 - \tau^k_{t+1})(1 - \theta)\frac{y_{t+1}}{k^p_{t+1}} + 1 - \delta^p)] \tag{10.4.20}$$

Take natural logs from both sides of the equation to obtain

$$\ln \lambda_t = \ln E_t[\lambda_{t+1}((1 - \tau^k_{t+1})(1 - \theta)\frac{y_{t+1}}{k^p_{t+1}} + 1 - \delta^p)] \tag{10.4.21}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln \lambda_t}{dt} = \frac{d \ln E_t[\lambda_{t+1}((1 - \tau^k_{t+1})(1 - \theta)\frac{y_{t+1}}{k^p_{t+1}} + (1 - \delta^p)]}{dt} \tag{10.4.22}$$

$$\frac{1}{\lambda}\frac{d\lambda_t}{dt} = E_t\Bigg\{\frac{1}{\lambda((1-\tau^k_{t+1})(1-\theta)\frac{y}{k^p}+1-\delta^p}\Bigg[((1-\tau^k)(1-\theta)\frac{y}{k^p}+1-\delta^p)\frac{d\lambda_{t+1}}{dt}\frac{\lambda}{\lambda}$$
$$+\frac{\lambda(1-\tau^k)(1-\theta)}{k^p}\frac{dy_{t+1}}{dt}\frac{y}{y}-\left[\frac{\lambda(1-\tau^k)(1-\theta)y}{(k^p)^2}\right]\frac{dk^p_{t+1}}{dt}\frac{k^p}{k^p}\Bigg]\Bigg\} \tag{10.4.23}$$

Pass to log-deviations to obtain

$$\hat{\lambda}_t = E_t\Bigg\{\hat{\lambda}_{t+1}+\Bigg[\frac{(1-\tau^k)(1-\theta)y}{((1-\tau^k)(1-\theta)\frac{y_{t+1}}{k^p_{t+1}}+1-\delta^p)k^p}\hat{y}_{t+1}$$
$$-\frac{(1-\tau^k)(1-\theta)y}{((1-\theta)\frac{y_{t+1}}{k^p_{t+1}}+1-\delta^p)k^p}\hat{k}^p_{t+1}\Bigg]\Bigg\} \tag{10.4.24}$$

Observe that

$$(1-\tau^k)(1-\theta)\frac{y}{k^p}+1-\delta^p \quad = \quad 1/\beta \tag{10.4.25}$$

Plug it into the equation to obtain

$$\hat{\lambda}_t \quad = \quad E_t\left[\hat{\lambda}_{t+1}+\frac{\beta(1-\tau^k)(1-\theta)y}{k^p}\hat{y}_{t+1}-\frac{\beta(1-\tau^k)(1-\theta)y}{k^p}\hat{k}^p_{t+1}\right] \tag{10.4.26}$$

$$\hat{\lambda}_t \quad = \quad E_t\hat{\lambda}_{t+1}+\frac{\beta(1-\tau^k)(1-\theta)y}{k^p}E_t\hat{y}_{t+1}-\frac{\beta(1-\tau^k)(1-\theta)y}{k^p}E_t\hat{k}^p_{t+1} \tag{10.4.27}$$

10.4.5 Linearized MRS(c_t, n^p_t)

$$\psi_2 c_t \quad = \quad \psi_1[1-n^p_t-n^g_t-\gamma(n_t)^2](1-\tau^l)\theta\frac{y_t}{n^p_t} \tag{10.4.28}$$

Take natural logs from both sides of the equation to obtain

$$\ln\psi_2 c_t \quad = \quad \ln\psi_1[1-n^p_t-n^g_t-\gamma(n_t)^2](1-\tau^l)\theta\frac{y_t}{n^p_t} \tag{10.4.29}$$

$$\ln\psi_2+\ln c_t \quad = \quad \ln\psi_1+\ln[1-n^p_t-n^g_t-\gamma(n_t)^2]+\ln(1-\tau^l_t)+\ln y_t-\ln n^p_t \tag{10.4.30}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln\psi_2}{dt}+\frac{d\ln c_t}{dt}=\frac{d\ln\psi_1}{dt}+\frac{d\ln[1-n^p_t-n^g_t-\gamma(n_t)^2]}{dt}$$
$$+\frac{d\ln(1-\tau^l_t)}{dt}+\frac{d\ln y_t}{dt}-\frac{d\ln n^p_t}{dt} \tag{10.4.31}$$

$$\frac{1}{c}\left[\frac{dc_t}{dt}\right] = -\frac{1}{1-n^p-n^g-\gamma(n^g)^2}\frac{d\left[n_t^p+n_t^g+\gamma(n_t)^2\right]}{dt} - \frac{d\tau_t^l}{dt}\frac{1}{1-\tau^l}+\frac{1}{y}\frac{dy_t}{dt}-\frac{1}{n^p}\frac{dn_t^p}{dt} \tag{10.4.32}$$

$$\frac{dc_t}{dt}\frac{1}{c} = -\frac{n^p}{1-n^p-n^g-\gamma(n^g)^2}\frac{dn_t^p}{dt}\frac{1}{n^p} - \frac{n^g(1+2\gamma n^g)}{1-n^p-n^g-\gamma(n^g)^2}\frac{dn_t^g}{dt}\frac{1}{n^g}-\frac{\tau^l}{1-\tau^l}\frac{d\tau_t^l}{dt}\frac{1}{\tau^l}+\frac{1}{y}\frac{dy_t}{dt}-\frac{1}{n^p}\frac{dn_t^p}{dt} \tag{10.4.33}$$

Pass to log-deviations to obtain

$$\hat{c}_t = -\frac{n^p}{1-n^p-n^g-\gamma(n^g)^2}\hat{n}_t^p - \frac{n^g(1+2\gamma n^g)}{1-n^p-n^g-\gamma(n^g)^2}\hat{n}_t^g - \frac{\tau^l}{1-\tau^l}\hat{\tau}_t^l+\hat{y}_t-\hat{n}_t^p \tag{10.4.34}$$

Group terms to obtain

$$\hat{c}_t = -\frac{1-n^g-\gamma(n^g)^2}{1-n^p-n^g-\gamma(n^g)^2}\hat{n}_t^p - \frac{n^g(1+2\gamma n^g)}{1-n^p-n^g-\gamma(n^g)^2}\hat{n}_t^g - \frac{\tau^l}{1-\tau^l}\hat{\tau}_t^l+\hat{y}_t \tag{10.4.35}$$

10.4.6 Linearized MRS(c_t, n_t^g)

$$\psi_2 c_t = \psi_1[1-n_t^p-n_t^g-\gamma(n_t)^2](1-\tau^l)w_t^g \tag{10.4.36}$$

Take natural logs from both sides of the equation to obtain

$$\ln\psi_2 c_t = \ln\psi_1[1-n_t^p-n_t^g-\gamma(n_t)^2](1-\tau^l)w_t^g \tag{10.4.37}$$

$$\ln\psi_2+\ln c_t = \ln\psi_1+\ln[1-n_t^p-n_t^g-\gamma(n_t)^2]+\ln(1-\tau_t^l)+\ln w_t^g \tag{10.4.38}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln\psi_2}{dt}+\frac{d\ln c_t}{dt} = \frac{d\ln\psi_1}{dt}+\frac{d\ln[1-n_t^p-n_t^g-\gamma(n_t)^2]}{dt} + \frac{d\ln(1-\tau_t^l)}{dt}+\frac{d\ln w_t^g}{dt} \tag{10.4.39}$$

$$\frac{1}{c}\left[\frac{dc_t}{dt}\right] = -\frac{1}{1-n^p-n^g-\gamma(n^g)^2}\frac{d\left[n_t^p+n_t^g+\gamma(n_t)^2\right]}{dt}-\frac{d\tau_t^l}{dt}\frac{1}{1-\tau^l}+\frac{dw_t^g}{dt}\frac{1}{w^g} \tag{10.4.40}$$

$$\frac{dc_t}{dt}\frac{1}{c} = -\frac{n^p}{1-n^p-n^g-\gamma(n^g)^2}\frac{dn_t^p}{dt}\frac{1}{n^p}$$
$$-\frac{n^g(1+2\gamma n^g)}{1-n^p-n^g-\gamma(n^g)^2}\frac{dn_t^g}{dt}\frac{1}{n^g} - \frac{\tau^l}{1-\tau^l}\frac{d\tau_t^l}{dt}\frac{1}{\tau^l} + \frac{1}{w^g}\frac{dw_t^g}{dt} \quad (10.4.41)$$

Pass to log-deviations to obtain

$$\hat{c}_t = -\frac{n^p}{1-n^p-n^g-\gamma(n^g)^2}\hat{n}_t^p - \frac{n^g(1+2\gamma n^g)}{1-n^p-n^g-\gamma(n^g)^2}\hat{n}_t^g - \frac{\tau^l}{1-\tau^l}\hat{\tau}_t^l + \hat{w}_t^g \quad (10.4.42)$$

