Endogenous Liquidity Providers and Exchange Rate Dynamics (Supplementary Material)

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Abstract

The high variance of exchange rates can be partially explained by the fact that traders with transitory demands can have temporary effects on the market rates. This paper explores theoretically the effect on market prices of these non-informational traders when the number of market makers providing liquidity to the traders is endogenous. A primary empirical implication of the model is that the expected reversion towards fundamentals will be proportionally greater when the deviation from fundamentals is large. This implication is then tested and verified using exchange rate data from the G7 countries. The results cast doubts on the random walk hypothesis of exchange rates.

JEL Classifications: F31, G12

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	Japan		Germany		France	
	OLS	Robust	OLS	Robust	OLS	Robust
α	-0.1455*	-0.1455*	-0.2263**	-0.2263**	-0.1753*	-0.1753**
	(0.0721)	(0.0643)	(0.0851)	(0.0617)	(0.0816)	(0.0532)
β	-11.3793	-11.3793*	-9.9227	-9.9227	-16.3929	-16.3929**
	(7.4228)	(5.6846)	(12.0286)	(7.2785)	(11.9684)	(5.8497)
p_α	0.095	0.061	0.649	0.531	0.261	0.085
p_β	0.323	0.197	0.614	0.405	0.292	0.031
D-W	1.61	1.61	1.76	1.76	1.77	1.77
$\bar{R}^2$	0.07	0.07	0.11	0.11	0.10	0.10
Q	58.06*	58.06*	35.22	35.22	39.59	39.59

	Britain		Canada		Italy	
	OLS	Robust	OLS	Robust	OLS	Robust
α	-0.1437*	-0.1437*	-0.3223**	-0.3223**	-0.1133	-0.1133
	(0.0721)	(0.0656)	(0.0981)	(0.0812)	(0.0667)	(0.0660)
β	-12.3951	-12.3951**	-131.6469	-131.6469	-14.1793*	-14.1793**
	(7.1277)	(4.2701)	(121.2570)	(121.2570)	(6.1293)	(3.2301)
p_α	0.090	0.062	(0.573)	(0.496)	0.022	0.021
p_β	0.216	0.039	0.361	0.102	0.099	0.022
D-W	1.64	1.64	1.74	1.74	1.62	1.62
$\bar{R}^2$	0.10	0.10	0.14	0.14	0.08	0.08
Q	36.05	36.05	50.58	50.58	41.23	41.23

Notes: Standard errors in parentheses. For each country, "OLS" denotes ordinary least squares results, and "Robust" has standard errors corrected for autocorrelations up to 12 lags. Q is the Box-Ljung Q-statistic against serial correlation up to 36 lags. A single asterisk for a coefficient estimate or the Q-statistic denotes significance at the 5% level, a double asterisk denotes significance at the 1% level. p_α and p_β are p-values from the hypothesis that the coefficients equal their Monte Carlo values.

Table 11: Monthly Data without Lagged Dependent Variables, High Pass Filter

4.3 Results with Alternative Filters

The above results have documented the result predicted by the model. The rate of return to fundamentals (expressed as a fraction of deviation from fundamentals) is positively correlated with the deviation from fundamentals. In order to show that this is a property of the underlying data and not a spurious result of the specific filter applied to the data, the above analysis can be repeated using alternative filters.

The filters chosen were the high-pass and the band-pass filters discussed in Baxter and King [3]. A high-pass filter separates out high-frequency elements of the data from more permanent

	Japan		Germany		France	
	OLS	Robust	OLS	Robust	OLS	Robust
α	-0.3534**	-0.3534**	-0.4680**	-0.4680**	-0.4369**	-0.4369**
	(0.0735)	(0.0731)	(0.0896)	(0.0490)	(0.0868)	(0.0410)
β	-17.5940*	-17.5940**	-8.7528	-8.7528	-11.3967	-11.3967*
	(6.8362)	(5.3111)	(11.2986)	(5.3392)	(11.2808)	(5.4392)
p_α	0.009	0.009	0.384	0.111	0.208	0.008
p_β	0.046	0.010	0.658	0.349	0.504	0.166
D-W	1.96	1.96	2.01	2.01	2.00	2.00
$\bar{R}^2$	0.26	0.26	0.23	0.23	0.23	0.23
Q	24.91	24.91	20.06	20.06	23.55	23.55

