Advanced Higher Physics: Assignment Support

Astronomy & Physics Education Group

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Determination of permittivity of free space, ε_0

Introduction

The capacitance C of an air-spaced parallel plate capacitor of plate overlap A and plate separation d is given by the following equation:

$$C = \varepsilon_0 \frac{A}{d}$$

[1]

where ε_0 is the permittivity of free space. This assumes we have an ideal capacitor, where the electrostatic field is entirely contained within the capacitor, dropping to zero immediately outside the plates

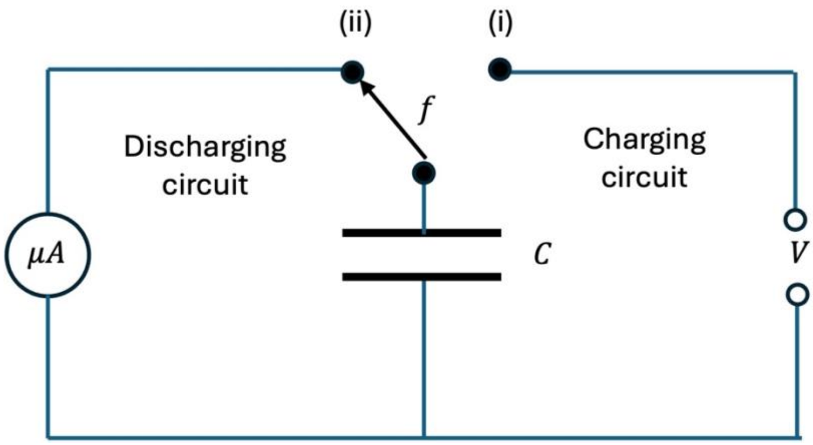

Figure 1: Charging/Discharging Circuit

Figure 1 shows a circuit that allows for an air-spaced parallel plate capacitor to be repeatedly charged and discharged.

The capacitor charges when the switch is connected to point (i); it discharges when the switch is connected to point (ii). If the charging voltage is V, then the capacitor will store a charge $Q = CV$, which it then loses when the switch moves to point (ii). In this circuit the switch works such that the capacitor is discharged f times per second. This means that the charge passing through the discharge circuit per second will be $f \times Q = f \times CV$ Coulombs. In other words, there is an effective current, I, flowing in the discharge circuit, where

$$I = fCV$$

[2]

(Remember: $I = \frac{Q}{t}$; here we have charge $Q = CV$ and f is a frequency, i.e. $f = \frac{1}{t}$.)

If the discharging current I is measured at a fixed V over a range of frequencies f, then a graph of I against f can be plotted. [2] then tells us that the gradient of such a plot is then equal to CV, which allows the capacitance to be determined. And if the dimensions of the capacitor are known, then [1] means we can determine a value for ε_0.

However, equation [1] does not take into account the "fringing error" due to the non-uniformity of the electric field at the edges of the capacitor plates. [1} assumes that the electrostatic field of the capacitor is uniform and contained entirely within the plates. In reality, this is not always the case. To take these effects into account, we need to adapt equation [1].

For a square plated capacitor of plate length L and plate separation d, to account for the fringing error a corrector factor of approximate value $\left[1 + \frac{2d}{\pi L}\left(1 + \ln\left\{\frac{\pi L}{d}\right\}\right)\right]$ has to be applied to the basic equation, so that we have an improved equation for capacitance:

$$C = \left[1 + \frac{2d}{\pi L}\left(1 + \ln\left\{\frac{\pi L}{d}\right\}\right)\right] \varepsilon_0 \frac{A}{d}$$

[3]

This correction factor is usually quite small ~5 %.

Appendix: Apparatus

Figure 2 shows the arrangement used to allow high frequency charging and discharging of the air-spaced capacitor.

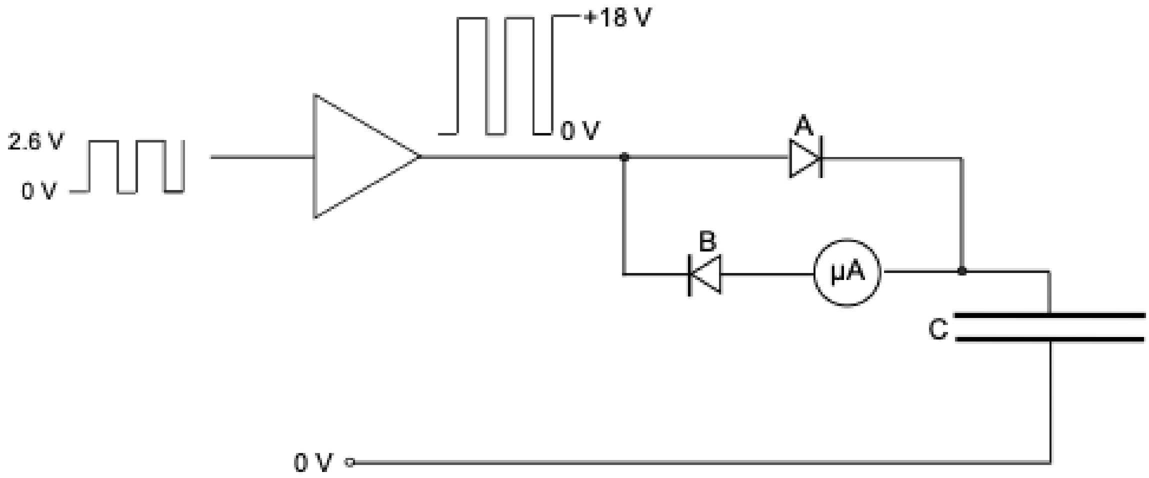

Figure 2

A square-wave from a waveform generator, offset to run between 0 Vand 2.6 V is fed to an amplifier which amplifies to a maximum of around 18 V.

- When the output of the amplifier goes high, the capacitor is charged through upper diode A.
- When the amplifier output goes low, the capacitor discharges through the microammeter and the lower diode B, the amplifier sinking the discharge current.

This process is repeated at the frequency of the waveform generator which operates over the range $1\ \mathrm{kHz} \rightarrow 22\ \mathrm{kHz}$. At these frequencies the microammeter gives a steady reading.

The system is arranged in two components:

(1) The **power supply box** which contains the stabilised dual rail voltage supply. The rail voltage can be altered over the approximate range $6\ \mathrm{V} \rightarrow 18\ \mathrm{V}$ by adjusting the control knob – turning it clockwise to increase the voltage. The output of the waveform generator is connected to the power supply box and a cable from the

power supply box feeds the waveform generator signal and the dual voltage supply to the amplifier and switching circuit board.

(2) The **amplifier and switching circuit board** which contains the amplifier and diode circuit of Figure 2. The air-spaced parallel plate capacitor and the digital microammeter are connected to the board circuit as shown in Figure 1. The upper position of the 3-way switch allows the amplified square wave to be monitored on the oscilloscope so that its peak value can be measured. The middle position isolates the oscilloscope from the square wave – this is the position for measuring the capacitor current. The lower position connects the rail voltage to the oscilloscope.

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