10.4.7 Linearized private physical capital accumulation

$$k_{t+1}^p = i_t + (1-\delta^p)k_t^p \quad (10.4.43)$$

Take natural logs from both sides of the equation to obtain

$$\ln k_{t+1}^p = \ln(i_t + (1-\delta^p)k_t^p) \quad (10.4.44)$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln k_{t+1}^p}{dt} = \frac{1}{i+(1-\delta)k^p}\frac{d(i_t+(1-\delta^p)k_t^p)}{dt} \quad (10.4.45)$$

Observe that since

$$i = \delta^p k^p, \text{ it follows that } \quad i + (1-\delta^p)k^p = \delta^p k^p + (1-\delta^p)k^p = k^p. \text{ Then} \quad (10.4.46)$$

$$\frac{dk_{t+1}^p}{dt}\frac{1}{k^p} = \frac{1}{k^p}\frac{di_t}{dt}\frac{i}{i} + \frac{k^p}{i+(1-\delta^p)k_t^p}\frac{dk_t^p}{dt}\frac{k^p}{k^p} \quad (10.4.47)$$

Pass to log-deviations to obtain

$$\hat{k}_{t+1}^p = \frac{\delta^p k^p}{k^p}\hat{i}_t + \frac{(1-\delta^p)k^p}{k^p}\hat{k}_t^p \quad (10.4.48)$$

$$\hat{k}_{t+1}^p = \delta^p\hat{i}_t + (1-\delta^p)\hat{k}_t^p \quad (10.4.49)$$

10.4.8 Linearized government physical capital accumulation

$$k_{t+1}^g = g_t^i + (1-\delta^g)k_t^g \quad (10.4.50)$$

Take natural logs from both sides of the equation to obtain

$$\ln k_{t+1}^g = \ln(g_t^i + (1-\delta^g)k_t^g) \quad (10.4.51)$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln k^g_{t+1}}{dt} = \frac{1}{g^i + (1-\delta^g)k^g}\frac{d(i_t + (1-\delta^g)k^g_t)}{dt} \tag{10.4.52}$$

Observe that since

$$g^i = \delta^g k^g, \text{ it follows that } \quad g^i + (1-\delta^g)k^g = \delta^g k^g + (1-\delta^g)k^g = k^g. \text{ Then} \tag{10.4.53}$$

$$\frac{dk^g_{t+1}}{dt}\frac{1}{k^g} = \frac{1}{k^g}\frac{dg^i_t}{dt}\frac{g^i}{g^i} + \frac{k^g}{^i + (1-\delta^g)k^g_t}\frac{dk^g_t}{dt}\frac{k^g}{k^g} \tag{10.4.54}$$

Pass to log-deviations to obtain

$$\hat{k}^g_{t+1} = \frac{\delta^g k^g}{k^g}\hat{g}^i_t + \frac{(1-\delta^p)k^g}{k^g}\hat{k}^g_t \tag{10.4.55}$$

$$\hat{k}^g_{t+1} = \delta^g \hat{g}^i_t + (1-\delta^g)\hat{k}^g_t \tag{10.4.56}$$

10.4.9 Linearized government budget constraint

$$(1-\tau^l_t)w^g_t n^g_t + g^i_t = \tau^k_t r_t k_t + \tau^l_t w^p_t n^p_t. \tag{10.4.57}$$

Take natural logarithms from both sides to obtain

$$\ln\left[(1-\tau^l_t)w^g_t n^g_t + g^i_t\right] = \ln\left[\tau^k_t r_t k_t + \tau^l_t w^p_t n^p_t\right]. \tag{10.4.58}$$

Totally differentiate with respect to time to obtain

$$\frac{d}{dt}\ln\left[(1-\tau^l_t)w^g_t n^g_t + g^i_t\right] = \frac{d}{dt}\ln\left[\tau^k_t r_t k_t + \tau^l_t w^p_t n^p_t\right]. \tag{10.4.59}$$

or

$$\frac{1}{(1-\tau^l)w^g n^g + g^i}\frac{d\left[(1-\tau^l_t)w^g_t n^g_t + g^i_t\right]}{dt} = \frac{1}{\left[\tau^k r k + \tau^l w^p n^p\right]}\frac{d\left[\tau^k_t r_t k_t + \tau^l_t w^p_t n^p_t\right]}{dt} \tag{10.4.60}$$

Note that

$$(1-\tau^l)w^g n^g + g^i = \tau^k r k + \tau^l w^p n^p \tag{10.4.61}$$

Hence

$$\frac{d\Big[(1-\tau_t^l)w_t^g n_t^g + g_t^i\Big]}{dt} = \frac{d\Big[\tau_t^k r_t k_t + \tau_t^l w_t^p n_t^p\Big]}{dt}. \tag{10.4.62}$$

or

$$\begin{aligned} & -w^g n^g \frac{d\tau_t^l}{dt}\frac{\tau^l}{\tau^l} + (1-\tau^l)n^g \frac{dw_t^g}{dt}\frac{w^g}{w^g} + (1-\tau^l)w^g \frac{dn_t^g}{dt}\frac{n^g}{n^g} + \frac{dg_t^i}{dt}\frac{g^i}{g^i} \\ = rk\frac{d\tau_t^k}{dt}\frac{\tau^k}{\tau^k} + \tau^k k \frac{dr_t}{dt}\frac{r}{r} + \tau^k r\frac{dk_t}{dt}\frac{k}{k} + w^p n^p \frac{d\tau_t^l}{dt} + \tau^l n^p \frac{dw_t^p}{dt}\frac{w^p}{w^p} + \tau^l w^p \frac{dn_t^p}{dt}\frac{n^p}{n^p} \end{aligned} \tag{10.4.63}$$

Pass to log-deviations to obtain

$$\begin{aligned} & -\tau^l w^g n^g \hat{\tau}_t^l + (1-\tau^l)w^g n^g \hat{w}_t^g + (1-\tau^l)w^g n^g \hat{n}_t^g + g^i \hat{g}_t^i \\ = \tau^k rk\hat{\tau}_t^k + \tau^k rk\hat{r}_t + \tau^k rk\hat{k}_t + \tau^l w^p n^p \hat{\tau}_t^l + \tau^l w^p n^p \hat{w}_t^p + \tau^l w^p n^p \hat{n}_t^p \end{aligned} \tag{10.4.64}$$

10.4.10 Total hours/employment

$$n_t = n_t^g + n_t^p \tag{10.4.65}$$

Take logs from both sides to obtain

$$\ln n_t = \ln(n_t^g + n_t^p) \tag{10.4.66}$$

Totally differentiate to obtain

$$\frac{d\ln n_t}{dt} = \frac{d\ln(n_t^g + n_t^p)}{dt} \tag{10.4.67}$$

$$\frac{dn_t}{dt}\frac{1}{n} = \left(\frac{dn_t^g}{dt} + \frac{dn_t^p}{dt}\right)\frac{1}{n} \tag{10.4.68}$$

$$\frac{dn_t}{dt}\frac{1}{n} = \left(\frac{dn_t^g}{dt}\frac{n^g}{n^g} + \frac{dn_t^p}{dt}\frac{n^p}{n^p}\right)\frac{1}{n} \tag{10.4.69}$$

$$\frac{dn_t}{dt}\frac{1}{n} = \frac{dn_t^g}{dt}\frac{1}{n^g}\frac{n^g}{n} + \frac{dn_t^p}{dt}\frac{1}{n^p}\frac{n^p}{n} \tag{10.4.70}$$

Pass to log-deviations to obtain

$$\hat{n}_t = \frac{n^g}{n}\hat{n}_t^g + \frac{n^p}{n}\hat{n}_t^p \tag{10.4.71}$$

10.4.11 Linearized private wage rate

$$w_t^p = \theta \frac{y_t}{n_t^p} \tag{10.4.72}$$

Take natural logarithms from both sides to obtain

$$\ln w_t^p = \ln \theta + \ln y_t - \ln n_t^p \tag{10.4.73}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln w_t^p}{dt} = \frac{d \ln \theta}{dt} + \frac{d \ln y_t}{dt} - \frac{d \ln n_t^p}{dt} \tag{10.4.74}$$

Simplify to obtain

$$\frac{dw_t^p}{dt}\frac{1}{w^p} = \frac{dy_t}{dt}\frac{1}{y} - \frac{dn_t^p}{dt}\frac{1}{n^p} \tag{10.4.75}$$

Pass to log-deviations to obtain

$$\hat{w}_t^p = \hat{y}_t - \hat{n}_t^p \tag{10.4.76}$$

10.4.12 Linearized real interest rate

$$r_t = \theta \frac{y_t}{k_t^p} \tag{10.4.77}$$

Take natural logarithms from both sides to obtain

$$\ln r_t = \ln \theta + \ln y_t - \ln k_t^p \tag{10.4.78}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln r_t}{dt} = \frac{d \ln \theta}{dt} + \frac{d \ln y_t}{dt} - \frac{d \ln k_t^p}{dt} \tag{10.4.79}$$

Simplify to obtain

$$\frac{dr}{dt}\frac{1}{r} = \frac{dy_t}{dt}\frac{1}{y} - \frac{dk_t^p}{dt}\frac{1}{k^p} \tag{10.4.80}$$

