Measurement, Units and Scale: A Learning Pathway

Making interdisciplinary connections to enhance understanding and skills

It is a very common complaint from employers and higher education that school leavers in general lack appropriate understanding and skills in handling and assessing quantitative information[1].

Learning in this area, in schools, is scattered over many years and across different subject disciplines. The ***learning pathway*** *described below can be used to plan a more coherent and more successful learner journey.*

The value of a **Learning Pathway** approach in key concept areas emerged from a funded project "**Connecting it up: Towards a Route Map for STEM Education in Scotland.**" [2] Five Learning Pathways were drafted in that project, of which ***Measurement, Units and Scale*** is one. A Pathway maps how learners' "core understanding" can grow in scope, depth and sophistication during their passage through school. The documentation is designed to support collaborative planning by teachers of curriculum delivery, so as to make the most of opportunities to use, reinforce and extend previous learning.

The whole Pathway extends from age 3 to 15, spanning Levels "Early" till "Fourth" in the Scottish Curriculum for Excellence (*CfE)* framework. There are many ways in which the Pathway can be fully accommodated in the course of addressing relevant "***Experiences and Outcomes***" (E&Os) mandated as the official curriculum guidance at each successive Level.

Concise overview of the Pathway

In the first few years of schooling, learning about **measurement** is linked with a growing understanding of **numbers**, through counting in whole numbers, introducing fractions, then decimals, and finally negative values. Any measurement, however, involves two features, a

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[1] This is evidenced, for instance, in STEM-ED Scotland research reports on *The School to University Transition in STEM Subjects*, and *Industry and the School Curriculum*. See http://www.gla.ac.uk/stem/publications/

[2] This project was led by STEM-ED Scotland, in collaboration with the Association for Science Education Scotland, the Scottish Mathematical Council and the Scottish Technology Teachers' Association, and funded by the Esmée Fairbairn Foundation. Further details can be found at http://www.gla.ac.uk/stem/routemapforstemeducation

numerical value and a unit. The idea of a **unit** develops, first by expressing a length as a number of multiples of a convenient reference object (eg multiples of the length of a paper clip), then introducing internationally standard units. It quickly becomes clear that different units are convenient for measurements of different scales (eg lengths of a shoe, a wall or a journey) and different but related units can be useful for this. In the metric system standard prefixes are used (as in km) to indicate the **scale** relationship to the basic unit. In later education, particularly in science, a considerable range of scale factors is met.

At earlier levels, the ideas of measurement are introduced in reference to length, weight[3], time, temperature and money. The list of such **properties** is gradually extended, first in relation to geometry (through measuring areas and volumes), then to speed of motion. In science the same ideas about measurement can be applied to further contexts, such as to energy, to electricity and to chemical reactions.

Areas and volumes give an early introduction to **"compound" units** (m^2 and m^3 respectively). Speed is an example involving two different base units (m/s). Many other compound units arise later in science and technology, for example density (kg/m^3) and energy ($kg\ m^2/s^2$).

A good learning progression in handling units provides a potentially very useful first step towards becoming comfortable in using **symbols**. Thus "m" and "s" are routinely used as shorthand for "metres" and "seconds". Subsequent manipulation of these when measuring compound quantities (as in using m^2 for area, or m/s for speed) actually presages the approach to the later broader challenge of learning to use algebra. Measurement learning can also provide a useful elementary grounding in using **equations**; for instance measuring the area of a rectangle as equal to its length multiplied by its breadth can be summarised by the "mnemonic" $A = L \times B$.

"Scale" is the third word included in the title of the Learning Pathway, and there are three different senses of that word that are important.

- The first usage highlights the vast range of **scales of magnitude** that one meets and must come to recognise in our world (eg intergalactic distances versus those at an atomic level, or the national GDP versus an individual's weekly budget). "Size matters!"
- The second usage refers to different **"unit scales"** (eg prices may be quoted in £s or in Euros, temperatures may be noted in °F or °C, and our own weight may be recorded in terms of kg or in stones and pounds).
- The third usage relates to **scale models** or diagrams, and to "scaling" a known result to apply to a smaller or a larger scenario than the one studied. The very important and widely applicable concepts of **ratio and proportion** are often relevant here.

[3] "Weight" is used here in it's everyday non-technical sense; this is the property that becomes known as "mass" during secondary science.

Measurement, Units and Scale: a full draft Learning Pathway

This is a revised version of a draft produced in a project "Connecting it up: towards a Route Map for STEM Education."[4] The Learning Pathway articulates a series of developing strands of conceptual understanding that can underpin a mature understanding of issues and practice involving measurement, units and scale. To achieve this requires collaboration by teachers across mathematics, sciences and technologies.

This conceptual development process is described in the tables that follow, set out for each of the Levels of the *CfE* for ages 3 – 15[5]. These tables also reference potentially relevant *Experiences and Outcomes* from the curriculum guidance that has been issued to schools. The tables are followed by a glossary of terms, lists of some common misconceptions at each level, and some examples of cross-curricular activities that could be used to support learning in this Pathway.

