A Nonlinear Panel Unit Root Test under Cross Section Dependence *

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May 5, 2011

Abstract

We propose a nonlinear heterogeneous panel unit root test for testing the null hypothesis of unit-roots processes against the alternative that allows a proportion of units to be generated by globally stationary ESTAR processes and a remaining non-zero proportion to be generated by unit root processes. The proposed test is simple to implement and accommodates cross sectional dependence. We show that the distribution of the test statistic is free of nuisance parameters as $(N, T) \longrightarrow \infty$. Monte Carlo simulation shows that our test holds correct size and under the hypothesis that data are generated by globally stationary ESTAR processes has a better power than the recent test proposed in Pesaran [2007]. Various applications are provided.

J.E.L. Classification: C12, C15, C22, C23; F31.

Keywords: Nonlinear panel unit root tests, cross sectional dependence.

*We wish to acknowledge Karim Abadir and Ron Smith for their useful comments and Alexander Kadow for brilliant research assistance on the final version. We also wish to express our gratitude for comments received from the conference participants of 2008 ESEM, 27th–31st August 2008, Milan, Italy; "Inference and Tests in Econometrics: A Tribute to Russel Davidson", GREQAM, Marseilles, France, 25–26 April 2008; 2008 Scottish Economic Society Annual Conference, Perth, 21st-23rd April 2008; and 2008 Studies in Nonlinear Dynamics and Econometrics (SNDE2008) conference, Federal Reserve Bank, San Francisco, 3rd–4th April 2008.

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1 Introduction

There is now a large literature on testing for the presence of unit roots in economic and financial variables employing a variety of time series and panel tests [1]. The growth in that area is mainly due to empirical applications on, for example, *Purchasing Power Parity* (PPP) and Growth (see Cerrato and Sarantis [2007a], Emerson and Kao [2006] amongst others).

A weakness of the existing univariate and panel unit root tests is that they are based on the assumption that the underlying variable follows a linear process. However economic theory suggests that generally financial economic variables exhibit nonlinear behaviour. For example, a number of theoretical models in international macroeconomics formalise the notion of nonlinear exchange rate behaviour due to transaction costs (*e.g.* Dumas [1992], Sercu and Uppal [1995], O'Connell [1998], Goswami et al. [2002]) [2], while others describe currency and financial crises as nonlinear processes (*e.g.* Jeanne and Masson [2000], Chang and Velasco [2001]). In growth economics, a number of theoretical models suggest that economic growth is a nonlinear process with the economy bouncing back and forth between different regimes (*e.g.* Zilibotti [1995], Peretto [1999], Matsuyama [1999], Galor and Weil [2000]) [3]. Theoretical models in finance highlight heterogeneous expectations (*e.g.* Brock and Hommes [1998], De Grauwe and Grimaldi [2005]), heterogeneity in investors' objectives (*e.g.* Peters [1994]), and herd behaviour (*e.g.* Lux [1995]) as some of the sources of nonlinearity in asset prices.

If economic and financial variables exhibit nonlinear behaviour, the standard unit root tests that are based on a linear AR process will have low power. Two recent papers, Sollis et al. [2002] and Kapetanios et al. [2003], address this issue by developing formal unit root tests against the alternative of nonlinear mean reversion. Both papers examine the unit root hypothesis against the nonlinear STAR (Smooth Transition AutoRegressive) alternative and show that, under the null hypothesis, the distribution of the respective tests is not normal. As a result the two papers employ Monte Carlo simulations to obtain critical values. The main difference between the two tests is that Sollis et al. [2002] use a logistic transition function (LSTAR) while Kapetanios et al. [2003] use an exponential transition function (ESTAR).

However both these nonlinear unit root tests are univariate and, consequently, will still suffer from low power in the case of small samples. In this paper we extend the Kapetanios et al. [2003] nonlinear unit root test to a panel context in order to address the low power problem of univariate tests. Since heterogeneous cross section dependence tends to be important in most empirical applications, we employ the Pesaran [2007] panel unit root framework that enables us to account for heterogeneous cross section dependence in a novel way. Pesaran [2007] shows that the individual CADF (Cross Augmented Dickey Fuller) and the panel statistic (CIPS) have non-normal distributions, so their critical values (for different N and T) are obtained by Monte Carlo simulations. The panel unit root test proposed by Pesaran [2007] differs from other tests such as Choi [2001] and Hadri [2000] in that the latter all assume that individual time series are independent, and thus, cross section dependence is not considered. Pesaran [2007] shows that cross sectional

[1] For a review of the various unit root tests see, for example, Breitung and Pesaran [2007] and Cerrato and Sarantis [2007b].

[2] For empirical studies on nonlinear exchange rate models, see Michael et al. [1997], Sarantis [1999], Taylor et al. [2001], Rapach and Wohar [2003] among others.

[3] A number of authors have also undertaken empirical investigations of nonlinear growth models; see, for example, Fiaschi and Lavezzi [2007], Liu and Stengos [1999], Durlauf and Johnson [1995].

dependence can be accounted for by augmenting the standard DF regression with the cross section averages of lagged levels and first differences of the individual series. [4]

In this paper we propose a novel nonlinear panel unit root test that extends both the univariate nonlinear tests and the linear panel unit root tests, thus filling an important gap in the existing literature. Our test also allows for cross section dependence, and can be computed using basic OLS linear regression, and thus does not require any programming. [5] Since the panel nonlinear statistic has a non-normal distribution, we use Monte Carlo simulations to analyse the size and power of the test under different scenarios, and we calculate critical values which can be used in future applications of the test. Theorems 1 and 2 show that the distribution of the proposed test statistics is free of nuisance parameters. We finally illustrate the applicability of our test.

The rest of the paper is organised as follows. Section 2 specifies the nonlinear dynamic panel model with cross section dependence. Section 3 derives the individual and panel nonlinear unit root tests, and then uses stochastic simulations to obtain the distributions of these statistics and critical values. Section 4 analyses the size and power of the panel nonlinear unit root test under alternative scenarios and compares the results to the performance of the linear Pesaran [2007] test. Section 5 reports the results from an application to real exchange rates, while section 6 concludes.

2 A Nonlinear Dynamic Panel with Cross Section Dependence

Suppose the observation y_{it} on the i^{th} cross section unit at time t is generated according to the dynamic nonlinear heterogeneous panel ESTAR model below:

$$y_{it} = \beta_i y_{i,t-1} + \nu_i y_{i,t-1} Z(\theta_i; y_{i,t-d}) + u_{it}, \qquad t = 1, \ldots, T, \qquad i = 1, \ldots, N, \tag{1a}$$

where the initial value, y_{i0}, is given, and the error term, u_{it}, has the one-factor structure:

$$\begin{aligned} u_{it} &= \gamma_i f_t + \varepsilon_{it}, && \text{(1b)} \\ (\varepsilon_{it})_t &\sim i.i.d.(0, \sigma_i^2), && \text{(1c)} \end{aligned}$$

in which f_t is the unobserved common effect, $(\gamma_i)_i$ are i.i.d. random variables and ε_{it} is the individual-specific (idiosyncratic) error. Following the literature on STAR models, the transition function adopted here is of the exponential form, *i.e.*,

$$Z\left(\theta_i; y_{i,t-d}\right) = 1 - \exp\left(-\theta_i y_{i,t-d}^2\right), \tag{1d}$$

where we assume that $\theta_i \geq 0$, and $d \geq 1$ is the delay parameter. To begin with we assume that y_{it} is a mean zero stochastic process. We discuss processes with non-zero mean later. To simplify the model and following the existing literature, the delay parameter d is set to be equal to one and Equation 1a–Equation 1d are rewritten in first difference form as:

$$\Delta y_{it} = \phi_i y_{i,t-1} + \nu_i y_{i,t-1}\left[1 - \exp\left(-\theta_i y_{i,t-1}^2\right)\right] + \gamma_i f_t + \varepsilon_{it}, \tag{2}$$

[4] Another way to account for cross section dependence can be found in Ng [2008]. However, Ng [2008] simply estimates the proportion of the panel that has a unit root and does not provide any information about the whole panel, conversely to the other tests. In addition, it is not a nonlinear test.

[5] Chiang et al. [2007] also proposes a nonlinear panel unit root test, but this test is based on an extension of the IPS test (Im et al. [2003]) and, therefore, does not account for cross section dependence which is so crucial in empirical applications.

where $\phi_i = -(1-\beta_i)$. Assuming $\phi_i = 0$ [6], Equation 2 can be rewritten as:

$$\Delta y_{it} = \nu_i y_{i,t-1}\left[1-\exp\left(-\theta_i y_{i,t-1}^2\right)\right] + \gamma_i f_t + \varepsilon_{it}. \tag{3}$$

Using Equation 3, we are interested in testing the hypothesis:

$$H_0 : \theta_i = 0 \forall i, \tag{4}$$

against the possibly heterogeneous alternatives:

$$H_1 : \begin{cases} \theta_i > 0 & \text{for } i = 1, \dots, N_1, \\ \theta_i = 0 & \text{for } i = N_1+1, \dots, N. \end{cases} \tag{5}$$

Remark 1: The alternative hypothesis above implies that some units are generated by a stationary ESTAR model but it also allows a proportion of units being a unit root process.

The following assumptions are introduced:

Assumption 1: $\frac{N_1}{N} \longrightarrow q$ as $N \longrightarrow \infty$, with $0 < q \leq 1$ under the alternative hypothesis 5 [7].

Assumption 2: (ε_{it}) are independently distributed for all $i = 1, \dots, N$ and $t = 1, \dots, T$, with zero mean, constant variance σ_i^2, and finite moments at least up to order 8.

Assumption 3: f_t is serially uncorrelated with zero mean, constant variance σ_f^2, and finite fourth moment. (Without loss of generality σ_f^2 will be set equal to unity.)

Assumption 4: ε_{it}, f_t, and γ_i are independently distributed for all i.

Assumption 5: Following Pesaran [2006], we define the weights (ω_i) having the following properties: $\omega_N = O\left(N^{-1}\right)$;

$$\sum_{i=1}^{N} \omega_i = 1; \quad \sum_{i=1}^{N} |\omega_i| < K < \infty; \quad \sum_{i=1}^{N} \omega_i^2 = O\left(N^{-1}\right), \quad \sum_{i=1}^{N} \omega_i^4 = O\left(N^{-3}\right).$$

Assumption 6: The distribution of γ_i has nonzero expectation and finite moments at least up to order 8.

Assumption 7: For all i, $-2 \leq \nu_i \leq 0$.

Assumption 8: Under the alternative hypothesis, as $N \rightarrow \infty$,

$$\overline{\nu\theta} \equiv N^{-1} \sum_{i=1}^{N} \nu_i \theta_i \rightarrow c_2 \neq 0.$$

Assumption 9: As $N \rightarrow \infty$,$N^{-1}\left|\sum_{i=1}^{N}\left(\nu_i\theta_i - \overline{\nu\theta}\right)^4\right| \rightarrow c_3 \neq 0$.

Assumption 10: As $N \rightarrow \infty$, γ_i converges in distribution to zero.

Assumption 11: As $N, T \rightarrow \infty$, $T/N \rightarrow 0$.

[6] It follows the practice in the literature (*e.g.* Balke and Fomby [1997], in the context of TAR models and Michael et al. [1997] in the context of ESTAR models). See also Donauer et al. [2010].

[7] As noted in Im et al. [2003], this condition is necessary for the consistency of the panel unit root tests.

3 Nonlinear Unit Root Tests with Serially Uncorrelated Errors

Assumptions 1 and 2 together imply that the composite error, u_{it}, is serially uncorrelated. This restriction will be relaxed in subsection 3.3. Assumption 7–11 are technical assumptions.

3.1 Individual NCADF Test

Testing the null hypothesis 4 directly is not feasible, since ν_i is not identified under the null. [8] To overcome this problem, we follow Luukkonen et al. [1988], and derive a t type test statistic. Using Taylor expansion on Equation 3, under the null hypothesis, the following auxiliary regression is obtained:

$$\Delta y_{it} = b_i y_{i,t-1}^3 + \gamma_i f_t + e_{it}. \tag{6}$$

However, at first we need the following lemma:

Lemma 1: Under assumptions 2–9 and 11, then as $N, T \to \infty$,

$$f_t = \frac{1}{\bar{\gamma}_\omega} \Delta \bar{y}_{\omega,t} - \frac{1}{\bar{\gamma}_\omega} \overline{(\nu\theta)(y^3)}_{\omega,t-1} + o_p(1) \tag{7}$$

where $\Delta \bar{y}_{\omega,t} = \sum_{i=1}^N \omega_i \Delta y_{i,t}$, $\overline{y_{\omega,t-1}^3} = \sum_{i=1}^N \omega_i y_{i,t-1}^3$, and $\bar{\gamma}_\omega = \sum_{i=1}^N \omega_i \gamma_i$. The $o_p(1)$ term tends to zero as $N, T \to \infty$.

Proof: see Appendix A.1.

Following Lemma 1, it follows that Equation 6 can be approximated by the following nonlinear cross sectionally augmented DF (NCADF) regression: [9]

$$\Delta y_{it} = \alpha_i + b_i y_{i,t-1}^3 + c_i \Delta \bar{y}_t + d_i \overline{y_{t-1}^3} + e_{it}, \tag{8}$$

where $\bar{y}_t = \sum_{i=1}^N y_{i,t}$ and $\overline{y_{t-1}^3} = \sum_{i=1}^N y_{i,t-1}^3$. The idea is, given the framework above, to develop a unit root test in the heterogeneous panel model based on Equation 8. Extending the idea in Kapetanios et al. [2003], we suggest using Equation 8 and the t-statistic on b_i, that is denoted by:

$$t_i(N,T) = \frac{\hat{b}_i}{s.e.\left(\hat{b}_i\right)}, \tag{9}$$

where $\hat{b}_i$ is the OLS estimate of b_i, and $s.e.\left(\hat{b}_i\right)$ its associated standard error. Denote the student statistic on the ratio of b_i in Equation 8 as:

$$t_i(N,T) = \frac{{y_{i,-1}^3}' M_i \Delta y_i}{\left(\Delta y_i' M_i \Delta y_i\right)^{\frac{1}{2}} \left({y_{i,-1}^3}' M_i y_{i,-1}^3\right)^{\frac{1}{2}}}, \tag{10}$$

where $\Delta y_i = (\Delta y_{i1}, \Delta y_{i2}, \ldots, \Delta y_{iT})'$, $y_{i,-1}^3 = (y_{i,0}^3, y_{i,1}^3, \ldots, y_{i,T-1}^3)'$, M_i the projection matrix onto $\delta(X_i)$, the orthogonal complement of the span of X_i, $M_i = I_T - X_i(X_i'X_i)^{-1}X_i'$,

[8] See for example Davies [1987].

