Advanced Higher Physics: Assignment Support

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Determination of $\frac{e}{m}$ of an electron

Introduction

The electron deflection tube is a modern variant of the tube first used by J. J. Thomson to determine the value of the charge-to-mass ratio, $\frac{e}{m}$ for an electron. The electron deflection tube and Helmholtz coils can be used in several ways to determine the value of $\frac{e}{m}$. Here the coils are used to produce a magnetic field that deflects a beam of the electrons into a circular arc. Careful measurements allow the radius of curvature of the arc to be determined and hence a value for $\frac{e}{m}$.

Basic theory

An electron beam can be deflected by a magnetic field $\vec{B}$ produced by the Helmholtz coils. The magnitude of $\vec{B}$ can be varied by changing the current I passing through them. The force, $\vec{F}$, experienced by an electron of charge e travelling with a velocity $\vec{v}$ is perpendicular to the plane defined by $\vec{B}$ and $\vec{v}$, and is given by

$$F = -e(\vec{v} \times \vec{B}).$$

When $\vec{v}$ is perpendicular to $\vec{B}$, the force $\vec{F}$ causes the electrons to move in a circular path of radius r. This is shown in Figure 1.

When an electron of charge of magnitude e is accelerated from rest by a potential difference V its kinetic energy increases to $\frac{1}{2}mv^2$. i.e.

$$eV = \frac{1}{2}mv^2$$

[1]

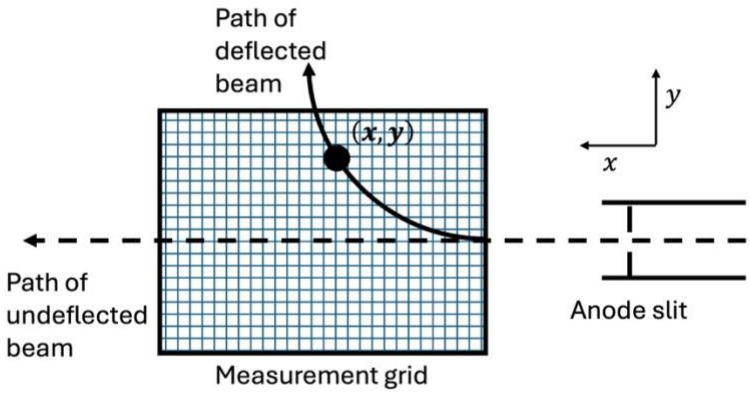

Figure 1: Path of electron beam

If electrons move so that their velocity $\vec{v}$ is at right angles to $\vec{B}$, then Newton's 2nd law tells us that the magnitude of the force, F ...

$$F = evB\sin 90 = evB = \frac{mv^2}{r}$$

[2]

Combining [1] and [2] we find

$$\frac{e}{m} = \frac{2V}{B^2r^2}$$

The magnetic field $\vec{B}$ is related to the current in the Helmholtz coils, I, by $B = kI$, where k is a constant:

$$k = \frac{8n\mu_0}{5a\sqrt{5}}$$

where n is the number of coils and a is their radius. (See appendix for derivation.). So ...

$$\boxed{\frac{e}{m} = \frac{2V}{k^2I^2r^2}}$$

By fixing the radius of curvature of the beam, r, we have a linear relationship between V^2 and I: $V = AI^2$, where

$$\boxed{A = \left(\frac{e}{m}\right)\frac{k^2r^2}{2}}$$

Hence a series of measurements of the current in the coils, I, versus different accelerating voltages V for a fixed radius of curvature of the electron beam should give a straight line graph, whose gradient is related to $\frac{e}{m}$.

Apparatus

In this experiment electrons are accelerated through a potential difference, V, and pass through a slit in the anode of an electron gun to form a beam of electrons of defined speed v. The electrons then enter a uniform magnetic field in a direction perpendicular to the magnetic vector $\vec{B}$ so that the force they experience causes t hem to move in a circular path. The components of the apparatus – illustrated in Figure 2 – are ...