	Britain		Canada		Italy	
	OLS	Robust	OLS	Robust	OLS	Robust
α	-0.4441**	-0.4441**	-0.5960**	-0.5960**	-0.3942**	-0.3942**
	(0.0835)	(0.0496)	(0.1043)	(0.0902)	(0.0738)	(0.0586)
β	-9.9460	-9.9460**	-42.6438	-42.6438	-10.7061	-10.7061**
	(6.6229)	(3.0362)	(114.3254)	(62.2798)	(5.6808)	(2.8432)
p_α	0.222	0.040	(0.625)	(0.572)	0.041	0.010
p_β	0.337	0.036	0.850	0.728	0.240	0.019
D-W	2.03	2.03	2.03	2.03	2.02	2.02
$\bar{R}^2$	0.23	0.23	0.29	0.29	0.26	0.26
Q	23.97	23.97	29.80	29.80	21.32	21.32

Notes: Twelve lagged values of the dependent variable included but not reported. Also see the notes for table 11

Table 12: Monthly Data with Lagged Dependent Variables, High Pass Filter

	Japan		Germany		France	
	OLS	Robust	OLS	Robust	OLS	Robust
α	-0.3636*	-0.3636*	-0.5658**	-0.5658**	-0.4383*	-0.4383**
	(0.1803)	(0.1606)	(0.2128)	(0.1542)	(0.2041)	(0.1331)
β	-177.80	-177.80*	-155.04	-155.04	-256.14	-256.14**
	(155.98)	(88.82)	(187.95)	(113.73)	(187.01)	(91.40)
p_α	0.092	0.059	0.638	0.516	0.269	0.090
p_β	0.449	0.183	0.606	0.394	0.301	0.034
D-W	1.61	1.61	1.76	1.76	1.77	1.77
$\bar{R}^2$	0.07	0.07	0.11	0.11	0.10	0.10
Q	58.06*	58.06*	35.22	35.22	39.59	39.59

	Britain		Canada		Italy	
	OLS	Robust	OLS	Robust	OLS	Robust
α	-0.3594*	-0.3594*	-0.8057**	-0.8057**	-0.2833	-0.2833
	(0.1802)	(0.1641)	(0.2542)	(0.2030)	(0.1667)	(0.1650)
β	-193.67	-193.67**	-2056.98	-2056.98	-221.55*	-221.55**
	(111.37)	(66.72)	(1894.64)	(1058.90)	(95.77)	(50.47)
p_α	0.091	0.063	(0.579)	(0.488)	0.021	0.020
p_β	0.228	0.044	0.362	0.103	0.093	0.001
D-W	1.64	1.64	1.74	1.74	1.62	1.62
$\bar{R}^2$	0.10	0.10	0.14	0.14	0.08	0.08
Q	36.05	36.05	50.58	50.58	41.23	41.23

Notes: See the notes for table 11

Table 13: Monthly Data without Lagged Dependent Variables, Band Pass Filter

	Japan		Germany		France	
	OLS	Robust	OLS	Robust	OLS	Robust
α	-0.8834**	-0.8834**	-1.1700**	-1.1700**	-1.0923**	-1.0923**
	(0.1837)	(0.1827)	(0.2239)	(0.1224)	(0.2169)	(0.1026)
β	-274.91*	-274.91**	-136.76	-136.76	-178.07	-178.07*
	(106.81)	(82.99)	(176.54)	(83.42)	(176.26)	(84.99)
p_α	0.009	0.009	0.389	0.115	0.214	0.009
p_β	0.046	0.010	0.657	0.347	0.513	0.174
D-W	1.96	1.96	2.01	2.01	2.00	2.00
$\bar{R}^2$	0.26	0.26	0.23	0.23	0.23	0.23
Q	24.91	24.91	20.06	20.06	23.55	23.55

	Britain		Canada		Italy	
	OLS	Robust	OLS	Robust	OLS	Robust
α	-1.1102**	-1.1102**	-1.4900**	-1.4900**	-0.9856**	-0.9856**
	(0.2087)	(0.1241)	(0.2608)	(0.2254)	(0.1845)	(0.1465)
β	-115.41	-115.41*	-666.31	-666.31	-167.28	-167.28**
	(103.48)	(47.44)	(1786.33)	(973.12)	(88.76)	(44.43)
p_α	0.226	0.042	(0.629)	(0.576)	0.041	0.010
p_β	0.571	0.216	0.851	0.731	0.237	0.018
D-W	2.03	2.03	2.03	2.03	2.02	2.02
$\bar{R}^2$	0.23	0.23	0.29	0.29	0.26	0.26
Q	23.97	23.97	29.80	29.80	21.32	21.32

Notes: Twelve lagged values of the dependent variable included but not reported. Also see the notes for table 11

Table 14: Monthly Data with Lagged Dependent Variables, Band Pass Filter

elements of the series. A band-pass filter is designed to separate out middle-frequency elements of the data, with the residual consisting of both short-term dynamics and the long-term dynamics.