[9] In the following analysis we include an intercept in the model.

$X_i = (\tau, \overline{\Delta y}, \overline{y^3_{-1}})$, $\tau = (1, 1, \ldots, 1)'$, $\overline{\Delta y} = (\overline{\Delta y}_1, \overline{\Delta y}_2, \ldots, \overline{\Delta y}_T)'$, $\overline{y^3_{-1}} = (\overline{y^3_{i,0}}, \overline{y^3_{i,1}}, \ldots, \overline{y^3_{i,T-1}})$. The critical values of the NCADF test can be computed by stochastic simulation for any fixed $T > 3$, and for given distributional assumptions for the random variables (ε, f).[10]

It should be noted that, as in Pesaran [2007], the exact null distribution of the individual test statistic $t_i(N, T)$ defined in Equation 10 is affected by the nuisance parameters. However, this distribution depends on the nuisance parameters only through their effects on the matrix M_i. The following theorem shows that this dependence vanishes as $N \longrightarrow \infty$.

Theorem 1 *Suppose the cross-section mean of the initial observations $\bar{y}_0$ is set to zero. Then,*

$$\bar{t}(N, T) = \frac{1}{N}\sum_{i=1}^{N} t_i(N, T) = \frac{1}{N}\sum_{i=1}^{N} \tau_i(N, T) + R,$$

where, under Assumptions 1–10, as $N \longrightarrow \infty$ and T is fixed, $\tau_i(N, T)$ converges to a distribution which is free of nuisance parameters, and where R converges to zero.

See proof in Appendix A.2.

Theorem 2 *Suppose that Assumptions 1–10 hold. Then $\tau_i(N, T)$ converges in distribution to*

$$\frac{\int W_i^3 dW_i - q'\Psi^{-1}h}{\sqrt{\int W_i^6 - h'\Psi^{-1}h}},$$

where

$$\begin{aligned} \Psi &= \begin{pmatrix} 1 & \int W_f^3 \\ \int W_f^3 & \int W_f^6 \end{pmatrix}, \\ q &= \begin{pmatrix} W_i(1) \\ \int W_f^3 dW_i \end{pmatrix}, \quad h = \begin{pmatrix} \int W_i^3 \\ \int W_f^3 W_i^3 \end{pmatrix}, \end{aligned}$$

using the short-hand notation

$$\begin{aligned} \int W_i^3 dW_i &= \int_0^1 W_i(t)^3 dW_i(t), \\ \int W_i^6 &= \int_0^1 W_i(t)^6 dt, \end{aligned}$$

etcetera, where W_i and W_f are standard Wiener processes that are independent of each other.

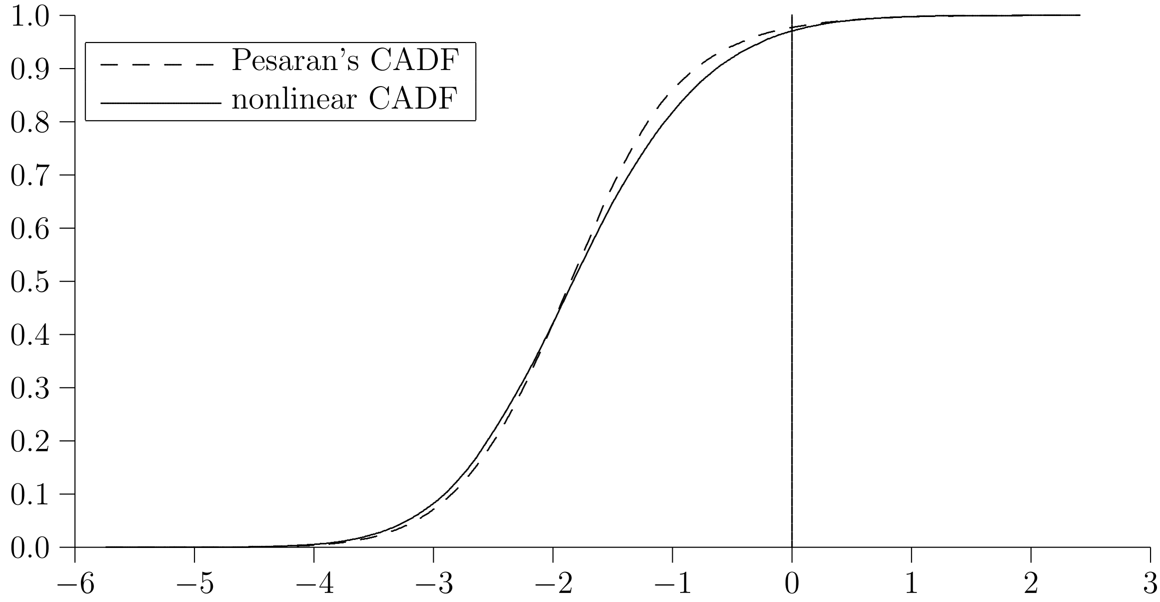
See proof in Appendix A.3.

Figure 1 displays the simulated cumulative distribution function of the individual NCADF statistic under the null hypothesis using 50,000 replications for $N = 100$ and

[10] To accommodate stochastic processes with non-zero means we follow Kapetanios et al. [2003]. In the case when the process has non-zero mean, we use demeaned and detrended data, i.e. when $x_t = \mu + \sigma t + y_t$, we use $y_t = x_t - \hat{\mu} + \hat{\sigma}_t$, where $\hat{\mu}$ and $\hat{\sigma}$ are the OLS estimators of μ and σ (see Kapetanios et al. [2003] for further details).

$T = 500$. For comparison the simulated cumulative distribution function of the Pesaran CADF statistic is also provided. The series $y_{it} = y_{i,t-1} + f_t + u_{it}$, for $i = 1, 2, \ldots, 100$, and $t = -50, -49, \ldots, 1, 2, \ldots, 500$ were first generated from $y_{i,-50} = 0$, with f_t and u_{it} as i.i.d. N(0,1). Then 50,000 NCADF regressions of Δy_{it} on $y^3_{i,t-1}$, $\Delta \bar{y}_t$, and $\bar{y}^3_{t-1}$. $\Delta \bar{y}_t$, and $\bar{y}^3_{t-1}$ were computed over the sample $t = 1, 2, \ldots, 500$. Figure 1 plots the ordered values of the OLS t-ratios of $y^3_{i,t-1}$ in these regressions.

Figure 1: Cumulative Distribution Function of Pesaran's Cross Sectionally Augmented DF and Nonlinear Cross Sectionally Augmented DF Statistics



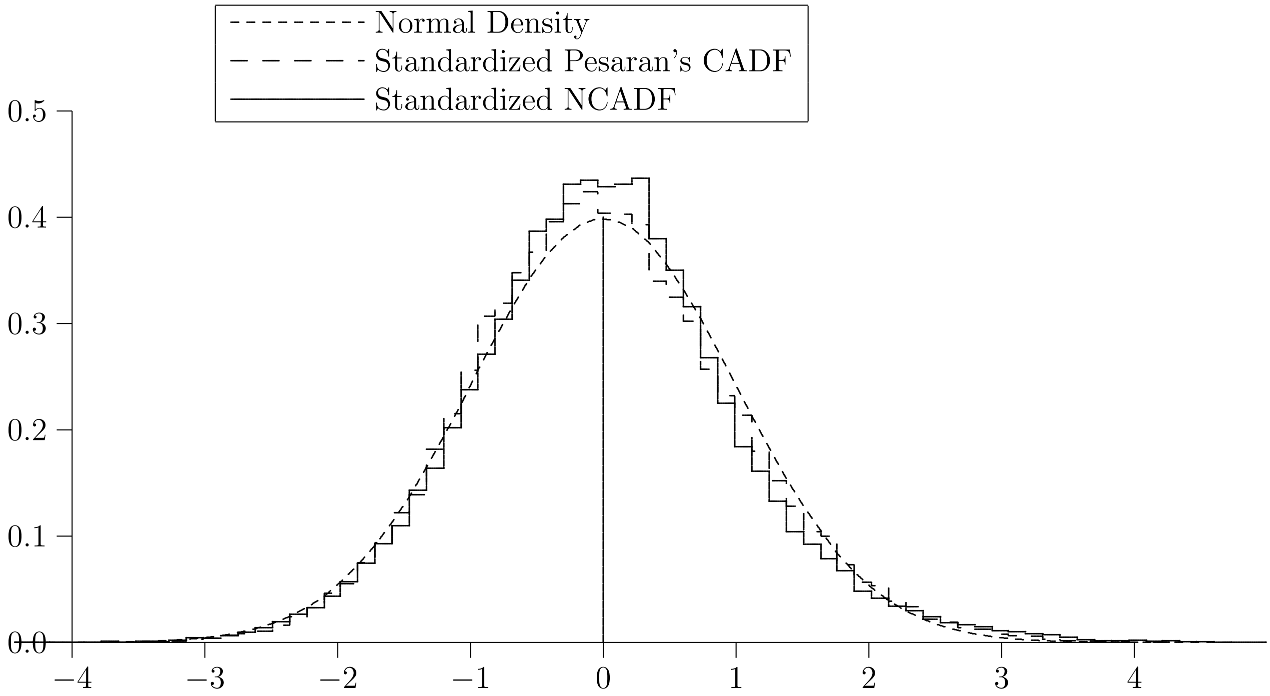
Not surprisingly the nonlinear CADF distribution, as the Pesaran's CADF distribution, is more skewed to the left as compared to the standard DF distribution. This is clearly reflected in the critical values of the distributions summarized in Table 1. Critical values of the individual nonlinear CADF distribution for values of T and N in the range of 10 to 200 are given Table 13 in Appendix B.1.

Table 1: Critical Values of the DF, Pesaran's CADF, and nonlinear CADF Distributions ($N = 100$, $T = 500$, 50,000 replications)

	1%	2.5%	5%	10%
DF	-2.60	-2.23	-1.94	-1.61
Pesaran's CADF	-3.80	-3.49	-3.22	-2.91
Nonlinear CADF	-3.72	-3.41	-3.15	-2.85

The nonlinear CADF distribution, like the Pesaran's CADF distribution and the standard DF distribution, departs from standard normality in two important respects: it has a substantially negative mean and its standard deviation is less than unity, although not by a large amount. The simulated density functions of the standardized NCADF, computed with $N = 100$, $T = 500$, and 50,000 replications are displayed in Figure 2. The mean, standard deviation, skewness and Kurtosis -3 coefficients of the NCADF and the

Figure 2: Simulated Density Function of the Standardized NCADF_i and the Standardized Pesaran's CADF_i Distributions as Compared to the Normal Density



Pesaran's CADF distributions are reported in Table 2. They are quite small, although

Table 2: Moments of the CADF distributions

	Pesaran's CADF	NCADF
Mean	-1.80	-1.83
Standard deviation	0.90	0.83
Skewness	0.20	0.28
Kurtosis -3	0.19	0.77

statistically highly significant.

Since cross sectional dependence in panel data is widely known now to be a serious problem, in the next sections we shall be using Equation 8 to develop a unit root test to test for the null hypothesis of unit root against an ESTAR stationary alternative.

3.2 Panel Nonlinear CADF Test

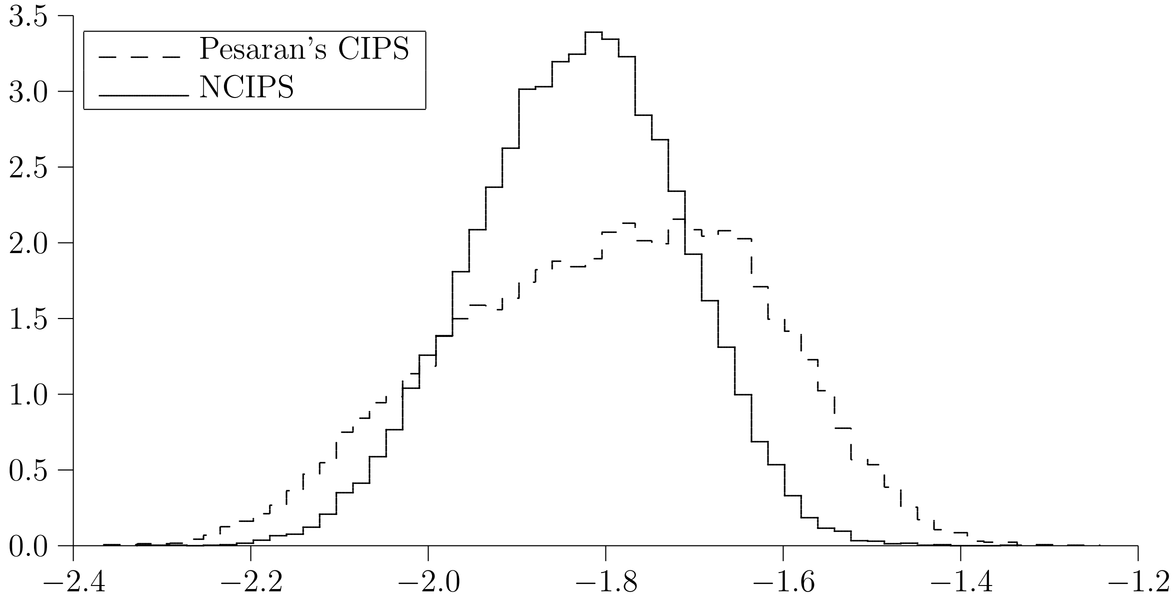
Following Pesaran [2007], we suggest using the t-statistic in Equation 10 to construct a panel unit root test by averaging the individual test statistics:

$$\bar{t}(N,T) = \frac{1}{N}\sum_{i=1}^{N} t_i(N,T) \tag{11}$$

This is a nonlinear cross sectionally augmented version of the IPS test (NCIPS). The test statistic defined in Equation 11 can also be extended to the case where serial correlation is present in the data. In this particular case, one may include, in the model, lags of the left hand side variable after using an information criteria to select the lag order.

