# Efficiency Wages, Unemployment and Macroeconomic Policy 

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#### Abstract

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We provide empirical evidence from a number of European countries, which shows that unemployment and output are positively related when unemployment is low and inversely related when unemployment is high. We then construct a stylised macro-model with goods and labour market imperfections to show that the economy can rationally operate at an inefficient equilibrium in the neighbourhood of which the relationship between output and unemployment is positive. Our results suggest that circumstances exist in which market imperfections pose serious obstacles to the smooth working of expansionary and/or stabilization policies whose final aim is to improve welfare.


## KEYWORDS:

Efficiency wages; effort supply; monopolistic competition; multiple equilibria; stability; fiscal multiplier

JEL CLASSIFICATION: E62, J41, H3

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## 1. Introduction

In the last few decades, the industrialised nations of Europe have been subjected to a variety of external and policy-induced demand shocks while simultaneously experiencing significant changes in their labour productivity and employment. Meanwhile, some leading economists have argued that persistent involuntary unemployment may give rise to externalities that could be exploited by economic agents. A much-discussed example in this context is for price-setting firms to use high or rising unemployment as a device to deter shirking. Lindbeck (1992) suggests that in this setting, macroeconomic policy interventions may produce unexpected consequences: "In the context of a nonmarket-clearing labour market, it is certainly reasonable to regard unemployment, in particular highly persistent unemployment, as a major macroeconomic distortion. There is therefore a potential case for policy actions, provided such actions do not create more problems than they solve. Experience in many countries suggests that the latter reservation is not trivial."

To more fully explore the extent to which these concerns are warranted, we first examine the aggregate data on unemployment and output for a number of European countries. Our empirical evidence shows that it is not uncommon for output and unemployment to be related according to a humped-shape relationship. More specifically, we find that changes in output and unemployment are correlated negatively (positively) when the latter is relatively high (low). It is when these variables co-vary positively that an expansionary shock may yield counterintuitive effects. We next develop a theoretical model that shows such a relationship can result when labour and goods market operate under certain plausible conditions. An important policy insight that emerges from this paper is that an exogenous stimulation of aggregate demand can only raise output and reduce unemployment provided the economy is operating relatively efficiently. The intuition for this lies in the supply side nonlinearities that could give rise to multiple equilibria. We show that when an
economy is trapped in an inefficient equilibrium, positive demand shocks can lead, perversely, to an increase in unemployment.

Other recent examples of related theoretical research, which examines the link between European unemployment and productivity, include Malley and Moutos (2001), Leith and Li (2001), Daveri and Tabellini (2000), Blanchard (1998), Caballero and Hammour (1998a,b), Gordon (1997) and Manning (1992). However, none of these studies explores the link between unemployment and output arising from both labour and product market imperfections.

On the empirical side, a large number of studies have examined the behaviour of labour productivity in the industrialised countries and provide indisputable evidence regarding the way in which labour productivity has changed over the last few decades. Recent examples include Disney, et al. (2000), Barnes and Haskel (2000), Marini and Scaramozzino (2000), Bart van Ark et al. (2000) and Sala-i-Martin (1996). The evidence provided in these studies is usually interpreted using one of the micro-theory based explanations underlying the behaviour of labour productivity. These may, in general, be divided into two categories. The first concentrates on the productivity gains that can be realised through: i) improved skill due to training; ii) increased efficiency due to progress in management and restructuring; and iii) rising physical productivity of other factors of production due to $\mathrm{R} \& \mathrm{D}$, etc.. The second category emphasises market forces and sees competition and market selection as the main motivation behind the rise in efficiency. The separating line between these two accounts is not very clear in the sense that the second will have to be achieved through the first when the economy is operating efficiently. However, if the economy happens to be in an inefficient phase, market forces can act directly without
having to induce any of the factors in the first category. The efficiency wage hypothesis is a typical example of this case and will be used in this paper to illustrate the point.

The rest of the paper proceeds as follows. Section 2 examines data from a number of European countries on output and unemployment and reports the evidence on how movements in output are matched with changes in unemployment. Section 3 outlines a theoretical model based on the efficiency wage hypothesis and shows that the production side of the economy - consisting of monopolistically competitive firms that set prices and offer efficiency wages to maximise profits - can give rise to a non-linear equilibrium relationship between output and unemployment, consistent with the evidence reported in Section 2. Section 3 concludes the paper and finally the Appendix outlines the derivation of the effort function we utilise in our theoretical model.

## 2. Output and Unemployment: Some Stylised Facts

Our aim in this section is to explore the way changes in unemployment and output are related to each other empirically. However, it would be helpful if, a priori, we postulate an equilibrium that sustains involuntary unemployment and allow for the latter to affect workers' effort supply. In this case, because there is a causal relationship between the level of unemployment and the productivity of the employed, a total change in output can be decomposed into changes due to employment and productivity. As a result, a sufficient condition for an expansionary demand shock to raise both output and employment is that the resulting fall in unemployment does not induce a fall in productivity of the employed to such an extent that it eliminates the effect of the rise in employment. Defining aggregate labour productivity as $q=Y / L$ where $Y, L$ and $q$ respectively denote output, employment and productivity, and noting that $d q=(d Y-q d L) / L$, it is clear that any of the six cases outlined in

Table 1 could, in principle, occur in the aggregate (see Barnes and Haskel, 2000, for evidence at plant level).

Table 1. Simultaneous Changes in Labour Productivity, Output and Employment.

| Change in <br> Labour <br> Productivity | Change in <br> Employment | Change in <br> Output | Change in <br> Unemployment <br> Rate |
| :---: | :---: | :---: | :---: |
|  | $d L<0$ | $d Y>0$ | $d u>0$ |
| Falling <br> Productivity <br> $d q<0$ | $d L>0$ | $d Y>0$ | $d u<0$ |
|  | $d L<0$ | $d Y<0$ | $d u>0$ |
|  | $d L>0$ | $d Y<0$ | $d u<0$ |

The last column of Table 1 shows the implied changes in the unemployment rate (based on the approximation that the labour force is constant). This discussion clearly suggests that it is a distinct possibility that output and unemployment can fall or rise simultaneously. While the cases in which changes in output and unemployment have opposite signs can be easily explained by a variety of standard theories, a convincing macroeconomic theory capable of predicting why these variables fall or rise simultaneously is more elusive. To obtain a more realistic indication of whether output and unemployment simultaneously move in the same direction, in Table 2 we examine quarterly data for a cross section of 10 European countries, chosen to reflect a wide range of industrial structures as well as macroeconomic and labour market experiences over the last few decades.