Early Level

Strand	Statement of core understanding	Relevant Experiences & Outcomes
0.1	***Measuring:*** Words and numbers can be used to describe and compare objects, and put them in order. Properties of different objects, including length and weight, can be measured and compared by counting multiples of a convenient smaller object as a ("non-standard") unit.	MNU 0 – 01a, 02a, 11a, 20a, 20b SCN 0 – 06a, 15a, 20b TCH 0 – 01a, 14a SOC 0 – 04a HWB 0 – 47a
0.2	***Time period:*** Passage of time is measured by counting weeks, days or hours.	MNU 0 – 10a SCN 0 – 06a, 20a
0.3	***Temperature:*** The property of temperature is a measure of warmness: colder objects or places have a lower temperature value than warmer ones.	MNU 0 – 10a, 11a SCN 0 – 05a SOC 0 – 07a, 12a

[4] This project was funded during 2010 -11 by the Esmée Fairbairn Foundation, and involved collaboration between STEM-ED Scotland and teachers drawn from the Association for Science Education Scotland, the Scottish Mathematical Council and the Scottish Technology Teachers Association: see www.gla.ac.uk/stem/routemapforscienceeducation . This revision has subsequently been developed by the project Steering Group, as a step following up the project, linked to an implementation strategy to involve groups of teachers collaborating within a single school or a local cluster of schools. This model of implementation is in tune with both the principles and the guidance of the Scottish *Curriculum for Excellence*

[5] For further information about the *CfE* see Education Scotland's website at www.educationscotland.gov.uk/thecurriculum/whatiscurriculumforexcellence/index.asp .

First Level

Strand	Statement of core understanding	Relevant Experiences & Outcomes
(revisit 0.1)	***Measuring:*** reinforcing earlier understanding, ordering objects by size, and measuring in multiples of a convenient smaller object.	***Reinforces strand 0.1 from previous level*** MNU 1 – 01a, 11a SCN 1 – 11a
1.1	***Standard units*** have been agreed, such as the centimetre, kilogram, litre and second, so that different people can make measurements in the same way and correctly use one another's results.	MNU 1 – 01a, 21a TCH 1 – 13a HWB 1 30b
1.2	***Measurements at different scales:*** Different units are useful in different contexts, depending on the size of the quantities being measured, eg it is sometimes more convenient to measure distances in metres or kilometres rather than in centimetres [6], and time periods can be described in years, months, weeks, days, hours or minutes.	***Builds on previous level, strands 0.1 and 0.2*** MNU 1 – 10a, 10b, 10c, 11a TCH 1 – 13a SCN 1 – 06a SOC 1 – 12a
1.3	***Rounding:*** Measurements do not generally come out as an exact whole number of the units used; one approach is to quote the "nearest" whole number, another is to include a fraction, eg "6½ cm long."	MNU 1 – 07a MTH 1 -07a, 11a, 15a TCH 1 – 13a HWB 1 – 30b SCN 1 – 06a
1.4	***Areas*** can be compared counting the number of standard tiles required to cover them[7].	MNU 1 – 11b TCH 1 – 13a
1.5	***Amounts of money*** can be counted using various coins[8].	MNU 1 – 09a, 09b
1.6	***Instruments*** have been developed to make measurements, eg rulers, thermometers[9], kitchen scales and clocks. Also, vending machines measure money inserted.	***Builds on previous level, strand 0.3*** MNU 1 – 10a, 10b, 11a MTH 1 – 10b, 12a TCH 1 – 01a, 13a SCN 1 – 04a, 06a, 15a HWB 1 – 30b SOC 1 - 12a,12b, 13b

[6] Recognising that different base units may be suitable for different contexts is a key stepping stone. Level 1 is an appropriate stage at which to introduce this for the case of length (could link this to TCH 1-13a, but can usefully also be related to everyday experience: eg lengths of a pencil, a corridor, and a journey.

[7] Area is a key new property included at this level, it is the first compound property introduced, as becomes clear when this is taken further at level 2.

[8] Linking learning about money to the Measurement Pathway at this level makes a very useful contribution to developing general understanding. This is the first quantity met where different units (£ and p) are combined in one measurement (counting), and where amounts are added or subtracted. (MNU 1-09a & 1.09b)

[9] Using a thermometer introduces numerical values for temperatures (and the Celsius unit C). There is a good opportunity to cross-refer to temperature measurements when addressing SCN 1-04a; the "types of energy" reviewed will include heat, and thermometers can be used to measure how hot something is, and can be linked to developing the idea that more energy is stored as heat when the temperature is higher. Also water freezes and boils at closely defined temperatures.