Pass to log-deviations to obtain

$$\hat{r}_t = \hat{y}_t - \hat{k}_t^p \tag{10.4.81}$$

10.4.13 Linearized government investment

$$g_t^i = g^{iy} y_t \tag{10.4.82}$$

Take natural logarithms from both sides to obtain

$$\ln g_t^i = \ln g^{iy} + \ln y_t \tag{10.4.83}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln g_t^i}{dt} = \frac{d \ln g^{iy}}{dt} + \frac{d \ln y_t}{dt} \tag{10.4.84}$$

or

$$\frac{dg_t^i}{dt} \frac{1}{g^i} = \frac{dy_t}{dt} \frac{1}{y} \tag{10.4.85}$$

Passing to log-deviations

$$\hat{g}_t^i = \hat{y}_t \tag{10.4.86}$$

10.4.14 Linearized government transfers

$$g_t^t = g^{ty} y_t \tag{10.4.87}$$

Take natural logarithms from both sides to obtain

$$\ln g_t^t = \ln g^{ty} + \ln y_t \tag{10.4.88}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln g_t^t}{dt} = \frac{d \ln g^{ty}}{dt} + \frac{d \ln y_t}{dt} \tag{10.4.89}$$

or

$$\frac{dg_t^t}{dt} \frac{1}{g^t} = \frac{dy_t}{dt} \frac{1}{y} \tag{10.4.90}$$

Passing to log-deviations

$$\hat{g}_t^t = \hat{y}_t \tag{10.4.91}$$

10.4.15 Linearized government services

$$s_t^g = (n_t^g)^\alpha (k_t^g)^{(1-\alpha)} \tag{10.4.92}$$

Take natural logarithms from both sides to obtain

$$\ln s_t^g = \alpha \ln n_t^g + (1-\alpha) \ln k_t^g \tag{10.4.93}$$

Totally differentiate with respect to time to obtain

$$\frac{ds_t^g}{dt}\frac{1}{s^g} = \alpha \frac{dn_t^g}{dt}\frac{1}{n^g} + (1-\alpha)\frac{dk_t^g}{dt}\frac{1}{k^g} \tag{10.4.94}$$

$$\hat{s}_t^g = \alpha \hat{n}_t^g + (1-\alpha)\hat{k}_t^g \tag{10.4.95}$$

10.4.16 Linearized technology shock process

$$\ln a_{t+1} \quad = \quad \rho^a \ln a_t + \epsilon_{t+1}^a \tag{10.4.96}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln a_{t+1}}{dt} \quad = \quad \rho^a \frac{d \ln a_t}{dt} + \frac{d\epsilon_{t+1}^a}{dt} \tag{10.4.97}$$

$$\frac{da_{t+1}}{dt} \quad = \quad \rho^a \frac{da_t}{dt} + \epsilon_{t+1}^a \tag{10.4.98}$$

where for $t = 1$ $\frac{d\epsilon_{t+1}^a}{dt} \approx \ln(e^{\epsilon_{t+1}^a}/e^{\epsilon^a}) = \epsilon_{t+1}^a - \epsilon^a = \epsilon_{t+1}^a$ since $\epsilon^a = 0$. Pass to log-deviations to obtain

$$\hat{a}_{t+1} \quad = \quad \rho^a \hat{a}_t + \epsilon_{t+1}^a \tag{10.4.99}$$

10.5 Log-linearized DCE system

$$0 \quad = \quad -\hat{y}_t + (1-\theta)\hat{k}_t^p + \hat{a}_t + \theta \hat{n}_t^p \tag{10.5.1}$$

$$k^p \hat{k}_{t+1}^p \quad = \quad y\hat{y}_t - c\hat{c}_t + (1-\delta^p)k^p\hat{k}_t^p - g^i\hat{g}_t^i \tag{10.5.2}$$

$$-\hat{c}_t = \hat{\lambda}_t \tag{10.5.3}$$

$$\hat{\lambda}_t = E_t\hat{\lambda}_{t+1} + \frac{\beta(1-\tau^k)(1-\theta)y}{k^p}E_t\hat{y}_{t+1} - \frac{\beta(1-\tau^k)(1-\theta)y}{k^p}E_t\hat{k}^p_{t+1} \tag{10.5.4}$$

$$\hat{c}_t = -\frac{1-n^g-\gamma(n^g)^2}{1-n^p-n^g-\gamma(n^g)^2}\hat{n}^p_t - \frac{n^g(1+2\gamma n^g)}{1-n^p-n^g-\gamma(n^g)^2}\hat{n}^g_t + \hat{y}_t \tag{10.5.5}$$

$$\hat{c}_t = -\frac{n^p}{1-n^p-n^g-\gamma(n^g)^2}\hat{n}^p_t - \frac{n^g(1+2\gamma n^g)}{1-n^p-n^g-\gamma(n^g)^2}\hat{n}^g_t + \hat{w}^g_t \tag{10.5.6}$$

$$\hat{k}^p_{t+1} = \delta^p\hat{i}_t + (1-\delta^p)\hat{k}^p_t \tag{10.5.7}$$

$$\hat{k}^g_{t+1} = \delta^g\hat{g}^i_t + (1-\delta^g)\hat{k}^g_t \tag{10.5.8}$$

$$\begin{aligned} &-\tau^l w^g n^g\hat{\tau}^l_t + (1-\tau^l)w^g n^g\hat{w}^g_t + (1-\tau^l)w^g n^g\hat{n}^g_t + g^i\hat{g}^i_t \\ &= \tau^k rk\hat{\tau}^k_t + \tau^k rk\hat{r}_t + \tau^k rk\hat{k}_t + \tau^l w^p n^p\hat{\tau}^l_t + \tau^l w^p n^p\hat{w}^p_t + \tau^l w^p n^p\hat{n}^p_t \end{aligned} \tag{10.5.9}$$

$$\hat{g}^i_t = \hat{y}_t \tag{10.5.10}$$

$$\hat{g}^t_t = \hat{y}_t \tag{10.5.11}$$

$$\hat{w}^p_t = \hat{y}_t - \hat{n}^p_t \tag{10.5.12}$$

$$\hat{r}_t = \hat{y}_t - \hat{k}^p_t \tag{10.5.13}$$

$$\hat{s}^g_t = \alpha\hat{n}^g_t + (1-\alpha)\hat{k}^g_t \tag{10.5.14}$$

$$\hat{a}_{t+1} = \rho^a\hat{a}_t + \epsilon^a_{t+1} \tag{10.5.15}$$

The model can be now solved by representing it in the following matrix form

$$\mathbf{A}E_{t+1}\mathbf{\hat{x}_t} = \mathbf{B}\mathbf{\hat{x}_t} + \mathbf{C}\varepsilon_\mathbf{t}, \tag{10.5.16}$$

where $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are coefficient matrices, $\varepsilon_\mathbf{t}$ is a matrix of innovations, and $\mathbf{\hat{x}_t}$ is the stacked vector of state (also called 'predetermined') variables, $\mathbf{\hat{s}_t} = \begin{bmatrix} \hat{a}_t & \hat{k}_t^p & \hat{k}_t^g \end{bmatrix}'$, and control variables, $\mathbf{\hat{z}_t} = \begin{bmatrix} \hat{y}_t & \hat{c}_t & \hat{i}_t & \hat{n}_t & \hat{n}_t^p & \hat{n}_t^g & \hat{w}_t^p & \hat{w}_t^g & \hat{\lambda}_t & \hat{g}_t^t & \hat{g}_t^i & \hat{s}_t^g \end{bmatrix}'$. Klein's (2000) generalized eigenvalue ("Schur") decomposition algorithm was used to solve the model. The MATLAB function to solve the above linear system is *solab.m*. The inputs are matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}$ defined above and $nk = 3$, which is the number of state variables. The outputs are the coefficient matrices $\mathbf{M}$ and $\mathbf{\Pi}$ which solve the linearized system. A solution to an RBC model is in the form of (approximate) policy, or transition rule, which describes the evolution of each variable. In particular, the predetermined and non-predetermined variables can be represented in the following form:

$$E_t\mathbf{\hat{s}_{t+1}} = \Pi\mathbf{\hat{s}_t} \tag{10.5.17}$$

$$\mathbf{\hat{z}_t} = M\mathbf{\hat{s}_t} \tag{10.5.18}$$

To simulate the model, one requires a sequence of normally distributed disturbances, $\{\epsilon_t\}_{t=0}^{\infty}$ for the three exogenous shocks with sample size T, the initial values of the endogenous predetermined variables, $\{k_0^p, k_0^g, a_0\}$ ($a_0 = 1$), and the evolution of the endogenous non-predetermined variables in model solution form

$$\mathbf{\hat{s}_{t+1}} = \Pi\mathbf{\hat{s}_t} + \mathbf{D}\varepsilon_\mathbf{t+1} \tag{10.5.19}$$

$$\mathbf{\hat{z}_t} = \mathbf{M}\mathbf{\hat{s}_t}, \tag{10.5.20}$$

where

$$\mathbf{D} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{10.5.21}$$

Based on the above representation, MATLAB code was written to simulate the model. The computation of impulse responses using the linearized model solution is straightforward.