Table 2 shows, for each country, the directions of quarterly changes as a proportion of the entire sample for which changes in output and unemployment have the same sign, that is $\left[d Y_{t \pm s}>0 \& d u_{t}>0\right]$ and $\left[d Y_{t \pm s}<0 \& d u_{t}<0\right]$ for $s=0,2,4$. The results corresponding to contemporaneous changes (middle column) indicate that for a substantial and statistically significant proportion of the sample (i.e. at least $35 \%$ of the sample for all countries) the sign combinations show output and unemployment moving in the same direction. It is also clear

Table 2. Directions of Quarterly Changes in Output, $\boldsymbol{Y}$, \& Unemployment Rate, $u$; (occurrences as percentage of the sample size, $n$ )

| $\begin{aligned} & \hat{O} \\ & \stackrel{\theta}{E} \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { N } \\ & \text { 苞 } \\ & \hline \end{aligned}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| BEL | $\begin{gathered} 60: 2-97: 4 \\ (n=151) \\ \hline \end{gathered}$ | 0.456 | 0.423 | 0.417 | 0.423 | 0.463 |
| DEU | $\begin{gathered} 60: 2-89: 4 \\ (n=119) \end{gathered}$ | 0.461 | 0.385 | 0.345 | 0.402 | 0.461 |
| ESP | $\begin{gathered} 61: 1-98: 4 \\ (n=152) \\ \hline \end{gathered}$ | 0.493 | 0.500 | 0.480 | 0.473 | 0.493 |
| FRA | $\begin{gathered} 65: 1-97: 4 \\ (n=132) \\ \hline \end{gathered}$ | 0.609 | 0.592 | 0.553 | 0.638 | 0.656 |
| GBR | $\begin{gathered} 60: 2-98: 3 \\ (n=154) \\ \hline \end{gathered}$ | 0.433 | 0.414 | 0.370 | 0.461 | 0.547 |
| IRE | $\begin{gathered} 60: 2-97: 4 \\ (n=151) \end{gathered}$ | 0.497 | 0.463 | 0.430 | 0.450 | 0.456 |
| ITA | $\begin{gathered} \hline 60: 2-98: 3 \\ (n=154) \\ \hline \end{gathered}$ | 0.487 | 0.559 | 0.513 | 0.546 | 0.560 |
| NLD | $\begin{gathered} 69: 2-97: 4 \\ (n=115) \\ \hline \end{gathered}$ | 0.414 | 0.416 | 0.400 | 0.416 | 0.414 |
| PRT | $\begin{gathered} \hline 60: 2-97: 4 \\ (n=151) \\ \hline \end{gathered}$ | 0.578 | 0.597 | 0.629 | 0.671 | 0.694 |
| SWE | $\begin{gathered} 60: 2-98: 3 \\ (n=154) \\ \hline \end{gathered}$ | 0.473 | 0.342 | 0.357 | 0.467 | 0.460 |

The number of observations, $n$, and the sample period correspond to the contemporaneous changes in the natural logarithm of real GDP (market prices), $Y$, and the unemployment rate, $u$. Based on a one-sample 2-tailed $t$-test the mean number occurrences of ( $d Y_{t} \pm_{s}>0 \& d u_{t}>0$ ) and $\left(d Y_{t} \pm_{s}<0 \& d u_{t}<0\right)$ are significantly different from zero at the 0.01 level of significance. This result also holds across all leads, lags and countries. Countries are defined as follows: Belgium (BEL), West Germany (DEU), Spain (ESP), France (FRA), UK (GBR), Ireland (IRE), Italy (ITA), Netherlands (NLD), Portugal (PRT), Sweden (SWE); $Y$ and $u$ are obtained from the OECD Business Sector Database.
from Table 2 that these findings hold for lagged and led changes, which we have considered in order to capture the variations over business cycle. Moreover, when the noncontemporaneous changes are considered, the proportion of periods where $Y$ and $u$ are positively related increases for virtually all other cases considered ${ }^{1}$. Given the results in Table 2, it seems fair to conclude that theories disregarding this possibility - by employing models with market structures which are only capable of generating the prediction that $Y$ and
$u$ are negatively related - can only be of limited use when analysing the potential effects of macroeconomic policies.

The evidence in Table 2 pertaining to changes in output and unemployment having the same sign is potentially consistent with a number of alternative explanations, e.g. i) an exogenous increase the labour force participation; ii) the net result of simultaneous exogenous shocks to both aggregate supply and demand which would move output and unemployment simultaneously in the same direction; or iii) the theories of creative destruction (see, for examples, Aghion and Howitt, 1992; 1994). However, if the periods in which output and unemployment are positively related occur on a systematic basis, the above explanations will prove inadequate since they all rely on random impulse mechanisms. We therefore next focus our analysis on this aspect of the evidence, i.e. a regular occurrence of positive and negative correlations between changes in output and unemployment. To do so, we first test to see if the occurrences of $\left[d Y_{t}>0 \& d u_{t}>0\right]$ and $\left[d Y_{t}<0 \& d u_{t}<0\right]$ reported in Table 2 are random. To this end we employ a simple 'runs test' which is a onesample nonparametric test for randomness in a dichotomous variable. Given that a run is defined as any sequence of cases having the same value, we assign 1 to those periods where [ $d Y_{t}>0 \& d u_{t}>0$ ] and $\left[d Y_{t}<0 \& d u_{t}<0\right.$ ] and 0 to all other periods. The total number of runs in the sample is a measure of randomness in the order of the cases; too many or too few runs can suggest a non-random, or dependent, ordering. The results are reported in Table 3 below where the rows in bold indicate the countries (i.e. Italy and Sweden) for which we are unable to reject the null hypothesis of randomness in the runs. Accordingly, these countries are excluded from further analysis below where we will focus on exploring the existence and

[^1]nature of the systematic relationship between changes in unemployment and output, as supported by the majority of cases.

Table 3. Runs Tests for the Significant Systematic Occurrences of $\left[d Y_{t}>0 \& d u_{t}>0\right]$ and $\left[d Y_{t}<0 \& d u_{t}<0\right]$ Cases.

| Country | T.V. | Cases < T.V. | Cases $\geq$ T.V. | Total <br> Cases | No. of <br> Runs | T.S.V. | A.S. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| BEL | 0.417 | 88 | 63 | 151 | 25 | -8.301 | 0.000 |
| DEU | 0.345 | 78 | 41 | 119 | 43 | -2.397 | 0.017 |
| ESP | 0.480 | 79 | 73 | 152 | 47 | -4.871 | 0.000 |
| FRA | 0.553 | 59 | 73 | 132 | 50 | -2.874 | 0.004 |
| GBR | 0.370 | 97 | 57 | 154 | 58 | -2.568 | 0.010 |
| IRE | 0.430 | 86 | 65 | 151 | 29 | -7.668 | 0.000 |
| ITA | $\mathbf{0 . 5 1 3}$ | $\mathbf{7 5}$ | $\mathbf{7 9}$ | $\mathbf{1 5 4}$ | $\mathbf{7 6}$ | $\mathbf{- 0 . 3 1 5}$ | $\mathbf{0 . 7 5 3}$ |
| NLD | 0.400 | 69 | 46 | 115 | 35 | -4.138 | 0.000 |
| PRT | 0.629 | 56 | 95 | 151 | 42 | -5.158 | 0.000 |
| SWE | $\mathbf{0 . 3 5 7}$ | $\mathbf{9 9}$ | $\mathbf{5 5}$ | $\mathbf{1 5 4}$ | $\mathbf{6 2}$ | $\mathbf{- 1 . 7 1 1}$ | $\mathbf{0 . 0 8 7}$ |

T.V. is the Test Value (mean cut point); T.S.V. is the value of the test statistic; A.S. is the 2-tailed asymptotic significance level.

