Economic Growth and Endogenous Fiscal Policy:

In Search of a Data Consistent General Equilibrium Model

Jim Malley
University of Glasgow

Apostolis Philippopoulos
Athens University of Economics and Business

December 7, 1999

Abstract: This paper searches for a general equilibrium model of optimal growth and endogenous fiscal policy with the aim of explaining the interaction between private agents and fiscal authorities in the U.S., West Germany, Japan and the U.K. over the period 1960-1996. Our search is conducted in the context of popular models with closed-form analytical solutions since this is necessary to formally test the models' theoretical restrictions. In West Germany and Japan there is evidence that the fiscal authorities act as optimizing Stackelberg leaders who are concerned about the current welfare of private agents. In contrast, the fiscal authorities in the U.S. and U.K. do not appear to act as optimizing agents; instead, they follow simple rule-of-thumb policy rules. In all countries, the tax smoothing model, according to which policymakers find it optimal not to react to the state of the economy, is rejected.

Keywords: Optimal fiscal policy and private agents. Economic growth.
JEL classification: H3. D9.

Acknowledgements: We thank Costas Azariadis, Fabrizio Carmignani, George Economides, Jim Kay, Tryphon Kollintzas, Ben Lockwood, Hassan Molana, Thomas Moutos, Ron Smith, Harald Uhlig and Ulrich Woitek for comments and suggestions. This usual disclaimer applies. Part of this work was undertaken when Jim Malley was visiting Athens University of Economics and Business. We would like to thank Athens and Glasgow Universities for the funding which made this visit possible.

Correspondence to: Jim Malley, Department of Economics, University of Glasgow, Glasgow G12 8RT, UK. Tel: +44-141-3304617. Email: j.malley@socsci.gla.ac.uk.

1. INTRODUCTION

It has now become standard practice in modern macroeconomics to conduct the analysis of endogenous fiscal policy within the context of optimal growth models. In these models, general equilibria are the outcome of the interaction between private agents and fiscal authorities that both solve explicit optimization problems. The standard theoretical setup is one where: (i) the representative private agent maximizes intertemporal utility by taking prices and fiscal policy as given; (ii) prices and quantities are determined in a competitive equilibrium for any given feasible policy; (iii) the government chooses its policy to maximize the representative agent's intertemporal utility by acting as a Stackelberg leader *vis-à-vis* the private sector. Then, these fiscal policy rules, together with the competitive equilibrium, give a general equilibrium in which allocations, prices and policies are functions of the "state" of the economy.

This paper searches for a general equilibrium model of optimal growth and endogenous fiscal policy to investigate the interaction between private agents and fiscal authorities in the U.S., West Germany, Japan and the U.K. (henceforth G-4)[1] over the period 1960-1996. In our search for a data consistent model, we focus on simple general equilibrium models which have been extensively used in the theoretical macroeconomics literature in the last three decades (see e.g. the papers collected in the two volumes edited by Persson and Tabellini [1994]). Moreover, our search is conducted in the context of models with closed-form analytical solutions. This is necessary to formally test the theoretical restrictions implied by these models against the data. All our general equilibrium models consist of behavioral reduced-form relations for private consumption-to-, private capital-to- and government

[1] That is, we test the G-3 countries plus the U.K.. It is widely believed that these four economies are somehow closer to the neoclassical paradigm.

consumption expenditures-to-output ratios.[2] Also, in all our models, fiscal policy is time consistent.

The format of the paper and the main results are as follows. In Section 2, we find it natural to start with a version of one of the most popular models of endogenous policy. Namely, a tax-smoothing model *a la* Barro [1979], in which it is not optimal for the government to follow state-contingent tax policy rules.[3] We therefore set up a basic optimal growth model augmented with a benevolent fiscal authority, where the latter finds it optimal to keep the income tax rate - and (since the government budget is balanced) the government expenditures-to-output ratio - constant over time. However, when we study the empirical validity of this general equilibrium model by testing its over-identifying restriction, data from the G-4 resoundingly reject it.[4] Our findings are consistent to those in e.g. Chari *et al*. [1994] for the U.S.. We do not find this result surprising since this model, as well as other versions of the tax-smoothing model, despite their theoretical appeal, are based on several unrealistic assumptions.

In Section 3, we relax some assumptions of the above simplistic framework. For instance, we deviate from the basic optimal growth model by postulating a simple rule-of-

---

[2] Note that we do not include public debt. That is, in all our models, the government balances its budget in each time period by using income taxes to finance its expenditures. This is to reduce the number of “state variables” and so obtain closed-form solutions. Also, by omitting public debt, we avoid well-known data measurement problems. In any case, omitting public debt is not unusual in the literature (see e.g. Barro and Sala-i-Martin [1992], Baxter and King [1993], McGrattan [1994], Ambler and Paquet [1996], Benhabib and Velasco [1996] and Devarajan *et al*. [1996]). Also, we do not include government production expenditures as in Barro [1990] that provide positive externalities to private firms and can generate long-term (i.e. endogenous) growth. If we add such expenditures, our main results do not change (see Malley *et al*. [1999]).

[3] The basic premise of tax smoothing is that tax policy is distorting and therefore the optimizing fiscal authorities allocate this policy over time to minimize its negative effects. Then, the tax rate changes only if there are unanticipated shocks, i.e. the tax rate follows a random walk independently of the state of the economy or the properties of the underlying shocks (for details, see e.g. Chari *et al*. [1994]).

[4] It is interesting that although many general equilibrium versions of the tax-smoothing result have been developed in the theoretical literature (see e.g. Chari *et al*. [1994], and for simpler models see e.g. Barro [1990], Barro and Sala-i-Martin [1992], Benhabib and Velasco [1996] and Devereux and Wen [1998]), it is not known how well the tax-smoothing result stands up to the empirical facts in a general equilibrium setup. This is surprising because there has been a tremendous amount of empirical interest in the partial equilibrium renditions of this model (see e.g. Serletis and Schorn [1999] and the references cited there). However, see

thumb for the behavior of private agents (see also Lettau and Uhlig [1999]). This rule simply says that private agents keep the total value of their wealth at its previous period level. Concerning fiscal policy, we assume that although policymakers act as optimizing agents, they heavily discount the future. Specifically, they only solve a static version of their full problem, when they act as benevolent Stackelberg leaders *vis-à-vis* private agents. Due to the assumptions that private agents follow a simple rule-of-thumb behavior and that policymakers are short-sighted, we are able to obtain a closed-form analytical solution for the general equilibrium.[5] In this new setting, the optimal income tax rate and the government expenditures-to-output ratio are not constant over time. That is, now it is optimal for the government to follow state-contingent policy rules. Data from West Germany and Japan do not reject this general equilibrium model, and they also give reasonable parameter estimates. However, the model is rejected for the U.S. and U.K.. Consequently, for West Germany and Japan, there is evidence that policymakers are optimizing agents, who act as Stackelberg leaders and are concerned with the current welfare of private agents.

In Section 4, we continue our search for a general equilibrium model that may be able to explain the U.S. and U.K. experiences. To this end, we opt for a simpler model of policy-making than the one presented in Section 3. In particular, we abandon the assumption that policymakers are optimizing agents. Instead, we assume that policymakers follow an *ad hoc* feedback rule according to which the government expenditures-to-output ratio reacts to a

---

Chari *et al*. [1994] for a test of a general equilibrium tax-smoothing model in the tradition of the RBC literature.