10.6 The Effect of a positive productivity shock: robustness check

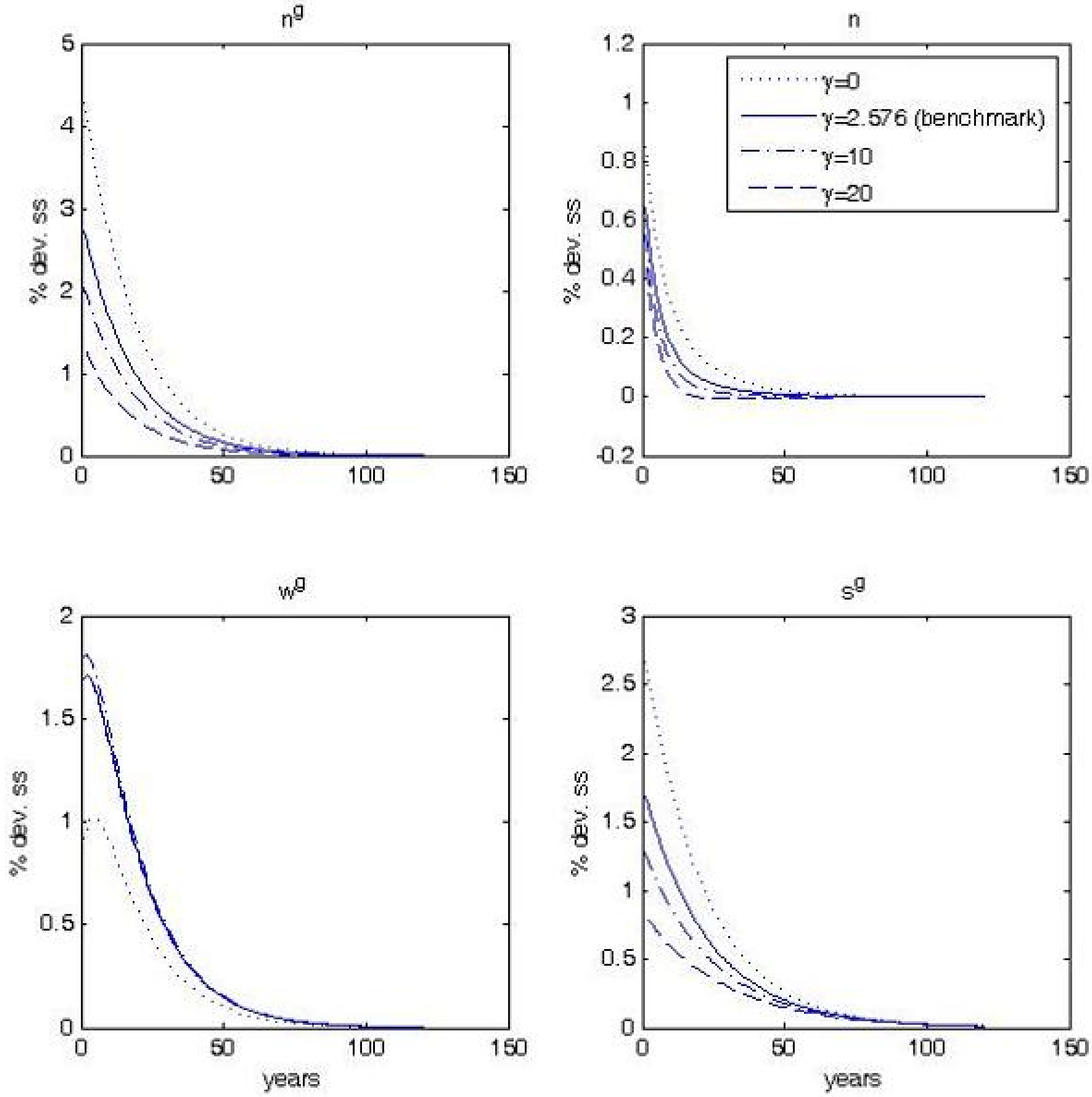
Given the quantitative importance of the new parameter γ for the steady-state values of the model variables, this subsection will examine the relevance of the transaction cost scale parameter for the transitional dynamics. The experiment in the next subsection will focus on the effect of a change in the transaction cost parameter γ on impulse responses.[41] When the value of the transaction cost parameter is changed, public sector labor variables are affected significantly. Other variables show no visible difference in terms of their impulse responses, and are thus not pictured in Fig. 3 on the next page. A higher transaction cost in the government sector is quantitatively important, as it affects the shape of the reaction function of public employment (hence total employment as well) and wage rate, as well as the level of public services provided, to unexpected technology innovation.

In particular, a higher value of γ significantly dampens public employment response in the face of a surprise TFP shock. For example, a higher transaction cost increases the disutility of work in the public section very quickly, which makes the household unwilling to reallocate hours between government work and leisure, as well as between the public and private sector work. Total employment, being a sum of private and public employment, is affected in a similar fashion. However, given the fact that private employment drives the dynamics of total employment, the quantitative effect of γ on the latter is rather small. Still, doubling the value of γ relative to the benchmark case produces a notable difference in the impulse responses that is present for more than 40 model years.

Next, through the mechanics of the public good production function, the dynamics of government services will be depressed as well. Since the impulse response of the public good provision depends on public hours and public capital, and given the fact that capital does not respond to changes in γ, the reaction in government services is proportional to the reaction in public hours (artifact of the linearization procedure implemented to solve the model).

[41] Note that different values of γ produce different steady-states. Thus, in what follows, variables fluctuate around different equilibria.

Figure 5: Impulse Responses: Sensitivity Analysis



In particular, the coefficient of proportionality equals the labor share α in the public sector.

Finally, the increase in transaction cost parameter γ above the benchmark calibration value strengthens the response of public wages upon the impact of the total factor productivity shock. The quantitative effect, however, is not large. Nevertheless, by driving the scale parameter down to zero, the government could decrease public wage volatility by a factor of two.

Overall, the small experiment performed in this section suggests that changes in γ parameter could significantly affect relative volatilities of public sector labor market variables as well. In particular, an increase in the transaction cost is expected to increase the relative volatility of the public wage rate, and decrease public employment variability. These conjectures are investigated further in the simulation section that follows.

10.7 Model simulation and goodness-of-fit

Using the model solutions, shock series were added to produce simulated data series. The length of the draws for the series of innovations is 138, and the simulation is replicated 1000 times. Natural logarithms are taken, and then all series are run through the Hodrick-Prescott filter with a smoothing parameter equal to 100. The first 100 observations are then excluded to decrease any dependence on the initial realizations of the innovations. Average standard deviation of each variable and its correlation of output of are estimated across the 1000 replications.The large number of replications implemented is to average out sampling error across simulations, before comparing model moments to the ones obtained from data.

10.7.1 Relative second moments evaluation

This section compares the theoretical second moments of the simulated data series with their empirical counterparts, with special attention paid to the behavior of public sector hours and wages. Table 3 on the next page summarizes the empirical and simulated business cycle statistics for the model calibrated for Germany.

In the German data, relative consumption volatility exceeds one, as the available series does not provide a breakdown into consumption of non-durables and consumption of durables. Another possible reason could be the presence of strong habits in consumption. Durable products behave like investment, and vary much more than non-durables, while model con-

Table 4: Business Cycle Statistics Germany, 1970-2007

	GE Data	Model
$\sigma(y)$	0.0154	0.0144
$\sigma(c)/\sigma(y)$	1.11	0.55 [0.52,0.58]
$\sigma(i)/\sigma(y)$	3.57	3.00 [2.94,3.07]
$\sigma(n^p)/\sigma(y)$	1.05	0.26 [0.25,0.28]
$\sigma(n^g)/\sigma(y)$	1.06	2.25 [2.24,2.26]
$\sigma(n)/\sigma(y)$	0.73	0.54 [0.53,0.55]
$\sigma(w^p)/\sigma(y)$	1.16	0.77 [0.76,0.79]
$\sigma(w^g)/\sigma(y)$	3.50	1.41 [1.40,1.42]
$corr(c, y)$	0.80	0.96 [0.95,0.97]
$corr(i, y)$	0.85	0.98 [0.97,0.99]
$corr(n^p, y)$	0.60	0.90 [0.86,0.94]
$corr(n^g, y)$	0.11	1.00 [1.00,1.00]
$corr(n, y)$	0.60	0.98 [0.97,0.99]
$corr(w^p, y)$	0.60	0.99 [0.99,0.99]
$corr(w^g, y)$	0.35	1.00 [1.00,1.00]
$corr(n, n^p)$	0.92	0.97 [0.96,0.98]
$corr(n, n^g)$	0.43	0.99 [0.98, 0.99]
$corr(n^p, n^g)$	0.12	0.92 [0.88,0.95]
$corr(n^p, w^p)$	0.21	0.83 [0.76,0.89]
$corr(n^g, w^g)$	-0.38	0.99 [0.99,1.00]
$corr(w^p, w^g)$	0.48	1.00 [1.00,1.00]

sumption corresponds to non-durable consumption. Since a major force in all the three models is consumption smoothing, as dictated by the Euler equation, the model under-predicts consumption volatility and investment variability. The lower variability in the model obtained is a new result in the literature. It is due to the fact that labor markets interaction are much more important quantitatively for the short-run dynamics of the model. After all, the simulation horizon in the annual model is only 38 periods, given the span of data.