At this stage it is helpful to compare summary measures of central tendency of the unemployment rate across the two different states, i.e. state 1: [ $d Y_{t}>0 \& d u_{t}>0$ ] and [dY $\left.Y_{t}<0 \& d u_{t}<0\right]$ and state 0: [ $\left.d Y_{t}>0 \& d u_{t}<0\right]$ and $\left[d Y_{t}<0 \& d u_{t}>0\right]$. These simple comparisons are reported in Table 4 below and show clearly that states 0 and 1 are likely to occur at high and low levels of unemployment, respectively. With the exception of German data, the evidence shows a clear tendency for unemployment to be lower when output and unemployment are positively related and higher when they are negatively related.

The clear pattern that emerges from the analysis, summarised as
State 0: [ $d Y_{t}>0 \& d u_{t}<0$ ] and [ $d Y_{t}<0 \& d u_{t}>0$ ] are more likely to occur when $u_{t}$ is relatively high

State 1: $\left[d Y_{t}>0 \& d u_{t}>0\right]$ and $\left[d Y_{t}<0 \& d u_{t}<0\right]$ are more likely to occur when $u_{t}$ is relatively low; and

Table 4. Comparing Mean and Median Unemployment Rates for the Periods when $Y$ and $u$ are positively and negatively related

| $\boldsymbol{O}$ | Mean for <br> State 1 <br> Periods | Mean for <br> State 0 <br> Periods | Difference <br> Between <br> Means | Median for <br> State 1 <br> Periods | Median for <br> State 0 <br> Periods | Difference <br> Between <br> Medians |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| BEL | 6.655 | 7.345 | -0.689 | 7.066 | 8.847 | -1.781 |
| DEU | $\mathbf{3 . 5 0 3}$ | $\mathbf{3 . 2 6 7}$ | $\mathbf{0 . 2 3 6}$ | $\mathbf{3 . 2 4 6}$ | $\mathbf{2 . 9 0 9}$ | $\mathbf{0 . 3 3 7}$ |
| ESP | 10.436 | 11.684 | -1.248 | 7.603 | 15.351 | -7.748 |
| FRA | 6.285 | 7.609 | -1.323 | 5.661 | 9.002 | -3.341 |
| GBR | 5.615 | 5.803 | -0.187 | 4.419 | 5.914 | -1.495 |
| IRE | 9.350 | 9.611 | -0.261 | 7.765 | 8.310 | -0.545 |
| NLD | 4.911 | 5.965 | -1.054 | 4.071 | 5.856 | -1.785 |
| PRT | 5.962 | 6.789 | -0.828 | 5.593 | 7.106 | -1.513 |

can be used to postulate a behavioural, or theoretical, structure that could embrace the above findings. For instance, as we shall see later, this type of systematic pattern in data is consistent with the theoretical implications of the efficiency wage hypothesis, which predicts a causal relationship between output and unemployment through the effect of the latter on productivity of workers. Of the 10 European countries examined above only Italy, Sweden and Germany do not match this prediction, even though as seen from Table 2 they too have a significant number of occurrences of output and unemployment moving in the same direction. This discrepancy may be due to the differences in the way labour markets function in these countries, e.g. use of guest labour, style of unionisation etc.. However, further investigation of these results is beyond the scope of this paper, as we do not attempt to empirically establish the general validity of any particular theory. Our primary purpose is to understand the regularities in data and to develop a theory capable of explaining them.

As a final attempt to further describe the stylised relationship between output and unemployment, suggested by the above data analysis, we estimate by GMM a quadratic relationship between de-trended output and unemployment. The results of this estimation for the remaining seven countries in Table 4 are given in Table 5 below. They show clear
support for a humped-shaped relationship between output and unemployment. As a tentative measure, we have also calculated the level of unemployment, denoted by $\bar{u}$, at which output reaches its maximum value. Note that, according to the behaviour postulated above, $\bar{u}$ is the threshold level of unemployment, which separates State 1 from State 0.

Table 5. GMM Estimates of $E_{t}\left[\left(\tilde{Y}_{t}-\left(\beta_{0}+\beta_{1} u_{t}+\beta_{2} u_{t}^{2}\right)\right) \mathbf{z}_{t}\right]=0$

| 8 8 8 8 | $\hat{\beta}_{1}$ | $\hat{\beta}_{2}$ | $\bar{u}=-\frac{\hat{\beta}_{I}}{2 \hat{\beta}_{2}}$ | $J$-Statistic |
| :---: | :---: | :---: | :---: | :---: |
| BEL | $\begin{gathered} \hline 0.0981 \\ (0.0258) \\ \hline \end{gathered}$ | $\begin{aligned} & \hline-0.0060 \\ & (0.0017) \\ & \hline \end{aligned}$ | 8.12 | $\begin{gathered} 5.98 \\ {[0.426]} \end{gathered}$ |
| ESP | $\begin{gathered} 0.0681 \\ (0.0310) \\ \hline \end{gathered}$ | $\begin{aligned} & \hline-0.0026 \\ & (0.0013) \\ & \hline \end{aligned}$ | 13.31 | $\begin{gathered} 8.67 \\ {[0.193]} \\ \hline \end{gathered}$ |
| FRA | $\begin{gathered} \hline 0.1221 \\ (0.0469) \\ \hline \end{gathered}$ | $\begin{aligned} & \hline-0.0074 \\ & (0.0035) \\ & \hline \end{aligned}$ | 8.27 | $\begin{gathered} 2.98 \\ {[0.812]} \end{gathered}$ |
| GBR | $\begin{gathered} 0.1483 \\ (0.0271) \\ \hline \end{gathered}$ | $\begin{aligned} & \hline-0.0106 \\ & (0.0021) \\ & \hline \end{aligned}$ | 7.01 | $\begin{gathered} 6.35 \\ {[0.385]} \\ \hline \end{gathered}$ |
| IRE | $\begin{gathered} 0.1676 \\ (0.0747) \\ \hline \end{gathered}$ | $\begin{aligned} & \hline-0.0068 \\ & (0.0034) \\ & \hline \end{aligned}$ | 12.31 | $\begin{gathered} 7.73 \\ {[0.258]} \\ \hline \end{gathered}$ |
| NLD | $\begin{gathered} 0.1094 \\ (0.0296) \\ \hline \end{gathered}$ | $\begin{aligned} & \hline-0.0093 \\ & (0.0027) \\ & \hline \end{aligned}$ | 5.87 | $\begin{gathered} 7.53 \\ {[0.274]} \\ \hline \end{gathered}$ |
| PRT | $\begin{gathered} \hline 0.3121 \\ (0.1302) \end{gathered}$ | $\begin{aligned} & \hline-0.0229 \\ & (0.0099) \end{aligned}$ | 6.83 | $\begin{gathered} 9.16 \\ {[0.165]} \end{gathered}$ |

$\widetilde{Y}$ is de-trended $\ln (Y)$. Four lags of $\widetilde{Y}$ and $u$ were used as instruments in vector $z$. Estimation method was based on Quadratic Barlett Kernel using 8 autocorrelation terms. Numbers in parentheses are the estimated $H A C$ standard errors. Numbers in square brackets are the corresponding $p$ values. The J-Statistic is for Hansen's test of the validity of the overidentifying restrictions.