We simulated the distribution of NCIPS setting $N = 100$, $T = 500$, and using 50,000 replications. The simulated density functions of the NCIPS and the Pesaran's CIPS Statistics are displayed in Figure 3. Both the densities show marked departures from

Figure 3: Simulated Density Function of the NCIPS Statistic and the Pesaran's CIPS Distributions



normality. The skewness and Kurtosis -3 coefficients of the NCIPS and the Pesaran's CIPS distributions are reported in Table 3. The critical values of the nonlinear CIPS test are given in Table 14 in Appendix B.2.

Table 3: Moments of the CIPS distributions

	Pesaran's CIPS	NCIPS
Mean	-1.80	-1.83
Standard deviation	0.17	0.12
Skewness	-0.10	-0.068
Kurtosis -3	-1.67	-1.45

3.3 The Serially Correlated Errors Case

Serial correlation can be incorporated in the model in a variety of different ways. In what follows, we use the model $\Delta y_{it} = b_i y_{i,t-1}^3 + e_{it}$ and specify the serial correlation structure as:

$$e_{it} = \rho_i e_{i,t-1} + \eta_{it}, \tag{12}$$

and thereafter cross section dependence as:

$$\eta_{it} = \gamma_i f_t + \varepsilon_{it}. \tag{13}$$

Using the model jointly with Equation 12 above we obtain:

$$\Delta y_{it} = a_i(1-\rho_i) + b_i(1-\rho_i)y_{i,t-1}^3 + \rho_i \Delta y_{i,t-1} + b_i \rho_i \Delta \left(y_{i,t-1}^3\right) + \eta_{it}. \tag{14}$$

And substituting Equation 13 into Equation 14

$$\Delta y_{it} = a_i(1-\rho_i) + b_i(1-\rho_i)y_{i,t-1}^3 + \rho_i \Delta y_{i,t-1} + b_i \rho_i \Delta \left(y_{i,t-1}^3\right) + \gamma_i f_t + \varepsilon_{it}. \tag{15}$$

Using Equation 15 and the same approach as in Appendix A.1, one can obtain a proxy for f_t using the following set of variables:

$$\left\{\overline{y_{t-1}^3}, \Delta \overline{y_{t-1}^3}\right\}$$

By generalizing this to an AR(p) error terms framework, we suggest using the following general nonlinear CADF regression:

$$\Delta y_{it} = a_i + b_{i0} y_{i,t-1}^3 + d_{i0} \overline{y_{t-1}^3} + \sum_{j=0}^{p} d_{ij} \Delta \overline{y_{t-j}^3} + \sum_{j=1}^{p} \delta_{ij} \Delta y_{i,t-j} + e_{it}. \tag{16}$$

Information criteria can be used to choose the length of p.

4 Small Sample Analysis

In this section we assess the size and power of the nonlinear panel test defined in Equation 11 under different scenarios. About the power of the test, we firstly look at it in the case of weak and strong cross sectional dependence, but no autocorrelation structure for the error term. In the next section, we generalise this scenario by allowing an autocorrelation specification for the error term and weak-strong cross sectional dependence. For comparison, in all the experiments we also report the size and power of Pesaran [2007] test when a nonlinear DGP is considered.

The data generating process (DGP) considered is the following Panel ESTAR:

$$\Delta y_{it} = \nu_i y_{i,t-1}\left[1 - \exp\left(-\theta_i y_{i,t-1}^2\right)\right] + \gamma_i f_t + \varepsilon_{it}, \tag{17a}$$
$$f_t \sim i.i.d.N(0,1), \tag{17b}$$
$$\varepsilon_{it} \sim i.i.d.N(0,\sigma_i^2), \tag{17c}$$
$$\sigma_i^2 \sim i.i.d.U[0.5,1.5]. \tag{17d}$$

with $i = 1, 2, \ldots, N$ and $t = -51, -50, \ldots, 1, 2, \ldots, T$. We fix $\nu_i = -1$ for all i. The choice of the cross sectional dependence parameters γ_i depends on whether we wish to impose Assumption 7 or not.

We consider two scenarios for cross sectional dependence, namely weak cross sectional dependence $\gamma_i \sim i.i.d.U[0,0.20]$, and strong cross sectional dependence $\gamma_i \sim i.i.d.U[1,3]$. [11]

4.1 Size Distortion Analysis

In our size analysis below, we generate data by setting $\theta = 0$ for all i. Size is computed at the 5% nominal significance level. The number of replications is set to 5,000. The standard error of the computed size is 0.0031. Results for the size are reported in Table 4. The test statistics seems to be correctly sized.

[11] We have also computed the size for the case when $\gamma_i \longrightarrow 0$. While for moderate N/T, results are

Table 4: Size: Case of no serial correlation

Sizes		Weak Cross Section Dependence				Strong Cross Section Dependence			
N	T	$CIPS_i$	$NCIPS_i$	CIPS	NCIPS	$CIPS_i$	$NCIPS_i$	CIPS	NCIPS
10	10	0.0482	0.0448	0.0554	0.0478	0.0456	0.0434	0.0594	0.0524
10	20	0.0502	0.0432	0.0488	0.0420	0.0470	0.0474	0.0650	0.0480
10	30	0.0544	0.0490	0.0544	0.0418	0.0504	0.0560	0.0622	0.0488
10	50	0.0562	0.0446	0.0476	0.0446	0.0446	0.0456	0.0630	0.0520
10	100	0.0456	0.0486	0.0492	0.0428	0.0486	0.0458	0.0670	0.0506
20	10	0.0502	0.0538	0.0476	0.0446	0.0468	0.0476	0.0544	0.0482
20	20	0.0462	0.0496	0.0504	0.0398	0.0428	0.0406	0.0558	0.0494
20	30	0.0542	0.0498	0.0548	0.0432	0.0504	0.0470	0.0592	0.0456
20	50	0.0480	0.0556	0.0524	0.0446	0.0474	0.0446	0.0612	0.0418
20	100	0.0448	0.0470	0.0572	0.0464	0.0510	0.0482	0.0554	0.0394
30	10	0.0440	0.0482	0.0512	0.0444	0.0554	0.0514	0.0568	0.0448
30	20	0.0524	0.0478	0.0486	0.0446	0.0454	0.0448	0.0530	0.0420
30	30	0.0542	0.0534	0.0608	0.0454	0.0508	0.0476	0.0616	0.0368
30	50	0.0480	0.0484	0.0554	0.0412	0.0510	0.0476	0.0612	0.0426
30	100	0.0486	0.0456	0.0646	0.0462	0.0474	0.0466	0.0620	0.0384
50	10	0.0478	0.0548	0.0488	0.0482	0.0530	0.0490	0.0524	0.0414
50	20	0.0516	0.0502	0.0438	0.0394	0.0460	0.0476	0.0554	0.0414
50	30	0.0602	0.0494	0.0530	0.0460	0.0492	0.0444	0.0530	0.0406
50	50	0.0502	0.0536	0.0486	0.0422	0.0484	0.0462	0.0570	0.0386
50	100	0.0506	0.0512	0.0506	0.0466	0.0476	0.0534	0.0538	0.0364
100	10	0.0464	0.0468	0.0520	0.0490	0.0474	0.0452	0.0480	0.0456
100	20	0.0512	0.0440	0.0558	0.0456	0.0532	0.0494	0.0470	0.0420
100	30	0.0548	0.0500	0.0474	0.0454	0.0444	0.0432	0.0532	0.0424
100	50	0.0500	0.0508	0.0430	0.0444	0.0478	0.0480	0.0598	0.0396
100	100	0.0592	0.0556	0.0472	0.0512	0.0486	0.0516	0.0564	0.0352

4.2 Power Analysis

In this section we assess the power of the test defined in Equation 11 under the same DGP as above but we consider the cases of weak and strong alternatives, namely we assume for the weak alternative:

$$\theta_i = \begin{cases} 0 & \text{for} \quad i = 1, \ldots, N/2, \\ 0.01 & \text{for} \quad i = N/2+1, \ldots, N, \end{cases} \tag{18a}$$

while for the strong alternative:

$$\theta_i = \begin{cases} 0 & \text{for} \quad i = 1, \ldots, N/2, \\ 0.05 & \text{for} \quad i = N/2+1, \ldots, N. \end{cases} \tag{18b}$$

The power is computed at the 5% nominal significance level. Results are reported in Table 5 (weak alternative) and Table 6 (strong alternative). The test we propose seems to have stronger power than the Pesaran [2007] test when the true DGP is nonlinear.

Finally, we assess the power of our test and the Pesaran [2007] test when the DGP is linear :

$$\begin{aligned} y_{it} &= \mu_i + \delta_i t + \phi_i y_{i,t-1} + \nu_i f_t + u_{it}, && (19) \\ \phi &= 1 \text{ for } i = 1, \ldots, N/2, && (20) \\ &\sim U[0,1] \text{ for } i = N/2+1, \ldots, N. && (21) \end{aligned}$$

To save space we only report the case of strong cross-sectional dependence. [12] Figure 4 shows the power results in the cases of $N = 10$, $T = 50$, and in several situations for the serial correlation.

The proposed test seems to have an acceptable good power, even when the true DGP is linear.

4.3 Serial Correlated Errors Case

In this section we analyze size and power of the proposed test when serial correlation is incorporated into the DGP. We consider positive serial correlation. The error terms (ε_{it}) were generated as:

$$\begin{aligned} \varepsilon_{it} &= \rho_i \varepsilon_{i,t-1} + \zeta_{it}, && (22a) \\ \zeta_{it} &\sim i.i.d.N\left(0, \sigma_i^2\right), && (22b) \\ \sigma_i^2 &\sim i.i.d.U[0.5; 1.5], && (22c) \\ \rho_i &\sim i.i.d.U[0.2; 0.4] \quad \text{in the case of positive correlation,} && (22d) \\ \rho_i &\sim i.i.d.U[-0.4; -0.2] \quad \text{in the case of negative correlation.} && (22e) \end{aligned}$$

We only consider here the power analysis for the case of strong alternative:

$$\theta_i = \begin{cases} 0 & \text{for} \quad i = 1, \ldots, N/2, \\ 0.05 & \text{for} \quad i = N/2+1, \ldots, N. \end{cases}$$

qualitatively the same as the ones reported in Table 4, for very large N size distortion appears to be a relevant issue.

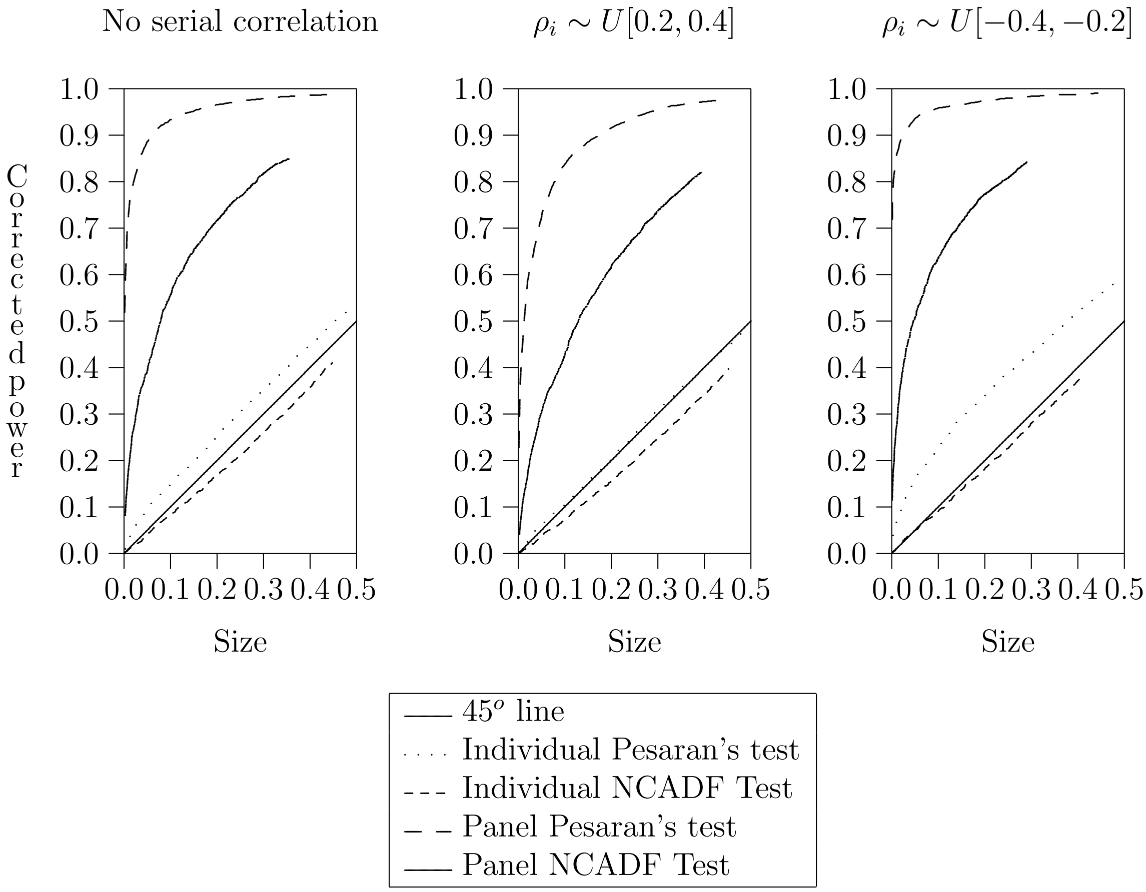
[12] Additional empirical results are avaialbe upon request.