Band-pass filters are often used in macroeconomic business-cycle applications, where the "business cycle" is taken to only include fluctuations with frequencies within the band. In the current theoretical application of endogenous arrival of liquidity providers, a high-pass filter seems more appropriate, and these results are presented first. However, use of band-pass filters can enable closer examination of the time frame of the endogeneity of the number of market makers. If the theoretical model is correct, bands outside this time frame will have little explanatory power.

The results of the high pass filter run on monthly data are shown in Tables 11 and 12. The limit of the high pass has been set to 12 months. The results clearly support the results for the Hodrick-Prescott filter. Much like the results with the Hodrick-Prescott filter, the results are of mixed significance for individual currencies. However, taken as a whole, they provide strong support for the results of the paper originally found with the HP filter.

The band-pass results where the limits of the band have been set at 4 and 12 months are presented in Tables 13 and 14. Although the difference in the magnitude of the filtered series results in coefficients of a different scale, the statistical significance of the coefficients is largely the same. Results not presented for bands of longer periods lose their predictive power (as measured by the adjusted R^2), with the signs of the coefficients of interest becoming less reliable. This is suggestive of liquidity provision in these markets being flexible at time frames of a few months rather than more than a year.

Tables 15 and 16 show the effects of running a high pass filter on weekly data, with a threshhold of 26 weeks. The results are once again strong support for the idea of non-linear mean reversion, with all coefficients having not only the expected sign, but also the expected relationship to their Monte Carlo values. Consistent with the Hodrick-Prescott evidence, the

	Japan		Germany		France	
	OLS	Robust	OLS	Robust	OLS	Robust
α	-0.1905**	-0.1905**	-0.2423**	-0.2423**	-0.2279**	-0.2279**
	(0.0379)	(0.0282)	(0.0381)	(0.0270)	(0.0403)	(0.0337)
β	-76.28**	-76.28**	-80.35**	-80.35**	-109.82**	-109.82**
	(25.78)	(22.70)	(25.30)	(22.69)	(29.32)	(29.85)
p_α	0.000	0.000	0.004	0.000	0.002	0.000
p_β	0.006	0.002	0.003	0.001	0.000	0.000
D-W	1.55	1.55	1.60	1.60	1.62	1.62
$\bar{R}^2$	0.12	0.12	0.16	0.16	0.16	0.16
Q	147.82**	147.82**	92.72**	92.72**	98.51**	98.51**

	Britain		Canada		Italy	
	OLS	Robust	OLS	Robust	OLS	Robust
α	-0.2948**	-0.2948**	-0.3265**	-0.3265**	-0.2569**	-0.2569**
	(0.0359)	(0.0396)	(0.0359)	(0.0432)	(0.0341)	(0.0404)
β	-34.14*	-34.14**	-241.85*	-241.85	-61.73**	-61.73**
	(16.02)	(11.18)	(97.57)	(259.06)	(15.34)	(7.76)
p_α	0.111	0.149	0.460	0.540	0.005	0.019
p_β	0.061	0.007	0.025	0.398	0.000	0.000
D-W	1.71	1.71	1.74	1.74	1.70	1.70
$\bar{R}^2$	0.16	0.16	0.18	0.18	0.16	0.16
Q	82.31**	82.31**	78.29**	78.29**	101.31**	101.31**

Notes: Standard errors in parentheses. For each country, “OLS” denotes ordinary least squares results, and “Robust” has standard errors corrected for autocorrelations up to 18 lags. Q is the Box-Ljung Q-statistic against serial correlation up to 36 lags. A single asterisk for a coefficient estimate or the Q-statistic denotes significance at the 5% level, a double asterisk denotes significance at the 1% level. p_α and p_β are p-values from the hypothesis that the coefficients equal their Monte Carlo values.