## 3. Output and Unemployment: Theory

In this section we examine whether the relationship deduced from data in the previous section can be derived from a stylised theoretical macro-model - e.g. of the type outlined by Blanchard and Kiyotaki (1987). We therefore construct a suitable version of the latter, which allows for unemployment to be sustained in equilibrium. To incorporate this modification we endow the model with market imperfections in both the labour and the goods market by
allowing for firms to be monopolistically competitive and to reward workers' effort by paying efficiency wages.

Following the work of Shapiro and Stiglitz (1984) and Yellen (1984), a number of models have employed some version of the efficiency wage hypothesis to study various aspects of macroeconomic activity. Examples can found in: Agénor and Aizenman (1999) and Rebitzer and Taylor (1995) on fiscal and labour market policies; Andersen and Rasmussen (1999), Pisauro (1991) and Carter (1999) on the role of the tax system; Leamer (1999) on specialisation; Albrecht and Vroman (1996), Fehr (1991) and Smidt-Sørensen (1990) on properties of labour demand; and Smidt-Sørensen (1991) on working hours. In this paper we employ a standard version of the hypothesis which postulates an effort supply function that is derived by workers maximising the expected utility from work as explained in the Appendix.

The model is static and describes an economy with three types of agents: firms, households and a government. Firms are monopolistically competitive and each produce a variety of a horizontally differentiated product using labour as input with an increasing returns to scale technology. Households are endowed with a unit of labour, which they supply inelastically. Unemployed households receive a benefit transfer from the government. The government revenue, raised by taxing the households, is used to subsidise the unemployed and to pay for government consumption. The final good in the economy is the Dixit-Stiglitz CES bundle of horizontally differentiated varieties produced by identical firms described later.

The demand side of the model consists of the households' and government's consumption. The latter is given by

$$
\begin{equation*}
\int_{j \in N} P_{j} g_{j} d j=P G \tag{1}
\end{equation*}
$$

where $j$ is the index denoting a variety of the differentiated good, $N$ is the mass of varieties on offer and $P_{j}$ and $g_{j}$ are the price and quantity of variety $j$. It is straightforward to show that $g_{j}=\left(\frac{P_{j}}{P}\right)^{-\varepsilon}\left(\frac{G}{N}\right)$,
where $G$ is the corresponding CES bundle with the constant elasticity of substitution ${ }^{2}$ between any two varieties $s>1$,

$$
G=\left(N^{-1 / s} \int_{j \in N} g_{j}^{l-(1 / s)} d j\right)^{l /[l-(1 / s)]}
$$

and $P$ is the price index dual to $G$; (2) maximises $G$ above subject to (1). The government expenditure comprises (1) and the unemployment benefit payments $B$ per unemployed worker/household. This expenditure is financed by a lump sum $\operatorname{tax}^{3}, T$, which, together with the normalisation of the number of households to unity - on the assumption that each household is endowed with one unit of labour - gives rise to the following budget constraint $P G+u B=T$.

Each household is endowed with initial money holdings $\bar{M}$ and receives distributed profits $\Pi$. In addition, it also supplies, inelastically, its unit of labour and at any point in time it can either be employed or be unemployed. When employed, a typical household works for a firm $j$, supplying the effort level $e_{j}>0$ and earning nominal wage $W_{j}$. If unemployed, it receives from the government the nominal unemployment benefit $B$ at no effort. Dropping the distinction between firms and setting profit income to zero (anticipating the symmetric

[^2]equilibrium and elimination of profits through a free entry and exit process), a household's budget constraint is

$P C+M=\left\{\begin{array}{l}W+\bar{M}-T, \quad \text { employed } \\ B+\bar{M}-T, \quad \text { unemployed }\end{array}\right.$
where $M$ is the desired stock of money, $C$ is the aggregate CES bundle of $N$ individual varieties $c_{j}$,
$c_{j}=\left(\frac{P_{j}}{P}\right)^{-\varepsilon}\left(\frac{C}{N}\right)$,
where
$C=\left(N^{-l / s} \int_{j \in N} c_{j}^{l-(1 / s)} d j\right)^{l /[l-(1 / s)]}$,
and $P$ is the corresponding price index dual to $C$; (5) maximises $C$ above subject to the
constraint $\int_{j \in N} P_{j} c_{j} d j=P C$.

## Household's utility is given by ${ }^{4}$

$V=v(C, M / P)-\lambda \cdot f(e)$,
where, in addition to the usual component $v$, which we assume to be a Cobb-Douglas function,
$v(C, M / P)=\frac{C^{\alpha}(M / P)^{l-\alpha}}{\alpha^{\alpha}(1-\alpha)^{l-\alpha}}$,

[^3]the utility function also depends on the level of effort, $e$, that an employed household will supply when working. $f(e) \geq 0$ captures the disutility of effort; $\lambda=1$ for an employed household; and $\lambda=0$ when the household is unemployed. Assuming that $f(e)$ is taken as given (see Appendix A. 1 for further details on the relevance of these and the derivation of the effort function) and maximising the utility function of an employed and an unemployed household subject to their respective budget constraint yields their consumption and money demand equations. Normalising the number of households to unity and using $L$ and $u$ to denote the proportions of employed and unemployed households, we have
$L+u=1$.
Using this normalisation, the household sector's aggregate consumption and money demand equations are
$C=\alpha\left(\frac{(1-u) W+u B+\bar{M}-T}{P}\right)$,
and
\[

$$
\begin{equation*}
\frac{M}{P}=(1-\alpha)\left(\frac{(1-u) W+u B+\bar{M}-T}{P}\right) . \tag{9}
\end{equation*}
$$

\]

Given the above, the aggregate demand for the CES bundle, facing the monopolistically competitive firms, is $Y=C+G$. On the assumption that each firm produces a distinct variety - given the incentive to specialise due to falling average costs explained below - the demand function facing firm $j$ is $y_{j}=c_{j}+g_{j}$ which is obtained by adding (5) and (2) ${ }^{5}$

$$
\begin{equation*}
y_{j}=\left(\frac{P_{j}}{P}\right)^{-s}\left(\frac{Y}{N}\right) \tag{10}
\end{equation*}
$$

[^4]since. It is a straightforward exercise to show that $P, Y$ and $N$ satisfy the following
\[

$$
\begin{align*}
& Y=\left(N^{-l / s} \int_{j \in N} y_{j}^{l-(l / s)} d j\right)^{1 /[l-(l / s)]},  \tag{11}\\
& \int_{j \in N} P_{j} y_{j} d j=P Y \tag{12}
\end{align*}
$$
\]

and

$$
\begin{equation*}
P=\left(\frac{1}{N} \int_{j \in N} P_{j}^{l-s} d j\right)^{l /(1-s)} . \tag{13}
\end{equation*}
$$

Labour is assumed to be the only factor of production, and to be perfectly mobile between firms. Firm $j$ 's technology is given by the following production function
$y_{j}=e_{j} L_{j}-\phi$,
where $L_{j}$ is the variable labour input, $e_{j}$ is labour productivity and $\phi$ is a constant parameter reflecting the fixed cost of production assumed to be identical across firms. The increasing returns to scale implied by falling average cost therefore gives rise to the incentive for full specialisation from which a one-to-one correspondence between the mass of varieties and firms in the market results.