Second Level

Strand	Statement of core understanding	Background and Relevant Experiences & Outcomes
2.1	***Standard units and metric scales:*** Three important internationally agreed standard units are the metre, the kilogramme and the second. Related smaller or larger units are convenient to use when measuring much smaller or much larger objects. A length of exactly 1 m long is 1000 mm long, and also 100 cm long. A distance of exactly 1 km is 1000 m long. The prefixes "milli", "centi" and "kilo" are used to imply these relationships, thus 1 kg is the same as 1000 g[10]. For time measurements, a block of 60 seconds is exactly 1 minute, and a period of 60 minutes is exactly 1 hour.	***Builds on strands 1.1 & 1.2*** MNU 2 – 02a, 10b, 11b TCH 2 – 02b, 13a SCN 2 – 04b, 06a, 12a
2.2	***Re-expression of metric measurements:*** A measurement stated in a metric unit can be re-expressed in a related unit with a different prefix *m*, *c*, or *k,* eg 1285 mm = 1.285 m. (This is often, rather misleadingly, described as "converting units".)[11] Times expressed in different units can also be inter-related (eg 90 min = 1½ hr).	***Makes use of strand 2.1*** MNU 2 – 11b TCH 2 – 02a, 13a SCN 2 – 04b, 06a
2.3	***Rounding:*** A length measurement can be evaluated, rounded to the nearest first decimal place, using a ruler labelled in cm where tenths divisions are also marked[12].	***Builds on strand 1.3*** MNU 2 – 01a, 03b TCH 2 – 13a
2.4	***Angles***: can be accurately drawn, or measured, in units of degrees, using an appropriate instrument.	MTH 2 - 17a, 17b, 17c SCN 2 – 11b TCH 2 – 12a, 13a
2.5	***Negative measurement values:*** Sometimes the value of a measurement may be stated as a negative number of units. Examples include Celsius temperatures below freezing and distances on a coordinate axis of a graph to the left of or below the "origin."	MNU 2 – 04a, 09c, 18a SCN 2 – 05a
2.6	***Areas, and a first introduction to a "formula":*** The area of a rectangle can be calculated by multiplying its length by its breadth, viz as L×B (where the formula is at this stage regarded as an aide memoire). If the length and breadth are measured in metres, the units of the answer are m×m, ie "square metres", or m^2 (perhaps this notation might again at first be treated just as a convenient shorthand). If the length and breadth measurements are stated in cm, the area from L×B will be in cm^2 or "sq cm").[13]	***Builds on strand 1.4*** MTH 2 – 11b, 11c, 15a

[10] This is a much more explicit statement of the understanding that is implied in MNU 2 – 11b, and matches that expressed separately for the case of time. This connection is important and it seems helpful to address it first, for a limited number of cases, at level 2. The "metric" scaling of length and mass units is in fact simpler than that for time!

[11] Using decimal numbers in measurements and recognising the meaning of standard unit prefixes are important steps on the way to comparing measurements of very different scales, and making valid use of expressions inter-relating different properties (as eg in calculating speed from distance travelled and time taken).

[12] Measuring a length, using a ruler, requires carefully aligning the "zero" on the ruler scale with one end of the length being measured. This point might have to be stressed.

[13] Re-expressing an area quoted in m^2 as a value in cm^2, or vice versa, might be mentioned, but probably is something to address more thoroughly at level 3. If this extension is tackled, it is important to note an anomaly in the way the notation cm^2 is used to imply $(cm)^2$ rather than $c(m^2)$. $1\ m^2 = 10{,}000\ cm^2$, viz 100 cm × 100 cm.

2.7	***Estimating the area of an irregular shape:*** Where the area of an irregular shape is estimated by "counting tiles" the area of the shape in standard units is given by multiplying the number of tiles by the area of a tile (ie L×B for a single tile).	***Builds on strand 1.4*** MTH 2 – 16a
2.8	***Volume*** is another property with compound units. The volume of a rectilinear object or space is given by multiplying its length by its breadth and then by its height (ie L×B×H). The units of the volume would then be expressed in cubic metres (m^3) or cubic centimetres (cm^3) depending on the units used for the length, breadth and height measurements. The volume units of ℓ and mℓ (introduced when measuring liquids using kitchen jugs) are related to these units: 1 m^3 = 1000 ℓ, and 1 cm^3 = 1 mℓ.	***Extends ideas from strands 2.2 & 2.6 to a new context*** MTH 2 – 15a SCN 2 – 08b, 16a, 16b TCH 2 – 13a
2.9	***Speed***: is yet a further example involving compound units. If an object is travelling at a steady speed, this speed can be calculated by dividing the distance travelled by the time taken. Using another mnemonic aid, the speed is calculated as "d/t". The units of the answer reflect those of the measured distance and time, for example metres per sec ("m/s") or kilometres per hour ("km/hr")[14].	***Builds on ideas in strands 2.6 and 2.8, to introduce a more general type of compound quantity*** SCN 2 – 06a MNU 2 – 07b, 10c MTH 2 – 15a
2.10	***Scale models*** of large objects or spaces can be very useful in describing, planning or designing. In the model every distance involved is reduced by a constant "scale factor" relative to the "real" situation. Examples are 1:25,000 OS maps, plans of rooms or buildings, and 3D models of cars, bridges, cranes etc. All angles in an accurate scale model are identical to those in the object modelled, whereas areas and volumes are scaled down by a much bigger factor than the distance scale factor[15].	***Builds on strand 1.2, and also on ideas implicit in 2.1*** MTH 2 – 17d, 18a, 21a TCH 2 – 14a, 15a SCN 2 – 03a, 06a
2.11	***Estimation and precision:*** In many contexts quantities may only be able to be estimated, and precise values might be subject to uncertainty or variability. In reporting or using such values it is important to be approximately aware of the degree of uncertainty, and the value quoted should be rounded appropriately.[16]	***Builds on the introduction of rounding in strand 2.3*** MNU 2 – 11a SCN 2 – 04a, 04b, 08a, 16b, 19a[17] TCH 2 - 02a, 02b, 11b, 12a, 13a, 14b[17]