Table 5: Power: Case of no serial correlation and weak alternative

Sizes		Weak Cross Section Dependence				Strong Cross Section Dependence			
N	T	$CIPS_i$	$NCIPS_i$	CIPS	NCIPS	$CIPS_i$	$NCIPS_i$	CIPS	NCIPS
10	10	0.0430	0.0454	0.0592	0.0662	0.3100	0.1396	0.5042	0.5278
10	20	0.0556	0.0456	0.0770	0.0870	0.7326	0.3546	0.8980	0.8884
10	30	0.0586	0.0586	0.1158	0.1370	0.8102	0.5646	0.9424	0.9354
10	50	0.0702	0.0482	0.2324	0.2702	0.8354	0.7240	0.9554	0.9498
10	100	0.1096	0.0704	0.6450	0.7514	0.8716	0.8210	0.9662	0.9650
20	10	0.0540	0.0490	0.0672	0.0680	0.1446	0.1280	0.2784	0.3992
20	20	0.0476	0.0478	0.0994	0.1180	0.5956	0.3064	0.8014	0.8222
20	30	0.0502	0.0502	0.1510	0.1868	0.7472	0.4798	0.9384	0.9528
20	50	0.0602	0.0514	0.2558	0.3584	0.8394	0.6780	0.9812	0.9844
20	100	0.0958	0.0678	0.7548	0.8972	0.8840	0.8286	0.9870	0.9872
30	10	0.0502	0.0490	0.0708	0.0734	0.0808	0.0932	0.1480	0.2496
30	20	0.0472	0.0398	0.0938	0.1348	0.3962	0.2250	0.6066	0.6760
30	30	0.0542	0.0434	0.1486	0.2128	0.6366	0.3870	0.8640	0.8960
30	50	0.0530	0.0508	0.3236	0.5038	0.7846	0.5968	0.9798	0.9860
30	100	0.0718	0.0504	0.8318	0.9704	0.8742	0.7802	0.9930	0.9938
50	10	0.0468	0.0450	0.0700	0.0808	0.0582	0.0602	0.0864	0.1228
50	20	0.0490	0.0484	0.1098	0.1700	0.1968	0.1268	0.3360	0.3974
50	30	0.0536	0.0472	0.1842	0.3010	0.4092	0.2448	0.6444	0.7162
50	50	0.0616	0.0528	0.3594	0.6578	0.6456	0.4162	0.9216	0.9642
50	100	0.0742	0.0632	0.9204	0.9970	0.8138	0.6646	0.9990	0.9992
100	10	0.0502	0.0428	0.0606	0.0838	0.0454	0.0512	0.0568	0.0782
100	20	0.0484	0.0446	0.1156	0.2338	0.0780	0.0662	0.1596	0.2064
100	30	0.0550	0.0518	0.1970	0.4484	0.1734	0.1174	0.3314	0.4312
100	50	0.0544	0.0534	0.4418	0.8510	0.3486	0.1864	0.7108	0.8500
100	100	0.0642	0.0534	0.9734	1.000	0.6278	0.3768	0.9990	0.9998

Table 6: Power: Case of no serial correlation and strong alternative

Sizes		Weak Cross Section Dependence				Strong Cross Section Dependence			
N	T	$CIPS_i$	$NCIPS_i$	CIPS	NCIPS	$CIPS_i$	$NCIPS_i$	CIPS	NCIPS
10	10	0.0548	0.0464	0.0818	0.0958	0.5288	0.1278	0.7820	0.6412
10	20	0.0640	0.0514	0.1904	0.2632	0.8148	0.3736	0.9510	0.9176
10	30	0.0948	0.0660	0.3760	0.4736	0.8492	0.5960	0.9608	0.9434
10	50	0.1410	0.0778	0.7734	0.8674	0.8730	0.7624	0.9628	0.9544
10	100	0.2028	0.1152	0.9968	0.9994	0.8768	0.8330	0.9632	0.9604
20	10	0.0458	0.0466	0.0896	0.1136	0.3960	0.1382	0.6510	0.6352
20	20	0.0578	0.0506	0.2270	0.3548	0.7926	0.3612	0.9722	0.9582
20	30	0.0778	0.0558	0.4306	0.6358	0.8542	0.5506	0.9832	0.9772
20	50	0.0912	0.0646	0.8684	0.9668	0.8860	0.7716	0.9870	0.9858
20	100	0.1312	0.0766	1.000	1.000	0.8950	0.8484	0.9882	0.9886
30	10	0.0572	0.0542	0.1018	0.1292	0.2792	0.1252	0.4846	0.5494
30	20	0.0552	0.0472	0.2492	0.4422	0.7350	0.3070	0.9576	0.9612
30	30	0.0606	0.0528	0.4874	0.7946	0.8402	0.5186	0.9916	0.9910
30	50	0.0788	0.0584	0.9250	0.9956	0.8764	0.7326	0.9946	0.9948
30	100	0.1276	0.0774	1.000	1.000	0.8986	0.8426	0.9966	0.9976
50	10	0.0504	0.0490	0.0998	0.1626	0.1428	0.0970	0.2610	0.3744
50	20	0.0576	0.0438	0.2712	0.5928	0.5546	0.2518	0.8492	0.9088
50	30	0.0640	0.0508	0.5684	0.9134	0.7440	0.4104	0.9872	0.9944
50	50	0.0760	0.0624	0.9714	1.000	0.8380	0.6052	0.9980	0.9988
50	100	0.1102	0.0742	1.000	1.000	0.8830	0.7830	0.9992	0.9994
100	10	0.0534	0.0502	0.1130	0.2044	0.0728	0.0632	0.1334	0.2124
100	20	0.0524	0.0538	0.3248	0.7712	0.2772	0.1256	0.5744	0.7972
100	30	0.0616	0.0538	0.6592	0.9916	0.4774	0.2256	0.9172	0.9912
100	50	0.0680	0.0614	0.9962	1.000	0.6750	0.3754	1.000	1.000
100	100	0.0962	0.0668	1.000	1.000	0.8050	0.6154	1.000	1.000



This plot uses the graphical technique of Davidson and MacKinnon [1998].

Figure 4: Power: Case of linear DGP and strong alternative

and strong cross sectional dependence $\gamma_i \sim i.i.d.U[1,3]$. The size and power are computed at 5% nominal significance level and they are based on the following nonlinear CADF regression:

$$\Delta y_{it} = \alpha_i + b_{i,0} y_{i,t-1}^3 + d_{i,0}\overline{y_{t-1}^3} + d_i \Delta \overline{y_t^3} + \delta_{i,j}\Delta y_{i,t-1} + e_{it}, \tag{23}$$

$$i = 1, 2, \ldots, N; t = 1, 2, \ldots, T; \bar{y}_t = \frac{1}{N}\sum_{i=1}^{N} y_{it}.$$

The test is computed as:

$$\bar{t}(N,T) = \frac{1}{N}\sum_{i=1}^{N} t_{iNL}(N,T) \tag{24}$$

where $\bar{t}(N,T)$ is the OLS t-ratio of b_i in the above nonlinear ADF regression. The number of simulations is set equal to 50,000. Table 7 shows the results for the size of the tests. Both tests have a good size with the Pesaran's test being consistently oversized.

In Table 8 we show results on the power of the test in the case when positive as well as negative serial correlation is present in the DGP. For panels of a moderate size, the gain in power from using the nonlinear panel unit root test with respect to the Pesaran's test is evident.

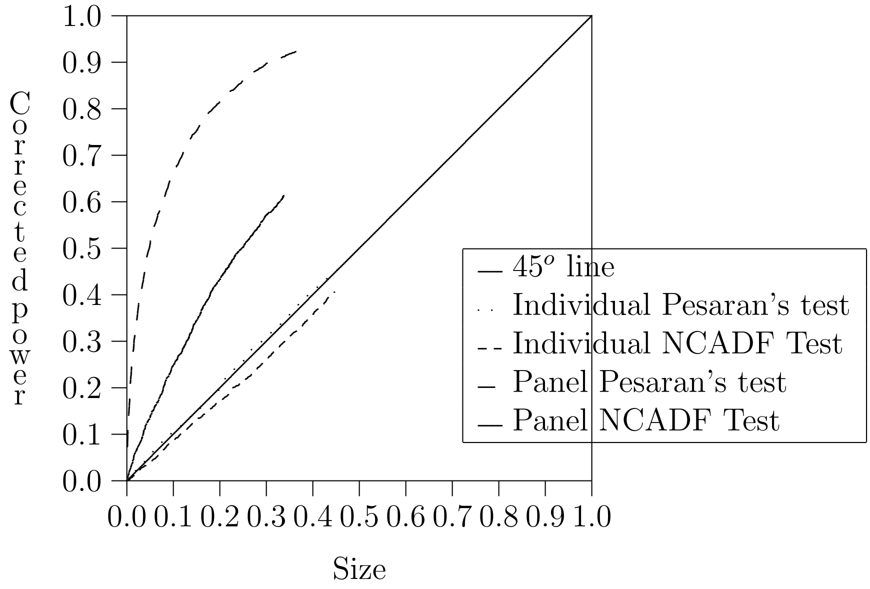
Table 7: Size: Case of serial correlation

Sizes		Weak Cross Section Dependence				Strong Cross Section Dependence			
N	T	$CIPS_i$	$NCIPS_i$	CIPS	NCIPS	$CIPS_i$	$NCIPS_i$	CIPS	NCIPS
10	10	0.0482	0.0448	0.0554	0.0378	0.0456	0.0434	0.0594	0.0524
10	20	0.0502	0.0432	0.0488	0.0420	0.0470	0.0474	0.0650	0.0480
10	30	0.0544	0.0490	0.0544	0.0418	0.0504	0.0560	0.0622	0.0488
10	50	0.0562	0.0446	0.0476	0.0446	0.0446	0.0456	0.0630	0.0520
10	100	0.0456	0.0486	0.0492	0.0428	0.0486	0.0458	0.0670	0.0506
20	10	0.0502	0.0538	0.0476	0.0446	0.0468	0.0476	0.0544	0.0482
20	20	0.0462	0.0496	0.0504	0.0398	0.0428	0.0406	0.0558	0.0494
20	30	0.0542	0.0498	0.0548	0.0432	0.0504	0.0470	0.0592	0.0456
20	50	0.0480	0.0556	0.0524	0.0446	0.0474	0.0446	0.0612	0.0418
20	100	0.0448	0.0470	0.0572	0.0464	0.0510	0.0482	0.0554	0.0394
30	10	0.0440	0.0482	0.0512	0.0444	0.0554	0.0514	0.0568	0.0448
30	20	0.0524	0.0478	0.0486	0.0446	0.0454	0.0448	0.0530	0.0420
30	30	0.0542	0.0534	0.0608	0.0454	0.0508	0.0476	0.0616	0.0368
30	50	0.0480	0.0484	0.0554	0.0412	0.0510	0.0476	0.0612	0.0426
30	100	0.0486	0.0456	0.0646	0.0462	0.0474	0.0466	0.0620	0.0384
50	10	0.0478	0.0548	0.0488	0.0482	0.0530	0.0490	0.0524	0.0414
50	20	0.0516	0.0502	0.0438	0.0394	0.0460	0.0476	0.0554	0.0414
50	30	0.0602	0.0494	0.0530	0.0460	0.0492	0.0444	0.0530	0.0406
50	50	0.0502	0.0536	0.0486	0.0422	0.0484	0.0462	0.0570	0.0386
50	100	0.0506	0.0512	0.0506	0.0466	0.0476	0.0534	0.0538	0.0364
100	10	0.0464	0.0468	0.0520	0.0490	0.0474	0.0452	0.048	0.0456
100	20	0.0512	0.0440	0.0558	0.0456	0.0532	0.0494	0.047	0.0420
100	30	0.0548	0.0500	0.0474	0.0454	0.0444	0.0432	0.0532	0.0424
100	50	0.0500	0.0508	0.0430	0.0444	0.0478	0.0480	0.0598	0.0396
100	100	0.0592	0.0556	0.0472	0.0512	0.0486	0.0516	0.0564	0.0352

Table 8: Power: Case of serial correlation and strong alternative

Sizes		Weak Cross Section Dependence				Strong Cross Section Dependence			
N	T	$CIPS_i$	$NCIPS_i$	CIPS	NCIPS	$CIPS_i$	$NCIPS_i$	CIPS	NCIPS
10	10	0.0548	0.0464	0.0818	0.0958	0.5288	0.1278	0.7820	0.6412
10	20	0.0640	0.0514	0.1904	0.2632	0.8148	0.3736	0.9510	0.9176
10	30	0.0948	0.0660	0.3760	0.4736	0.8492	0.5960	0.9608	0.9434
10	50	0.1410	0.0778	0.7734	0.8674	0.8730	0.7624	0.9628	0.9544
10	100	0.2028	0.1152	0.9968	0.9994	0.8768	0.8330	0.9632	0.9604
20	10	0.0458	0.0466	0.0896	0.1136	0.3960	0.1382	0.6510	0.6352
20	20	0.0578	0.0506	0.2270	0.3548	0.7926	0.3612	0.9722	0.9582
20	30	0.0778	0.0558	0.4306	0.6358	0.8542	0.5506	0.9832	0.9772
20	50	0.0912	0.0646	0.8684	0.9668	0.8860	0.7716	0.9870	0.9858
20	100	0.1312	0.0766	1.000	1.000	0.8950	0.8484	0.9882	0.9886
30	10	0.0572	0.0542	0.1018	0.1292	0.2792	0.1252	0.4846	0.5494
30	20	0.0552	0.0472	0.2492	0.4422	0.7350	0.3070	0.9576	0.9612
30	30	0.0606	0.0528	0.4874	0.7946	0.8402	0.5186	0.9916	0.9910
30	50	0.0788	0.0584	0.9250	0.9956	0.8764	0.7326	0.9946	0.9948
30	100	0.1276	0.0774	1.000	1.000	0.8986	0.8426	0.9966	0.9976
50	10	0.0504	0.0490	0.0998	0.1626	0.1428	0.0970	0.2610	0.3744
50	20	0.0576	0.0438	0.2712	0.5928	0.5546	0.2518	0.8492	0.9088
50	30	0.0640	0.0508	0.5684	0.9134	0.7440	0.4104	0.9872	0.9944
50	50	0.0760	0.0624	0.9714	1.000	0.8380	0.6052	0.9980	0.9988
50	100	0.1102	0.0742	1.000	1.000	0.8830	0.7830	0.9992	0.9994
100	10	0.0534	0.0502	0.1130	0.2044	0.0728	0.0632	0.1334	0.2124
100	20	0.0524	0.0538	0.3248	0.7712	0.2772	0.1256	0.5744	0.7972
100	30	0.0616	0.0538	0.6592	0.9916	0.4774	0.2256	0.9172	0.9912
100	50	0.0680	0.0614	0.9962	1.000	0.6750	0.3754	1.000	1.000
100	100	0.0962	0.0668	1.000	1.000	0.8050	0.6154	1.000	1.000

Finally, the power of our test is assessed using different autoregressive orders. An additional lag is included in Equation 22a: $\varepsilon_{it} = \rho_{i1}\varepsilon_{i,t-1} + \rho_{i2}\varepsilon_{i,t-2} + \zeta_{it}$ (AR(2) error terms). Figure 5 shows the power results in the case of linear DGP with half unit root series and half stationary series with random AR coefficient, and in the case of strong cross sectional dependence. We choose $N = 10$, $T = 50$, and $\rho_{i1} = 0.4$ for all i, $\rho_{i2} = 0.2$ for all i. Our test has a good power, regardless of the number of lags in Equation 22a. [13]



This plot uses the graphical technique of Davidson and MacKinnon [1998].