We assume that $e_{j}$ is determined by workers' attitude towards shirking and represents their optimal effort supply function which depends on: $i$ ) the real value of the wage paid by the firm $w_{j}=W_{j} / P$; ii) the real value of the unemployment benefit, $b=B / P$, which the government transfers to the unemployed household; and iii) the extent of unemployment in
the economy captured by the unemployment rate ${ }^{6} u$. Thus, we postulate the following effort supply function for a worker employed by firm $j$

$$
\begin{equation*}
e_{j}=e\left(w_{j}, b, u\right), \tag{15}
\end{equation*}
$$

which is assumed to satisfy the following properties,
$e\left(w_{j}, b, u\right) \geq 0$ as $w_{j} \geq b$; and $e\left(w_{j}, b, u\right)=0$ as $w_{j}=b$.
$e_{j w}^{\prime}=\frac{\partial e_{j}}{\partial w_{j}}>0, e_{j b}^{\prime}=\frac{\partial e_{j}}{\partial b}<0$ and $e_{j u}^{\prime}=\frac{\partial e_{j}}{\partial u}>0$,
and to have plausible second and cross partial derivatives. In particular, we shall assume

$$
e_{j w w}^{\prime \prime}=\frac{\partial^{2} e_{j}}{\partial w_{j}^{2}}<0, e_{j b w}^{\prime \prime}=\frac{\partial^{2} e_{j}}{\partial b \partial w_{j}}>0 \text { and } e_{j u w}^{\prime \prime}=\frac{\partial^{2} e_{j}}{\partial u \partial w_{j}}>0 .
$$

An effort function, which satisfies the above properties, is explicitly derived in Appendix A1.

Each individual firm takes $P, Y, N, u$ and $B$ as given and chooses its 'efficiency wage' $W_{j}$ and its price $P_{j}$ so as to maximise its profit
$\pi_{j}=P_{j} y_{j}-W_{j} L_{j}$,
subject to the demand function in (10) and the production function in (14) as well as taking account of its workers' reaction to the choice of $W_{j}$ which is given by the effort function in (15). The first order conditions are $\partial \pi_{j} / \partial W_{j}=0$ and $\partial \pi_{j} / \partial P_{j}=0$ whose solution imply the following wage and price setting rules ${ }^{7}$

$$
\begin{equation*}
W_{j}=\frac{e_{j}}{e_{j w}^{\prime}}, \tag{17}
\end{equation*}
$$

and

[^5]$P_{j}=\left(\frac{s}{s-1}\right) \frac{W_{j}}{e_{j}}$.

Equation (17) is a well-known result in the efficiency wage literature and implies that firms raise their wage rate up to the point where the effort function is unit elastic. Equation (18) is the usual mark-up pricing rule for a monopolistically competitive firm. In a symmetric equilibrium where all firms are identical, we drop the subscript $j$ and write the above equations as
$e_{w}^{\prime}(w, b, u) \cdot w=e$,
and
$e(w, b, u)=\sigma w$,
where $\sigma=s /(s-1)$. To see the (partial equilibrium) implications of these, first note that together they yield
$e_{w}^{\prime}(w, b, u)=\sigma>1$.

Totally differentiating (18') and taking account of (19) implies

$$
\begin{equation*}
\frac{d u}{d b}=-\frac{e_{b}^{\prime}}{e_{u}^{\prime}}>0 \tag{20}
\end{equation*}
$$

which shows that an increase in the benefit rate raises the unemployment rate. Totally differentiating (17) and ( $18^{\circ}$ ) and solving using (20) to eliminate $d b$ and $d w$ yields (see Appendix A2)

$$
\begin{equation*}
\frac{d e}{d u}=\left(\frac{\sigma}{e_{w w}^{\prime \prime}}\right) \cdot\left[\frac{e_{u}^{\prime} e_{b w}^{\prime \prime}}{e_{b}^{\prime}}-e_{u w}^{\prime \prime}\right] \tag{21}
\end{equation*}
$$

Thus, under our assumptions regarding the shape of the effort function, (21) implies $d e / d u>0$ which is consistent with the theoretical consensus that the net result of an increase in unemployment rate is to raise workers' effort level.

We can use the above results to examine the way in which equilibrium output and unemployment are related to each other on the supply side. The symmetric equilibrium of the industry is obtained when entry eliminates profits,
$\Pi=\int_{j \in N} \pi_{j} d j=P Y-W L$,
where $L=\int_{j \in N} L_{j} d j$ is total employment. Thus, through free entry and exit process $N$ adjusts to ensure $\Pi=0$. Imposing this on (22) and solving for $Y$ gives $Y=w L$, from which upon substitution for $w$ from (18) we obtain
$Y=\frac{1}{\sigma} e L$.

Equation (23) may be interpreted as a 'quasi-aggregate' production function. It traces the combinations of aggregate employment and output $(L, Y)$ which satisfy the supply side equilibrium in which labour productivity is determined by an effort supply function and firms pay wages to induces workers to supply the effort level that maximises their profits. Or, put differently, these combinations give the equilibrium locus that describes how $Y$ changes as firms and workers respond to changes in $u$ while the wage is adjusted to ensure the resulting effort supply maximises profits. Totally differentiating (23) and noting that $d L=-d u$ from (7), we obtain

$$
\begin{equation*}
\frac{d Y}{d u}=\left(\frac{e}{\sigma}\right)\left[\left(\frac{1-u}{u}\right)\left(\frac{u}{e} \frac{d e}{d u}\right)-1\right] . \tag{24}
\end{equation*}
$$

Thus, provided that $d e / d u$ in (21) is finite as $u \rightarrow 1$, we would expect the right-hand-side of (24) to be negative for sufficiently large levels of $u$. Conversely, starting from very low levels of $u$, we would expect the right-hand-side of (24) to be positive as long as $d e / d u$ in (21)
is positive, as explained above. Given these and assuming that $d e / d u$ in (21) is continuous in $u$, the equilibrium locus in $(u, Y)$ space may be depicted as in Figure 1 below.