Table 15: Weekly Data without Lagged Dependent Variables, High Pass Filter

	Japan		Germany		France	
	OLS	Robust	OLS	Robust	OLS	Robust
α	-0.4949**	-0.4949**	-0.5625**	-0.5625**	-0.5595**	-0.5595**
	(0.0379)	(0.0277)	(0.0398)	(0.0282)	(0.0425)	(0.0351)
β	-82.76**	-82.76**	-86.24**	-86.24**	-95.39**	-95.39**
	(22.73)	(27.05)	(22.61)	(22.93)	(26.49)	(30.16)
p_α	0.000	0.000	0.000	0.000	0.000	0.000
p_β	0.001	0.004	0.000	0.000	0.001	0.003
D-W	1.99	1.99	1.98	1.98	1.97	1.97
$\bar{R}^2$	0.35	0.35	0.34	0.34	0.35	0.35
Q	25.22	25.22	22.59	22.59	23.75	23.75

	Britain		Canada		Italy	
	OLS	Robust	OLS	Robust	OLS	Robust
α	-0.6540**	-0.6540**	-0.6782**	-0.6782**	-0.5974**	-0.5974**
	(0.0407)	(0.0333)	(0.0414)	(0.0438)	(0.0387)	(0.0360)
β	-9.30	-9.30	-312.20**	-312.20	-32.37*	-32.37**
	(14.60)	(12.51)	(88.99)	(220.79)	(13.95)	(5.46)
p_α	0.177	0.099	0.442	0.468	0.004	0.002
p_β	0.699	0.652	0.001	0.187	0.037	0.000
D-W	1.98	1.98	1.96	1.96	1.98	1.98
$\bar{R}^2$	0.33	0.33	0.33	0.33	0.34	0.34
Q	16.74	16.74	25.37	25.37	29.73	29.73

Notes: Eighteen lagged values of the dependent variable included but not reported. Also see the notes for table 15. Due to round-off problems with RATS, the lire has been computed for robusterrors to 16 lags.

Table 16: Weekly Data with Lagged Dependent Variables, High Pass Filter

	Japan		Germany		France	
	OLS	Robust	OLS	Robust	OLS	Robust
α	0.0568	0.0568	0.0380	0.0380	0.0095	0.0095
	(0.0713)	(0.0444)	(0.0829)	(0.0656)	(0.0880)	(0.0856)
β	-292.43*	-292.43**	-462.89*	-462.89**	-494.57	-494.57
	(144.42)	(85.01)	(213.79)	(167.16)	(263.64)	(273.07)
p_α	0.008	0.000	0.043	0.010	0.110	0.101
p_β	0.091	0.004	0.049	0.012	0.087	0.098
D-W	1.78	1.78	1.85	1.85	1.89	1.89
$\bar{R}^2$	0.00	0.00	0.01	0.01	0.01	0.01
Q	69.28**	69.28**	45.56**	45.56**	53.15*	53.15*

	Britain		Canada		Italy	
	OLS	Robust	OLS	Robust	OLS	Robust
α	-0.0972	-0.0972*	-0.1995**	-0.1995*	-0.1041	-0.1041*
	(0.0656)	(0.0452)	(0.0769)	(0.0786)	(0.0620)	(0.0420)
β	-64.05	-64.05	416.25	416.25	-51.24	-51.24
	(90.01)	(45.52)	(1014.41)	(919.59)	(82.29)	(36.53)
p_α	0.617	0.468	(0.366)	(0.377)	0.676	0.537
p_β	0.758	0.543	(0.538)	(0.437)	0.799	0.567
D-W	1.97	1.97	2.04	2.04	1.96	1.96
$\bar{R}^2$	0.01	0.01	0.01	0.01	0.01	0.01
Q	34.60	34.60	37.28	37.28	54.65*	54.65*

Notes: See the notes to table 15.

Table 17: Weekly Data without Lagged Dependent Variables, Band Pass Filter

	Japan		Germany		France	
	OLS	Robust	OLS	Robust	OLS	Robust
α	0.0899	0.0899	-0.0093	-0.0093	-0.0356	-0.0356
	(0.0807)	(0.0589)	(0.0938)	(0.0731)	(0.0984)	(0.0993)
β	-279.33	-279.33**	-454.00*	-454.00*	-438.68	-438.68
	(143.58)	(81.52)	(213.54)	(185.55)	(264.19)	(278.06)
p_α	0.000	0.000	0.004	0.000	0.015	0.016
p_β	0.110	0.005	0.055	0.028	0.136	0.157
D-W	1.99	1.99	1.94	1.94	1.94	1.94
$\bar{R}^2$	0.03	0.03	0.02	0.02	0.03	0.03
Q	18.29	18.29	26.68	26.68	26.30	26.30