[5] This is not the only way to obtain an analytical solution. For instance, we could take an approximation around the steady-state. Although this is easy when fiscal policy is exogenous (see e.g. Campbell [1994] and the references cited there), it becomes problematic in a setup where both private agents and policymakers are optimizing and interact with each other. In that case, one should make sure that the steady-state values of policy variables in the approximated competitive equilibrium and the approximated policymakers' problem converge to each other so that the approximation is model-consistent. As far as we know, this is still an open issue, and anyway it is unlikely that this would give a closed-form analytical solution. In any event, we can defend our modeling approach by noting that: (i) there are good theoretical reasons, as well as empirical support, for rule of thumb behavior (see e.g. Ellison and Fudenberg [1993], Lettau and Uhlig [1999] and the

menu of arbitrarily chosen predetermined variables. When we incorporate this policy rule into the growth model of Section 3, the data cannot reject this less restrictive general equilibrium model. Consequently, the evidence from the U.S. and U.K. indicates that policymakers follow simple rule-of-thumb feedback policy rules.

Comparing the results for the U.S. and U.K. with the results for West Germany and Japan, a possible explanation is that acting optimally requires too much coordination for a government, like that in the U.S., which is essentially divided between two political parties. Concerning the U.K. case, its post-war period has been characterized by to many abrupt, policy regime changes to be explained by a single model of optimal policymaking. In contrast, in Japan there is much more centralization and unitary decision-making, while in Germany, although there is fragmentation and coalition formation, such coalitions (once formed) are rather stable and (more importantly) ideologically homogenous.

Before we move on, we wish to say that the literature on optimal growth and fiscal policy is vast. For instance, there are Real Business Cycle (RBC) models that reproduce the observed features of the business cycle by adding a government sector. The latter can either follow various types of exogenous fiscal policy (see e.g. Christiano and Eichenbaum [1992], McGrattan [1994] and Baxter and King [1993]) or can choose fiscal policy optimally (see e.g. Chari *et al*. [1994] and Ambler and Paquet [1996] for the U.S.). However, most of these models are "tested" with the use of calibration techniques in the RBC tradition (for a notable exception, see Christiano and Eichenbaum [1992]). In other words, most of these models are evaluated by their ability to match the observed moments of the data. In contrast, here we obtain closed-form analytical solutions and hence can *formally* test the theoretical cross-equation restrictions implied by the interaction between private agents and fiscal authorities.

---

references cited in the latter) and (ii) there is empirical support for short-sighted policymakers (see e.g. Laver and Hunt [1992] and Lockwood *et al.* [1996]).

2. GROWTH AND FISCAL POLICY: A TAX-SMOOTHING MODEL

Consider a closed economy consisting of a representative private agent and a government. The private agent chooses time-paths of consumption and capital to solve an intertemporal optimization problem as in the basic optimal growth model. The government finances public consumption services by taxing the private agent's income. Public consumption services are endogenous and provide direct utility. The government is benevolent and acts as a Stackelberg leader *vis-à-vis* the private agent.[6] Time is discrete, the time-horizon is infinite and, for simplicity, there is no uncertainty.

We solve for Markov strategies, i.e. optimal strategies are functions of the current value of the relevant state variables. Hence, we solve for Markov-perfect general equilibria, which are sub-game perfect and so time consistent. This is important because when taxes are distorting, optimal policy can be time inconsistent (see Chamley [1986]).[7]

2.1 Private Sector

The representative agent maximizes intertemporal utility:

$$\sum_{t=0}^{\infty} b^t [\log c_t + d \log g_t]$$

(1)

---

[6] Throughout the paper, when policy is optimally chosen, we assume the government is benevolent. This is a usual assumption in the neoclassical paradigm. As Stokey [1991, p. 629] points out "assuming a benevolent government is a useful setting because it does not require any interpersonal comparisons and the preferences of the government do not have to be specified in an *ad hoc* way". Also Chari *et al*. [1989, p. 273] say that "…although one could argue that policymakers are self-interested, it is not clear why the preferences of society do not reflect the preferences of its constituents". On the other hand, Persson and Tabellini [1999] argue that even with fully informed voters, political equilibria exhibit failures (e.g. some public goods are under-provided because politicians need to please a subset of voters, or politicians earn rents for themselves at the voters' expense). We believe that in the medium-run, and in the context of a growth model, it makes sense to assume that the preferences of policymakers reflect the preferences of voters. Therefore, following most of the literature, we assume that, when the government acts optimally, it is benevolent.

[7] On the other hand, Markov perfect equilibria exclude reputational strategies that can lead to better outcomes (see e.g. Benhabib and Velasco [1996]).

where c_t is private consumption at time t, g_t is public consumption services at t, $0 < b < 1$ is the discount rate and $d \geq 0$ is the weight given to public consumption services relative to private consumption. For simplicity, the utility function is logarithmic and additively separable.

The flow budget constraint of the representative agent is:

$$k_{t+1} + c_t = (1 - q_t) A k_t^a$$

(2)

where k_{t+1} is the end-of-period capital stock, k_t is the beginning-of-period capital stock, $y_t = A k_t^a$ is current output produced via a Cobb-Douglas technology (where $A > 0$ and $0 < a < 1$), and $0 \leq q_t < 1$ is the income tax rate. The initial capital stock is given. Notice that in (2), we have assumed full capital depreciation within a single period.

We formulate the problem as a dynamic programming one. From the competitive private agent's point of view, the state at time t can be summarized by the predetermined capital stock, k_t, and the current tax rate, q_t. Let $U(k_t; q_t)$ denote the value function of the private agent at time t. Using (2) for c_t, the value function satisfies the Bellman equation:

$$U(k_t; q_t) = \max_{k_{t+1}} \left[\log\left[(1 - q_t) A k_t^a - k_{t+1} \right] + d \log g_t + b U(k_{t+1}; q_{t+1}) \right].$$

(3)

Appendix A shows that, for given Markov tax strategies, optimal private consumption, c_t, and the end-of-period capital stock, k_{t+1}, are respectively:[8]

[8] The fact that the competitive private agent's decisions are obtained as the policy solutions to a dynamic programming problem, in combination with the requirement that fiscal policy variables are Markov, makes the competitive equilibrium a recursive one, i.e. allocations and prices are functions of the current value of the relevant state variables. In turn, the problem of the government becomes also recursive and its strategies are Markov. See Appendices A and B, and for details see Kollintzas *et al*. [1999].

$$c_t = (1 - ab)(1 - q_t) Ak_t^a \tag{4}$$

$$k_{t+1} = ab\ (1 - q_t) Ak_t^a \ . \tag{5}$$

That is, with a log-linear utility function, a Cobb-Douglas production function and full capital depreciation, we obtain a closed-form solution for the private agent's problem (see Stokey and Lucas [1989] and for applications Sargent [1987]).

2.2 The Government Sector and Competitive Equilibrium (given economic policy)

The government balances its budget in each time period. Thus,

$$g_t = q_t Ak_t^a \ . \tag{6}$$

Equations (4), (5) and (6) can give a recursive competitive equilibrium in c_t, k_{t+1} and one of the two policy instruments, q_t and g_t.[9] Looking ahead at the empirical work below, it is convenient to express this equilibrium in terms of g_t. Thus, solving (6) for q_t and substituting into (4) and (5), we obtain:

$$c_t = (1 - ab)(Ak_t^a - g_t) \tag{7}$$

$$k_{t+1} = ab\ (Ak_t^a - g_t)\ . \tag{8}$$

so that (7) and (8) give a recursive competitive equilibrium for any feasible level of government expenditures, g_t.[10]

2.3 Endogenous Fiscal Policy and General Equilibrium

We now endogenize fiscal policy by assuming that the government is benevolent and acts as a Stackelberg leader *vis-à-vis* the private agent. That is, at any time t, the government

---

[9] Because of (6), only one of the two policy instruments, q_t and g_t, can be set independently.

chooses g_t to maximize (1) subject to the private agent's decision rules (7) and (8).[11] The resulting Markov strategy for g_t, in combination with (7) and (8), will give a Markov-perfect general equilibrium.