Figure 1. The relationship between output and unemployment with efficiency wages and monopolistic competition


The shape depicted in Figure 1 is consistent with our empirical evidence reported in the previous section which showed that, in seven out of ten countries, $d Y / d u$ is likely to change from positive to negative as the level of unemployment passes a certain threshold. Within the context of the model developed above, the maximum level of output $Y=\bar{Y}$ is achieved at this point where $u=\bar{u}$. Prior to this point, effort - and hence productivity - is relatively lower but is rising with $u$. Also, the positive effect of a rise in productivity dominates the negative effect of loss of labour as firms shed workers. Beyond $\bar{u}$ the opposite holds and shedding labour that is working efficiently results in a reduction in the level of output.

It is clear that in the economy described above firms will (rationally) produce the same level of output, $Y_{l}$ say, employing either $\left(1-u_{l}\right)$ inefficient workers or (1-u$)_{h}$ ) efficient workers. Thus, multiple equilibria can arise given the nonlinear nature of the relationship
between output and employment ${ }^{8}$. As a result, the effect of a policy shock on employment and output depends on the initial equilibrium and unemployment can fall in the event of a positive shock only if the economy is operating in the efficient region where $u \geq \bar{u}$. Finally, it can be easily shown that the (tax financed) fiscal multiplier is given by the following expression (see Appendix A3 for details) where $Y_{D}^{\prime}$ and $Y_{S}^{\prime}$ are the slopes of the aggregate demand and aggregate supply functions in $(P, Y)$ space, respectively,
$\frac{d Y}{d G}=\frac{1}{1-\frac{Y_{D}^{\prime}}{Y_{S}^{\prime}}}$.
While $Y_{D}^{\prime}<0$ always holds, $Y_{S}^{\prime}$ can be positive or negative within the framework developed above. Therefore, the effect of a fiscal expansion depends on the size and sign of the ratio of the slopes of the two functions. In particular, in the efficient situation when $Y_{S}^{\prime}>0$ the multiplier will - as in most recent new Keynesian studies of the effect of fiscal policy - lie between zero and unity. But when the economy is operating inefficiently and $Y_{S}^{\prime}<0$, the multiplier can in fact exceed unity - as in the typical Keynesian case - or even become negative - like in models when more than full crowding out occurs. However, the former case, in which the rise in output will be accompanied by a fall in employment, corresponds to an unstable initial equilibrium where $-Y_{S}^{\prime}>-Y_{D}^{\prime}>0$ whereas in the latter case $-Y_{D}^{\prime}>-Y_{S}^{\prime}>0$ and the initial equilibrium is stable.

## 4. Summary and Conclusions

The main motivating factor underlying our study has been to explore the circumstances in which positive policy shocks might give rise to adverse employment effects.

[^6]Accordingly our objective has been two fold. First, we have sought to describe the stylised facts regarding the co-movement of output and unemployment changes using data from ten West European countries. Our simple data analysis shows that while in all countries unemployment changes and output changes can be either positively or negatively related, in seven of these countries we find a clear (and statistically significant) tendency for unemployment to be lower when output and unemployment are positively related and higher otherwise. This suggests a humped-shape relationship between output and unemployment. Our second objective has been to outline a theoretical setting, which could give rise to such a relationship. We have therefore constructed a stylised macro-model with goods and labour market imperfections and have shown that the equilibrium relationship between output and unemployment can in fact be positive when a rise in unemployment induces workers to supply a higher effort such that the positive effect of the gain in productivity outweighs the negative effect of the reduction in employment; otherwise we obtain the conventional result that output and unemployment are negatively related.

Clearly, our results, which complement those of the literature on the effects of contractionary fiscal policy (see, for details, Barry and Devereux, 1995), suggest that plausible circumstances do exist in which market imperfections pose serious obstacles to the smooth working of expansionary and/or stabilization policies whose final aim is to improve welfare. We show that the economy can rationally operate at an inefficient equilibrium, and that positive demand shocks in such circumstances will have perverse effects. Accordingly, we conclude by stressing Lindbeck's (1992) concerns about the effectiveness of macroeconomic stabilisation policy in the presence of labour and product market imperfection, which are echoed by our empirical and theoretical results.
where the aggregate demand curve is flatter than the (downward sloping) aggregate supply curve will be unstable. See Appendix A3 for further details on the slopes of aggregate demand and supply curves.

## 5. References

Agénor, P-R and J. Aizenman (1999), Macroeconomic adjustment with segmented labor markets, Journal of Development Economics, 58, 277-296.

Aghion, P. and P. Howitt (1992), A model of growth through creative destruction, Econometrica, 60, 323-351.

Aghion, P. and Howitt, P. (1994), Growth and unemployment, Review of Economic Studies, 61, 477-94.

Albrecht, J.W and S.B. Vroman (1996), A note on the long-run properties of the shirking model, Labour Economics, 3, 189-195.

Andersen, T.M. and B.S. Rasmussen (1999), Efforts, taxation and unemployment, Economics Letters, 62, 97-103.

Barnes, M. and J. Haskel (2000), Productivity in the 1990s: evidence from British plants, тітео.

Barry, F. and M. Devereux (1995), The 'expansionary fiscal contraction' hypothesis: a neoKeynesian analysis, Oxford Economic Papers, 47, 249-264.

Blanchard, O.J. and N. Kiyotaki (1987), Monopolistic competition and the effects of aggregate demand, American Economic Review, 77, 647-66.

Blanchard, O.J. (1998), Revisiting European unemployment: unemployment, capital accumulation and factor prices, mimeo, (MIT).

Caballero, R. and M.L. Hammour (1998a), Jobless growth: appropriability, factor substitution, and unemployment. Carnegie-Rochester Series on Public Policy, 48, 5194.

Caballero, R. and M.L. Hammour (1998b), The macroeconomics of specificity, Journal of Political Economy, 103, 724-767.

Carter, T.J. (1999), The effect of taxes on labour in efficiency wage models: a comment, Journal of Public Economics, 72, 325-327.

Disney, R., Haskel, J. and Y. Helden (2000), Restructuring and productivity growth in UK manufacturing, mimeo.

Dixon, H. and P. Lawler (1996), Imperfect competition and the fiscal multiplier, Scandinavian Journal of Economics, 98, 219-231.

Fehr, E. (1991), Wages and labor demand: a note, Journal of Institutional and Theoretical Economics, 147, 539-546.

Heijdra, B.J. and F. Van der Ploeg (1996), Keynesian multipliers and the cost of public funds under monopolistic competition, Economic Journal, 106, 1284-96.

Heijdra, B.J., Ligthart J.E., and F. Van der Ploeg (1998), Fiscal policy, distortionary taxation, and direct crowding out under monopolistic competition, Oxford Economic Papers, 50, 79-88.

Leamer, E.E. (1999), Efforts, wages, and the international division of labour, Journal of Political Economy, 107, 1127-1162.