In terms of labor market fluctuations, private sector employment and private wage in the model also vary less compared to data. Total employment in German data varies less than either private or public employment due to smaller variation in the number of self-employed individuals. It is evident from Table 3 above that the model underestimates public wage volatility. Still, this simple model generates public wage that varies twice as much as the private sector wage. Therefore, the introduction of the transaction cost in this paper is definitely a step in the right direction. However, public employment in the model varies too much in the model, as compared to the volatility exhibited in German data. Overall, the transaction cost mechanism presented in this paper seems to have an important quantitative effect in the German economy, especially when describing public sector labor market fluctuations.

The model also captures relatively well the high contemporaneous correlations of main variables with output. Moreover, public sector variables are also pro-cyclical, but not as much as the models predict. Lastly, the model captures quite well the co-movement between labor market variables. The dimension where the model fails, however, is the correlation between public sector hours and wages: in German data, it is negative, while the model predicts an almost perfect positive linear relationship. A possible reason could be that the government may have a target for the wage bill share in output, so any increase in employment has to be matched with a corresponding decrease in the public sector wage rate. A fiscal rule of that sort (*e.g.* as in Economides *et al.* 2011) could generate the observed negative correlation in data.[42]

Overall, the model with transaction cost captures relatively well the labor market dynamics in Germany. Furthermore, the setup addresses dimensions that were ignored in earlier RBC models. Thus, the existence of such frictions in the public sector proves to be an important ingredient in RBC models when studying German labor markets.

10.7.2 Sensitivity Analysis

The results from the analysis performed so far are all contingent on the particular values of the parameters in the model calibration. Even though the benchmark calibration was justified either from previous studies, or from data averages, there might be still concerns

[42]However, Economides *et al.* (2011) model government wage bill share in output as a stochastic process. In addition, public employment is also fixed. Public sector workers choose their working hours, and public sector wage is determined residually from the wage bill. Thus, the correlation between hours and wages is matched by construction, and not due to any interesting mechanism in the model.

that the results are sensitive to particular values chosen for the parameters. First, there are no previous studies and estimates of the scale parameter of the transaction cost. The problem in the model in this paper was avoided by setting γ to match the average public/private wage ratio. It was shown that a change in the transaction cost parameter affects impulse responses of the public sector labor variables. Thus, it will be investigate further whether this result shows up in the relative second moments of public wage and hours as well.

Second, there might be uncertainties about the relative weights on different components of utility. Since there are limited micro econometric studies and no reliable estimates, the model was made consistent with the assumption used in the RBC literature that private consumption is twice more valuable than public consumption in household's utility, and that the household puts twice higher weight on utility from leisure as compared to utility derived from private consumption. Again, the empirical evidence for those assumptions is scarce.

In light of the limited empirical evidence and/or conflicting studies, those parameters will be allowed to deviate significantly from their benchmark values. In the following, robustness checks will be performed with the value of the transaction cost parameter, as well as with the utility weights attached to leisure and public services. Relative second moments and contemporaneous correlations with output will be compared across cases, and compared against the benchmark calibration and data.

The simulated sample second moments generated by a lower ($\gamma = 0$) and higher ($\gamma = 10$) transaction cost scale parameter are reported in Table 4 on the next page against German data. As expected, the only significant changes are in the relative volatility of public employment and government wages. In line with the evidence from the earlier section dealing with impulse responses, higher transaction costs lead to lower variability in public hours, but higher volatility in government wages. In both cases, a higher γ brings simulated moments a bit closer to the moments in data. Nevertheless, no significant changes in terms of correlation are observed.

Table 5: Moments of the model and data (alternative transaction cost parameter)

	GE Data	$\gamma = 0$	$\gamma = 2.576$	$\gamma = 10$
$\sigma(y)$	0.0154	0.0142	0.0144	0.0145
$\sigma(c)/\sigma(y)$	1.11	0.55 [0.52,0.57]	0.55 [0.52,0.58]	0.55 [0.52,0.58]
$\sigma(i)/\sigma(y)$	3.57	3.01 [2.94,3.07]	3.00 [2.94,3.07]	3.00 [2.93,3.06]
$\sigma(n^p)/\sigma(y)$	1.05	0.24 [0.24,0.25]	0.26 [0.25,0.28]	0.27 [0.26,0.27]
$\sigma(n^g)/\sigma(y)$	1.06	3.61 [3.59,3.62]	2.25 [2.24,2.26]	1.68 [1.67,1.69]
$\sigma(n)/\sigma(y)$	0.73	0.72 [0.70,0.73]	0.54 [0.53,0.55]	0.46 [0.45,0.47]
$\sigma(w^p)/\sigma(y)$	1.16	0.79 [0.77,0.81]	0.77 [0.76,0.79]	0.77 [0.75,0.79]
$\sigma(w^g)/\sigma(y)$	3.50	0.79 [0.77,0.81]	1.41 [1.40,1.42]	1.49 [1.48,1.50]
$corr(c, y)$	0.80	0.96 [0.95,0.97]	0.96 [0.95,0.97]	0.96 [0.95,0.97]
$corr(i, y)$	0.85	0.98 [0.97,0.99]	0.98 [0.97,0.99]	0.98 [0.97,0.99]
$corr(n^p, y)$	0.60	0.88 [0.84,0.93]	0.90 [0.86,0.94]	0.90 [0.87,0.94]
$corr(n^g, y)$	0.11	1.00 [1.00,1.00]	1.00 [1.00,1.00]	1.00 [1.00,1.00]
$corr(n, y)$	0.60	0.99 [0.98,0.99]	0.98 [0.97,0.99]	0.97 [0.96,0.98]
$corr(w^p, y)$	0.60	0.99 [0.98,0.99]	0.99 [0.99,0.99]	0.99 [0.99,0.99]
$corr(w^g, y)$	0.35	0.99 [0.98,0.99]	1.00 [1.00,1.00]	1.00 [1.00,1.00]
$corr(n, n^p)$	0.92	0.95 [0.93,0.97]	0.97 [0.96,0.98]	0.98 [0.97,0.99]
$corr(n, n^g)$	0.43	0.99 [0.99,0.99]	0.99 [0.98, 0.99]	0.98 [0.98, 0.99]
$corr(n^p, n^g)$	0.12	0.90 [0.86,0.94]	0.92 [0.88,0.95]	0.92 [0.89,0.95]
$corr(n^p, w^p)$	0.21	0.81 [0.74,0.87]	0.83 [0.76,0.89]	0.83 [0.77,0.89]
$corr(n^g, w^g)$	-0.38	0.98 [0.98,0.99]	0.99 [0.99,1.00]	1.00 [0.99,1.00]
$corr(w^p, w^g)$	0.48	1.00 [1.00,1.00]	1.00 [1.00,1.00]	1.00 [1.00,1.00]

Next, the results from simulations using higher- ($\psi_1/\psi_3 = 3$) and lower ratios ($\psi_1/\psi_3 = 1.5$) are reported in Table 5 on the next page. These two alternative calibrations are compared against the benchmark values and data. As ψ_3 increases, public good consumption becomes more valuable relative to the private consumption good. In addition, given the fixed value of the utility weight attached to the latter (ψ_1), the weight attached to leisure (ψ_2) decreases. The fall in ψ_2 makes private consumption more valuable relative to leisure. Therefore, hours will be expected to vary less, as leisure becomes a less important component of household's welfare. Indeed, as a result of the change in utility parameters, absolute output volatility, as well as relative public and total hours slightly fall, while private and public wage volatility increase a bit. However, the differences in volatilities are minute. Furthermore, there is no change in the other variables: consumption and investment vary the same across the three cases, and all contemporaneous correlations are virtually the same.

In summary, the sensitivity analysis performed above considered significant variations in the values chosen for some of the model parameters, *i.e.*, varying the scale parameter of the transaction cost function, or changing the weights on household's utility and thus affecting the preference for the mix of the private and the public consumption good. It was shown that the transaction cost scale parameter, γ, has important quantitative role in the model, as an increase in the level of transaction costs increases the relative volatility of public, and total hours, while depressing public wage rates. However, since γ was calibrated to match the average public/private wage ratio in data, those effects are not relevant in the current setup. In addition, changes in utility weights proved to be of no quantitative importance. Thus the benchmark model could be considered a plausible case which adequately approximates household's preferences in the real world. Furthermore, the model was shown to be robust to such changes in the preferences for consumption relative to leisure. In other words, there is no significant undervaluation or overvaluation of components of utility, and the public good in particular.

