Advanced Higher Physics: Assignment Support

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Determination of permeability of free space, μ_0

Introduction

Consider the experimental set up shown in Figure 1. This shows a solenoid connected in series with a variable resistor and a waveform generator. A solenoid is a helical winding of wire on a cylinder, usually circular in cross-section. Next to the solenoid is a second coil, which is referred to as the Search Coil.

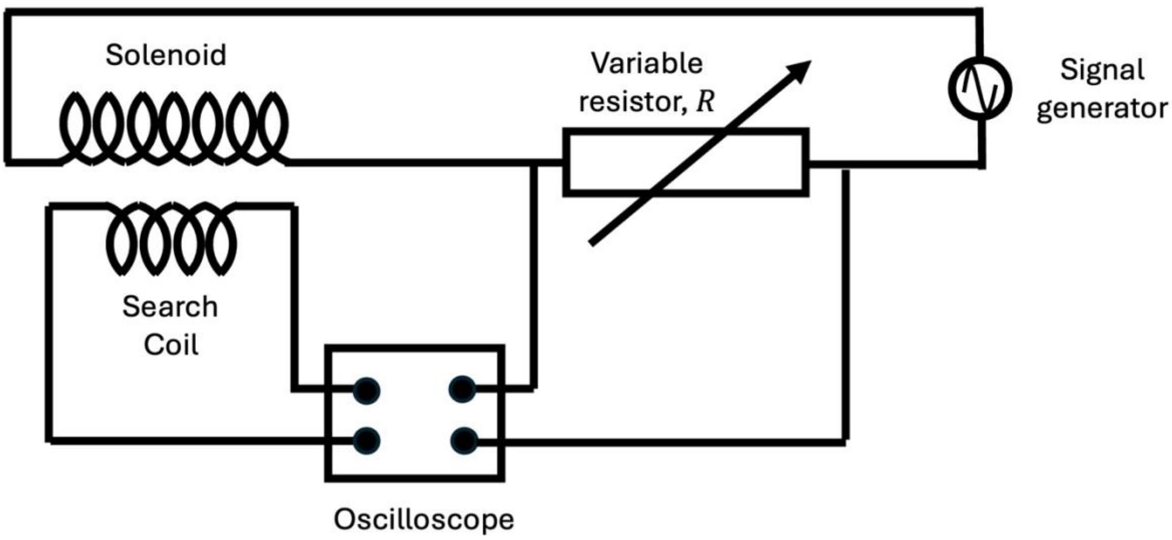

Figure 1: Experimental set up

If a current I is passed through this solenoid, a magnetic field of magnitude B will be induced. If there are n turns per metre in the solenoid, then we can express the magnetic field as

$$B = \mu_0 n I$$

[1]

where μ_0 is the permeability of free space. The accepted value is

$$\boxed{\mu_0 = 1.257 \times 10^{-6}\ \mathrm{WbA^{-1}m^{-1}} \approx 4\pi \times 10^{-7}\ \mathrm{WbA^{-1}m^{-1}}}$$

Consider passing an a.c. current, with frequency f, through the solenoid. The current can be expressed as

$$I = I_0 \sin(2\pi f t)$$

[2]

We can then say that the induced magnetic field is

$$B = \mu_0 n I_0 \sin(2\pi f t)$$

[3]

If a second coil, which is referred to here as the "Search Coil", is placed close to the solenoid, then the alternating magnetic field on the solenoid will create an alternating induced voltage in the Search Coil, V_{SC}.

$$V_{SC} = \frac{dB}{dt} \times N \times A$$

[4]

where A is the cross-section area of the Search Coil, and N is the number of turns in the Search Coil. Substituting [3] into [4] we find

$$V_{SC} = 2\pi f I_0 \cos(2\pi f t) \times \mu_0 n \times N \times A$$

from which we can see that the peak voltage is

$$V_{SC}^{peak} = 2\pi f I_0 \mu_0 n N A$$

[5]

For the variable resistor, meanwhile, we can write

$$V_R = IR = I_0 \sin(2\pi f t)\, R$$

and we can see that

$$V_R^{Peak} = I_0 R$$

[6]