Figure 5: Power: Case of serial correlation of order 2

[13] Additional empirical results are available upon request.

5 Empirical Applications

5.1 Real Exchange Rates

In this section we apply our test to two different real exchange rates data sets against the US dollar for twenty OECD countries over the period 1973Q1–1998Q2, and 20 black market exchange rates. The first data set is the same used by Murray and Papell [2002, 2005]. The second data set consists of 20 black market exchange rates. Data are obtained from Reinhart and Rogoff [2004] and span the period 1973M1–1998M12.

Since the long run *Purchasing Power Parity* (PPP) relationship is one of the main components of theoretical international macroeconomic models, a large number of studies have tested this relationship by applying unit root tests to real exchange rates. Most of these studies show evidence of unit root behaviour in real exchange rates, which has become a puzzle in international finance. The growing literature on nonlinear exchange rates argues that transaction costs and frictions in financial markets may lead to nonlinear convergence in real exchange rates. Consequently, the non-mean reversion reported by linear unit root tests may be due to the fact that these tests are based on a mis-specified stochastic process.

We start with the first data set. The individual statistics for our unit root test are shown in Table 9.

For comparison purposes, we also report the statistics for the Pesaran [2007] test which accounts for cross section dependence but not for nonlinearity.

The Pesaran [2007] test rejects the unit root null hypothesis in only 1 out of 20 cases at all levels of significance. By contrast, the nonlinear test rejects the null in 2 cases at the 1% significance level, and in 5 cases at the 5% and 10% level. Hence our test rejects the unit root null more frequently and therefore yields stronger support for the long-run PPP.

As we argued above, univariate tests have low power and this problem is overcome by employing panel unit root tests. The results for our panel unit root test and the Pesaran panel unit root test are shown in Table 10.

The contrast between the two panel statistics is rather strong. The Pesaran [2007] test fails to reject the unit root null at all levels of significance, thus implying non-mean reversion. On the other hand, our nonlinear panel test rejects the unit root null for the panel of real exchange rates at all levels of significance, giving support to the long-run PPP.

5.2 Black market exchange rates

We now consider the second data set. Table 11 shows the results. The frequency of rejections using our test is higher than the Pesaran [2007] test. Our panel unit root test finds evidence supporting PPP, while the Pesaran [2007] does not. This evidence of nonlinear mean reversion in the real exchange rates may suggest that previous evidence of non-mean reversion in real exchange rates might be due to using linear unit root tests.

5.3 Nominal interest rates stationarity

The dataset used is taken from Moon and Perron [2007]. The dataset consists of (monthly) interest rates of different maturity and risk for Canada and US over the period 1985:01 until 2004:04. The Canadian rates are 1, 3 and 6 month T-bills, federal government

Table 9: Individual Unit Root Tests for Real Dollar Exchange Rates

Country	Lag	NCADF	CADF
Australia	3	-2.1765	-1.6501
Austria	4	-2.2085	-2.1432
Belgium	4	-2.4220	-1.2380
Canada	6	-1.1528	-1.3575
Denmark	3	-3.3390**	-2.8699
Finland	7	-1.7015	-2.4148
France	4	-0.9386	-2.1170
Germany	4	-3.3166**	-2.6044
Greece	4	-0.1449	-2.1730
Ireland	6	-0.1855	-1.0970
Italy	4	-2.6717	-2.0218
Japan	3	-2.5943	-1.9477
Netherlands	4	-2.7076	-1.9930
N Zealand	3	-3.7296**	-3.8758***
Norway	7	-2.2595	-1.8869
Portugal	8	-1.9120	-0.6359
Spain	8	-1.6911	-2.1622
Sweden	8	-3.8830***	-1.5888
Switzerland	4	-5.1263***	-2.7768
UK	7	-2.5354	-2.0689
Critical Values ($N = 20$, $T = 100$):			
1%		-3.74	-3.87
5%		-3.09	-3.24
10%		-2.80	-2.92

** Statistics significant at 5% level.
*** Statistics significant at 1% level.

Table 10: Panel Unit Root Tests for Real Dollar Exchange Rates

Country	NCADF	CADF
Panel	-2.3348***	-2.0311
Critical Values ($N = 20$, $T = 100$):		
1%	-2.24	-2.36
5%	-2.11	-2.20
10%	-2.03	-2.11

*** Statistic significant at 1% level.

bonds with maturity 1, 2, 3, 4, 7 and 10 years, commercial paper with maturity 1 month and 3 months, and Scotia indices of yields on corporate bonds (short-medium and long term). The US dataset consists of 3, 6 months Treasury securities and Treasury bonds with maturities 1, 2, 3, 5, 7, 10 years. 1 month commercial paper, and Moody's indices of yields on corporate bonds with AAA and BAA ratings.

Testing for the stationarity of nominal interest rates is an important issue since tests for term structure relationships, generally use cointegration. Therefore, they assume that nominal interest rates are integrated processes (see for example Campbell and Clarida [1987] and Newbold et al. [2001], amongst the others.) Moon and Perron [2007] use the dataset cited above and after decomposing the data into common and idiosyncratic components, they report the presence of a single nonstationary factor in the panel and with the presence of a stationary idiosyncratic component, they conclude that the time series are cointegrated.

Table 12 report the individual CADF and NCADF as well as panel statistics for US and Canada. The CADF test rejects the null hypothesis of a unit root more often than the NCDF. The bottom of the table reports the panel CADF and NCADF. The panel tests show evidence that nominal interest rates are stationary, with the exception of the panel NCADF in the case of US. This empirical result may invalidate the use of cointegration tests when testing for the Fisher effect or also term structure relationships.

6 Conclusion

A number of panel unit root tests allowing for cross section dependence have been proposed in the literature. In this paper we propose a nonlinear heterogeneous panel unit root test for testing the null hypothesis of unit-root processes against the alternative that allows a proportion of units to be generated by globally stationary ESTAR processes and a remaining non-zero proportion to be generated by unit root processes. The proposed test is simple to apply and accommodates both nonlinearity and cross sectional dependence. Our test is compared to the Pesaran [2007] linear test via Monte Carlo simulation exercises, and it is found that our test holds correct size and under the hypothesis that data are generated by globally stationary ESTAR processes has a better power than the Pesaran test. We also calculate critical values for varying cross section and time dimensions which can be used in future applications of our test.

We provide empirical applications to a panel of bilateral real exchange rate series

Table 11: Individual and panel unit root tests for black market exchange rates

Country	Lag	NCADF	CADF
Algeria	5	-1.24	-1.25
Argentina	4	-3.74**	-1.43
Bolivia	7	-2.09	-1.75
Chile	5	-3.31**	-3.49**
Colombia	6	-3.75**	-3.6**
C. Rica	6	-0.89	-0.66
D.Republic	6	-4.25*	-4.18*
Equador	4	-3.8*	-3.58**
Egypt	5	-2.1	-1.75
Ethyopia	4	-1.48	-0.91
Salvador	1	-3.11**	-3.09
Hungary	6	-1.57	-1.69
Ghana	6	-3.2**	-3.38**
India	0	-1.19	-1.21
Indonesia	5	-1.17	-0.9
Kenya	7	-2.71	-2.87
Korea	7	-1.75	-3.83**
Kuwait	4	-1.16	-1.22
Malaysia	7	-0.95	-0.51
Mexico	6	-2.93	-1.67
Critical Values ($N = 20$, $T = 30$):			
1%		-3.77	-3.84
5%		-3.14	-3.23
10%		-2.84	-2.91
Country	NCADF	CADF	
Panel	-2.3195***	-2.1485	
Critical Values ($N = 20$, $T = 30$):			
1%	-2.26	-2.36	
5%	-2.13	-2.2	
10%	-2.06	-2.11	

** Statistics significant at 5% level.
*** Statistics significant at 1% level.

Table 12: Individual and panel CADF and NCADF statistics for US and Canada

Canada

Interest rates	Lags	CADF		NCADF	
1 month	9	-2.89		-2.86	*
3 month	9	-4.09	*	-2.46	
6 month	9	-0.69		-2.37	
1 year	8	-3.54	**	-2.89	*
2 years	4	-4.83	*	-3.65	**
3 years	0	-4.19	*	-2.42	
5 years	7	-3.43	**	-2.06	
7 years	0	-3.17	*	-1.52	
10 years	0	-2.87		-2.05	
1-month com. paper	11	-3.24	**	-3	*
3-month com. paper	9	-2.15		-2.37	
1-month bank acc.	9	-3.2	*	-1.38	
Long-corporate	7	-2.75		-3.41	**
Mid.-corporate	7	-1.27		-2.49	
panel	-	-3.022142857	*	-2.495	*

US

Interest rates	Lags	CADF		NCADF	
3 month	2	-3.21	*	-0.68	
6 month	2	-2.67		-1.08	
1 year	12	-1.52		-3.21	**
2 years	2	-2.85		-2.32	*
3 years	12	-0.67		-1.43	
5 years	12	-2.94	**	-0.31	
7 years	3	-5.17	*	-1.54	
10 years	9	-4.29	*	-3.16	**
1-m com. Paper	12	-4.31	*	-1.06	
AAA	3	-3.48	**	-2.83	***
BAA	12	-3.74	**	-1.13	
panel	-	-3.168181818	*	-1.704545455	

*: 1% significance
**: 5% significance
***: 10% significance

with the US dollar from the 20 major OECD countries, and a panel of black market exchange rates. In contrast to the evidence obtained by linear tests, we find evidence of nonlinear mean-reversion in the real exchange rates for the whole panel that gives support to the long run PPP hypothesis. Given the importance of the PPP in international macroeconomic models, our evidence suggests that the employment of nonlinear panel unit root tests may provide a solution to the PPP puzzle.

We also provide a further empirical application on testing for interest rate stationarity. We report strong evidence suggesting that interest rates in Canada are stationary, whilst in the case of the US the evidence is mixed. Given the growing literature of nonlinear models, we believe that the development of panel nonlinear unit root tests has large potential in macroeconomic and financial applications. Evidence indicates that different time series may follow different nonlinear specifications. Consequently, one could consider unit root tests with different types of transition functions that allow for asymmetric dynamic adjustment. Another extension would be to allow for different transition variables.

Appendix

A Proofs

A.1 Lemma 1

Proof. By Equation 3,

$$\begin{aligned}\Delta y_{it} &= \nu_i y_{i,t-1}\left[\theta_i y_{i,t-1}^2 + \left\{1-\exp\left(-\theta_i y_{i,t-1}^2\right) - \theta_i y_{i,t-1}^2\right\}\right] + \gamma_i f_t + \varepsilon_{it} \\ &= \nu_i \theta_i y_{i,t-1}^3 + \gamma_i f_t + e_{it}, \qquad\qquad \text{(A.1.25)}\end{aligned}$$

where

$$\begin{aligned}e_{it} &= \nu_i y_{i,t-1}\widetilde{e}_{it} + \varepsilon_{it}, \\ \widetilde{e}_{it} &\equiv 1-\exp\left(-\theta_i y_{i,t-1}^2\right) - \theta_i y_{i,t-1}^2.\end{aligned}$$

Here, because ε_{it} is independent of $y_{i,t-1}$,

$$Var\left(e_{it}\right) = \nu_i^2 Var\left(y_{i,t-1}\widetilde{e}_{it}\right) + \sigma_i^2. \qquad \text{(A.1.26)}$$

Now, because of the inequality $1 \geq \exp(-x) \geq 1-x$ for $x \geq 0$, we have $-x \leq 1-\exp(-x) - x \leq 0$ i.e. $|\widetilde{e}_{it}| \leq \theta_i y_{i,t-1}^2$. Hence, by the Cauchy-Schwarz inequality,

$$Var\left(y_{i,t-1}\widetilde{e}_{it}\right) \leq E\left(y_{i,t-1}^2\widetilde{e}_{it}^2\right) \leq \sqrt{E\left(y_{i,t-1}^4\right)E\left(\widetilde{e}_{it}^4\right)} \leq \theta_i^2\sqrt{E\left(y_{i,t-1}^4\right)E\left(y_{i,t-1}^8\right)}. \qquad \text{(A.1.27)}$$

Next, following Donauer et al (2010), p.21, rewrite Equation 3 as

$$y_{it} = \left\{1+\nu_i - \nu_i \exp\left(-\theta_i y_{i,t-1}^2\right)\right\} y_{i,t-1} + \gamma_i f_t + \varepsilon_{it}.$$

Here, because for $\theta_i \geq 0$, $0 \leq \exp\left(-\theta_i y_{i,t-1}^2\right) \leq 1$, and by assumption 7, we obtain

$$-1 \leq 1+\nu_i \leq 1+\nu_i-\nu_i\exp\left(-\theta_i y_{i,t-1}^2\right) \leq 1.$$

Consequently, for $k = 1, 2, ...,$ since all cross moments vanish,

$$\begin{aligned} E\left(y_{it}^{2k}\right) &= E\left[\left\{1+\nu_i-\nu_i \exp\left(-\theta_i y_{i,t-1}^2\right)\right\}^{2k} y_{i,t-1}^{2k}\right] + E\left(\gamma_i^{2k} f_t^{2k}\right) + E\left(\varepsilon_{it}^{2k}\right) \\ &\le E\left(y_{i,t-1}^{2k}\right) + E\left(\gamma_i^{2k} f_t^{2k}\right) + E\left(\varepsilon_{it}^{2k}\right), \end{aligned}$$

and it follows by recursion that

$$E\left(y_{it}^{2k}\right) \le y_{i,0}^{2k} + E\left(\gamma_i^{2k}\right) \sum_{j=1}^{t} E\left(f_j^{2k}\right) + \sum_{j=1}^{t} E\left(\varepsilon_{ij}^{2k}\right). \tag{A.1.28}$$