Leith, C. and , C.W. Li (2001), Unemployment and the Productivity Slowdown: A Labour Supply Perspective, University of Glasgow Discussion Paper in Economics 2001-13.

Lindbeck, A. (1992), Macroeconomic theory and the labour market, European Economic Review, 36, 209-235.

Marini G. and P. Scaramozzino (2000), Endogenous growth and social security, CeFiMS DP 02/00/02, Centre for Financial and Management Studies, SOAS, University of London

Malley, J. and T. Moutos (2001), Capital accumulation and unemployment: a tale of two continents, Scandinavian Journal of Economics, 103, 79-99.

Molana, H. and T. Moutos (1989), Useful government expenditure in a simple model of imperfect competition in Reider, U., Gessner, P. Peyerimoff, A. and Radermacher, F.J. (eds), Methods of Operations Research, 63, Verlag Anton Hain, Frankfurt.

Molana, H. and T. Moutos (1992), A note on taxation, imperfect competition, and the balanced-budget multiplier, Oxford Economic Papers, 44, 737-52.

Molana, H. and J. Zhang (2001), Market Structure and Fiscal Policy Effectiveness, Scandinavian Journal of Economics, 103, 147-164.

Moutos, T. (1991), Turnover costs, unemployment and macroeconomic policies, European Journal of Political Economy, 7: 1-16.

Pisauro, G. (1991), The effect of taxes on labour in efficiency wage models, Journal of Public Economics, 46, 329-345.

Rebitzer, J.B. and L.J. Taylor (1995), The consequences of minimum wage laws; some new theoretical ideas, Journal of Public Economics, 56, 245-255.

Reinhorn, L.J. (1998), Imperfect competition, the Keynesian cross, and optimal fiscal policy, Economics Letters, 58, 331-337.

Sala-i-Martin, X.X. (1996), A positive theory of social security, Journal of Economic Growth, 1, 277-304.

Shapiro, C. and J.E. Stiglitz (1984), Equilibrium unemployment as a worker discipline device, American Economic Review, 74: 433-44.

Smidt-Sørensen, J.B. (1990), The equilibrium effort-wage elasticity in efficiency-wage models, Economics Letters, 32, 365-369.

Smidt-Sørensen, J.B. (1991), An efficiency-wage-hours model and shorter working hours, Scottish Journal of Political Economy, 38, 113-131.

Startz, R. (1989), Monopolistic competition as a foundation for Keynesian macroeconomic models, Quarterly Journal of Economics, 104, 737-52.
van Ark, B., Kuipers, S., and G. Kuper (2000), Productivity, Technology and Economic Growth, Kluwer Academic Publishers.

Yellen, J.L. (1984), Efficiency wage models of unemployment, American Economic Review, 74, 200-205.

## 6. Appendix

## A1. Derivation of the Effort Supply Function $e(w, u, b)$

This appendix explains how a specific effort supply function such as that in equation (15) can be derived within the framework of the efficiency wage hypothesis where, following common practice, the agent is assumed to maximise the expected utility of remaining in employment.

We assume that all households participate in the labour market and at any point in time a household can be in one of the following states: (i) employed (working); (ii) being fired (when caught shirking at work); (iii) unemployed (being without a job); or (iv) being hired (finding a job). Let the utility indices corresponding to of the above states be denoted as follows:
(i) employed (working): $V^{E}$
(ii) being fired (losing one's job): $V^{F}$
(iii) unemployed (being without a job): $V^{U}$
(iv) being hired (finding a job): $V^{H}$
$V^{U}$ and $V^{E}$ can be obtained as follows. Disregarding the money holdings and taxes (which are the same for all states) and focusing on unemployment benefit or wage as the only source of work-related income, the utility function in equation (6) implies that the indirect utility of an unemployed and an employed household is, respectively,
$V^{U}=B / P$,
and
$V^{E}=(W / P)-f(e)$.
While (A1.1) is straightforward, (A1.2) needs some explanation regarding $f(e)$. We shall assume $f^{\prime}>0$ and $f^{\prime \prime} \geq 0$ which implies that the disutility of effort rises with a non-
decreasing rate. In particular, we shall use the explicit form $f(e)=k e^{2}$ where $k>0$ is a scaling factor.

Finally, we need to specify $V^{H}$, which is the satisfaction a household attaches to finding a job or being hired. But the utility associated with this state is in principle not distinguishable from $V^{E}$ and for simplicity we let
$V^{H}=V^{E}$,

The probabilities associated with moving from one state to another are assumed to be determined as follows:

## (a) Probability associated with being fired when shirking, $F$.

We assume that shirking is the only reason for being fired (we do not explicitly model the monitoring technology). Therefore, ceteris paribus, $F$ is a monotonic function of the effort level, $e$. Thus,

$$
F=F(e) ; F(0)=1 ; \quad F(1)=0 ; \frac{d F}{d e}<0 .
$$

For simplicity, normalise the maximum possible effort to unity and let
$F=1-e$.

## (b) Probability associated with finding a job, or being hired, when unemployed, $H$.

We assume that the labour force is homogeneous and, ceteris paribus, $H$ is a monotonic function of the unemployment rate, $u$ (we do not explicitly model the search technology). Thus,
$H=H(u) ; H(0) \leq 1 ; H(1)=0 ; \frac{d H}{d u}<0$.
For simplicity we let
$H=1-u$.

We define the optimal level of effort as that which maximises a household's expected utility of remaining in employment. The latter is denoted by $R(e)$ and is, by definition, given by
$R(e)=(1-F) V^{E}+F V^{F}$.
Also, given that a 'fired' worker can either be hired or remain unemployed, we let $V^{F}$ be a weighted average of $V^{H}$ and $V^{U}$. Thus,
$V^{F}=H V^{H}+(1-H) V^{U}$.

Equations (A1)-(A7) yield
$R(e)=e\left(w-k e^{2}\right)+(1-e)\left((1-u)\left(w-k e^{2}\right)+u b\right)$,
where $w=W / P$ and $b=B / P$. This equation can be rearranged as

$$
\begin{equation*}
R(e)=-u k e^{3}-(1-u) k e^{2}+u(w-b) e+((1-u) w+u b) . \tag{A1.8}
\end{equation*}
$$

The agent takes ( $w, b, u$ ) as given and chooses $e$ to maximise $R(e)$. The first order condition for this is $-3 u k e^{2}-(2(1-u) / 3 u) e+(1 / 3 k)(w-b)=0$. This has two roots of which only one is positive which also satisfies the second order for a maximum and can, after some normalisation, be written as
$e=\left[\gamma(w-b)+\frac{l-u}{u}\right]^{\frac{1}{2}}-\frac{l-u}{u}$,
where $\gamma \equiv 3 / k$. It is clear that equation (A1.9) satisfies our specified conditions, since $e(w, b, u) \geq 0 \quad$ as $\quad w \geq b ; \quad e(w, b, u)=0 \quad \forall u \in(0,1) \quad$ as $\quad w=b ; \quad e_{w}^{\prime}=\frac{\partial e}{\partial w}>0 ;$
$e_{b}^{\prime}=\frac{\partial e}{\partial b}<0 ; \quad e_{u}^{\prime}=\frac{\partial e}{\partial u}>0 ; \quad e_{w w}^{\prime \prime}=\frac{\partial^{2} e}{\partial w^{2}}<0 ; \quad e_{b w}^{\prime \prime}=\frac{\partial^{2} e}{\partial b \partial w}>0 ;$ and $e_{u w}^{\prime \prime}=\frac{\partial^{2} e}{\partial u \partial w}>0$.