	Britain		Canada		Italy	
	OLS	Robust	OLS	Robust	OLS	Robust
α	-0.0930	-0.0930	-0.4017**	-0.4017**	-0.0802	-0.0802
	(0.0841)	(0.0831)	(0.0915)	(0.1266)	(0.0781)	(0.0785)
β	-75.31	-75.31	467.28	467.28	-44.96	-44.96
	(91.96)	(45.82)	(1008.58)	(743.70)	(83.26)	(41.90)
p_α	0.030	0.029	(0.176)	(0.324)	0.013	0.014
p_β	0.684	0.413	(0.512)	(0.374)	0.875	0.754
D-W	1.95	1.95	1.87	1.87	1.96	1.96
$\bar{R}^2$	0.01	0.01	0.03	0.03	0.02	0.02
Q	16.35	16.35	65.50**	65.50**	25.49	25.49

Notes: Eighteen lagged values of the dependent variable included but not reported. Also see the notes for table 15.

Table 18: Weekly Data with Lagged Dependent Variables, Band Pass Filter

major currencies (Japan and Germany) show a tendency to have strong results at the weekly frequency, while the other currencies (Britain, Canada, and Italy) generally showed stronger results at the monthly frequency.

Tables 17 and 18 show results using a band pass filter on weekly data with the band set at 13 to 26 weeks. The results for the major currencies remain largely intact. In fact, the point estimates of α often tchange sign (although insignificantly so) suggesting that the non-linear term is the only mean reversion term having an effect. Britain, Canada, and Italy, however, are no longer statistically distinguishable from the Monte Carlo values of a random walk. This, together with the low values of R^2 throughout the weekly band pass results, suggests that the endogeneity of the number of liquidity providers is occuring at a higher frequency than this band allows.

Appendix A Univariate ECM Representations

A.1 The Case of Moving-Average Filters

This section shows that the univariate error-correction model can provide a valid alternative data representation to the standard ARIMA time-series model. Throughout the discussion, the underlying data generating process (DGP) is assumed to be a random walk, which can be expressed as an ARIMA(1,1,0) process: $\Delta y = \epsilon$, where ϵ is a normally-distributed, random error term. To motivate the univariate error-correction specification, it is useful to consider the simple case of the following moving-average filter for the level of the series:

$$\bar{y}_t = \gamma_o y_{t-1} \gamma_1 y_t + \gamma_2 y_{t+1} \tag{24}$$

where the (positive) weights sum to one. Applying this simple centered moving-average filter[7] to the data, one could estimate the following error-correction model:

$$\Delta y_t = \alpha z_t + u_t \tag{25}$$

where $z_t \equiv (y - \bar{y}_{t-1})$. In this formulation, the regressor z represents an error-correction mechanism (ECM) relating the change in the series y as a function of the last period's deviation from the "equilibrium" level. In view of the underlying DGP, one might expect that the estimate of α should equal zero (with $u = \epsilon$). But this will not be the case.

Intuitively, the coefficient on the ECM term z will be statistically significant since the "equilibrium" level here derives from a measure of central tendency. Hence, by construction the series will fluctuate about this level and thus exhibit error-correcting behavior when away from this benchmark (i.e., after a shock). In econometric terms, the ECM term will be correlated by construction with the residuals. Consequently, the OLS estimate of α will be inconsistent and its asymptotic bias can be computed as follows:

$$\begin{aligned}
\hat{\alpha} &= \left(\sum_{i=1}^{T} z_i^2\right)^{-1} \left(\sum_{i=1}^{T} z_i \Delta y_i\right) && (26)\\
&= \left(\frac{1}{T}\sum_{i=2}^{T} \gamma_o^2 \epsilon_{i-1}^2 - 2\gamma_0\gamma_1\epsilon_{i-1}\epsilon_i + \gamma_1^2\epsilon_i^2\right)^{-1} \left(\frac{1}{T}\sum_{i=2}^{T} \gamma_0\epsilon_{i-1}\epsilon_i - \gamma_2\epsilon_i^2\right) && (27)\\
&\to \frac{-\gamma_2}{\gamma_0^2 + \gamma_1^2} < 0 && (28)
\end{aligned}$$

Another way to view this result is to note that the OLS estimator can be written as:

$$\hat{\alpha} = \alpha + \left(\frac{1}{T}\sum_{i=1}^{T} z_i^2\right)^{-1} \left(\frac{1}{T}\sum_{i=1}^{T} z_i \epsilon_i\right) \tag{29}$$

where the true value for α is zero (if u equals ϵ), but where the second (bias) term does not

[7] The case of a 3-period, centered moving average filter is considered for mathematical convenience. The arguments presented herein can be straightforwardly generalized to the broad case of linear moving-average filters.

vanish in the limit as $T \to \infty$ due to the correlation between z and ϵ. Note that the exogeneity of the regressor is violated provided we have a *two-sided* moving average filter which includes a lead term (γ_2 is non-zero). Since the first lag of the ECM term is used, the lead term in the filter assures contemporaneous correlation between the regressor and the error term ($E[z_t\epsilon_t] \neq 0$).