Now, taking weighted averages in (A.1.25), we have

$$\Delta\overline{y}_{\omega,t} = \overline{(\nu\theta y^3)}_{\omega,t-1} + \overline{\gamma}_\omega f_t + \overline{e}_{\omega,t}, \tag{A.1.29}$$

where

$$\Delta\overline{y}_{\omega,t} \equiv \sum_{i=1}^{N} \omega_i \Delta y_{it},$$

Hence, if $\overline{\gamma}_\omega \neq 0$ (which holds with probability 1), (A.1.29) yields

$$f_t = \frac{1}{\overline{\gamma}_\omega}\Delta\overline{y}_{\omega,t} - \frac{1}{\overline{\gamma}_\omega}\overline{(\nu\theta y^3)}_{\omega,t-1} - \frac{1}{\overline{\gamma}_\omega}\overline{e}_{\omega,t}.$$

But

$$Var\left(\overline{e}_{\omega,t}\right) = \sum_{i=1}^{N} \omega_i^2 Var\left(e_{it}\right),$$

where, via (A.1.26) and (A.1.27)

$$Var\left(e_{it}\right) \le \nu_i^2\theta_i^2\sqrt{E\left(y_{i,t-1}^4\right) E\left(y_{i,t-1}^8\right)} + \sigma_i^2,$$

and since by (A.1.28), $E\left(y_{it}^{2k}\right)$ is at most $O(t)$, and we have

$$Var\left(e_{it}\right) \le O(t),$$

implying

$$Var\left(\overline{e}_{\omega,t}\right) \le O\left(\frac{t}{N}\right) \to 0$$

as $N, T \to \infty$ and $T/N \to 0$. We also have

$$\overline{(\nu\theta y^3)}_{\omega,t-1} = \sum_{i=1}^{N}\left(\nu_i\theta_i - \overline{\nu\theta}\right)\omega_i y_{i,t-1}^3 + \overline{(\nu\theta)}\overline{(y^3)}_{\omega,t-1}, \tag{A.1.30}$$

where

$$\begin{aligned} \overline{\nu\theta} &\equiv N^{-1}\sum_{i=1}^{N}\nu_i\theta_i, \\ \overline{(y^3)}_{\omega,t-1} &\equiv \sum_{i=1}^{N}\omega_i y_{i,t-1}^3. \end{aligned}$$

By assumption 8, $\overline{\nu\theta}$ is $O_p(1)$. Moreover, since $E\left(y_{i,t-1}^3\right)$ is constant over i, it is clear that

$$E\left\{\sum_{i=1}^{N}\left(\nu_i\theta_i-\overline{\nu\theta}\right)\omega_i y_{i,t-1}^3\right\}=\sum_{i=1}^{N}\left(\nu_i\theta_i-\overline{\nu\theta}\right)\omega_i E\left(y_{i,t-1}^3\right)=0,$$

and furthermore,

$$Var\left\{\sum_{i=1}^{N}\left(\nu_i\theta_i-\overline{\nu\theta}\right)\omega_i y_{i,t-1}^3\right\}=\sum_{i=1}^{N}\left(\nu_i\theta_i-\overline{\nu\theta}\right)^2\omega_i^2 Var\left(y_{i,t-1}^3\right), \tag{A.1.31}$$

where by (A.1.28), $Var\left(y_{i,t-1}^3\right)\leq O(t)$. Further, by the Cauchy-Schwarz' inequality,

$$\left(\sum_{i=1}^{N}\left(\nu_i\theta_i-\overline{\nu\theta}\right)^2\omega_i^2\right)^2\leq\sum_{i=1}^{N}\left(\nu_i\theta_i-\overline{\nu\theta}\right)^4\sum_{i=1}^{N}\omega_i^4$$

which by assumptions 5 and 9 is $O\left(N^{-2}\right)$. This shows that as $N,T\to\infty$ and $T/N\to 0$, the r.h.s. of (A.1.31) tends to zero, and so, we may neglect the first term on the r.h.s. of (A.1.30), which completes the proof of Lemma 1. ■

A.2 Theorem 1

Consider the auxiliary regression:

$$\begin{aligned}\Delta y_{it} &= a_i+b_i y_{i,t-1}^3+c_i\overline{\Delta y_t}+d_i\overline{y_{t-1}^3}+\epsilon_{it}\\ &= b_i y_{i,t-1}^3+\gamma_i' x_t+\epsilon_{it},\end{aligned} \tag{A.2.32}$$

where

$$\begin{aligned}\gamma_i &\equiv \left(a_i,c_i,d_i\right)',\\ x_t &\equiv \left(1,\overline{\Delta y_{it}},\overline{y_{i,t-1}^3}\right)',\end{aligned}$$

and ϵ_{it} is an error term. Then, in matrix formulation, (A.2.32) is:

$$\Delta y_i=b_i y_{i,-1}^3+X_i\gamma_i+\epsilon_i,$$

where

$$\begin{aligned}\Delta y_i &\equiv \left(\Delta y_{i1},\Delta y_{i2},...,\Delta y_{iT}\right)',\\ X_i &\equiv \left(\tau,\overline{\Delta y}_i,\overline{y_{i,-1}^3}\right),\\ \tau &\equiv (1,1,...,1)',\\ \overline{\Delta y}_i &\equiv \left(\overline{\Delta y_{i1}},\overline{\Delta y_{i2}},...,\overline{\Delta y_{iT}}\right)',\\ \overline{y_{i,-1}^3} &\equiv \left(\overline{y_{i0}^3},\overline{y_{i1}^3},...,\overline{y_{i,T-1}^3}\right)',\\ \epsilon_i &\equiv \left(\epsilon_{i1},\epsilon_{i2},...,\epsilon_{iT}\right)'.\end{aligned}$$

The t-statistic for testing that $b_i=0$ is

$$t_i(N,T)=T^{1/2}\frac{y_{i,-1}^{3\prime}M_i\Delta y_i}{\left(\Delta y_i' M_i\Delta y_i\right)^{1/2}\left(y_{i,-1}^{3\prime}M_i y_{i,-1}^3\right)^{1/2}}, \tag{A.2.33}$$

where (from now on suppressing the index i on M and X)

$$M \equiv I_T - X\left(X'X\right)^{-1}X'.$$

To find an expression of (A.2.33) in terms of nuisance parameters, we may assume that

$$\Delta y_{it} = \gamma_i f_t + \varepsilon_{it}, \tag{A.2.34}$$

where ε_{it} are $iid\,(0,\sigma^2)$. In matrix formulation, this is

$$\Delta y_i = f\gamma_i + \varepsilon_i, \tag{A.2.35}$$

where

$$f \equiv (f_1, f_2, ..., f_T)',$$

As in Pesaran [2007], with $\xi_i \equiv \varepsilon_i - \delta_i\overline{\varepsilon}$, $\delta_i \equiv \gamma_i/\overline{\gamma}$,

$$\begin{aligned}\Delta y_i &= f\gamma_i + \varepsilon_i = \left(\frac{\gamma_i}{\overline{\gamma}}\overline{\gamma}f + \frac{\gamma_i}{\overline{\gamma}}\overline{\varepsilon}\right) + \left(\varepsilon_i - \frac{\gamma_i}{\overline{\gamma}}\overline{\varepsilon}\right)\\ &= \delta_i\overline{\Delta y} + \xi_i,\end{aligned}$$

where

$$\xi_i \equiv \varepsilon_i - \delta_i\overline{\varepsilon}.$$

Hence, because $M\overline{\Delta y} = 0$,

$$M\Delta y_i = M_i\xi_i = \omega_i M\upsilon_i, \tag{A.2.36}$$

where $\omega_i^2 = \mathrm{Var}(\xi_i)$ and υ_i is standard normal. Similarly, defining

$$\begin{aligned}s_{f,t} &\equiv \sum_{s=1}^{t} f_s,\\ s_{i,t} &\equiv \sum_{s=1}^{t} \varepsilon_{is},\end{aligned}$$

we have

$$\begin{aligned}y_{i,t-1} &= \gamma_i s_{f,t-1} + s_{i,t-1} + y_{i,0}\\ &= \left(\frac{\gamma_i}{\overline{\gamma}}\overline{\gamma}s_{f,t-1} + \frac{\gamma_i}{\overline{\gamma}}\overline{s}_{t-1}\right) + \left(s_{i,t-1} - \frac{\gamma_i}{\overline{\gamma}}\overline{s}_{t-1}\right) + y_{i,0}\\ &= \delta_i\overline{y}_{t-1} + \eta_{i,t-1} + y_{i,0},\end{aligned}$$

where

$$\eta_{i,t} \equiv s_{i,t} - \delta_i\overline{s}_t.$$

In the following, we will neglect the term $y_{i,0}$, since it is clear that it only induces terms of negligible order.

The corresponding expression for $y_{i,t-1}^3$ is

$$\begin{aligned}y_{i,t-1}^3 &= \left(\delta_i\overline{y}_{t-1} + \eta_{i,t-1}\right)^3\\ &= \delta_i^3\overline{y}_{t-1}^3 + 3\delta_i^2\overline{y}_{t-1}^2\eta_{i,t-1} + 3\delta_i\overline{y}_{t-1}\eta_{i,t-1}^2 + \eta_{i,t-1}^3.\end{aligned} \tag{A.2.37}$$

Hence, stacking the $\overline{y}^j_{i,t-1}$ in vectors $\overline{y}^j_{t-1}$, for $j = 1, 2, 3$, we have, because $M\overline{y}^3_{-1} = 0$,

$$My^3_{i,-1} = 3\delta_i^2 M\left(\overline{y}^2_{-1} \odot \eta_{i,-1}\right) + 3\delta_i M\left(\overline{y}_{-1} \odot \eta^2_{i,-1}\right) + M\eta^3_{i,-1},$$

where $\odot$ is the Hadamard (elementwise) product. Moreover, because

$$\overline{y}_{t-1} = \overline{\gamma} s_{f,t-1} + \overline{s}_{t-1}, \tag{A.2.38}$$

the tth element of $\overline{y}^2_{-1}$ is

$$\overline{y}^2_{t-1} = \overline{\gamma}^2 s^2_{f,t-1} + 2\overline{\gamma} s_{f,t-1}\overline{s}_{t-1} + \overline{s}^2_{t-1},$$

and it follows that

$$\begin{aligned} My^3_{i,-1} &= 3\delta_i^2\overline{\gamma}^2 M\left(s^2_{f,-1} \odot \eta_{i,-1}\right) + 6\delta_i^2\overline{\gamma} M\left(s_{f,-1} \odot \overline{s}_{-1} \odot \eta_{i,-1}\right) \\ &\quad + 3\delta_i^2 M\left(\overline{s}^2_{-1} \odot \eta_{i,-1}\right) + 3\delta_i\overline{\gamma} M\left(s_{f,-1} \odot \eta^2_{i,-1}\right) \\ &\quad + 3\delta_i M\left(\overline{s}_{-1} \odot \eta^2_{i,-1}\right) + M\eta^3_{i,-1} \\ &= \omega_i^3 M\mathfrak{s}^3_{i,-1} + R_{i1} + R_{i2}, \end{aligned} \tag{A.2.39}$$

where

$$\begin{aligned} R_{i1} &\equiv 3\gamma_i^2\omega_i M\left(s^2_{f,-1} \odot \mathfrak{s}_{i,-1}\right) + 3\gamma_i\omega_i^2 M\left(s_{f,-1} \odot \mathfrak{s}^2_{i,-1}\right), \\ R_{i2} &\equiv 6\gamma_i^2\overline{\gamma}^{-1}\omega_i M\left(s_{f,-1} \odot \overline{s}_{-1} \odot \mathfrak{s}_{i,-1}\right) + 3\gamma_i^2\overline{\gamma}^{-2}\omega_i M\left(\overline{s}^2_{-1} \odot \mathfrak{s}_{i,-1}\right) \\ &\quad + 3\gamma_i\overline{\gamma}^{-1}\omega_i^2 M\left(\overline{s}_{-1} \odot \mathfrak{s}^2_{i,-1}\right) \end{aligned}$$

with $\mathfrak{s}_{i,-1} = \eta_{i,-1}/\omega_i$. (Observe that in the corresponding proof in Pesaran, 2007, no rest terms arise when pre-multiplying $y_{i,-1}$ with M. Having $y^3_{i,-1}$ in place of $y_{i,-1}$, in order for our Taylor series expansion below to work, we need Assumption 10.) Hence, via Equation A.2.36, the nominator of Equation A.2.33 is

$$y^{3\prime}_{i,-1} M\Delta y_i = \omega_i^4 \upsilon_i' M\mathfrak{s}^3_{i,-1} + \omega_i\upsilon_i' R_{i1} + \omega_i\upsilon_i' R_{i2} \tag{A.2.40}$$

Similarly, via Equation A.2.39,

$$\begin{aligned} y^{3\prime}_{i,-1} My^3_{i,-1} &= \left(My^3_{i,-1}\right)' My^3_{i,-1} \\ &= \omega_i^6\mathfrak{s}^{3\prime}_{i,-1} M\mathfrak{s}^3_{i,-1} + \omega_i^3\mathfrak{s}^{3\prime}_{i,-1} MR_{i1} + \omega_i^3 R'_{i1} M\mathfrak{s}^3_{i,-1} \\ &\quad + \omega_i^3\mathfrak{s}^{3\prime}_{i,-1} MR_{i2} + \omega_i^3 R'_{i2} M\mathfrak{s}^3_{i,-1} \\ &\quad + \left(R_{i1} + R_{i2}\right)'\left(R_{i1} + R_{i2}\right). \end{aligned} \tag{A.2.41}$$