From the government's viewpoint, the state at time t is the predetermined capital stock, k_t. Let $V(k_t)$ denote the value function of the government at time t. The value function must satisfy the Bellman equation:

$$V(k_t) = \max_{g_t} \left[\log c_t + d \log g_t + bV(k_{t+1}) \right].$$

(9)

where c_t and k_{t+1} follow (7) and (8) respectively.

Appendix B shows that the solution to (9) implies that it is optimal for the government to keep the income tax rate q_t, and equivalently (since the budget is balanced) the government expenditures-to-output ratio $\frac{g_t}{y_t}$, constant over time. Thus, the government's strategy is:[12]

$$0 < q_t = \frac{g_t}{y_t} = \frac{d(1 - ab)}{(1 + d)} < 1 \tag{10}$$

which is a form of the classic tax-smoothing result in general equilibrium.[13] Note that since the optimal tax rate is constant over time, and the government balances its budget in each time

---

[10] Equation (7) plus (8) give $c_t + g_t + k_{t+1} = Ak_t^a = y_t$, which is the aggregate economy's resource constraint.

[11] This is equivalent to saying that the government moves first in each time-period. In contrast, when the government moves simultaneously with private agents (or competitive private agents move first), this would correspond to a Nash game. In that case, the government would take into account the economy's resource constraint instead of the private agent's optimal decision rules (see Stokey [1991]). We study Stackelberg equilibria following the literature on optimal fiscal policy.

[12] Barro [1990], Barro and Sala-i-Martin [1992], Benhabib and Velasco [1996] and Devereux and Wen [1998] have also derived closed-form solutions for the optimal tax rate in similar setups. However, Barro [1990] and Barro and Sala-i-Martin [1992] use a highly stylised model. Benhabib and Velasco [1996] study more types of equilibria than here, but they use a small open economy model in which the return to capital is determined by the exogenous world interest rate. Devereux and Wen [1998] use the *AK* model in which the capital return, *A*, is a parameter. In contrast, here all returns are endogenously determined and we have government consumption services.

period, the level of endogenous government expenditures inherits the properties of the state of the economy (here, the state is the beginning-of-period capital stock, k_t).[14]

To summarise, (7), (8) and (10) give a general equilibrium growth model with endogenous fiscal policy. In equilibrium, it is optimal to keep the tax rate (and the government consumption-to-output ratio) constant over time.

2.4 The Econometric Model

To test whether the general equilibrium model given by (7), (8) and (10) is data consistent over the period 1960-1996[15], we first rewrite it as the following stochastic system, where all variables are expressed as shares of output, $y_t = Ak_t^a$:[16]

$$b_{11}y_{1t} - b_{12}y_{2t} - b_{13}y_{3t} - g_{11}x_{1t} = u_{1t} \tag{11a}$$

---

[13] In models like this, the result that the optimal tax rate is constant over time is not general. Although a survey of the literature on what model specification can give a constant tax rate is beyond the scope of this paper, we wish to say that in Barro [1990] and Barro and Sala-i-Martin [1992] the optimal tax rate under commitment is constant over time and hence there are no time-inconsistency issues. Benhabib and Velasco [1996] have shown that optimal tax rates under commitment are no longer constant once we use more general production functions. That is, as in Chamley [1986], it is optimal to tax capital heavily in the short-run and reduce its taxation in the future. However, when Benhabib and Velasco [1996] solve for equilibria without commitment, the optimal tax rate is constant. Recall that here we also solve for equilibria without commitment (i.e. Markov-perfect equilibria).

[14] Barro [1979] first derived the tax smoothing result in a partial equilibrium model. He showed that when government expenditures are exogenous, and if it is optimal to keep tax revenues constant over time, the public debt inherits the properties of the state of the economy. That is, in Barro [1979], the public debt smoothes out intertemporal tax distortions. In Lucas and Stokey [1983], the smoothing device is returns to bonds. In Chari *et al*. [1994], it is revenues from capital income taxes and returns to bonds (this paper also surveys the literature). In our model, the smoothing device is endogenous government expenditures. We believe that the important thing is whether it is optimal for policymakers to keep the tax rate constant. What is the specific device that smoothes out tax distortions over time and across states of nature is less critical.

[15] Data on private final consumption, *C*, public general consumption, *G*, and gross fixed capital formation, *I*, are from OECD Statistical Compendium 98(1). Output, *Y* is equal to *C+I+G*. The end-of-period capital stock, *K* is calculated for each country using a perpetual inventory and a constant 7% rate of depreciation. Note that the results reported in all of our empirical work do not change when alternative deprecation rates ranging from 5 to 10% are employed. Note that this range encompasses the one (7 to 9%) recently reported for the U.K., U.S. and Germany by O'Mahoney, 1999 who calculates constant rates for manufacturing by taking weighted averages of equipment & structures rates.

[16] To introduce a multiplicative stochastic shock (for instance, in the production function) in the theoretical model above is straightforward and does not change any of our results, if agents make their decisions after the current shock is realized (see e.g. Sargent [1987] and Stokey and Lucas [1989]). However, when the shock enters additively (for instance, when the budget constraint in (2) is subject to an additive stochastic shock) the results change because the model is not linear-quadratic and hence certainty equivalence does not hold. For a similar problem in a linear-quadratic setup, see Lockwood *et al*. [1996]). Nevertheless, we can show that, even when the shock enters additively, our main results do not change if we take an approximation around the deterministic version of the model. However, since this would unnecessarily complicate the theoretical model, we follow usual practice and introduce shocks in the econometric model (11a)-(11c) in an *ad hoc* fashion.

$$-b_{21}y_{1t} + b_{22}y_{2t} - b_{23}y_{3t} - g_{21}x_{1t} = u_{2t} \tag{11b}$$
$$-b_{31}y_{1t} - b_{32}y_{2t} + b_{33}y_{3t} - g_{31}x_{1t} = u_{3t} \tag{11c}$$

where $y_{1t}=c_t/y_t$, $y_{2t}=k_{t+1}/y_t$, $y_{3t}=g_t/y_t$, $x_{1t}=1$ and u_{it} for $i=1,2,3$ is the stochastic error term.

The normalization and exclusion restrictions implied by (7), (8) and (10) can be expressed, using (11a)-(11c) as, $b_{11} = b_{22} = b_{33} = 1$ & $b_{12} = b_{13} = b_{21} = b_{23} = b_{31} = b_{32} = 0$ respectively. Additionally, the single cross-equation restriction is $g_{11} = (1-ab) = (1-g_{21})$, where $g_{21} = ab$.

2.5 *Identification*

Since estimation and testing using an under-identified model is meaningless, we next need to establish how many restrictions implied by theory are required to identify the model.[17] Once we obtain these, we will proceed to examine the empirical validity of the model by testing whether the remaining, i.e. the overidentifying, restrictions are data consistent.

We now describe the procedure we employ to determine the number of overidentifying restrictions. First, to more succinctly set out the conditions required for identification in the presence of cross equation restrictions,[18] we re-express (11a)–(11c) in matrix terms:

$$\mathbf{B}y_t + \Gamma x_t = u_t, \quad t = 1,\cdots T, \tag{12}$$

where y_t is a $G \times 1$ vector of endogenous variables, x_t is a $K \times 1$ vector of predetermined variables, $\mathbf{B}$ is a $G \times G$ matrix of coefficients, Γ is a $G \times K$ matrix of coefficients and u_t is a $G \times 1$ vector of unobserved disturbances.