However, the notion of exogeneity, which is violated in this example, is meaningful only for a given parameterization. In other words, whether exogeneity fails or not depends explicitly upon the form of the equation or representation being analyzed. The source of this ambiguity stems from the fact that the definition of exogeneity relies upon the correlation of an observed variable with an unobservable error term – which itself depends upon the exact parameterization (see Hendry [11] for a discussion).

An alternative representation of the data in our example would be to treat the true value of α as non-zero (equal to the *plim* for its OLS estimator shown above). In that case, the "true" error term u in the ECM model is given by:

$$u_t = (1 + \alpha\gamma_2)\epsilon_t - \alpha\gamma_0\epsilon_{t-1} \tag{30}$$

Under this definition for the stochastic errors, the regressor is (weakly) exogenous – $E[z_t u_t] = 0$ – and the OLS estimator is (by definition) consistent, although not efficient due to serially correlated errors.

Note that the explanatory power (R^2) of the ECM equation would depend on the choice of γ's in this example, which determines how much of the variation in Δy is explained in terms of the regressor z and how much is left over in the residual. The important point to observe here is that the data representation can be transformed from one in which the dependent variable is explained only in terms of an unobserved noise term to one in which some if not most of the variation is defined in terms of an observable variable(s). Conversely, a random walk specification with normal errors can be obtained from reparameterizing a time series exhibiting error-correction behavior consistent with particular univariate ECM representation. There are a multiplicity of such representations (see Hendry [11]).

For example, another representation which avoids the serial correlation problem in the preceding example includes the lagged innovation Δy_{t-1} as an additional regressor. This variable under the DGP is the optimal instrument for ϵ_{t-1}, which was subsumed in the previous [MA(1)] error process. Under this alternative specification, we have the following reparameterization:

$$\Delta y_t = -\frac{1}{\gamma_2} z_t + \frac{\gamma_0}{\gamma_2} \Delta y_{t-1} + u_t \tag{31}$$

where $u = 0$ and is thus free from serial correlation and clearly uncorrelated with z and Δy_{t-1}. Estimating this OLS regression would yield the "true" coefficients above and $R^2 = 1$ (perfect fit). In other words, with this representation we can explain *all* of the variation in the dependent variable Δy as a function of observable variables $(z, \Delta y_{t-1})$ using a univariate ECM specification.

A.2 Other Filters and Non-Linear ECMs

We can extend the discussion of univariate ECM representations to consider a broader class of filters beyond simple moving-average filters. One such filter, often used to examine underlying trends in output and other series, is the Hodrick-Prescott (HP) filter. The HP filter decomposes a discrete time series $\{y_t\}$ into a permanent and cyclical component:

$$y_t = y_t^p + y_t^c \tag{32}$$

by minimizing a loss function which seeks to produce a smooth growth rate for the fundamental component:

$$\min_{y^p} \left[\sum_t (y_t - y_t^p)^2 + \lambda \sum_t (\Delta y_t^p - \Delta y_{t-1}^p)^2 \right] \tag{33}$$

	$\lambda = 400$	$\lambda = 1600$	$\lambda = 7200$	$\lambda = 14400$
α	-0.392**	-0.284**	-0.197**	-0.164**
(OLS)	(0.081)	(0.071)	(0.061)	(0.057)
(MCSD)	(0.075)	(0.067)	(0.058)	(0.054)
β	-2.684	-2.093	-1.840	-1.700
(OLS)	(15.036)	(9.460)	(5.827)	(4.661)
(MCSD)	(14.049)	(8.855)	(5.616)	(4.492)
R^2	0.18	0.14	0.10	0.09
Q	53.70	47.99	43.62	42.17

Notes: OLS standard errors and sample standard errors reported in parentheses. Asterisk denotes 5% significance and ** denotes 1% significance.