Table 6: Moments of the model and data (alternative utility parameters)

	Germany Data	$\psi_2 = 0.59$ $\psi_3 = 0.10$	$\psi_2 = 0.53$ $\psi_3 = 0.16$	$\psi_2 = 0.49$ $\psi_3 = 0.20$
$\sigma(y)$	0.0154	0.0147	0.0144	0.0144
$\sigma(c)/\sigma(y)$	1.11	0.55 [0.52,0.57]	0.55 [0.52,0.58]	0.55 [0.52,0.58]
$\sigma(i)/\sigma(y)$	3.57	3.01 [2.94,3.08]	3.00 [2.94,3.07]	3.00 [2.94,3.07]
$\sigma(n^p)/\sigma(y)$	1.05	0.28 [0.27,0.29]	0.26 [0.25,0.28]	0.26 [0.25,0.28]
$\sigma(n^g)/\sigma(y)$	1.06	2.27 [2.25,2.28]	2.25 [2.24,2.26]	2.25 [2.24,2.26]
$\sigma(n)/\sigma(y)$	0.73	0.56 [0.54,0.57]	0.54 [0.53,0.55]	0.54 [0.53,0.55]
$\sigma(w^p)/\sigma(y)$	1.16	0.76 [0.74,0.77]	0.77 [0.76,0.79]	0.77 [0.76,0.79]
$\sigma(w^g)/\sigma(y)$	3.50	1.40 [1.38,1.41]	1.41 [1.40,1.42]	1.41 [1.40,1.42]
$corr(c,y)$	0.80	0.96 [0.95,0.97]	0.96 [0.95,0.97]	0.96 [0.95,0.97]
$corr(i,y)$	0.85	0.98 [0.97,0.99]	0.98 [0.97,0.99]	0.98 [0.97,0.99]
$corr(n^p,y)$	0.60	0.91 [0.87,0.94]	0.90 [0.86,0.94]	0.90 [0.86,0.94]
$corr(n^g,y)$	0.11	1.00 [1.00,1.00]	1.00 [1.00,1.00]	1.00 [1.00,1.00]
$corr(n,y)$	0.60	0.98 [0.97,0.99]	0.98 [0.97,0.99]	0.98 [0.97,0.99]
$corr(w^p,y)$	0.60	0.99 [0.99,0.99]	0.99 [0.99,0.99]	0.99 [0.99,0.99]
$corr(w^g,y)$	0.35	1.00 [1.00,1.00]	1.00 [1.00,1.00]	1.00 [1.00,1.00]
$corr(n,n^p)$	0.92	0.97 [0.96,0.98]	0.97 [0.96,0.98]	0.97 [0.96,0.98]
$corr(n,n^g)$	0.43	0.99 [0.98, 0.99]	0.99 [0.98, 0.99]	0.99 [0.98, 0.99]
$corr(n^p,n^g)$	0.12	0.92 [0.88,0.95]	0.92 [0.88,0.95]	0.92 [0.88,0.95]
$corr(n^p,w^p)$	0.21	0.83 [0.77,0.89]	0.83 [0.76,0.89]	0.83 [0.76,0.89]
$corr(n^g,w^g)$	-0.38	0.99 [0.99,1.00]	0.99 [0.99,1.00]	0.99 [0.99,1.00]
$corr(w^p,w^g)$	0.48	1.00 [0.99,1.00]	1.00 [1.00,1.00]	1.00 [1.00,1.00]

10.8 Ramsey problem derivation

In this section, the public finance approach, pioneered by Atkinson and Stiglitz (1980) in a static context with many consumption goods, is applied here in a dynamic setting with a single consumption good. In particular, the intertemporal framework, as first outlined in Ramsey (1927), will consist of a social planner (government), who desires to maximizes agent's utility subject to the constraints that describe the competitive economy. In other words, in the optimal fiscal policy (Ramsey) framework, the government assumes the role of a benevolent planner, who takes into account that the representative household and the firm behave in their own best interest, taking fiscal policy variables as given. The instruments under government's control in this section are labor and capital tax rates, next-period public capital, public employment and public sector wage rate. It is assumed that only linear taxes are allowed, and that the government can credibly commit to those. Importantly, lump-sum taxation is prohibited, as the professional literature assumes that the government cannot redistribute income lump-sum. Thus, given the restriction to a set of linear distortionary tax rates, only a second-best outcome is feasible. However, the emphasis on the second-best theory makes the setup more realistic, and thus can be taken as a better approximation to the environment in which policymakers decide on a particular fiscal policy.

Another assumption crucial to the Ramsey approach is that the activities of all agents in the economy are observable, as information in this setup is perfect. Lastly, the government has access to a commitment technology, which effectively will tie its own hands and prevent it from reneging on an initially-made promise at a later point in time. Thus, in the full-commitment scenario, the time inconsistency problem, as described by Kydland and Prescott (1977), cannot arise; The government will made a "once-and-for-all" decision in period 0 and no further re-optimization will be allowed. In fact, under full commitment, such re-optimization will never be desired by the government: whatever policy is announced by the government in period 0 is the one that is implemented afterwards.

It is important to emphasize that each set of fiscal policy instruments implies a feasible allocation that fully reflects the optimal behavioral responses of the household and firm. Alternatively, each set of fiscal policy instruments can be thought of generating a different competitive equilibrium allocation, i.e. allocations and prices are contingent on the particular values chosen for the fiscal instruments. The difference from the analysis performed so far in the paper, is that in Ramsey framework, the government chooses all instruments, instead of taking them as being exogenous. At the same time, the government also picks optimally the allocations of agents, as dictated by the dual approach to the Ramsey problem as in Chamley (1986).[43] It will also be assumed that the government discounts time at the same rate as the representative household. The constraints which the government takes into account when maximizing household's welfare include the government budget constraints, and the behavioral responses of both the household, and the firm. These are summarized in the DCE of the exogenous fiscal policy case (13.2.1)-(13.2.14).[44] In other words, in the dual approach of Ramsey problem, which will be utilized in this section, the choice variables for the government are $\{C_t, N_t^p, N_t^g, K_{t+1}^p, K_{t+1}^g, w_t^p, w_t^g, r_t\}_{t=0}^{\infty}$ plus the two tax rates $\{\tau_t^l, \tau_t^k\}_{t=0}^{\infty}$.[45] The initial conditions for the state variable $\{K_0^p, K_0^g\}$ is taken as given.

Following the procedure in Chamley (1986) and Ljungqvist and Sargent (2004), the Ramsey problem will be transformed and simplified, so that the government chooses after-tax interest rate $\tilde{r}_t$ and wage rates $\tilde{w}_t^p$ and $\tilde{w}_t^p$ directly, instead of setting tax rates and prices separately, where

$$\tilde{r}_t \equiv (1-\tau_t^k) r_t \tag{10.8.1}$$

$$\tilde{w}_t^p \equiv (1-\tau_t^l) w_t^p \tag{10.8.2}$$

$$\tilde{w}_t^g \equiv (1-\tau_t^l) w_t^g. \tag{10.8.3}$$

Next, the government budget constraint is rewritten as follows

$$\begin{aligned} \tau_t^k r_t K_t + \tau_t^l w_t^p N_t^p &= \\ r_t K_t - (1-\tau_t^k) r_t K_t + w_t^p N_t^p - (1-\tau_t^l) w_t^p N_t^p &= \\ (1-\tau_t^l) w_t^g N_t^g + K_{t+1}^g - (1-\delta^g) K_t^g + G_t^t&. \end{aligned} \tag{10.8.4}$$

From the constant-returns-to-scale production function it follows that

$$r_t K_t + w_t^p N_t^p = A_t (N_t)^{\theta} K_t^{1-\theta}. \tag{10.8.5}$$

[43]In contrast, the primal approach all the policy variables and prices are solved as functions of the allocations, thus the government decides only on the optimal allocation.

[44]Stockman (2001) shows that the absence of debt and thus the inability of the government to run surpluses and deficits has no dramatic effect on the optimal policies in the full commitment case.

[45]Note that by choosing next-period public capital, the planner is choosing public investment optimally.

Substitute out (13.8.5) into (13.8.4) and using (13.8.1)-(13.8.3), the transformed government budget constraint becomes

$$A_t(N_t)^{\theta}K_t^{1-\theta} - \tilde{r}_t K_t - \tilde{w}_t^p N_t^p = \tilde{w}_t^g N_t^g + K_{t+1}^g - (1-\delta^g)K_t^g + G_t^t. \qquad (10.8.6)$$

Once the optimal after-tax returns are solved for, the expression for the before-tax real interest rate and private wage can be obtained from the DCE system. Solving for optimal capital and labor tax rates is then trivial.