As in Pesaran [2007], the terms involving $\overline{s}_{-1}$, which are collected in R_{i2}, will tend to zero as $N \to \infty$. Hence, as $N \to \infty$, using Equation A.2.36-Equation A.2.41, Equation A.2.33 is asymptotically equivalent to

$$\begin{aligned} & t_i^*(N,T) \\ \equiv\ & T^{1/2}\frac{\omega_i^4\upsilon_i' M\mathfrak{s}^3_{i,-1} + \omega_i\upsilon_i' R_{i1}}{\sqrt{\omega_i^2\upsilon_i' M\upsilon_i}\sqrt{\omega_i^6\mathfrak{s}^{3\prime}_{i,-1} M\mathfrak{s}^3_{i,-1} + \omega_i^3\left(\mathfrak{s}^{3\prime}_{i,-1} MR_{i1} + R'_{i1} M\mathfrak{s}^3_{i,-1}\right) + R'_{i1}R_{i1}}} \\ =\ & T^{1/2}\frac{\upsilon_i' M\mathfrak{s}^3_{i,-1} + \omega_i^{-3}\upsilon_i' R_{i1}}{\sqrt{\upsilon_i' M\upsilon_i}\sqrt{\mathfrak{s}^{3\prime}_{i,-1} M\mathfrak{s}^3_{i,-1} + \omega_i^{-3}\left(\mathfrak{s}^{3\prime}_{i,-1} MR_{i1} + R'_{i1} M\mathfrak{s}^3_{i,-1}\right) + \omega_i^{-6} R'_{i1}R_{i1}}}, \end{aligned}$$

and by Taylor expansion,

$$t_i^*(N,T) = T^{1/2}\frac{\upsilon_i' M \mathfrak{s}_{i,-1}^3}{\sqrt{\upsilon_i' M \upsilon_i}\sqrt{\mathfrak{s}_{i,-1}^{3\prime} M \mathfrak{s}_{i,-1}^3}}(1+R_i),$$

where

$$R_i \equiv \frac{\omega_i^{-3}\upsilon_i' R_{i1}}{\upsilon_i' M \mathfrak{s}_{i,-1}^3} - \frac{1}{2}\frac{\omega_i^{-3}\left(\mathfrak{s}_{i,-1}^{3\prime} M R_{i1} + R_{i1}' M \mathfrak{s}_{i,-1}^3\right)}{\mathfrak{s}_{i,-1}^{3\prime} M \mathfrak{s}_{i,-1}^3} + O\left(\left\|R_{i1}^2\right\|\right).$$

Now, as $N \to \infty$, the mean statistic

$$\bar{t}(N,T) \equiv \frac{1}{N}\sum_{i=1}^{N} t_i(N,T),$$

is asymptotically equivalent to

$$\overline{t^*}(N,T) \equiv \frac{1}{N}\sum_{i=1}^{N} t_i^*(N,T) = \frac{T^{1/2}}{N}\sum_{i=1}^{N}\frac{\upsilon_i' M \mathfrak{s}_{i,-1}^3}{\sqrt{\upsilon_i' M \upsilon_i}\sqrt{\mathfrak{s}_{i,-1}^{3\prime} M \mathfrak{s}_{i,-1}^3}} + R, \tag{A.2.42}$$

where

$$\begin{aligned} R \equiv {} & \frac{T^{1/2}}{N}\sum_{i=1}^{N}\frac{\upsilon_i' M \mathfrak{s}_{i,-1}^3}{\sqrt{\upsilon_i' M \upsilon_i}\sqrt{\mathfrak{s}_{i,-1}^{3\prime} M \mathfrak{s}_{i,-1}^3}} \\ & \left\{\frac{\omega_i^{-3}\upsilon_i' R_{i1}}{\upsilon_i' M \mathfrak{s}_{i,-1}^3} - \frac{1}{2}\frac{\omega_i^{-3}\left(\mathfrak{s}_{i,-1}^{3\prime} M R_{i1} + R_{i1}' M \mathfrak{s}_{i,-1}^3\right)}{\mathfrak{s}_{i,-1}^{3\prime} M \mathfrak{s}_{i,-1}^3} + O\left(\left\|R_{i1}^2\right\|\right)\right\}. \end{aligned} \tag{A.2.43}$$

Here,

$$\frac{T^{1/2}}{N}\sum_{i=1}^{N}\frac{\upsilon_i' M \mathfrak{s}_{i,-1}^3}{\sqrt{\upsilon_i' M \upsilon_i}\sqrt{\mathfrak{s}_{i,-1}^{3\prime} M \mathfrak{s}_{i,-1}^3}}\frac{\omega_i^{-3}\upsilon_i' R_{i1}}{\upsilon_i' M \mathfrak{s}_{i,-1}^3} = \frac{3}{N}\sum_{i=1}^{N}\kappa_i,$$

where

$$\kappa_i \equiv T^{1/2}\frac{\omega_i^{-2}\upsilon_i'\left\{\gamma_i^2 M\left(s_{f,-1}^2 \odot \mathfrak{s}_{i,-1}\right) + \omega_i\gamma_i M\left(s_{f,-1} \odot \mathfrak{s}_{i,-1}^2\right)\right\}}{\sqrt{\upsilon_i' M \upsilon_i}\sqrt{\mathfrak{s}_{i,-1}^{3\prime} M \mathfrak{s}_{i,-1}^3}},$$

which, by Assumption 10, tends to zero as $N \to \infty$. The same argument may be applied to all terms composing Equation A.2.43, and so, Equation A.2.42 implies that as $N \to \infty$, $\bar{t}(N,T)$ is asymptotically equivalent to

$$\frac{T^{1/2}}{N}\sum_{i=1}^{N}\frac{\upsilon_i' M \mathfrak{s}_{i,-1}^3}{\sqrt{\upsilon_i' M \upsilon_i}\sqrt{\mathfrak{s}_{i,-1}^{3\prime} M \mathfrak{s}_{i,-1}^3}}.$$

Following the lines of proof as in Pesaran [2007], we have

$$T^{-2}\upsilon_i' M \mathfrak{s}_{i,-1}^3 = T^{-2}\upsilon_i' \mathfrak{s}_{i,-1}^3 - \left(\upsilon_i' X D\right)\left(D X' X D\right)^{-1}\left(T^{-2} D X' \mathfrak{s}_{i,-1}^3\right),$$

where

$$D = \begin{pmatrix} T^{-1/2} & 0 & 0 \\ 0 & T^{-1/2} & 0 \\ 0 & 0 & T^{-2} \end{pmatrix}.$$

Moreover,

$$
\begin{aligned}
DX'\upsilon_i &= \begin{pmatrix} T^{-1/2}\tau'\upsilon_i \\ T^{-1/2}\overline{\Delta y}'\upsilon_i \\ T^{-2}\overline{y^3_-}'\upsilon_i \end{pmatrix}, \\
T^{-2}DX'\mathfrak{s}^3_{i,-1} &= \begin{pmatrix} T^{-5/2}\tau'\mathfrak{s}^3_{i,-1} \\ T^{-5/2}\overline{\Delta y}'\mathfrak{s}^3_{i,-1} \\ T^{-4}\overline{y^3_-}'\mathfrak{s}^3_{i,-1} \end{pmatrix}, \\
DX'XD &= \begin{pmatrix} 1 & T^{-1}\tau'\overline{\Delta y} & T^{-5/2}\tau'\overline{y^3_-} \\ T^{-1}\tau'\overline{\Delta y} & T^{-1}\overline{\Delta y}'\overline{\Delta y} & T^{-5/2}\overline{\Delta y}'\overline{y^3_-} \\ T^{-5/2}\tau'\overline{y^3_-} & T^{-5/2}\overline{\Delta y}'\overline{y^3_-} & T^{-4}\overline{y^3_-}'\overline{y^3_-} \end{pmatrix},
\end{aligned}
$$

where, because from Equation A.2.35 and Equation A.2.37, with obvious notation,

$$
\begin{aligned}
\overline{\Delta y} &= \overline{\gamma} f + \overline{\varepsilon} \\
\overline{y^3_-} &= \overline{\delta^3}\overline{y}^3_{-1} + 3\overline{(\delta^2\odot\eta_{-1})}\overline{y}^2_{-1} + 3\overline{\left(\delta\odot\eta^2_{-1}\right)}\overline{y}_{-1} + \overline{\eta^3_{-1}}
\end{aligned}
$$

we have

$$
\begin{aligned}
T^{-1}\tau'\overline{\Delta y} &= \overline{\gamma}T^{-1}\tau' f + T^{-1}\tau'\overline{\varepsilon}, \\
T^{-1}\overline{\Delta y}'\overline{\Delta y} &= \overline{\gamma}^2 T^{-1} f' f + 2\overline{\gamma}T^{-1} f'\overline{\varepsilon} + T^{-1}\overline{\varepsilon}'\overline{\varepsilon}, \\
T^{-5/2}\tau'\overline{y^3_-} &= \overline{\delta^3}T^{-5/2}\tau'\overline{y}^3_{-1} + 3\overline{(\delta^2\odot\eta_{-1})}T^{-5/2}\tau'\overline{y}^2_{-1} \\
&\quad +3\overline{\left(\delta\odot\eta^2_{-1}\right)}T^{-5/2}\tau'\overline{y}_{-1} + T^{-5/2}\tau'\overline{\eta^3_{-1}}, \\
T^{-5/2}\overline{\Delta y}'\overline{y^3_-} &= \overline{\gamma}\overline{\delta^3}T^{-5/2} f'\overline{y}^3_{-1} + 3\overline{\gamma}\overline{(\delta^2\odot\eta_{-1})}T^{-5/2} f'\overline{y}^2_{-1} \\
&\quad +3\overline{\gamma}\overline{\left(\delta\odot\eta^2_{-1}\right)}T^{-5/2} f'\overline{y}_{-1} + \overline{\gamma}T^{-5/2} f'\overline{\eta^3_{-1}} \\
&\quad +\overline{\delta^3}T^{-5/2}\overline{\varepsilon}'\overline{y}^3_{-1} + 3\overline{(\delta^2\odot\eta_{-1})}T^{-5/2}\overline{\varepsilon}'\overline{y}^2_{-1} \\
&\quad +3\overline{\left(\delta\odot\eta^2_{-1}\right)}T^{-5/2}\overline{\varepsilon}'\overline{y}_{-1} + T^{-5/2}\overline{\varepsilon}'\overline{\eta^3_{-1}}, \\
T^{-4}\overline{y^3_-}'\overline{y^3_-} &= T^{-4}\overline{\delta^3}^2\overline{y}^{3\prime}_{-1}\overline{y}^3_{-1} + 6\overline{\delta^3}\overline{(\delta^2\odot\eta_{-1})}T^{-4}\overline{y}^{3\prime}_{-1}\overline{y}^2_{-1} \\
&\quad +6\overline{\delta^3}\overline{\left(\delta\odot\eta^2_{-1}\right)}T^{-4}\overline{y}^{3\prime}_{-1}\overline{y}_{-1} + 2\overline{\delta^3}T^{-4}\overline{y}^{3\prime}_{-1}\overline{\eta^3_{-1}} \\
&\quad +9\overline{(\delta^2\odot\eta_{-1})}^2 T^{-4}\overline{y}^{2\prime}_{-1}\overline{y}^2_{-1} + 18\overline{(\delta^2\odot\eta_{-1})}\,\overline{\left(\delta\odot\eta^2_{-1}\right)}T^{-4}\overline{y}^{2\prime}_{-1}\overline{y}_{-1} \\
&\quad +6\overline{(\delta^2\odot\eta_{-1})}T^{-4}\overline{y}^{2\prime}_{-1}\overline{\eta^3_{-1}} + 9\overline{\left(\delta\odot\eta^2_{-1}\right)}^2 T^{-4}\overline{y}'_{-1}\overline{y}_{-1} \\
&\quad +6\overline{\left(\delta\odot\eta^2_{-1}\right)}T^{-4}\overline{y}'_{-1}\overline{\eta^3_{-1}} + T^{-4}\overline{\eta^3_{-1}}'\overline{\eta^3_{-1}}.
\end{aligned}
$$

As in Pesaran [2007], as $N\to\infty$, terms involving averages over i may be neglected. In particular, it follows from (A.2.38) that $\overline{y}_{-1}$ asymptotically behaves like $\overline{\gamma}s_{f,-1}$. Hence, denoting asymptotic equivalence by $\sim$, and cancelling out lower order terms we have, because $\mathfrak{s}_{i,-1}\sim\sigma_i^{-1}s_{i,-1}$,

$$
T^{-2}\upsilon_i'\mathfrak{s}^3_{i,-1}\sim\sigma_i^{-3}T^{-2}\varepsilon_i' s^3_{i,-1},
$$

and moreover,

$$\begin{aligned}
T^{-1}\tau'\overline{\Delta y} &\sim \overline{\gamma}T^{-1}\tau' f, \\
T^{-1}\overline{\Delta y}'\overline{\Delta y} &\sim \overline{\gamma}^2 T^{-1} f' f, \\
T^{-5/2}\tau'\overline{y_-^3} &\sim \overline{\gamma}^3\overline{\delta^3}T^{-5/2}\tau' s_{f,-1}^3, \\
T^{-5/2}\overline{\Delta y}'\overline{y_-^3} &\sim \overline{\gamma}^4\overline{\delta^3}T^{-5/2} f' s_{f,-1}^3, \\
T^{-4}\overline{y_-^3}'\overline{y_-^3} &\sim \overline{\gamma}^6\overline{\delta^3}^2 T^{-4} s_{f,-1}^{3\prime} s_{f,-1}^3,
\end{aligned}$$

and so, denoting the limits of $\overline{\gamma}$ and $\overline{\delta^3}$ by γ_* and δ_*,

$$DX'XD \sim \Gamma\widetilde{\Psi}_T\Gamma,$$

where

$$\begin{aligned}
\Gamma &\equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & \gamma_* & 0 \\ 0 & 0 & \gamma_*^3\delta_* \end{pmatrix}, \\
\widetilde{\Psi}_T &\equiv \begin{pmatrix} 1 & T^{-1}\tau' f & T^{-5/2}\tau' s_{f,-1}^3 \\ T^{-1}\tau' f & T^{-1} f' f & T^{-5/2} f' s_{f,-1}^3 \\ T^{-5/2}\tau' s_{f,-1}^3 & T^{-5/2} f' s_{f,-1}^3 & T^{-4} s_{f,-1}^{3\prime} s_{f,-1}^3 \end{pmatrix}.
\end{aligned}$$