Table 19: Monthly Monte Carlo Simulations Without Lagged Dependent Variables

	$\lambda = 400$	$\lambda = 1600$	$\lambda = 7200$	$\lambda = 14400$
α	-0.780**	-0.562**	-0.382**	-0.314**
(OLS)	(0.087)	(0.079)	(0.070)	(0.065)
(MCSD)	(0.080)	(0.072)	(0.067)	(0.065)
β	-2.563	-2.365	-2.028	-1.850
(OLS)	(13.880)	(9.086)	(5.732)	(4.679)
(MCSD)	(13.538)	(8.906)	(5.712)	(4.724)
R^2	0.39	0.30	0.22	0.19
Q	28.50	26.61	26.79	27.21

Notes: Twelve lags of the dependent variable included but not reported. OLS standard errors and sample standard errors reported in parentheses. Asterisk denotes 5% significance and ** denotes 1% significance.

Table 20: Monthly Monte Carlo Simulations With Lagged Dependent Variables

The parameter λ specifies the smoothness of the filter: a low value of λ will create a filtered series y^p that closely matches the initial series, while a high value of λ will create a smooth filtered series.[8] At the extreme, as $\lambda \to \infty$ the filtered series converges on a linear trend.

In effect, the HP filter acts as a linear filter which smooths the series in first-differences using a two-sided moving average filter, from which the level of the smoothed series is obtained. Cogley and Nason [5] show this to be the effect of the filter in the case of an integrated series. As a result, they find that applying the HP filter to difference-stationary series induces spurious cycles.[9] In the case of a trend stationary series, Cogley and Nason [5] show that the HP filter effectively acts to linearly detrend the data and smooth the resulting deviations from trend. Consequently, applying the HP filter to trend stationary series works like a high pass filter – smoothing fluctuations of a certain frequency – without creating spurious time-series properties.

The basic lesson here is that some care must be taken when drawing economic inferences based on filtered data. In particular, we must distinguish between time-series characteristics of the empirical data attributable to underlying economic phenomena and those that are simple artifacts of the filtering method used. Thus, we proceed by conducting Monte Carlo experiments in the case of a pure random walk to determine the relevant benchmark parameters with the HP filter, as well as experiments with the the high-pass and band-pass filters described in Baxter and King [3].

Monte Carlo simulations to calculate the asymptotic parameter values one would obtain from estimating the univariate ECM model on a random walk passed through an HP filter are shown in Tables 19 through 22. The parameter values reported in the tables are based on OLS regressions of the ECM model (using 300 or 960 observations and 10,000 replications) where the underlying data were generated from a random walk process with normal errors. Note

[8] The variability of the underlying trend compared to the actual series clearly involves some judgement regarding the source and nature of shocks. Hodrick and Prescott [12] suggest using $\lambda = 1600$ in the case of quarterly GDP data. Applications of the filter to annual data generally fix λ at 100 or 400, and a conventional rule of thumb involves setting λ equal to 100 times the square of the number of observations per year.

[9] See also Harvey and Jaeger [10]

	$\lambda = 14400$	$\lambda = 28800$	$\lambda = 57600$	$\lambda = 230400$
α	-0.166**	-0.139**	-0.117**	-0.083**
(OLS)	(0.030)	(0.028)	(0.026)	(0.022)
(MCSD)	(0.029)	(0.027)	(0.025)	(0.022)
β	-2.446	-2.240	-2.021	-1.539
(OLS)	(9.516)	(7.483)	(5.939)	(3.735)
(MCSD)	(9.088)	(7.211)	(5.787)	(3.742)
R^2	0.08	0.07	0.06	0.04
Q	57.33	53.40	49.68	43.83

Notes: OLS standard errors and sample standard errors reported in parentheses. Asterisk denotes 5% significance and ** denotes 1% significance.

Table 21: Weekly Monte Carlo Simulations Without Lagged Dependent Variables

	$\lambda = 14400$	$\lambda = 28800$	$\lambda = 57600$	$\lambda = 230400$
α	-0.327**	-0.272**	-0.224**	-0.146**
(OLS)	(0.034)	(0.032)	(0.030)	(0.025)
(MCSD)	(0.031)	(0.030)	(0.029)	(0.026)
β	-2.910	-2.422	-2.291	-1.729
(OLS)	(9.282)	(7.359)	(5.893)	(3.737)
(MCSD)	(9.071)	(7.284)	(6.032)	(3.896)
R^2	0.18	0.15	0.13	0.09
Q	24.88	25.19	24.48	21.23

Notes: Eighteen lags of the dependent variable included but not reported. OLS standard errors and sample standard errors reported in parentheses. Asterisk denotes 5% significance and ** denotes 1% significance.