The transformed Ramsey problem then becomes:

$$\max_{C_t, N_t^p, N_t^g, K_{t+1}^p, K_{t+1}^g, \tilde{w}_t^p, \tilde{w}_t^g, \tilde{r}_t} E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \psi_1 \ln C_t + \psi_2 \ln\left[1 - N_t^p - N_t^g - \gamma(N_t^g)^2\right]$$
$$+ (1-\psi_1-\psi_2)\ln\left[(N_t^g)^{\alpha}(K_t^g)^{1-\alpha}\right]\Bigg\} \qquad (10.8.7)$$

s.t

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}}\left[1 - \delta^p + (1-\tau_{t+1}^k)(1-\theta)\frac{Y_{t+1}}{K_{t+1}^p}\right] \qquad (10.8.8)$$

$$\psi_2 C_t = \psi_1\left[1 - N_t^p - N_t^g - \gamma(N_t^g)^2\right](1-\tau_t^l)w_t^p \qquad (10.8.9)$$

$$\psi_2 C_t[1 + 2\gamma N_t^g] = \psi_1\left[1 - N_t^p - N_t^g - \gamma(N_t^g)^2\right](1-\tau_t^l)w_t^g \qquad (10.8.10)$$

$$A_t(N_t^p)^{\theta}K_t^{(1-\theta)} = C_t + K_{t+1}^g - (1-\delta^g)K_t^g + K_{t+1}^p - (1-\delta^p)K_t^p \qquad (10.8.11)$$

$$A_t(N_t)^{\theta}K_t^{1-\theta} - \tilde{r}_t K_t - \tilde{w}_t^p N_t^p = \tilde{w}_t^g N_t^g + K_{t+1}^g - (1-\delta^g)K_t^g + G_t^t \qquad (10.8.12)$$

$$K_{t+1}^p = I_t + (1-\delta^p)K_t^p \qquad (10.8.13)$$

$$r_t = (1-\theta)\frac{Y_t}{K_t^p} \qquad (10.8.14)$$

$$w_t^p = \theta\frac{Y_t}{N_t^p} \qquad (10.8.15)$$

$$S_t^g = (N_t^g)^\alpha (K_t^g)^{(1-\alpha)} \tag{10.8.16}$$

$$K_{t+1}^g \;\; = \;\; G_t^i + (1-\delta^g)K_t^g \tag{10.8.17}$$

The Lagrangian function of the government thus becomes

$$\begin{aligned}
&\mathcal{L}^g(C_t, N_t^p, N_t^g, K_{t+1}^p, K_{t+1}^g, \tilde{w}_t^p, \tilde{w}_t^g, \tilde{r}_t, \lambda_t^1, \lambda_t^2, \lambda_t^3, \lambda_t^4, \lambda_t^5) = \\
&= E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \psi_1 \ln C_t + \psi_2 \ln \Big[1 - N_t^p - N_t^g - \gamma (N_t^g)^2\Big] \\
&\quad + (1-\psi_1-\psi_2)\ln \Big[(N_t^g)^\alpha (K_t^g)^{1-\alpha}\Big] \\
&+\lambda_t^1 \Big[- C_{t+1} + \beta C_t (1 + \tilde{r}_{t+1} - \delta^g)\Big] + \lambda_t^2 \Big[\psi_2 C_t - \psi_1 [1 - N_t^p - N_t^g - \gamma (N_t^g)^2]\tilde{w}_t^p\Big] \qquad (10.8.18) \\
&+\lambda_t^3 \Big[\psi_2 C_t [1 + 2\gamma N_t^g] - \psi_1 [1 - N_t^p - N_t^g - \gamma (N_t^g]^2)\tilde{w}_t^g\Big] \\
&+\lambda_t^4 \Big[A_t (N_t^p)^\theta (K_t^p)^{(1-\theta)} + (1-\delta^p)K_t^p - C_t - K_{t+1}^p + (1-\delta^p)K_t^g - K_{t+1}^g\Big] \\
&+\lambda_t^5 \Big[A_t (N_t)^\theta (K_t^p)^{1-\theta} - \tilde{r}_t K_t^p - \tilde{w}_t^p N_t^p - \tilde{w}_t^g N_t^g - K_{t+1}^g + (1-\delta^g)K_t^g + G_t^t\Big]\Bigg\}
\end{aligned}$$

where $\lambda_t^i, i = 1,2,3,4,5$ is the Lagrangian multiplier associated with each constraint. The FOCs of some of the control variables differ in the initial period, as compared to following periods.[46] Therefore, the economy will be studied starting from period 1, assuming that period-0 optimality conditions do not affect equilibrium results. FOCs for $t \geq 1$:

$$C_t : \frac{\psi_1}{C_t} + \lambda_t^1 \beta (1 + \tilde{r}_{t+1} - \delta^p) + \frac{\lambda_{t-1}^1}{\beta} + \lambda_t^2 \psi_2 + \lambda_3 \psi_2 [1 + 2\gamma N_t^g] - \lambda_t^4 = 0 \tag{10.8.19}$$

$$N_t^p : \frac{\psi_2}{1 - N_t^p - N_t^g - \gamma (N_t^g)^2} = \lambda_t^2 \psi_1 \tilde{w}_t^p + \lambda_t^3 \psi_1 \tilde{w}_t^g + \lambda_t^4 \theta \frac{Y_t}{N_t^p} + \lambda_t^5 [\theta \frac{Y_t}{N_t^p} - \tilde{w}_t^p] \tag{10.8.20}$$

$$\begin{aligned}
N_t^g : \frac{\psi_2}{1 - N_t^p - N_t^g - \gamma (N_t^g)^2}[1 + 2\gamma N_t^g] = \\
\lambda_t^2 \psi_1 (1 + 2\gamma N_t^g)\tilde{w}_t^p + 2\gamma \lambda_t^3 \psi_2 C_t + \lambda_t^3 \psi_1 (1 + 2\gamma N_t^g)\tilde{w}_t^g - \lambda_t^5 \tilde{w}_t^g
\end{aligned} \tag{10.8.21}$$

[46] Those are the choice variables in the consumption Euler equation, i.e. C_t, N_t^p, N_t^g at $t = 0$ are different from those when $t \geq 1$. This is because when $t \geq 1$, the FOCs of those variables also include $t-1$ variables. However, at $t = 0$, the FOCs only include time 0 variables.

$$K_{t+1}^p: -\lambda_{t-1}^4 + \beta\lambda_t^4[(1-\theta)\frac{Y_t}{K_t^p} + 1 - \delta^p] + \beta\lambda_t^5[(1-\theta)\frac{Y_t}{K_t^p} - \tilde{r}_t] = 0 \qquad (10.8.22)$$

$$K_{t+1}^g: -\frac{\beta(1-\psi_1-\psi_2)(1-\alpha)}{K_{t+1}^g} - \lambda_t^4 - \lambda_t^5 + \beta(1-\delta^g)(\lambda_{t+1}^4 + \lambda_{t+1}^5) = 0 \qquad (10.8.23)$$

$$\tilde{w}_t^p: -\lambda_t^2\psi_1\left[1 - N_t^p - N_t^g - \gamma(N_t^g)^2\right] - \lambda_t^5 N_t^p = 0 \qquad (10.8.24)$$

$$\tilde{w}_t^g: -\lambda_t^3\psi_1\left[1 - N_t^p - N_t^g - \gamma(N_t^g)^2\right] - \lambda_t^5 N_t^g = 0 \qquad (10.8.25)$$

$$\tilde{r}_t: \lambda_{t-1}^1 C_{t-1} = \lambda_t^5 K_t \qquad (10.8.26)$$

$$\lambda_t^1: \frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}}\left[1 - \delta^p + \tilde{r}_{t+1}\right] \qquad (10.8.27)$$

$$\lambda_t^2: \psi_2 C_t = \psi_1(1 - N_t^p - N_t^g - \gamma(N_t^g)^2)\tilde{w}_t^p \qquad (10.8.28)$$

$$\lambda_t^3: \psi_2 C_t[1 + 2\gamma N_t^g] = \psi_1\left[1 - N_t^p - N_t^g - \gamma(N_t^g)^2\right]\tilde{w}_t^g \qquad (10.8.29)$$

$$\lambda_t^4: A_t(N_t^p)^\theta(K_t^p)^{(1-\theta)} = C_t + K_{t+1}^g - (1-\delta^g)K_t^g + K_{t+1}^p - (1-\delta^p)K_t^p \qquad (10.8.30)$$

$$\lambda_t^5: A_t(N_t^p)^\theta(K_t^p)^{1-\theta} - \tilde{r}_t K_t - \tilde{w}_t^p N_t^p = \tilde{w}_t^g N_t^g + K_{t+1}^g - (1-\delta^g)K_t^g + G_t^t \qquad (10.8.31)$$

$$I_t = K_{t+1}^p - (1-\delta)K_t^p \qquad (10.8.32)$$

$$r_t = (1-\theta)\frac{Y_t}{K_t^p} \qquad (10.8.33)$$

$$w_t^p = \theta\frac{Y_t}{N_t^p} \qquad (10.8.34)$$

$$S_t^g = (N_t^g)^\alpha(K_t^g)^{(1-\alpha)} \qquad (10.8.35)$$

$$G_t^i = K_{t+1}^g - (1-\delta^g)K_t^g \qquad (10.8.36)$$

In contrast to previous studies, *e.g.* Chari, Christiano and Kehoe (1994, 1999), who investigate the behavior of taxes over the business cycle, next section will concentrate on the steady-state values of the fiscal policy instruments, as in Judd (1985), Chamley (1986) and Ljungqvist and Sargent (2004).[47]

[47] In addition, in business cycles literature, capital tax rate is on average zero, and labor income tax rate is essentially constant over the cycle. As noted by Wyplosz (2001), since the 1980s, the countercyclical use of fiscal policies in OECD countries has declined.