The reduced form of the structural system in (12) can be written as:

[17] Although the results of the rank test for the tax-smoothing model appear obvious for this model, the relevance of the ensuing discussion becomes more apparent when we test the more complicated models which follow (see Sections 3.3 and 4).

[18] This is based on the "theory of estimable functions" (see Richmond [1974] and Hsiao [1983]).

$y_t = \Pi x_t + n_t$,

(13)

where $\mathbf{B}\Pi + \Gamma = 0$ and $n_t = \mathbf{B}^{-1}u_t$. Equation (13) can be rewritten in stacked form as:

$\Theta \boldsymbol{d} = 0$,

(14)

where Θ is a $GK \times (G+K)G$ matrix defined as $\Theta = (I_G \otimes \Pi' : I_{GK})$; $\boldsymbol{d}$ is a $(G + K)G \times 1$ constant vector containing the endogenous and predetermined coefficients; I_G is a $G \times G$ identity matrix; $\otimes$ is the kronecker product; $\Pi = -\mathbf{B}^{-1}\,\Gamma$; ':' is the concatenation operator and I_{GK} is the $GK \times GK$ identity matrix.

The normalization, exclusion and cross-equation linear restrictions, denoted as *R*, on the elements of $\boldsymbol{d}$ implied by (7), (8) and (10) can be re-expressed in matrix form as:

$\Phi \boldsymbol{d} = d$

(15)

where Φ is an $R \times G(G+K)$ matrix and d is an $R \times 1$ vector whose elements are obtained via the restrictions.

Collecting (14) and (15) and defining $\boldsymbol{W} = \begin{bmatrix} \Theta \\ \Phi \end{bmatrix}$, (15) can be re-expressed as

$\boldsymbol{W}\delta = \begin{bmatrix} 0 \\ d \end{bmatrix} = \tilde{d}$.

(16)

Equation (16) represents a set of *(GK+R)* linear equations with *G(G+K)* unknowns, $\boldsymbol{d}$. Hsiao [1983, see *Theorem 3.4.1*] shows that the vector $\boldsymbol{d}$ can be uniquely identified if and only if *rank(**W**)=G(G+K).* Equivalently, if we form a matrix $\boldsymbol{M}(= I_G \otimes \mathbf{B} : I_G \otimes \Gamma)$ from the coefficients of the structural system in (12), Hsiao [*op cit*., see *Theorem 3.4.2*] shows that the vector $\boldsymbol{d}$ can be uniquely identified if and only if $rank(\boldsymbol{M\Phi'})=G^2$. Since we need to establish the minimum number of restrictions required for identification, it is convenient to work with the latter rank condition.[19] To determine whether the normalisation and exclusion restrictions in (11a)-(11c) are sufficient to identify the coefficients vector,[20] we calculate the rank of $\boldsymbol{M\Phi'}$.[21] Given that G^2=9, the normalization and exclusion restrictions are indeed sufficient.

2.6 Estimation and Testing

Imposing the restrictions required for identification, we now estimate (11a)-(11c) and check whether the overidentifying restrictions are data consistent. Estimation is carried out by using the Full Information Maximum Likelihood (FIML) estimator.[22] Relative to single equation estimators, the advantages of FIML in this context are that (i) it is generally more efficient; (ii) cross-equation restrictions can be implemented and tested; and (iii) it allows direct estimation of an auto-regressive process for the errors to remove the serial correlation inherent in annual macroeconomic time-series relationships.[23] Furthermore, in the estimations reported in Table 2 below, FIML has the advantage that the potential inconsistency of the parameter estimates due to simultaneous equation bias is circumvented. In contrast to GMM estimators which "instrument out" the regressors which are correlated with the error term, FIML maximises a likelihood function that involves a Jacobian term (see the discussion in Davidson and MacKinnon ([1993**,** pp. 637-643] for details).

The first column of Table 1 below provides information pertaining to both the value and significance of the restricted and estimated model parameters. The second column reports

---

[19] The advantage of dividing the restrictions in this context is that the overidentfying restrictions are econometrically testable.

[20] Note that neither the normalisation nor the exclusion restrictions separately, are sufficient to identify the model, e.g. the rank of $\mathbf{M\Phi'}$ is 4 and 7 in each case.

[21] These calculations are undertaken by using Maple V, Release 4.

[22] Here we use TSP, Version 4.4.

[23] Treating serial correlation, as a problem of specification, is not relevant here since our aim is to directly test the implications of the theory. Accordingly, instead of using *ARDL* specifications, we estimate an *AR(2)*

the Wald test of whether the single cross-equation restriction, implied by (7), (8) and (10), is valid. The results in Table 1 reveal that some implications of the theoretical model are supported by the data for all countries, e.g. both $\frac{d(1-ab)}{(1+d)} = g_{31}$ and $ab = 1 - g_{11}$ are less than unity.[24] However, the single cross-equation restriction imposed by the theory is uniformly rejected.[25]

Table 1: Parameter Estimates of (11a)-(11c) and Wald Test of the Overidentifying Restriction

FIML Estimates												Wald Tests	
U.S. (1962-95)												U.S.	
β_{11}	β_{12}	β_{13}	β_{21}	β_{22}	β_{23}	β_{31}	β_{32}	β_{33}	γ_{11}	γ_{21}	γ_{31}	Restriction	$\chi^2(1)$
1	0	0	0	1	0	0	0	1	0.78	2.23	0.15	(1) $\gamma_{11} = (1-\gamma_{21})$	29.45
n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	9.19	7.54	1.40		
West Germany (1962-1993)												West Germany	
β_{11}	β_{12}	β_{13}	β_{21}	β_{22}	β_{23}	β_{31}	β_{32}	β_{33}	γ_{11}	γ_{21}	γ_{31}	Restriction	$\chi^2(1)$
1	0	0	0	1	0	0	0	1	0.77	3.01	0.20	(1) $\gamma_{11} = (1-\gamma_{21})$	87.5
n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	4.52	18.8	49.0		
Japan (1962-1995)												Japan	
β_{11}	β_{12}	β_{13}	β_{21}	β_{22}	β_{23}	β_{31}	β_{32}	β_{33}	γ_{11}	γ_{21}	γ_{31}	Restriction	$\chi^2(1)$
1	0	0	0	1	0	0	0	1	0.66	3.93	0.09	(1) $\gamma_{11} = (1-\gamma_{21})$	48.4
n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	24.0	7.82	5.35		
U.K. (1962-95)												U.K.	
β_{11}	β_{12}	β_{13}	β_{21}	β_{22}	β_{23}	β_{31}	β_{32}	β_{33}	γ_{11}	γ_{21}	γ_{31}	Restriction	$\chi^2(1)$
1	0	0	0	1	0	0	0	1	0.80	2.63	0.21	(1) $\gamma_{11} = (1-\gamma_{21})$	38.9
n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	2.30	12.4	7.32		

Notes: (i) the first row of numbers in column 1 for each country gives the parameter estimate and the next row the asymptotic *t-ratio*; (ii) the symbol n/a indicates that the *t-ratio* is not applicable since the parameter value is imposed using the *a-priori* theoretical restrictions required for identification; (iii) the critical value of the Wald test (which is distributed χ^2) for one degree of freedom at the 5% level of significance is 3.84.