Table 22: Weekly Monte Carlo Simulations With Lagged Dependent Variables

that a non-linear (cubic) error correction term is also included in the regressions, and the basic equation estimated in the simulations – not including possible lagged values of Δy – is of the form

$$\Delta y_t = \alpha z_t + \beta z_t^3 + u_t \tag{34}$$

Including the cubic ECM term z^3 is intended to capture any potential non-linearities in the rate of mean reversion depending on the size of the deviation from central tendency or equilibrium. In our earlier example of the moving average filter, the OLS estimate of the coefficient β on the cubic ECM term can be shown to be non-zero in the case of normally-distributed errors, and in general depends importantly on the *fourth* moment of the distribution of error process.[10]

Since it is behavior away from fundamentals that endogenous liquidity seeks to explain, the variances used in the Monte Carlo simulations were based on the estimated sample fourth moments of the exchange rate data, which were shown to exhibit excess kurtosis relative to normality (see Table 1). Tables 19 through 22 show the results across different choices of the smoothing parameter λ, and Tables 3, 6, 8, and 10 show the results across different choices of the variance σ^2, calibrated according to the sample moment of each exchange rate series.

From the results of the Monte Carlo simulations, we see that the coefficient α on the linear error correction term is always significant. Again, this finding obtains by construction as the HP filter tracks the random walk series as a dynamic measure of central tendency. Correspondingly, the error correction coefficient, reflecting mean reversion, should be significant. Changes to variance σ^2 have little impact on the point estimate of α. Also, the estimated speed of ad-

[10]In the case of our moving average filter, for an OLS regression on z^3 alone the asymptotic coefficient on the cubic ECM term is given by:

$$plim\hat{\beta} = \frac{-\gamma_2 \left[3\gamma_0^2 E[\epsilon_i^2]^2 + \gamma_2^2 E[\epsilon_i^4]\right]}{[(\gamma_0^6 + \gamma_2^6)E[\epsilon_i^6] + 15\gamma_0^2\gamma_2^2(\gamma_0^2 + \gamma_2^2)E[\epsilon_i^4]E[\epsilon_i^2]]} \tag{35}$$

and hence would depend on the second, fourth, and sixth moments of the distribution. With Gaussian (normal) errors, the fourth and sixth moments are direct functions of the variance (proportional to σ^4 and σ^6, respectively), implying that the absolute value of $\hat{\beta}$ is *inversely* related to variance σ^2 and thus kurtosis $3\sigma^4$.

justment (point estimate on α) increases monotonically with a reduction in λ. This is because a smaller value of λ implies an underlying filtered or fundamental value which tracks the actual series more closely (higher R^2), and thus leads to smaller and shorter-lived deviations. Overall, including lag terms in the ECM model also improves the empirical fit and lowers the amount of serial correlation in the error terms (as indicated by the Q-statistic).[11]

In the case of normally-distributed errors, the OLS estimator of the coefficient β on the cubic error-correction term is *not* significant under the various specifications. Although the point estimates are non-zero, the asymptotic standard errors (based on the OLS estimator standard errors or the Monte Carlo sample variance of $\hat{\beta}$) are much larger. Intuitively, since the non-linearities captured by cubic ECM terms reflect disproportionately large adjustments, the action in the series necessary to pin down β involve observations in the tails of the distribution. However, in the case of normality of the errors, thickness of the tails can only be achieved by raising variance, which by itself acts to reduce β (e.g. see Table 6), and does not improve the signal-to-noise ratio. Consequently, an assumption of normality and a random walk fail to generate significant non-linear dynamics.

The robustness of the results to alternative filtering strategies is examined using the high pass and band pass filtering algorithms of Baxter and King [3]. A high pass filter separates out the cyclic components of the data series having high frequencies, leaving the lower frequency data as the permanent component. A band pass filter defines the cyclic component as consisting of only a certain "band" of frequencies. Frequencies greater and lesser than the band then accrue to the "permanent" component.

Either type of filter requires an infinite moving average to calculate precisely, so we use the approximation given by Baxter and King [3], which uses a two-sided symmetric moving average filter. Clearly, a greater number of elements in the moving average will result in a

[11] Monte Carlo experiments (not shown) including lead as well as lag terms would further improve the fit (R^2) by using more information subsumed in the filter and the residuals, but induces a greater amount of serial correlation.

closer approximation. However, in addition to increasing computational costs, it results in the loss of additional data. A filter consisting of $2K + 1$ elements will necessarily result in the loss of the first K and last K elements from the filtered series. Using U.S. macroeconomic data, Baxter and King find quite robustly that increases in K above 12 do not result in a material improvement in the filter. Accordingly, the present paper sets K at 12.

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