## A2. Derivation of Equations (20) and (21).

We use the following equations (15), (17), (18') and (19) which are reproduced below as (A2.1)-(A2.4), respectively,
$e=e(w, b, u)$,
$e_{w}^{\prime}(w, b, u) \cdot w=e$,
$e(w, b, u)=\sigma w$,
$e_{w}^{\prime}(w, b, u)=\sigma$.

First, totally differentiating (A2.3) yields
$e_{w}^{\prime} d w+e_{b}^{\prime} d b+e_{u}^{\prime} d u=\sigma d w$,
and substituting from (A2.4), i.e. $e_{w}^{\prime}=\sigma$, in (A2.5) we obtain
$e_{b}^{\prime} d b+e_{u}^{\prime} d u=0$,
which is solved to yield equation (20).

Next, totally differentiating (A2.4) implies
$e_{w w}^{"} d w+e_{b w}^{"} d b+e_{u w}^{"} d u=0$,
and using (A2.6) to eliminate $d b$ we have

$$
\begin{equation*}
e_{w w}^{\prime \prime} d w+e_{b w}^{\prime \prime}\left(-\frac{e_{u}^{\prime}}{e_{b}^{\prime}}\right) d u+e_{u w}^{\prime \prime} d u=0 \tag{A2.8}
\end{equation*}
$$

which can be solved for $d w$ to give
$d w=\left(\frac{1}{e_{w w}^{\prime \prime}}\right)\left[e_{b w}^{\prime \prime}\left(\frac{e_{u}^{\prime}}{e_{b}^{\prime}}\right)-e_{u w}^{\prime \prime}\right] d u$,
Substituting from (A2.9) into $d e=\sigma d w$ implied by (A2.3), we obtain equation (21).

## A3. Derivation of the Fiscal Multiplier, Equation (25).

We derive the fiscal multiplier as follows. First, the aggregate demand function (AD) is derived by noting that $Y=C+G$, where $C$ is obtained by solving equations (8) and (9), i.e. $C=[\alpha /(1-\alpha)](\bar{M} / P)$. Hence,
$Y=G+\left(\frac{\alpha}{l-\alpha}\right) \frac{\bar{M}}{P}$
Totally differentiating (A3.1), for any give $\bar{M}$, then implies that on AD,
$d Y=d G-\left(\frac{\alpha \bar{M}}{(1-\alpha) P^{2}}\right) d P$.
Next, recalling from equations (23) and (7) that $Y=w L$ and $L=1-u$, we obtain, on the aggregate supply (AS) side, $Y=w(1-u)$, which can be totally differentiated to yield $d Y=(1-u) d w-w d u$,
which upon substitution from (A2.9) can be written as,
$d Y=\left\{\left(\frac{1-u}{e_{w w}^{\prime \prime}}\right)\left[e_{b w}^{\prime \prime}\left(\frac{e_{u}^{\prime}}{e_{b}^{\prime}}\right)-e_{u w}^{\prime \prime}\right]-w\right\} d u$.
Also, using (A2.6) and the fact that for any given $B, d b=-\frac{B}{P^{2}} d P$, we obtain

$$
\begin{equation*}
d u=\left(\frac{e_{b}^{\prime}}{e_{u}^{\prime}}\right)\left(\frac{B}{P^{2}}\right) d P \tag{A3.5}
\end{equation*}
$$

which can be substituted in (A3.4) to give the reaction of output to a change in the price level on the aggregate supply ( $\mathbf{A S}$ ),

$$
\begin{equation*}
d Y=\left\{\left(\frac{1-u}{e_{w w}^{\prime \prime}}\right)\left[e_{b w}^{\prime \prime}\left(\frac{e_{u}^{\prime}}{e_{b}^{\prime}}\right)-e_{u w}^{\prime \prime}\right]-w\right\}\left(\frac{e_{b}^{\prime}}{e_{u}^{\prime}}\right)\left(\frac{B}{P^{2}}\right) d P . \tag{A3.6}
\end{equation*}
$$

Simplifying notation and writing (A3.2) and (A3.6) as
$d Y=Y_{D}^{\prime} d P+d G$
$d S=Y_{S}^{\prime} d P$
where $Y_{D}^{\prime}$ and $Y_{S}^{\prime}$ are the slopes of $\mathbf{A D}$ and $\mathbf{A S}$ in $(P, Y)$ space. Solving the above to eliminate $d P$ we obtain the fiscal multiplier in equation (25), namely, $\frac{d Y}{d G}=\frac{1}{1-\frac{Y_{D}^{\prime}}{Y_{S}^{\prime}}}$.


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[^1]:    ${ }^{1}$ In the empirical analysis, which follows, we shall concentrate on the contemporaneous changes only since this case clearly does not over-record the proportion of periods when $Y$ and $u$ are moving in the same direction.

[^2]:    ${ }^{2}$ See Molana and Zhang (2001) for a study of the role of a variable elasticity of substitution in the context of fiscal policy effectiveness. Note also that, following the common practice the CES bundle is normalised by the mass of varieties, $N$, to switch off the variety effect in the aggregate.

[^3]:    ${ }^{3}$ The use of a lump-sum tax is a common simplification in the literature which reduces the distortionary role of the government. For further explanations see Molana and Moutos (1991), Heijdra and Van der Ploeg (1996) and Heijdra, et al. (1998) among others.
    ${ }^{4}$ For simplicity, like most studies we assume complete separation between households' and government's consumption. Therefore, government consumption does not appear in households' utility function. For some exceptions see, for example, Molana and Moutos (1989), Heijdra et al (1998) and Reinhorn (1998) who extend the original results by allowing for some substitution between the public and private consumption.

[^4]:    5 We have followed the existing studies in assuming that $G$ and $C$ are similar CES bundles. See Startz (1989), Dixon and Lawler (1996), Heijdra and Van der Ploeg (1996) and Heijdra, et al. (1998) for further details.

[^5]:    ${ }^{6}$ Given that the number of households is normalised to $1, u$ is simply the proportion of unemployed households and is equivalent to the unemployment rate.
    7 The second order conditions are satisfied as long as $s>1$ and $e_{j \text { inw }}^{\prime \prime}<0$.

[^6]:    ${ }^{8}$ It is a straightforward exercise to show that the aggregate supply function in $(P, Y)$ space is nonlinear and can have more than one intersection with the aggregate demand. In such a situation, the equilibrium occurring