Therefore, although the result that it is optimal for policymakers to keep the tax rate constant over time has been one of the most popular models of endogenous fiscal policy, perhaps due to its clarity and algebraic convenience, its empirical relevance appears limited in a general equilibrium setting. Rejection of the overidentifying restriction is not surprising given the very restrictive set of assumptions required to obtain this general equilibrium model.

process in all equations for all countries to ensure that all serial correlation is eliminated. Further, note that the conclusions we draw throughout our empirical analysis are not altered if we employ an *AR(1)* specification.

[24] Note, that in the U.S. case, γ_{31} is not significantly different from zero.

For instance, this model assumes fully rational behaviour and full capital depreciation within a single time-period.

Our next natural step is to search for a more realistic model. Before we proceed, it is worth emphasising two lessons from the above analysis. First, the policy recipe to keep the tax rate constant over time, although popular to theorists, does not appear to be the one that policymakers have been following since the 1960s. This is consistent with the results of the RBC model of Chari *et al*. [1994] for the U.S.. Second, our findings do not necessarily suggest sub-optimality on the part of policymakers. This is because the tax-smoothing result relies on some rather unrealistic assumptions about the functioning of the underlying economy.

3. GROWTH AND FISCAL POLICY: A MORE GENERAL MODEL

Given the results in Section 2, we now generalize the above model first by relaxing the assumption of full capital depreciation. This is the most obvious assumption to relax since it is the one most obviously at odds with the data.

In this new environment, the budget constraint of the private agent (2) changes to:

$$k_{t+1} - (1-g)k_t + c_t = (1-q_t)Ak_t^a \tag{17}$$

where $0 \le g \le 1$ is the rate of capital depreciation.

Instead of (3), the private agent's dynamic programming problem is now:

$$U(k_t;q_t) = \max_{k_{t+1}}\left[\log\left[(1-q_t)Ak_t^a + (1-g)k_t - k_{t+1}\right] + d\log g_t + bU(k_{t+1};q_{t+1})\right]. \tag{17a}$$

As known, when $g < 1$, a closed-form analytical solution is not available. As we said in the Introduction, we will circumvent this technical difficulty by relinquishing the notion of full

[25] Note that application of recursive FIML estimation by using a variable start date with a fixed end date; a variable end date with a fixed start date; and a moving fixed window of 20 observations does not alter our main findings. To preserve space, these results are not presented here but can be made available on request.

rationality, and instead employ an exogenously specified, and simple, rule of thumb on the part of private agents.[26] In particular, we assume that the private agent simply keeps his total value of assets at the previous period level, i.e., $U_k(k;q_t)k_t = U_k(k_{t+1};q_{t+1})k_{t+1}$.[27] Then, as Appendix C shows, we get the following closed-form expressions for c_t and k_{t+1}:[28]

$$c_t = (1-b)(1-g)k_t + (1-ab)(1-q_t)Ak_t^a \tag{18}$$

$$k_{t+1} = b(1-g)k_t + ab(1-q_t)Ak_t^a .$$

(19)

3.1 Competitive Equilibrium (Given Economic Policy)

Working as above, we use (6) for q_t and substitute into (18) and (19). Thus,

$$c_t = (1-b)(1-g)k_t + (1-ab)(Ak_t^a - g_t) \tag{20}$$

$$k_{t+1} = b(1-g)k_t + ab(Ak_t^a - g_t) .$$

(21)

which are closed-form expressions for c_t and k_{t+1} for any feasible level of government expenditures, g_t.[29]

3.2 Endogenous Economic Policy and General Equilibrium

[26] Although here we use rule-of-thumb behavior for technical expedience, it is also widely recognized that fully rational behavior requires calculations on the part of private agents that are sometimes too complicated and too costly to be realistic (see Ellison and Fudenberg [1993] and Lettau and Uhlig [1999]). For instance, to support this argument, Lettau and Uhlig [1999] point out the difficulty of the dynamic programming solution of the intertemporal consumption-saving paradigm to explain the data (see e.g. the literature on the risk-premium puzzle). Accordingly, one way to proceed is to deviate from full rationality. In fact, there is empirical support for rule-of-thumb behavior within this paradigm (see the references cited in Lettau and Uhlig [1999]). This is exactly what we also find here (see our empirical results below). From a theory point of view, Ellison and Fudenberg [1993] and Lettau and Uhlig [1999] go further by providing a theory of learning about rules of thumb. In particular, Lettau and Uhlig [1999] obtain the fully rational solution (which exactly solves our dynamic programming problem (17a)) as a special case of behavior.

[27] $U_k(.;.)$ is the marginal valuation of capital k_t at time t, so that $U_k(.;.)k_t$ is the total value of capital.

[28] Obviously, (18) and (19) do not exactly solve the dynamic programming problem (17a).

[29] Equations (20) plus (21) give $c_t + g_t + [k_{t+1} - (1-g)k_t] = Ak_t^a = y_t$, i.e. the aggregate economy's resource constraint.

We again assume that the government is benevolent and acts as a Stackelberg leader *vis-à-vis* the private agents. Thus, the government chooses g_t to maximize (1) subject to (20) and (21). Then, the resulting strategy for g_t, in combination with (20) and (21), will give us a general equilibrium.

The dynamic programming problem for the government is now:

$$V(k_t) = \max_{g_t}\left[\log c_t + d \log g_t + bV(k_{t+1})\right]$$

(22)

where c_t and k_{t+1} follow (20) and (21) respectively.

For the same reasons as in the private agents' problem above, we cannot obtain a closed-form analytical solution to the problem in (22). However, we can study a special case for which we can get a closed-form solution, and which appears to be consistent not only with general public perception and the political science literature (see e.g. Laver and Hunt [1992]), but also with formal econometric testing (see e.g. Lockwood *et al*. [1996]).[30] In particular, we consider the special case in which policy-makers do not care about the future (see also Grossman and Helpman [1998]). This means $b = 0$ in the above dynamic programming problem, (22). In this case, as we show in detail in Appendix D, the optimal income tax rate q_t, and equivalently the government expenditures-to-output ratio $\frac{g_t}{y_t}$, is:

$$0 < q_t = \frac{g_t}{y_t} = \frac{d}{1+d} + \frac{d(1-g)(1-b)}{(1+d)(1-ab)}\frac{k_t}{y_t} < 1 \tag{23}$$

[30] Specifically, Lockwood *et al*. [1996] show that when there is a nonzero probability of losing the coming election, this reduces the "effective" discount rate of the political party in power (i.e. how much policymakers care about the future). Their model generates data consistent electoral cycles.

This is a state-contingent rule. In contrast with the tax-smoothing model where it was optimal to keep the tax rate constant over time, now the properties of the tax rate are the properties of the aggregate economy.[31]

3.3 The Econometric Model

As in Section 2, the general equilibrium given by equations (20), (21) and (23) can be expressed in stochastic form as:

$$b_{11}y_{1t} - b_{12}y_{2t} - b_{13}y_{3t} - g_{11}x_{1t} - g_{12}x_{2t} = u_{1t} \qquad (24a)$$
$$-b_{21}y_{1t} + b_{22}y_{2t} - b_{23}y_{3t} - g_{21}x_{1t} - g_{22}x_{2t} = u_{2t} \qquad (24b)$$
$$-b_{31}y_{1t} - b_{32}y_{2t} + b_{33}y_{3t} - g_{31}x_{1t} - g_{32}x_{2t} = u_{3t} \qquad (24c)$$

where $y_{1t}=c_t/y_t$, $y_{2t}=k_{t+1}/y_t$, $y_{3t}=g_t/y_t$, $x_{1t}=1$, $x_{2t}=k_t/y_t$ and u_{it} for $i=1,2,3$ is the stochastic error term.