Similarly, because $\upsilon_i \sim \sigma_i^{-1}\varepsilon_i$,

$$\begin{aligned}
DX\upsilon_i &\sim \sigma_i^{-1}\Gamma\widetilde{q}_T, \\
T^{-2}DX'\mathfrak{s}_{i,-1}^3 &\sim \sigma_i^{-3}\Gamma\widetilde{h}_T,
\end{aligned}$$

where

$$\widetilde{q}_T = \begin{pmatrix} T^{-1/2}\tau'\varepsilon_i \\ T^{-1/2} f'\varepsilon_i \\ T^{-2} s_{f,-1}^{3\prime}\varepsilon_i \end{pmatrix}, \ \widetilde{h}_T = \begin{pmatrix} T^{-5/2}\tau' s_{i,-1}^3 \\ T^{-5/2} f' s_{i,-1}^3 \\ T^{-4} s_{f,-1}^{3\prime} s_{i,-1}^3 \end{pmatrix},$$

and so, as $N \to \infty$,

$$T^{-2}\upsilon_i' M\mathfrak{s}_{i,-1}^3 \sim \sigma_i^{-4}\left(T^{-2}\varepsilon_i' s_{i,-1}^3 - \widetilde{q}_T'\widetilde{\Psi}_T^{-1}\widetilde{h}_T\right).$$

In the same fashion,

$$T^{-1}\upsilon_i' M\upsilon_i \sim \sigma_i^{-2}\left(T^{-1}\varepsilon_i'\varepsilon_i - T^{-1}\widetilde{q}_T'\widetilde{\Psi}_T^{-1}\widetilde{q}_T\right),$$

and

$$T^{-4}\mathfrak{s}_{i,-1}^{3\prime} M\mathfrak{s}_{i,-1}^3 \sim \sigma_i^{-6}\left(T^{-4} s_{i,-1}^{3\prime} s_{i,-1}^3 - \widetilde{h}_T'\widetilde{\Psi}_T^{-1}\widetilde{h}_T\right),$$

implying that for all i, as $N \to \infty$,

$$\begin{aligned}
& \tau_i(N,T) \\
\equiv\ & T^{1/2}\frac{\upsilon_i' M\mathfrak{s}_{i,-1}^3}{\sqrt{\upsilon_i' M\upsilon_i}\sqrt{\mathfrak{s}_{i,-1}^{3\prime} M\mathfrak{s}_{i,-1}^3}} \\
\sim\ & \frac{T^{-2}\varepsilon_i' s_{i,-1}^3 - \widetilde{q}_T'\widetilde{\Psi}_T^{-1}\widetilde{h}_T}{\sqrt{T^{-1}\varepsilon_i'\varepsilon_i - T^{-1}\widetilde{q}_T'\widetilde{\Psi}_T^{-1}\widetilde{q}_T}\sqrt{T^{-4} s_{i,-1}^{3\prime} s_{i,-1}^3 - \widetilde{h}_T'\widetilde{\Psi}_T^{-1}\widetilde{h}_T}}.
\end{aligned}$$

Note that the r.h.s. of this expression does not depend on any nuisance parameters. This completes the proof of Theorem 1.

A.3 Theorem 2

Without loss of generality, set $\sigma_i = 1$. As in Pesaran [2007], as $T \to \infty$ (we use short-hand notation for the integrals so that e.g. $\int W_i^3 dW_i = \int_0^1 W_i(t)^3 dW_i(t)$ and $\int W_i^6 = \int_0^1 W_i(t)^3 dt$, and from now on, $\to$ denotes convergence in distribution)

$$\begin{aligned}
T^{-2}\varepsilon_i' s_{i,-1}^3 &\to \int W_i^3 dW_i, \\
T^{-1}\varepsilon_i'\varepsilon_i &\to 1, \\
T^{-4} s_{i,-1}^{3\prime} s_{i,-1}^3 &\to \int W_i^6, \\
\widetilde{\Psi}_T &\to \begin{pmatrix} 1 & 0 & \int W_f^3 \\ 0 & 1 & 0 \\ \int W_f^3 & 0 & \int W_f^6 \end{pmatrix} \equiv \widetilde{\Psi}, \\
\widetilde{q}_T &\to \begin{pmatrix} W_i(1) \\ W_{fi}(1) \\ \int W_f^3 dW_i \end{pmatrix} \equiv \widetilde{q},\ \widetilde{h}_T \to \begin{pmatrix} \int W_i^3 \\ 0 \\ \int W_f^3 W_i^3 \end{pmatrix} \equiv \widetilde{h},
\end{aligned}$$

where W_i,W_f and W_{fi} are standard Brownian motions. (Moreover, W_i and W_f are mutually independent.) Hence, it follows that $T^{-1} q_T' \Psi_T^{-1} h_T$ may be neglected, and moreover,

$$\begin{aligned}
\widetilde{q}_T' \widetilde{\Psi}_T^{-1} \widetilde{h}_T &\to \widetilde{q}' \widetilde{\Psi}^{-1} \widetilde{h} = q' \Psi^{-1} h, \\
\widetilde{h}_T' \widetilde{\Psi}_T^{-1} \widetilde{h}_T &\to \widetilde{h}' \widetilde{\Psi}^{-1} \widetilde{h} = h' \Psi^{-1} h,
\end{aligned}$$

where

$$\begin{aligned}
\Psi &= \begin{pmatrix} 1 & \int W_f^3 \\ \int W_f^3 & \int W_f^6 \end{pmatrix}, \\
q &= \begin{pmatrix} W_i(1) \\ \int W_f^3 dW_i \end{pmatrix},\ h = \begin{pmatrix} \int W_i^3 \\ \int W_f^3 W_i^3 \end{pmatrix},
\end{aligned}$$

and we find the sequential limit result

$$T^{1/2} \frac{\upsilon_i' M \mathfrak{s}_{i,-1}^3}{\sqrt{\upsilon_i' M \upsilon_i} \sqrt{\mathfrak{s}_{i,-1}^{3\prime} M \mathfrak{s}_{i,-1}^3}} \to \frac{\int W_i^3 dW_i - q' \Psi^{-1} h}{\sqrt{\int W_i^6 - h' \Psi^{-1} h}}. \tag{A.3.44}$$

Along the lines of Pesaran [2007], it may be proved that this is also the joint limit result as $N, T \to \infty$ simultaneously such that $N/T \to k$, where k is a finite and positive constant.

B Critical Values

B.1 Individual NCADF Distribution

Table 13: Critical Values of Individual NCADF Distribution

N	T	1 %	2.5 %	5 %	10 %	N	T	1 %	2.5 %	5 %	10 %
10	10	-5.18	-4.17	-3.50	-2.87	50	10	-5.16	-4.17	-3.52	-2.91
	15	-4.19	-3.60	-3.16	-2.69		15	-4.21	-3.57	-3.15	-2.68
	20	-3.93	-3.44	-3.07	-2.67		20	-4.10	-3.47	-3.11	-2.69
	30	-3.79	-3.38	-3.05	-2.70		30	-3.75	-3.33	-3.00	-2.69
	50	-3.81	-3.41	-3.11	-2.78		50	-3.68	-3.35	-3.04	-2.76
	70	-3.67	-3.39	-3.12	-2.80		70	-3.70	-3.36	-3.07	-2.75
	100	-3.71	-3.39	-3.12	-2.80		100	-3.59	-3.31	-3.09	-2.79
	200	-3.73	-3.40	-3.12	-2.82		200	-3.72	-3.36	-3.10	-2.81
15	10	-5.35	-4.22	-3.52	-2.92	70	10	-5.17	-4.23	-3.52	-2.92
	15	-4.21	-3.64	-3.15	-2.67		15	-4.32	-3.64	-3.22	-2.74
	20	-3.96	-3.42	-3.06	-2.68		20	-3.97	-3.47	-3.10	-2.65
	30	-3.81	-3.36	-3.06	-2.69		30	-3.79	-3.41	-3.06	-2.71
	50	-3.69	-3.32	-3.06	-2.75		50	-3.73	-3.41	-3.11	-2.76
	70	-3.75	-3.41	-3.11	-2.78		70	-3.68	-3.37	-3.05	-2.76
	100	-3.70	-3.38	-3.13	-2.76		100	-3.71	-3.40	-3.10	-2.81
	200	-3.67	-3.37	-3.09	-2.78		200	-3.62	-3.34	-3.11	-2.83
20	10	-5.05	-4.20	-3.47	-2.89	100	10	-4.89	-3.99	-3.39	-2.81
	15	-4.27	-3.63	-3.13	-2.73		15	-4.04	-3.53	-3.16	-2.75
	20	-3.94	-3.39	-3.04	-2.67		20	-3.91	-3.45	-3.05	-2.66
	30	-3.71	-3.39	-3.09	-2.74		30	-3.76	-3.36	-3.06	-2.70
	50	-3.70	-3.28	-3.04	-2.73		50	-3.63	-3.33	-3.04	-2.75
	70	-3.66	-3.35	-3.07	-2.75		70	-3.64	-3.31	-3.01	-2.74
	100	-3.74	-3.38	-3.09	-2.80		100	-3.74	-3.35	-3.10	-2.79
	200	-3.77	-3.40	-3.14	-2.84		200	-3.69	-3.40	-3.11	-2.82
30	10	-5.62	-4.37	-3.55	-2.95	200	10	-5.21	-4.17	-3.42	-2.84
	15	-4.22	-3.62	-3.14	-2.68		15	-4.30	-3.67	-3.21	-2.78
	20	-3.87	-3.42	-3.09	-2.70		20	-3.91	-3.44	-3.11	-2.70
	30	-3.86	-3.42	-3.14	-2.73		30	-3.69	-3.34	-3.04	-2.73
	50	-3.69	-3.37	-3.06	-2.75		50	-3.77	-3.40	-3.10	-2.77
	70	-3.71	-3.32	-3.07	-2.75		70	-3.66	-3.28	-3.08	-2.75
	100	-3.77	-3.32	-3.10	-2.79		100	-3.70	-3.38	-3.11	-2.79
	200	-3.68	-3.37	-3.11	-2.84		200	-3.64	-3.38	-3.14	-2.81

B.2 Panel NCADF Distribution

Table 14: Critical values of Panel NCADF Distribution

N	T	1%	2.5%	5%	10%	N	T	1%	2.5%	5%	10%
10	200	-2.50	-2.40	-2.33	-2.25	50	200	-2.14	-2.09	-2.04	-1.99
10	100	-2.42	-2.31	-2.22	-2.11	50	100	-2.10	-2.05	-2.01	-1.96
10	70	-2.39	-2.27	-2.19	-2.10	50	70	-2.08	-2.03	-1.99	-1.94
10	50	-2.36	-2.26	-2.16	-2.05	50	50	-2.05	-2.00	-1.96	-1.91
10	30	-2.31	-2.20	-2.12	-2.01	50	30	-2.00	-1.95	-1.90	-1.84
10	20	-2.32	-2.20	-2.09	-1.97	50	20	-1.96	-1.90	-1.85	-1.79
10	15	-2.34	-2.19	-2.08	-1.94	50	15	-1.95	-1.88	-1.82	-1.75
10	10	-2.53	-2.34	-2.17	-1.98	50	10	-2.01	-1.91	-1.83	-1.75
15	200	-2.33	-2.25	-2.18	-2.09	70	200	-2.11	-2.06	-2.02	-1.98
15	100	-2.30	-2.22	-2.14	-2.06	70	100	-2.07	-2.03	-1.99	-1.95
15	70	-2.26	-2.19	-2.13	-2.04	70	70	-2.05	-2.00	-1.97	-1.92
15	50	-2.24	-2.16	-2.08	-2.00	70	50	-2.02	-1.98	-1.94	-1.89
15	30	-2.20	-2.11	-2.03	-1.95	70	30	-1.96	-1.91	-1.87	-1.83
15	20	-2.17	-2.09	-2.00	-1.90	70	20	-1.92	-1.87	-1.83	-1.77
15	15	-2.19	-2.08	-1.98	-1.88	70	15	-1.91	-1.84	-1.80	-1.73
15	10	-2.34	-2.18	-2.04	-1.90	70	10	-1.95	-1.88	-1.80	-1.72
20	200	-2.26	-2.19	-2.13	-2.06	100	200	-2.08	-2.04	-2.01	-1.97
20	100	-2.24	-2.16	-2.11	-2.03	100	100	-2.05	-2.01	-1.97	-1.93
20	70	-2.20	-2.13	-2.08	-2.00	100	70	-2.02	-1.99	-1.95	-1.91
20	50	-2.18	-2.11	-2.05	-1.98	100	50	-1.99	-1.95	-1.92	-1.88
20	30	-2.14	-2.07	-2.00	-1.92	100	30	-1.94	-1.89	-1.86	-1.81
20	20	-2.11	-2.03	-1.95	-1.86	100	20	-1.89	-1.84	-1.81	-1.76
20	15	-2.10	-2.00	-1.93	-1.84	100	15	-1.87	-1.82	-1.77	-1.72
20	10	-2.22	-2.09	-1.97	-1.84	100	10	-1.92	-1.85	-1.78	-1.70
30	200	-2.20	-2.14	-2.09	-2.02	200	200	-2.05	-2.01	-1.99	-1.95
30	100	-2.18	-2.11	-2.06	-2.00	200	100	-2.01	-1.98	-1.96	-1.92
30	70	-2.15	-2.09	-2.03	-1.97	200	70	-2.00	-1.96	-1.93	-1.89
30	50	-2.11	-2.05	-2.00	-1.94	200	50	-1.96	-1.93	-1.90	-1.86
30	30	-2.07	-2.00	-1.95	-1.88	200	30	-1.90	-1.87	-1.84	-1.80
30	20	-2.02	-1.95	-1.90	-1.83	200	20	-1.86	-1.81	-1.78	-1.73
30	15	-2.02	-1.94	-1.87	-1.79	200	15	-1.82	-1.78	-1.74	-1.69
30	10	-2.13	-2.00	-1.90	-1.80	200	10	-1.87	-1.80	-1.75	-1.68

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