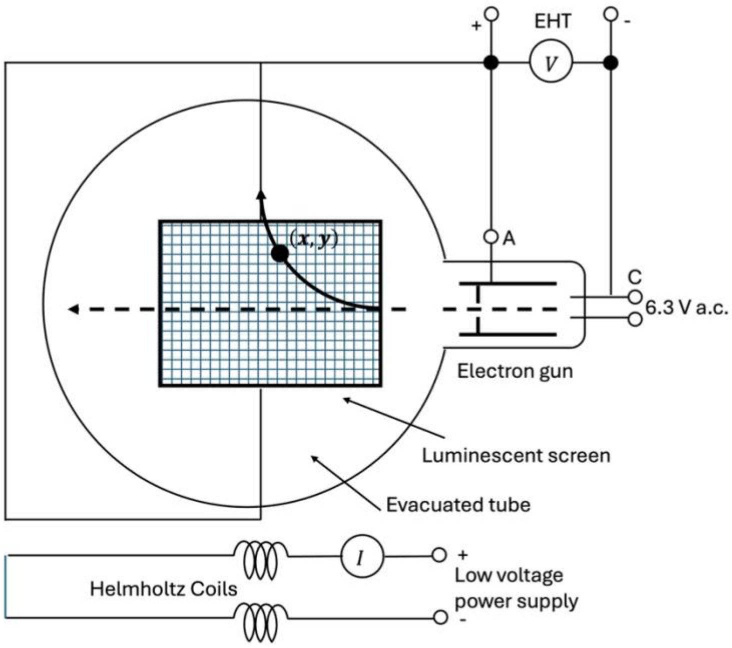

Figure 2: Equipment set up

- An evacuated electron beam tube: an electron gun (that emits a ribbon beam of electrons) and a luminescent screen. The electrons can be accelerated by applying a positive potential between the cathode C and the anode A.
- Helmholtz coils: used to produce the magnetic field. For the kit used here, the number of turns in the coils is $n = 320$ and the radius of the coils is 6.8 cm.
- A power supply with an integrated analogue voltmeter: used for heating the filament in the electron gun and supply a high voltage to accelerate the electrons.
- A low voltage supply: controls the current in the Helmholtz coils. This has integrated digital meters for reading the supplied voltage and current.

Checking systematic uncertainty

The applied magnetic field causes the electron beam to describe part of a circular path whose radius we want to determine. Three points are required to specify a circle uniquely. We know a point O which is at the exit aperture of the anode slit. We require to measure only one point to calculate the radius, r, since the third point can be anywhere along a tangent to the circle at O. This is determined by reading a coordinate (x, y) on the scale of the luminescent screen. The radius of curvature r of the electron path is then given by

$$r = \frac{x^2 + y^2}{2y}$$

[4]

However, the radius r cannot simply be obtained from [4] because of a feature of the electron beam tube: due to the difficulty in manufacture of aligning the electron gun there is usually a systematic error in the apparatus arising from the positioning of the scale with respect to the anode slit.

To measure this uncertainty so that we can correct for it, the following must be done:

- Check the apparatus is connected as in Figure 2
- Turn on the power supply for the electron beam tube and adjust the high voltage to obtain a straight beam on the screen.
- Turn on the power supply to the coils and adjust the voltage supplied to obtain a curved beam.
- Although it is the current in the coils which is the variable of interest, this is controlled by altering the voltage.
- Check any asymmetry in the y deflection by selecting a point (x_1, y_1) through which the beam passes. Reverse the direction of I, making sure that the magnitude of the current is the same. This should produce a deflection through $(x_1, -y_1)$.
- From the $\pm y$ values determine where the x-axis should be for this value of y_1. You should now be able to correct the observed value of y_1 to give Y_1, the value corrected for the systematic error in y at position x_1.
- [Note the contention used here is that uncorrected position variables are in lower-case, whilst the corrected position variables are in upper case.]

- Repeat this procedure for another point (x_2, y_2) on the same electron trajectory. It is better to choose the second point as far away from the first point as possible. This will give you Y_2. The x position measurements, x_1 and x_2 will both be affected by the same offset Δx which is the difference between the origin of the scale on the screen and the slit in the anode. By inspecting the apparatus this must only be a few mm and positive.

- For this situation.

$$r = \frac{(x_1 + \Delta x)^2 + Y_1^2}{2Y_1} = \frac{(x_2 + \Delta x)^2 + Y_2^2}{2Y_2}$$

This will give you a quadratic in Δx – solve this and choose the smaller of the two roots. Add Δx to all readings of the coordinate x to give X.

- From the corrected values X_1 and Y_1 (say) you can calculate r:

$$r = \frac{X_1^2 + Y_1^2}{2Y_1}$$

Appendix – Helmholtz Coils

The following analysis shows that placing the two Helmholtz coils a distance apart exactly equal to the radius of the coils produces a nearly constant field between them.

The magnetic field $\vec{B}$ on the axis of a circular coil (of mean radius a, number of turns n and carrying current I) at a distance z from the centre of the coil is

$$B = \frac{\mu_0 n I a^2}{2\sqrt{a^2 + z^2}} = \frac{C}{\sqrt{a^2 + z^2}}$$

where $C = \frac{\mu_0 n I a^2}{2}$ and $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{Hm}^{-1}$.

The direction of this magnetic field is along the z-axis.

We wish to find the separation of a pair of similar coils, parallel and coaxial and carrying equal currents in the same direction, so as to give nearly constant magnetic field between them.

Let the combined magnetic field of the two coils be B_T.

$$B_T = B(z_1) + B(z_2)$$

It can be shown that $\frac{dB_T}{dz} = 0$ at $z = 0$ where $z_1 = -z_2$

Also, $\frac{d^2 B_T}{dz^2} = 0$ at $z = 0$ where $|z_1| = |z_2| = \frac{a}{2}$

In otherwords, the separation of the coils should be equal to their radius a. Hence for a pair of such coils connected in series, the combined magnetic field on their axes at a distance of $\frac{a}{2}$ from each of their centres is

$$B_T = \frac{8 n \mu_0 I}{5a\sqrt{5}} = kI$$

where $k = \frac{8 n \mu_0}{5a\sqrt{5}}$.