By varying the frequency of the current, it is possible to alter the voltages across the Search Coil and the resistor until they are equal. At that point

$$V_{SC}^{peak} = V_R^{Peak}$$

$$\Rightarrow 2\pi f I_0 \mu_0 n N A = I_0 R$$

$$\Rightarrow R = 2\pi f \mu_0 n N A$$

[7]

In [7] the only variables are R and f, so a graph of R against f will have gradient

$$\boxed{gradient = 2\pi\mu_0 n[NA]}$$

[8]

from which a value for μ_0 can be determined. $[NA]$ is known as the "effective value" of NA. Our model assumes that every coil in the search coil has identical radii, but in reality they vary a bit. $[NA]$ is then a truer representation of the apparatus. For the search coil used in this experiment, the value for $[NA]$ has been determined as (0.92 ± 0.01) turns $\times$ m^2. This was determined by measuring the e.m.f. induced in the coil by a known change in magnetic induction. The search coil has 600 turns and the long solenoid has ~1270 turns per metre.

[8] represents the gradient expression for an infinitely long solenoid. In reality, adjustments need to be made to this relationship to take into account the fact that the solenoid used is very much finite. Figure 2 below shows a finite cylindrical solenoid of length L and diameter d.

The magnetic induction at any pint P (i.e. at any point along the central axis), B_P is given by

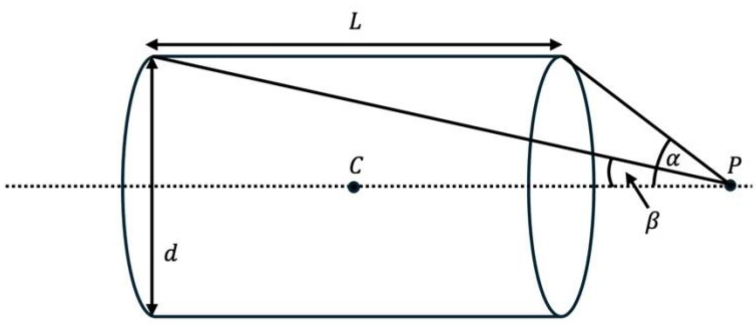

Figure 2: A finite real solenoid

$$B_P = \frac{\mu_0 n I}{2}[(\cos\beta) - \cos(\alpha)]$$

where α and β are the angles subtended at P by the two ends of the solenoid as shown.

At the centre of the solenid, C, we find that $\alpha = 180° - \beta$. Figure 3 shows this.

Substituting for α in the above equation gives us

$$B = \frac{\mu_0 n I}{2}[\cos(\beta) - \cos(180° - \beta)]$$

$$= \frac{\mu_0 n I}{2} 2\cos(\beta)$$

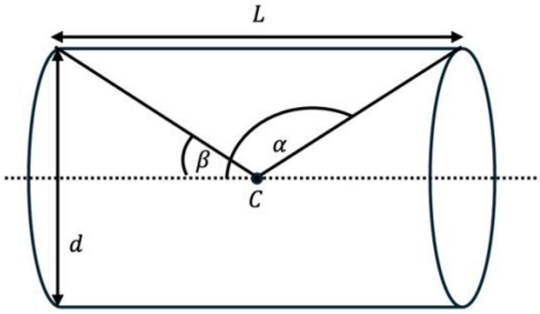

Figure 3: Angles subtended from centre

$$\Rightarrow B = \mu_0 n I \cos\beta$$

[1*]

If we let $L \to \infty$, we see that $\beta \to 0$ and [1*] becomes [1]. In other words as the solenoid gets larger, the correction factor becomes smaller.

If we use [1*] instead of [1], and work through the earlier analysis, we find that for a real solenoid,

$$R = 2\pi f \mu_0 n \cos(\beta)\,[NA]$$

[7*]

and so a graph of R against f will have ...

$$\boxed{gradient = 2\pi\mu_0 n \cos(\beta)\,[NA]}$$

TO determine the value of $\cos(\beta)$, we note that as shown in Figure 3

$$\tan(\beta) = \frac{d/2}{L/2} = \frac{d}{L}$$

[8]

So to determine the correction factor needed for a given set of equipment we must measure the length of the solenoid L and its diameter d. [8] then gives us β, which in turn gives us our correction factor $\cos(\beta)$.

Notes on equipment

Equipment list

- Oscilloscope
- Signal generator
- Solenoid kit

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