[14] **There is a challenge and an opportunity for science and technology, linked to several of the ideas developed on measurement and units at level 2.** In general the science E&Os are written very descriptively, but it would be greatly beneficial, for all subjects concerned, if opportunities are taken to use and reinforce concepts being introduced at this stage through mathematics. SCN 2-04b and 2-06a are particularly relevant here, but so are all instances where effects can be measured. Being relentless in recognising that all measurements involve both a number and a unit is a start. Science addresses situations that involve a wider range of scales and more properties which have compound standard units, but there are very important emerging conceptual strands to support here, including manipulating units, beginning to get a sense of order of magnitude and significant figures, beginning to use symbols and hinting at equations.

[15] Of course areas and volumes scale by the square and cube of the distance scale factor, but this may not be addressed at level 2.

[16] This is a first introduction to ideas of uncertainty, aiming simply to raise awareness of the basic issue, without giving specific rules on how to present values and estimate errors.

[17] The science and technology E&O references here suggest numerous contexts where uncertainties or broad estimates in data could, in principle at least, be flagged. See note 11.

Third Level[18]

Strand	Statement of core understanding	Background and Relevant Experiences & Outcomes
(revisit 2.10)	***Scale models:*** reinforcing earlier understanding	***Reinforces strand 2.10, deepening understanding.*** MNU 3 – 04a, 08a, 17b, 17c, MTH 3 -18a TCH 3 – 12a, 14a, 15a SCN 3 – 03a, 13a, 19b
3.1	The range of ***unit prefixes*** in common use goes well beyond the examples of *m*, *c*, and *k* used at level 2, and their meanings can be looked up, and correctly interpreted when they arise (eg 1 MV = 1,000,000 Volts, 1 nm = 0.000 000 001 m).	***Builds on earlier strand 2.1*** SCN 3 – 04b, 11b, 13a MTH 3 – 06a
3.2	***Order of magnitude and scientific notation[19]:*** Decimal numbers representing very large or very tiny values compared with 1.0 can be alternatively expressed in scientific notation. The power of 10 in a value expressed in scientific notation gives what is known as the "order of magnitude" of the value, immediately identifying the level of scale involved.	***Subtly links with strand 3.3 below, and in effect builds on 2.1 & 3.1*** MTH 3 – 06a, MTH 4 – 06b SCN 3 – 03a, 04b, 08a, 11b
3.3	***Unit prefixes linked to scientific notation:*** The prefix used with a metric unit is an alternative way of specifying a power of ten. Thus a length of 3.0 mm = 3.0×10^{-3} m and a power station output of 2.5 GW = 2,500,000,000 W = 2.5×10^{9} W.	***Builds on and connects 3.1 & 3.2*** MTH 3 – 06a Can reinforce in many SCN and TCH contexts
3.4	***Using measurements to derive related quantities:*** When different measurements are combined, using a rule or formula to calculate some other quantity, it is helpful to express all measurements in their base unit (eg in metres rather than in mm or km) so the result for the derived quantity will come out in its appropriate base unit. (examples are: area of a triangle = ½×base × altitude; electric current in a circuit = voltage applied / circuit resistance = V/R; average speed = distance travelled / time taken = d/t)	***Builds on 2.6, 2.8 & 2.9*** MNU 3 – 10a, 11a, MTH 3 – 15a, 15b SCN 3- 04a, 07a, 08a, 09a, 19a TCH 3 – 12a

[18] **At the earlier levels, progress in developing understanding of measurement has been fundamentally led by mathematics education, notably linked to increasingly sophisticated use of numbers. Sciences and technologies benefitted from being able to use and to reinforce these ideas. From Level 3 onwards it can be argued that the lead in concept development passes on the whole to the sciences and technologies, where a broader range of physical quantities, and a broader range of scales, are addressed. Here, mathematics can often benefit by taking account of, and reinforcing, ideas introduced elsewhere. On the other hand, all STEM subjects have interests in increasing use of symbols, and in manipulating expressions and equations, and opportunities should be taken to coordinate how progress is achieved in these areas. All of this depends on choosing to address the SCN and TCH E&Os in a quantitative way**.