The normalization and exclusion restrictions implied by the theory can be expressed, using (24a)-(24c), as $b_{11} = b_{22} = b_{33} = 1$ and $b_{12} = b_{21} = b_{31} = b_{32} = 0$ respectively. Additionally the within- and cross-equation restrictions can be written as follows: $g_{11} = (1 - ab) = -b_{13} = 1 - g_{21}$, $g_{21} = ab = -b_{23}$, $g_{12} = (1-b)(1-g) = (1-g-g_{22})$, where g is the constant rate of capital depreciation, $g_{22} = b(1-g)$ and $g_{32} = \dfrac{d(1-b)(1-g)}{(1+d)(1-ab)} = \dfrac{g_{31}g_{12}}{g_{11}}$, where $g_{31} = \dfrac{d}{(1+d)}$. Unlike the tax-smoothing model, this model is still underidentified in the presence of the normalisation and exclusion restrictions.[32] Using the method of the previous section, we can show that imposing any one of the remaining two linear restrictions is sufficient to identify the model. Accordingly, the

---

[31] In particular, (23) implies that g_t should react positively to the beginning-of-period capital stock, k_t. This is typical in neoclassical growth models of this type where government consumption behaves similarly to private consumption (compare (23) and (20) above), see also Ambler and Paquet [1996].
[32] In this case, the rank of **M** is 7 which is less than G^2 (=9) required for exact identification.

remaining or overidentifying restrictions in this case include one linear and one nonlinear restriction, e.g. $g_{12} = (1-g - g_{22})$ and $g_{32} = \frac{g_{31}g_{12}}{g_{11}}$.[33]

The results of Table 2 reveal that the two restrictions implied by (24a)-(24c) cannot be rejected for the West Germany and Japan, while they are rejected for the U.K and the U.S..[34] Therefore, (20), (21) and (23) appear to constitute a useful general equilibrium model to explain the interaction between private agents and fiscal authorities in the former two countries (see below in the next section for an interpretation of these results).

Table 2: Parameter Estimates of (24a)-(24c) & Wald Tests of Overidentifying Restrictions

FIML Estimates														
U.S. (1962-95)														
β_{11}	β_{12}	β_{13}	β_{21}	β_{22}	β_{23}	β_{31}	β_{32}	β_{33}	γ_{11}	γ_{21}	γ_{31}	γ_{12}	γ_{22}	γ_{32}
1	0	-0.71	0	1	-0.29	0	0	1	0.71	0.29	0.27	0.04	0.90	-0.06
n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	9.69	8.42	2.55	61.6	-3.0
West Germany (1962-1993)														
β_{11}	β_{12}	β_{13}	β_{21}	β_{22}	β_{23}	β_{31}	β_{32}	β_{33}	γ_{11}	γ_{21}	γ_{31}	γ_{12}	γ_{22}	γ_{32}
1	0	-0.46	0	1	-0.54	0	0	1	0.46	0.54	-0.02	0.09	0.99	0.09
n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	6.37	-0.24	3.11	2.61	3.10
Japan (1962-1995)														
β_{11}	β_{12}	β_{13}	β_{21}	β_{22}	β_{23}	β_{31}	β_{32}	β_{33}	γ_{11}	γ_{21}	γ_{31}	γ_{12}	γ_{22}	γ_{32}
1	0	-0.66	0	1	-0.34	0	0	1	0.66	0.34	-0.02	0.11	0.93	0.01
n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	4.15	-0.19	2.08	30.6	0.72
U.K. (1962-95)														
β_{11}	β_{12}	β_{13}	β_{21}	β_{22}	β_{23}	β_{31}	β_{32}	β_{33}	γ_{11}	γ_{21}	γ_{31}	γ_{12}	γ_{22}	γ_{32}
1	0	-0.69	0	1	-0.31	0	0	1	0.69	0.31	-0.01	0.03	0.90	0.10
n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	5.15	-0.06	1.30	31.8	2.99

Wald Tests			
Restrictions	(1) $\gamma_{12} = (1-\gamma) - \gamma_{22}$	(2) $\gamma_{32} = \frac{\gamma_{31}\gamma_{12}}{\gamma_{11}}$	(1) and (2) Jointly
U.S.	0.18	9.15	9.37
West Germany	0.16	4.63	5.29
Japan	3.22	0.50	3.61
U.K.	0.02	7.85	7.86

Note: The test statistics for restrictions (1) and (2) separately and for (1) and (2) jointly are distributed $\chi^2(1)$ and $\chi^2(2)$ respectively. The critical values of the Wald test for one and two degrees of freedom at the 5% level of significance are 3.84 and 5.99 respectively.

[33] Note that imposing different combinations of linear or non-linear restrictions to identify the model does not alter the conclusions drawn from the Wald tests reported in Table 2. Further, note that application of recursive FIML estimation, using a variable start date with a fixed end date; a variable end date with a fixed start date; and a moving fixed window of 20 observations also does not alter our conclusions. To preserve space they are not presented here but can be made available on request.

[34] Our results for the U.S. are consistent with those of Ambler and Paquet [1996] who obtain mixed results when they attempt to calibrate an RBC model with optimal fiscal policy on U.S. data.

Note that one of the main assumptions of the model is that policymakers are optimizing agents who set fiscal policy by acting as benevolent Stackelberg leaders *vis-à-vis* the private agents. Although we managed to test formally only the special case in which policymakers are myopic, we cannot exclude the possibility that the general case is also not rejected by the data. In other words, it is theoretically possible that, in the West Germany and Japan, policymakers are benevolent Stackelberg leaders who are also far-sighted. However, we have experimented with several alternative models, and in particular with more complicated structures that are closer to the general, far-sighted case. For instance, we considered a different special case in which policymakers are far-sighted, but government expenditures provide very little direct utility (namely, d is close to zero). This special case is the model of e.g. Lucas and Stokey [1983]. We again managed to obtain a closed-form analytical solution and hence are able to formally test the model. This general equilibrium model is easily rejected[35]. Hence, we can claim that the data do not appear to support the general case. However, since we cannot formally test the general case directly, we do not wish to speculate further against, or for, it. At present, we are content with the reasonable result that in West Germany and Japan, policymakers appear to be optimizing agents who consider the welfare of private agents.

4. GROWTH AND AN ARBITRARY MODEL OF FISCAL POLICY

Since the U.S. and the U.K. data have rejected the model in Section 3, we now search for a different general equilibrium model that may be consistent with policymaking experience in these two countries. To this end, we opt for a simpler model of policymaking than the one presented in Section 3. Specifically, instead of searching for alternative models of endogenous fiscal policy, we abandon the assumption that policymakers are optimizing agents. We assume

[35] All results will be made available upon request.

instead that policymakers follow an *ad hoc* feedback rule (or a rule-of-thumb)[36] according to which the current government expenditures-to-output ratio (i.e. the policy instrument) reacts to a menu of predetermined variables. For instance, we assume that fiscal policy reacts to the aggregate state of the economy (i.e. the beginning of period capital-to-output ratio) and the previous period choice variables (i.e. the lagged-once values of the government expenditures-to-output ratio and the private consumption-to-output ratio).[37] Note that these three variables represent the most parsimonious set of contemporaneous and lagged predetermined regressors that are model consistent.[38] Of course, this is only one of many possible specifications for such an *ad hoc* policy rule. A comprehensive investigation of such fiscal policy rules would be interesting but it is not our aim here. Instead, we want to test whether a reasonable simple *ad hoc* policy rule, as part of a general equilibrium model, can explain the U.S. and the U.K. data.