10.9 Steady state analysis of Ramsey equilibrium

In this section, the focus is on the long-run effect of optimal government behavior. Thus, all time subscripts and uncertainty are eliminated to obtain:

$$\frac{\psi_1}{c} + \lambda^1\beta(1+\tilde{r}-\delta) + \frac{\lambda^1}{\beta} + \lambda^2\psi_2 + \lambda_3\psi_2[1+2\gamma n^g] - \lambda^4 = 0 \tag{10.9.1}$$

$$\frac{\psi_2}{1-n^p-n^g-\gamma(n^g)^2} = \lambda^2\psi_1\tilde{w}^p + \lambda^3\psi_1\tilde{w}^g + \lambda^4\theta\frac{y}{n^p} + \lambda^5[\theta\frac{y}{n^p} - \tilde{w}^p] \tag{10.9.2}$$

$$\begin{aligned}\frac{\psi_2}{1-n^p-n^g-\gamma(n^g)^2}[1+2\gamma n^g] &= \lambda_t^2\psi_1(1+2\gamma n^g)\tilde{w}^p \\ &+2\gamma\lambda^3\psi_2 c + \lambda^3\psi_1(1+2\gamma n^g)\tilde{w}^g - \lambda^5\tilde{w}^g\end{aligned} \tag{10.9.3}$$

$$-\lambda^4 + \beta\lambda^4[(1-\theta)\frac{y}{k} + 1 - \delta] + \beta\lambda^5[(1-\theta)\frac{y}{k} - \tilde{r}] = 0 \tag{10.9.4}$$

$$-\frac{\beta(1-\psi_1-\psi_2)(1-\alpha)}{k^g} - \lambda^4 - \lambda^5 + \beta(1-\delta^g)(\lambda^4+\lambda^5) = 0 \tag{10.9.5}$$

$$-\lambda^2\psi_1[1-n^p-n^g-\gamma(n^g)^2] - \lambda^5 n^p = 0 \tag{10.9.6}$$

$$-\lambda^3\psi_1[1-n^p-n^g-\gamma(n^g)^2] - \lambda^5 n^g = 0 \tag{10.9.7}$$

$$\lambda^1 c = \lambda^5 k \tag{10.9.8}$$

$$1 \;\; = \;\; \beta[1-\delta^p+\tilde{r}] \tag{10.9.9}$$

$$\psi_2 c = \psi_1[1-n^p-n^g-\gamma(n^g)^2]\tilde{w}^p \tag{10.9.10}$$

$$\psi_2 c[1+2\gamma n^g] = \psi_1[1-n^p-n^g-\gamma(n^g)^2]\tilde{w}^g \tag{10.9.11}$$

$$y = c + \delta^g k^g + \delta^p k^p \tag{10.9.12}$$

$$y - \tilde{r}k - \tilde{w}^p n^p = \tilde{w}^g n^g + \delta^g k^g + g^t \tag{10.9.13}$$

$$i = \delta^p k^p \tag{10.9.14}$$

$$r = (1-\theta)\frac{y}{k} \tag{10.9.15}$$

$$w^p = \theta\frac{y}{n^p} \tag{10.9.16}$$

$$g^i = \delta^g k^g \tag{10.9.17}$$

Next, an important analytical result in the optimal policy framework will be demonstrated: First, simplify the FOC for private capital (13.5.4) to obtain

$$\lambda^4 = \beta\lambda^4[(1-\theta)\frac{y}{k} + 1 - \delta^p] + \beta\lambda^5[(1-\theta)\frac{y}{k} - \tilde{r}_t]. \tag{10.9.18}$$

Next, substitute out the expression for the interest rate from (13.9.9) into (13.9.18) to obtain

$$\lambda_4 = \beta\lambda^4[r + (1-\delta)] + \beta\lambda^5[r - \tilde{r}]. \tag{10.9.19}$$

Now plug in the expression for $(1-\delta)$ from (13.9.9) into (13.9.19) to obtain

$$\lambda_4[1 - \beta(r + \frac{1}{\beta} - \tilde{r})] = \beta\lambda^5[r - \tilde{r}], \tag{10.9.20}$$

or

$$-\lambda_4\beta[r - \tilde{r}] = \beta\lambda^5[r - \tilde{r}]. \tag{10.9.21}$$

Hence

$$(\lambda^4 + \lambda_5)[r - \tilde{r}] = 0. \tag{10.9.22}$$

Since $\lambda^4, \lambda^5 > 0$ by construction, it follows that $\tilde{r} = r$. Therefore, in steady-state Ramsey equilibrium, the optimal steady-state capital tax is zero. This result is consistent with earlier findings in Judd (1985), Chamley (1986), Zhu (1992), Ljungqvist and Sargent (2004) and Kocherlakota (2010).

10.10 Measuring conditional welfare

In steady state

$$u(c, 1-n) = \psi_1 \ln c + \psi_2 \ln[1 - n^p - n^g - \gamma(n^g)^2] + \psi_3 \ln(n^g)^{\alpha}(k^g)^{(1-\alpha)}. \tag{10.10.1}$$

Let A and B denote two different regimes. The welfare gain, ξ, is the fraction of consumption that is needed to complement household's steady-state consumption in regime B so that the household is indifferent between the two regimes. Thus

$$\begin{aligned}\psi_1 \ln c^A + \psi_2 \ln(1 - n^A - \gamma(n^{g,A})^2) + \psi_3 \ln(n^{g,A})^\alpha + \psi_3 \ln(k^{g,A})^{(1-\alpha)} = \\ \psi_1 \ln[(1+\xi)c^B] + \psi_2 \ln(1 - n^B - \gamma(n^{g,B})^2) + \psi_3 \ln(n^{g,B})^\alpha + \psi_3 \ln(k^{g,B})^{(1-\alpha)},\end{aligned} \quad (10.10.2)$$

where $n^A = n^{p,A} + n^{g,A}$ and $n^B = n^{p,B} + n^{g,B}$.

Next, expand the right-hand side to obtain

$$\begin{aligned}\psi_1 \ln c^A + \psi_2 \ln(1 - n^A - \gamma(n^{g,A})^2) + \psi_3 \ln(n^{g,A})^\alpha + \psi_3 \ln(k^{g,A})^{(1-\alpha)} = \psi_1(1+\xi) \\ +\psi_1 \ln c^B + \psi_2 \ln(1 - n^B - \gamma(n^{g,B})^2) + \psi_3 \ln(n^{g,B})^\alpha + \psi_3 \ln(k^{g,B})^{(1-\alpha)}.\end{aligned} \quad (10.10.3)$$

Rearrange to obtain

$$\begin{aligned}\psi_1 \ln(1+\xi) = \psi_1 \ln c^A + \psi_2 \ln(1 - n^A - \gamma(n^{g,A})^2) + \psi_3 \ln(n^{g,A})^\alpha \\ +\psi_3 \ln(k^{g,A})^{(1-\alpha)} - \psi_1 \ln c^B - \psi_2 \ln(1 - n^B - \gamma(n^{g,B})^2) \\ -\psi_3 \ln(n^{g,B})^\alpha - \psi_3 \ln(k^{g,B})^{(1-\alpha)}.\end{aligned} \quad (10.10.4)$$

Divide throughout by ψ_1 to obtain

$$\begin{aligned}\ln(1+\xi) = \ln c^A + \frac{\psi_2}{\psi_1} \ln(1 - n^A - \gamma(n^{g,A})^2) + \frac{\psi_3}{\psi_1} \ln(n^{g,A})^\alpha + \frac{\psi_1}{\psi_{\ 3}} \ln(k^{g,A})^{(1-\alpha)} \quad (10.10.5) \\ - \ln c^B - \frac{\psi_2}{\psi_1} \ln(1 - n^B - \gamma(n^{g,B})^2) - \frac{\psi_3}{\psi_1} \ln(n^{g,B})^\alpha - \frac{\psi_3}{\psi_1} \ln(k^{g,B})^{(1-\alpha)}.\end{aligned}$$

Raise to the exponent both sides of the equation and rearrange terms to obtain

$$(1+\xi) = \frac{c^A}{c^B} \left[\frac{1 - n^A - \gamma(n^{g,A})^2}{1 - n^B - \gamma(n^{g,B})^2}\right]^{\frac{\psi_2}{\psi_1}} \left[\frac{n^{g,A}}{n^{g,B}}\right]^{\frac{\alpha\psi_3}{\psi_1}} \left[\frac{k^{g,A}}{k^{g,B}}\right]^{\frac{(1-\alpha)\psi_3}{\psi_1}}. \quad (10.10.6)$$

Thus

$$\xi = \frac{c^A}{c^B} \left[\frac{1 - n^A - \gamma(n^{g,A})^2}{1 - n^B - \gamma(n^{g,B})^2}\right]^{\frac{\psi_2}{\psi_1}} \left[\frac{n^{g,A}}{n^{g,B}}\right]^{\frac{\alpha\psi_3}{\psi_1}} \left[\frac{k^{g,A}}{k^{g,B}}\right]^{\frac{(1-\alpha)\psi_3}{\psi_1}} - 1. \quad (10.10.7)$$

Note that if $\xi > 0 (< 0)$, there is a welfare gain (loss) of moving from B to A. In this paper E is the exogenous policy case, while A will be the Ramsey policy scenario.