[19] The outcome introducing scientific notation is officially set at Level 4 (MTH 4-06b). It is both logical and extremely valuable, however, to address this at level 3. It is a key threshold tool to understand scale in the real world. There are many benefits to being able to use this number format in sciences and technologies. For instance it is a key tool in quantitative computing, and it is vital to effective public (let alone specialist) understanding of science on many fronts. In reality, using this notation adds relatively little additional intellectual substance to MTH 3-06a as it involves the arithmetically simplest application of "whole number powers" and represents by far the most important of "the advantages of writing numbers in this form." It should be noted that electronic calculators routinely display small and large answers in scientific notation.

3.5	***Algebra***[20] can be used very effectively, in scientific manipulations that can arise in many different contexts. These include expressing the units of a derived quantity.	***Builds on 3.4*** MTH 3 – 14a, 15a, 15b Can usefully be applied and reinforced in very many SCN and TCH contexts
3.6	***Using experimental data, accuracy and precision:*** When measurements are made during science experiments, or in building artefacts in technology, it is meaningful to express these rounded appropriately to the precision of the measurement technique used. When measurements are used in a calculation of the value of some other property, the result should also be rounded to the same number of "significant figures" as the least precise measurement used.	***Builds on 2.11*** MNU 3 – 01a, 11a, 16a, 22a SCN 3 – 01a, 03a, 04a, 07a, 09a, 11b, 15b, 16a, 16b, 17b TCH 3 – 12a, 14a
3.7	***Proportion***[21]***:*** In scale models all distances are reduced by a constant "proportionality factor" relative to the full scale subject modelled. There are many other situations in science and technology where, if one property is changed, another changes "in direct proportion" (ie by the same factor)	***Builds on 2.10*** MNU 3 – 08a, 20b TCH 3 – 02a, 11a, 12a, 14a SCN 3 – 04a, 04b, 05b, 08a, 16b TCH 3 – 15a
3.8	***Non-linear scales of measurement:*** In some circumstances the scales of measurement that are most useful are "non-linear." These include sound loudness measured in decibels, wind strength given on the Beaufort scale, earthquake magnitude reported on the Richter scale, and the hydrogen ion concentration in water stated as a pH value[22]. It is valuable for learners to understand that such scales have a firm quantitative basis, and to appreciate why they might be of practical use in some contexts.	***Taking the case of pH, this is fundamentally a measure of the order of magnitude of hydrogen ion concentration, and understanding builds on 3.2 above.*** SCN 3 – 12b, 18a

[20] "Algebra" is formally referenced under the "Expressions and Equations" organiser (MTH strands 13 and 14), but it clearly has much wider application, including in this Measurement, Units and Scale Pathway. Mastering basic algebraic skills is of central importance to learning across the STEM disciplines. At earlier levels in this Pathway, we have been careful to highlight opportunities to seed the development of understanding the usefulness of "generalising" quantities and procedures by using a symbol (a "place-holder"), or a formula or even equation (as a "mnemonic"), and we have explicitly drawn to attention the "algebra" involved in the form of compound units (eg of area and speed). Level 3 is the opportunity to pull this together more comprehensively, and to apply this in the sciences and technologies. Under-development of these skills is often commented on as the most significant learning deficiency among entrants to university level.

[21] Many learners traditionally have regarded "proportion" as a difficult or confusing concept. It is suggested that at this level application of this term might be restricted to cases of direct proportion (formulae and equations can be used where inverse or square law relationships are involved). That said, it would help to use the term wherever possible (eg in chemistry, reaction equations define a number of changes related by proportion, and analysis by sampling relies on proportion). There are many other examples in all subjects.

[22] The pH is a useful value to quote as it best represents the enormous influence that "acidity" has on the properties of solutions, including in biological systems. Its precise value is critical, for instance, to the proper functioning of biological systems in animals and plants. pH is a "logarithmic" measurement, a concept that cannot be fully understood at this level. However a basic understanding can be achieved by relating pH to the order of magnitude as expressed in scientific notation. If pH = 4.0, the H^+ concentration = 0.0001 mol/ ℓ = 1.0×10^{-4} mol/ℓ. Neutral water at 25^oC has pH = 7.0 corresponding to the H^+ concentration = 1.0×10^{-7} mol/ℓ. A pH of 4.5 implies a concentration smaller than 1.0×10^{-4} mol/ℓ but not as small as 1.0×10^{-5} mol/ℓ. This Learning Pathway opens the way to a much better understanding of this measure than has commonly been achieved in the past.