Augmenting (20) and (21) with the above policy rule, leads to the following system of stochastic equations:

$$b_{11}y_{1t} - b_{12}y_{2t} - b_{13}y_{3t} - g_{11}x_{1t} - g_{12}x_{2t} - g_{13}x_{3t} = u_{1t} \tag{25a}$$

$$-b_{21}y_{1t} + b_{22}y_{2t} - b_{23}y_{3t} - g_{21}x_{1t} - g_{22}x_{2t} - g_{23}x_{3t} = u_{2t} \tag{25b}$$

$$-b_{31}y_{1t} - b_{32}y_{2t} + b_{33}y_{3t} - g_{31}x_{1t} - g_{32}x_{2t} - g_{33}x_{3t} = u_{3t} \tag{25c}$$

where all variables are defined as in (24a)-(24c) and $x_{3t}=c_{t-1}/y_{t-1}$.

The normalisation and exclusion restrictions implied by this model are $b_{11} = b_{22} = b_{33} = 1$ and $b_{12} = b_{21} = b_{31} = b_{32} = g_{13} = g_{23} = 0$ respectively. Additionally, the within- and cross-equation restrictions can be expressed as follows:

---

[36] We choose an *ad hoc* policy rule because adding more structure in the form of a sophisticated optimal policy would imply more restrictions. Given the previous U.S. and U.K results, more restrictions would only serve to make the new general equilibrium model far easier to reject.

[37] These lagged-once values of choice variables can capture e.g. habit persistence, slow adjustment or lags in policy implementation.

[38] Note that the lagged influence of government expenditure is captured via the *AR(2)* error process.

$g_{11} = (1-ab) = -b_{13} = 1-g_{21}$, $g_{21} = ab = -b_{23}$, $g_{12} = (1-b)(1-g) = (1-g-g_{22})$, where $g_{22} = b(1-g)$. Similar to the tax-smoothing model, the rank test reveals that this model can be identified by using only the normalisation and exclusion restrictions. Hence, we can test the validity of the four remaining overidentifying restrictions. The results in Table 3 (below) indicate each of the overidentifying restrictions individually are not rejected by the data and the joint set are also not rejected for each country.

Therefore, there is *prima fascia* evidence that the U.S. and U.K. fiscal authorities do not conduct policy by acting as optimising agents. Their behaviour appears instead to be consistent with the use of *ad hoc* feedback policy rules according to which fiscal policy instruments react to the recent state of the economy. Concerning the U.S. case, one possible explanation for this rule-of-thumb behavior is that acting optimally requires too much coordination. In the U.S., the government is essentially divided, in the sense that one political party can be in control of the presidency and the other party in control of the congress. This means that decision-making and implementation do not happen automatically. Concerning the U.K. case, we believe that a model with benevolent optimizing fiscal authorities cannot explain the UK data, mainly because the post-war period has been characterized by discontinuities in policymaking and sharp regime changes (see e.g. Begg [1987]). In particular, fiscal policy was used for demand management in the two decades before 1970. During the 1970s, the emphasis changed and fiscal policy was generally sound and broadly neutral. The Thatcher regime after 1979 made fiscal policy tighter so as to make it consistent with tight monetary policy in the fight against inflation. Furthermore, unlike its post-Keynesian predecessors, the Thatcher governments believed that output was close to its natural level and so emphasized the importance of supply side policies.

Table 3: Estimates of Individual Overidentifying Restrictions from
(25a)-(25c) & Wald Test of all Restrictions Jointly

FIML Estimates and Wald Test – U.S. (1962-95)		
Restrictions:	Estimate	Wald Test of (1)-(4) Jointly
(1) $\gamma_{11} + \beta_{13}$	0.35 (0.73)	2.59
(2) $\gamma_{11} - 1 + \gamma_{21}$	-0.38 (-1.0)	
(3) $\gamma_{21} + \beta_{23}$	0.59 (0.67)	
(4) $\gamma_{12} + \gamma_{22} - (1-\gamma)$	0.09 (1.27)	
FIML Estimates and Wald Test – U.K. (1962-95)		
Restrictions:	Estimate	Wald Test of (1)-(4) Jointly
(1) $\gamma_{11} + \beta_{13}$	-0.10 (-0.24)	0.35
(2) $\gamma_{11} - 1 + \gamma_{21}$	-0.06 (-0.05)	
(3) $\gamma_{21} + \beta_{23}$	-6.71 (-0.15)	
(4) $\gamma_{12} + \gamma_{22} - (1-\gamma)$	0.70 (0.17)	

Note: The numbers in parentheses are the asymptotic *t-ratios*. The joint restriction is distributed $\chi^2(4)$. The critical value of the Wald test for four degrees of freedom at the 5% level of significance is 9.49.

In contrast, in Japan there is much more centralization and unitary decision-making, while in Germany, although there is fragmentation and coalition formation, such coalitions (once formed) are rather stable and (more importantly) ideologically homogenous. All this seems to suggest that in Japan and Germany, it is easier for policymakers to behave as optimizing agents. However, possible effects of the politico-economic system on the conduct of fiscal policy is something that remains to be formally tested in richer models.

5. CONCLUSIONS AND EXTENSIONS

In this paper we have searched for a general equilibrium model of optimal growth and endogenous fiscal policy to investigate the interaction between private agents and fiscal authorities in the U.S., West Germany, Japan and the U.K. during 1960-1996. We have found evidence that the West German and Japanese fiscal authorities behave as optimizing agents, while the U.S. and U.K. fiscal authorities follow rule-of-thumb feedback policy rules. Our search was conducted in the context of simple general equilibrium models. This enabled us to obtain closed-form analytical solutions and so directly test the models' theoretical restrictions by using formal econometric techniques.

We close with a possible extension. As we argued above, it is interesting to introduce politico-economic considerations and *formally* investigate whether policymakers across countries and over time have their own political agendas, which might be systematically different from those of benevolent optimal policymakers (for a survey see e.g. Persson and Tabellini [1999]). We leave this for future work.

6. APPENDICES

Appendix A: Proof of equations (4)-(5)

A conjecture for the value function is $U(k_t;q_t) = u_0 + u_1 \log k_t + u_2 q_t + u_3 \log q_t$, where u_0, u_1, u_2, u_3 are undetermined coefficients. Then, (3) becomes:

$$u_0 + u_1 \log k_t + u_2 q_t + u_3 \log q_t = \max_{k_{t+1}} \left[\log c_t + d \log g_t + b\left[u_0 + u_1 \log k_{t+1} + u_2 q_{t+1} + u_3 \log q_{t+1}\right]\right]$$

where $c_t = (1-q_t) A k_t^a - k_{t+1}$.

The first-order condition for k_{t+1} is $\frac{1}{c_t} = \frac{b u_1}{k_{t+1}}$, and the envelope condition for k_t is $\frac{1}{c_t} = \frac{u_1}{a(1-q_t)Ak_t^a}$. These two optimality conditions combined give (5) in the text, and in turn (4) follows from (2) and (5). To verify that our conjecture is correct, we use (4) and (5) back into the Bellman equation above. Then, by equating coefficients, we get $u_1 = \frac{a}{1-ab} > 0$. The values of u_2 and u_3 depend on the value of the next period tax rate, q_{t+1}.[39] In a general equilibrium setup where policy is endogenously chosen (like here), the values of u_2 and u_3 are determined from the Markov properties of the tax strategy. See Appendix B below.