Fourth Level

Strand	Statement of core understanding	Background and Relevant Experiences & Outcomes
(revisit 3.1, 3.2 and 3.3)	***Dealing with small and large scales, unit prefixes and scientific notation:*** There are many opportunities to extend the range and depth of application of skills in this area. The introductions of whole number roots would, for instance, allow a little more scope in understanding pH (eg if pH=4.5 the hydrogen concentration = $10^{-4.5}$ mol/ℓ = $10^{½} \times 10^{-5}$ mol/ℓ = 3.2×10^{-5} mol/ℓ)[23].	***Reinforces and deepens skills introduced in3.1, 3.2 & 3.3*** MTH 4 – 06a, 06b SCN 4 – 04a, 04b, 05a, 05b, 06a, 10b, 11b, 13a, 15a, 18a TCH 4 – 01a, 01c, 02a, 03b, 12b, 15a, 15d
4.1	***Dealing with and manipulating algebraic relationships and equations:*** Within Mathematics, this is a major theme with a perspective much broader than measurement alone. However, the learning process can build on the earlier work described above, across the STEM subjects. In sciences and technologies relationships and equations are vital in deriving the values of one property, from experimental measurements of other quantities. Importantly, manipulations can be freely applied in equations: **units** must "balance" (viz. be the same) in every term and on both sides of an equation[24].	***Builds on 3.4 & 3.5*** MNU 4 – 03a, 03b, 07a, MTH 4-07b, 10a&b, 11b&c, 13a-d, 14a&b, 15a, 16a&b SCN 4 – 02b, 03a, 04a, 05a, 05b, 07a, 07b, 08a, 08b, 09a, 09c, 10b, 11b, 13b, 16b, 18a, 19a, 20b TCH 4 – 02a, 03a, 04a, 07b, 09a, 10b, 12a, 12b, 14a,
4.2	***Reinforcing proportion, and linking this to mathematical similarity:*** Where two properties are related by (direct) proportion, measuring a change in the value of one of the properties can be used to deduce the changed value of the other property. This reasoning can be widely applied in science and in technology. The lengths of corresponding sides of objects described as mathematically "similar" are related by proportion. Studies of relationships between different sides, and between sides and angles, in right angled triangles, can be used to deepen understanding of gradients (in straight line graphs or in maps) and calculating bearings (in navigation by map).	***Builds on 3.7 & 2.10*** MNU 4 – 07a, 08a, MTH 4 – 16a, 17b, 18a SCN 4 – 04a, 05a, 07b, 09a, 16b, 17a, 19a TCH 4 – 11a, 12a, 12b, 14a, 15d

[23] An experimental pH titration plot for an acid-base reaction can be referenced later in mathematics when introducing the idea of logarithms.
[24] This process becomes recognised as "dimensional analysis" in the physical sciences.

4.3	***<u>Variability and Uncertainty in measurements</u>***: Achieve greater experience and depth in dealing with inaccuracy, variability, tolerance and rounding, which are issues to address whenever practical exercises are performed or where predictions are made using data. Where variability is an issue, its nature can be explored by evaluating the summary measures such as the mean, median, mode and spread of sampled data. In appropriate contexts the probability of different outcomes can be estimated and discussed.	***Builds on 3.6*** MNU 4 – 01a, 11a, MTH 4 – 20a, 20b, 21a, 22a SCN – 02a, 02b, 04a, 07a, 09a, 13b, 16b, 18a, 19a, 20b TCH 4 – 02a, 10a, 11a, 12b, 13a
4.4?	***<u>Measurement in Graphs</u>:*** It might be thought useful to introduce this extra strand. If so, there would be a case for adding a first stage at level 3. Alternatively, it has been proposed to describe a separate pathway on ***Using Graphs***. If that is done, it would be useful to highlight inter-connections. For example, in analysing the spread of a set of repeat measurements (as in 4.3 above), drawing error bars on graphs of experimental values is a very useful estimation and visualisation tool.	Eg MTH 4-18a explicitly opens up full use of a 4-quadrant coordinate grid, opening a basis for discussing 2D vectors, etc.

<u>Beyond Level 4</u>

In all STEM subjects the applications of measurement become more sophisticated and in many cases may depend on exploiting more advanced mathematical techniques[25]. However, it is argued that Level 4 might be a suitable stage to end the description of the Measurement, Units and Scale learning pathway as a broad and united theme in its own right. On the other hand, the full range of understanding reached by the end of level 4 provides a vey valuable platform for studies in all areas.

The strand of understanding on variability and accuracy, culminating in 4.3 above, could feed in as a starting point to a new Learning Pathway on Probability and Statistics, important across the STEM subjects, which might usefully be proposed for levels 5 – 7.

<u>Glossary of terms, lists of common learner misconceptions and potential cross-curricular activities</u>

The lists given in the remainder of this document are not claimed to be at all comprehensive. These are presented as starting drafts which might usefully be amplified by groups working with the Measurement, Units and Scale pathway.

[25] For example, to continue the description of using the measurement and interpretation of pH, this becomes fully understood only after the use and interpretation of logarithms (to base 10) has been mastered.

Glossary of terms

- A **measurable property** is a property that can be quantified using some kind of unit as a basis, e.g. length, weight and time are measurable, since they can be measured as iterations of a single defined unit such as a metre, kilogram or second.
- **The metric system** is an international standard set of base units that are used to derive inter-related units differing in scale by multiple factors of 10. For example. the millimetre, centimetre and kilometre are linked to the base unit metre by scale factors of 1/1000, 1/100 and 1000 respectively.
- Measurements can be made in **non-standard units**: for example we can measure height using hand spans or texture by quantifying the roughness of a surface by counting the bumps in a defined area. Non-standard measurements require new units and methods which makes it harder for these measurements to be replicated, checked and compared by other people.
- Measurements are approximations. The **precision** of a measurement tells us how finely and accurately it was made. The precision of a measurement is determined by the measuring device and the units.
- The **accuracy** of a measurement is determined by how closely the measured value agrees with the true value. Accuracy can be affected by a number of factors, for example the measurement tool.
- . . . . . the above list is deliberately short and illustrative. It is recommended that groups of teachers, collaborating to implement the Learning Pathway through their classroom practice, should discuss and agree their own version of a fuller Glossary of terms. It is important that the whole team use consistent terminology.

Useful IT Measurement Tools

- Scientific Calculators e.g. rounding and scientific notation
- Graphing calculators/software e.g. understanding scale, zoom facility (for finding roots of equations etc), data logging and analysis. Sample software - TI Nspires, Geogebra, Excel
- Drawing software e.g. setting scales, constructing scale diagrams, measuring distances, resizing, rescaling. Sample Software - Autocad Inventor (up to 3D), Serif DrawPlus (2D only), Excel
- Data logging (sensors) and Experimental Design e.g. measuring temperature, light, digital microscope, length of areas of cells etc. Sample software - phet-edu website for simulations, yenka, West Point (Bridge design) (USA site), Basic Stamp
- Geometry Software e.g. measuring length/angle and enlargement. Sample software - Geometer Sketchpad, Gegebra, Cabri, Cabri 3D
- . . . . . consider these and other possibilities.

Common Misconceptions – early level

i. Difficulty with some comparative terms when using bipolar pairs, e.g. if both objects feel heavy to hold may not go on to decide which is heavier – both are heavy.
ii. May distinguish between the different attributes of an object, e.g. height and weight, but not understand that the two attributes may lead to objects being ranked differently expecting the order for height and weight to be the same.

Common Misconceptions – first level

i. Children and young people may not 'conserve' measures e.g. moving an object changes its length, pouring changes 'how much' and cutting up paper makes more surface area.
ii. Using a visual comparison without making an effort to match the items e.g. stating which sheet of paper is bigger without superimposing one on top of the other.
iii. If objects are out of alignment, children and young people will usually consider the one pushed forward as being longer – they judge relative position rather than comparing length using a common baseline.
iv. Children and young people may compare measurements from objects that have been measured using different units. If comparing objects measured using different units they may let the measured number override their perceptual judgement.
v. Children and young people may be careless in their repetition of units and not notice gaps or overlaps when measuring e.g. not filling containers to the top, not laying the units end to end.
vi. Comparing time spans without taking into account different starting times e.g. the TV program that finished last was on the longest.
vii. Children and young people often do not think to use counting to say how big or how much bigger; e.g. they may 'weigh' something by putting it to one side of a balance and smaller objects into the other side, but do not count the objects.
viii. Confusion between *capacity* - how much an object or place can contain of a defined unit, and *volume* - the quantity of three-dimensional space enclosed by an object or place.
ix. Confusion between Centigrade and Celsius when discussing temperature. Centigrade can be defined as – a unit of angle equal to one hundredth of a grade or a scale divided into 100 degrees, and temperature was referred to as the degree centigrade before the Celsius scale (degree Celsius - °C) was formally adopted.
x. Children and young people may be influenced by the more dominant perceptual properties even when they know that ordering objects by different properties may lead to different orders, e.g. may still think that the tallest container holds the most.
xi. Children and young people measuring the same object do not necessarily expect the same answer each time when deciding how many fit.
xii. When using a ruler do not recognise that centimetres and millimetres are repeated units.

Common Misconceptions – second level

i. Confusion between square metres and metres squared. One metre squared is a square one metre on each side with an area of one square metre and 10 square metres are ten 'one square metres'. Ten metres square (or ten metres squared) is a square ten 'one square metres' on each side.
ii. Converting times using the decimal system, e.g. 0.5 hours = 50 minutes
iii. Confusion between magnitude - the relative size of an object, and the actual size of the object.
iv. Doubling the length does not imply that the area and volume have doubled.
v. Often mix up measuring the area of a shape with measuring the perimeter.
vi. Mixing up measuring with mm and cm.
vii. Taking the measurement from the end of the ruler and not from the 0 cm mark. (Using rulers with inches on the other side can cause further confusion.)
viii. The desire to match the objects closely may override the need for consistency of unit while matching objects, e.g. covering a shape with a mixture of different units to ensure the shape is covered.
ix. May not understand that having no gaps or overlaps means the measurement is consistent and fair.
x. May still think of a 'unit' as an object and that measuring is 'fitting' e.g. how many marbles fit in the jar? How many squares fit into the shape?, and therefore have difficulty when combining part units, e.g. when finding the area of a shape with squares.
xi. May confuse the unit with the instrument used to represent it, e.g. a square metre has to be a square with sides of one metre, or not recognise that the markings on measuring instruments are the units.
xii. Confused by the same instrument being called a different name in another subject, e.g. a protractor (Science and Mathematics) is also called an angle measurer (Technological Studies).
xiii. Scientific notation is taught differently in Mathematics, Science, Technological Studies, and Computing