Appendix B: Proof of equation (10)

It is convenient to express the problem in terms of the tax rate, q_t. Thus, the government chooses q_t to solve (9) subject to (4), (5) and (6). The Bellman equation is:

$$V(k_t) = \max_{q_t}\left[\log c_t + d \log g_t + bV(k_{t+1})\right]$$

where $c_t = (1-ab)(1-q_t)Ak_t^a$, $g_t = q_t A k_t^a$ and $k_{t+1} = ab(1-q_t)Ak_t^a$. We conjecture a value function of the form $V(k_t) = e_0 + e_1 \log k_t$, where e_0, e_1 are undetermined coefficients. By using this conjecture into the Bellman equation, the first-order condition for k_{t+1} and the envelope condition for k_t give (10) in the text. It is easy to verify that the conjecture for the value function is correct. In doing so, we get values for e_0 and e_1. Finally, the resulting

Markov strategy for the tax rate also completes the solution for u_2 and u_3 in Appendix A above.

Appendix C: Derivation of equations (18) and (19)

It is not possible to find a conjecture for the value function which can solve the problem in (17a). We therefore arbitrarily set $U_k(k;q_t)k_t = U_k(k_{t+1};q_{t+1})k_{t+1}$. Using this rule into the first-order condition for k_{t+1} and the envelope condition for k_t, we get (19) in the text. In turn, (18) follows from (19) and (17).

Appendix D: When Policymakers are short-sighted

In the general case, the government's dynamic programming problem is:

$$V(k_t) = \max_{g_t}\left[\log\left[(1-g)(1-b)k_t + (1-ab)\left(1-\frac{g_t}{Ak_t^a}\right)Ak_t^a\right] + d\log g_t + b^g V\left[b(1-g)k_t + ab\left(1-\frac{g_t}{Ak_t^a}\right)Ak_t^a\right]\right]$$

Consider the special case in which the government does not care about the future, i.e. $b^g = 0$. This is a static problem. The first-order condition for g_t is simply $\frac{1}{c_t} = \frac{d}{(1-ab)g_t}$. Using (18) for c_t, we get $\frac{g_t}{Ak_t^a} = \frac{d}{1+d} + \frac{d(1-g)(1-b)}{(1+d)(1-ab)}\frac{k_t}{Ak_t^a}$, which is (23) in the text.

---

[39] If policy were exogenous, u_2 and u_3 would depend on the properties of the process for the tax rate (see e.g. Sargent [1987, chapter 1]).

7. REFERENCES

Ambler S. and A. Paquet [1996]: "Fiscal spending shocks, endogenous government spending and real business cycles", *Journal of Economic Dynamics and Control*, 20, 237-256.

Barro R. [1979]: "On the determination of public debt", *Journal of Political Economy,* 87, 940-971.

Barro R. [1990]: "Government spending in a simple model of economic growth", *Journal of Political Economy,* 89, S103-S125.

Barro R. and X. Sala-i-Martin [1992]: "Public finance in models of economic growth", *Review of Economic Studies*, 99, 645-661.

Baxter M. and R. King [1993]: "Fiscal policy in general equilibrium", *American Economic Review,* 83, 315-334.

Begg D. [1987]: "Fiscal policy", in "*The Performance of the British Economy"*, edited by R. Dornbusch and R. Layard. Oxford University Press, Oxford.

Benhabib J. and A. Velasco [1996]: "On the optimal and best sustainable taxes in an open economy", *European Economic Review*, 40, 135-154.

Campbell J. [1994]: "Inspecting the mechanism: An analytical approach to the stochastic growth model", *Journal of Monetary Economics*, 33, 463-506.

Chamley C. [1986]: "Optimal taxation of capital income in general equilibrium with infinite lives", *Econometrica*, 54, 607-622.

Chari V.V., P. Kehoe and E. Prescott [1989]: "Time consistency and policy", in "*Modern Business Cycle Theory*", edited by R. Barro. Blackwell, Oxford.

Chari V.V., L. Christiano and P. Kehoe [1994] "Optimal fiscal policy in a business cycle model", *Journal of Political Economy*, 102, 617-652.

Christiano L. and M. Eichenbaum [1992]: "Current real-business-cycle theories and aggregate labor-market fluctuations", *American Economic Review*, 82, 430-450.

Davidson R. and J. MacKinnon [1993]: "*Estimation and Inference in Econometrics"*. Oxford University Press, Oxford.

Devarajan S., V. Swaroop and H. Zou [1996]: "The composition of public expenditure and economic growth", *Journal of Monetary Economics*, 37, 313-344.

Devereux M. and J-F. Wen [1998]: "Political instability, capital taxation and growth", *European Economic Review*, 42, 1635-1651.

Dunne J. , P. Pashardes and R. Smith [1984]: "Needs, costs and bureaucracy: the allocation of public consumption in the UK", *Economic Journal,* 94, 1-15.

Ellison G. and D. Fudenberg [1993]: "Rules of thumb and social learning", *Journal of Political Economy*, 101, 612-643.

Grossman G. and E. Helpman [1998]: "Intergenerational redistribution and short-lived governments", *Economic Journal*, 108, 1299-1329.

Hsiao C. [1983]: “Identification”, in “*Handbook of Econometrics”*, Vol. 1, edited by Z. Griliches and M. Intriligator. North Holland, Amsterdam.

Kollintzas T., A. Philippopoulos and V. Vassilatos [1999]: Is tax policy coordination necessary? *Center for Economic Policy Research, Discussion Paper*, London.

Laver M. and B. Hunt [1992]: “*Policy and Party Competition*”. Routledge, London.

Lettau M. and H. Uhlig [1999]: “Rules of thumb versus dynamic programming”, *American Economic Review*, 89, 148- 174.

Lockwood B., A. Philippopoulos and A. Snell [1996]: “Fiscal policy, public debt stabilization and politics: theory and UK evidence”, *Economic Journal*, 106, 894-911.

Lucas R. and N. Stokey [1983]: “Optimal fiscal and monetary policy in an economy without capital”, *Journal of Monetary Economics*, 12, 55-93.

Malley J., A. Philippopoulos and G. Economides [1999]: “Testing for tax-smoothing in a general equilibrium model of growth”, *University of Glasgow, Discussion Paper*, no. 9918.

McGrattan E. [1994]: “The macroeconomic effects of distortionary taxation”, *Journal of Monetary Economics*, 33, 573-601.

O’Mahoney M. [1999]: “Measures of fixed capital stocks in the post-war period: a five country study”, in “*Quantitative Aspects of Post-war European Growth*”, edited by N.F.R. Crafts and B. van Ark. Cambridge University Press, Cambridge.

Persson T. and G. Tabellini [1994], editors: “*Monetary and Fiscal Policy*”, volumes 1 and 2, MIT Press, Cambridge, Mass.

Persson T. and G. Tabellini [1999]: “The size and scope of government: Comparative politics with rational politicians”, *European Economic Review*, 43, 699-735.

Richmond J. [1974]: “Identifiability in linear models”, *Econometrica*, 42, 731-736.

Sargent T. [1987]: “*Dynamic Macroeconomic Theory”*. Harvard University Press. Cambridge, Mass.

Serletis A. and R. Schorn [1999]: “International evidence on the tax-and revenue-smoothing hypotheses”, *Oxford Economic Papers*, 51, 387-396.

Stokey N. and R. Lucas R.[1989]: “*Recursive Methods in Economic Dynamics”*. Harvard University Press. Cambridge, Mass.

Stokey N. [1991]: “Credible public policy”, *Journal of Economic Dynamics and Control*, 15, 627-656.