Common Misconceptions – third level

i. The scale of the molecules in solids, liquids and gases.
ii. Do not recognise that the equation distance = speed x time is an algebraic equation (d = s x t) that can be rearranged and the letters are only symbols that could be substituted by any symbol There is also confusion with the science formula for distance = speed x time (d =v x t)
iii. May not connect the partitioning of a rectangle into appropriate squares and using the array to find its area with multiplying the lengths of the sides of a rectangle to find its area.
iv. Although children and young people understand the inverse relationship between the unit and the number of units required, they may still be distracted by the size of the numbers in the measurement and ignore the units, e.g. the larger number is always bigger.
v. While converting between international standard units may not see the significance of the decimal structure built into all metric measures.

Cross curricular activities could include – early level

- Sequencing time activity, such as a photograph diary of major events/activities throughout a year and discuss changes in height, appearance, seasons, etc.
- Recognising, describing, comparing and measuring the similarities and differences between individuals e.g. drawing round shoes, cutting out the template then comparing and ranking shoe sizes from largest to smallest (use the templates to make a graph), measuring heights and lengths using hand spans, feet, arm lengths.
- Identifying the sun, moon and stars through observation and through creative play and science stories linking them to daily patterns and routines, and the tools used to measure them e.g. calendars, clocks.
- Identifying the different seasons and relating them temperature through science stories and observations e.g. discussing the different types of clothing people wear in different seasons.
- Looking at how people measured in the past through creative play e.g. length was first measured with the forearm, hand, or finger, time was measured by the sun and moon, and seeds or grains were used to measure capacities of containers and weight

Cross curricular activities could include – first level

- Linking shadows to light, measuring shadows at different times throughout the day and recognising that the change in measurement is due to movement of sun. *(By exploring reflections, the formation of shadows and the mixing of coloured lights, I can use my knowledge of the properties of light to show how it can be used in a creative way.* ***SCN 2-11b****)*
- Estimating, comparing and measuring ingredients when preparing simple healthy foods and drinks.
- Measuring how much sugar can be dissolved in water and comparing the sugar content of different drinks. Practical measurements weighing 100ml of water and comparing the weight of 100ml of water with 100 ml of different drinks with varying sugar contents. Measuring out actual weights of sugar to show how much is in each drink. *(I have participated in practical activities to separate simple mixtures of substances and can relate my findings to my everyday experience.* ***SCN 2-16a****)*
- Reinforce measurement ideas in activities, e.g. dissolving substances – how precise are the measurements of the substances to be dissolved?
- Investigating different aspects of the history of measurement e.g. researching the development of weighing scale; building simple scales using a ruler and pivot point or plastic cups, string, and cardboard/corex; and estimating and comparing weights.

Cross curricular activities could include – second level

- Making an instrument to measure displacement using two plastic cups and marking the water line. *(By investigating floating and sinking of objects in water, I can apply my understanding of buoyancy to solve a practical challenge.* ***SCN 2-08b****)*
- Planning events and journeys using timetables.
- Discussing and identifying where measurement is commonly used in science and technology.
- Discussing standard scientific prefixes through from 'nano' to 'giga' and contexts for use - technology, computers, medicine, everyday, memory and power.
- Discussing changes in scale by talking about: if you were as small as or as tall as…what would…look like?; if you were as light as or as heavy as…what would ... feel like? Set a challenge to measure very small, large, light, tall things.

Cross curricular activities could include: - third level

- Comparing measurements taken in different Olympic events.
- Project management and enterprise skills – defining the importance of accurate measurement in different enterprise activities.
- Rates of change in chemical reactions / acceleration
- Ratio to compare quantities
- Orders of magnitude, cells in the boy, atoms etc.

Reference Websites

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Gatsby Science Enhancement Programme www.sep.org.uk

Nrich Maths Project (Cambridge) www.nrich.maths.org and www.nrich.maths.org/stemnrich

Scottish Qualifications Arrangements Documents. SQA *www.sqa.org.uk*

Some relevant reference sourses

A Framework for K-12 Science Education: Practices, Crosscutting Concepts, and Core Ideas: NSF, USA 2012. See www.nap.edu/catalog.php?record_id=13165

Learning Progressions in Science: An Evidence-based Approach to Reform, Corcoran, Mosher & Rogat, Center on Continuous Instructional Improvement, Columbia University, 2009.

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