What does it all mean?

Uncovering the research that decorates the Maths & Stats Building

The Writing on the Wall

The School of Mathematics & Statistics building is covered in complicated equations, diagrams and even a data set, but what do they mean?

These aren't just random formulas; they are all taken from the cutting-edge research that is being done right now by the mathematicians and statisticians working inside

From the deepest questions in pure mathematics to the mathematical and statistical models that help us understand the world we live in, each part of our building tells a story of curiosity, discovery, and the ongoing quest for knowledge.

Topological quantum field theory

The plumbing of spacetime

What it means: The pictures show the axioms (rules) of a two-dimensional topological quantum field theory. Each diagram should be interpreted as the worldlines (trajectories) of closed strings. Topological quantum field theories arose in high energy and mathematical physics in order to describe the quantum nature of spacetime while neglecting any of its metric properties (which are controlled by gravity). Today they are used to compute topological invariants in mathematics and in quantum computing.

Fisher's Iris Dataset

A classic dataset from statistics.

Species $\approx f$(Sep. L, Sep. W, Pet. L, Pet. W)

What it means: The patterns around our entrance show one of the most famous datasets in all of statistics: Ronald A. Fisher's Iris Dataset. The data consists of four measurements (sepal length, sepal width, petal length, and petal width) for 150 iris flowers, belonging to three different species. This 4-dimensional data is shown as a scatterplot matrix — a 4x4 grid of plots where each panel shows the relationship between two of the measurements, with each dot representing a single flower. The distinct clusters you can see in the plots are the three species, showing how certain pairs of measurements make it easy to statistically distinguish between the species, which is why it remains a classic example for classification in statistics and machine learning.

Braid Groups

Mathematical notation for studying groups.

What it means: Displayed are braid diagrams which graphically encode the mathematical structure of a "group" by gluing two diagrams vertically. The diagram on the very left shows the braiding of two strands and the pictures on the right show more complicated braids obtained by gluing. (There is also a "flag pole" around which the braids can wind.) These groups have their origin in topology and form a central part of an area of pure maths, called geometric group theory, but they also have applications in mathematical physics and quantum computing.

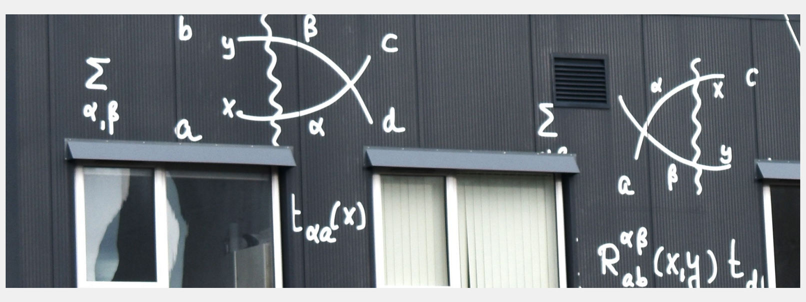
Magnetic helicity

Understanding plasma behaviour

$\Upsilon(\boldsymbol{B}) = \int_\Omega \boldsymbol{A}\cdot\boldsymbol{B} - \sum_{i=1}^n \oint_{\gamma_i} \boldsymbol{A}\cdot\boldsymbol{t}_i \int_{\Sigma_i} \boldsymbol{B}\cdot\boldsymbol{n}_{\Sigma_i}$

What it means: Magnetic helicity is an important invariant of magnetohydrodynamics, a theory of plasmas that combines fluid dynamics and electromagnetism. Helicity is closely linked to the topology of the magnetic field, i.e. the connectivity of field lines. The topology of the domain Ω also plays a role in the definition of helicity and the summation term above encodes the domain topology. Nontrivial domain topologies have important practical applications, such as plasma confinement in a tokamak, an experimental machine for harnessing the energy of fusion.

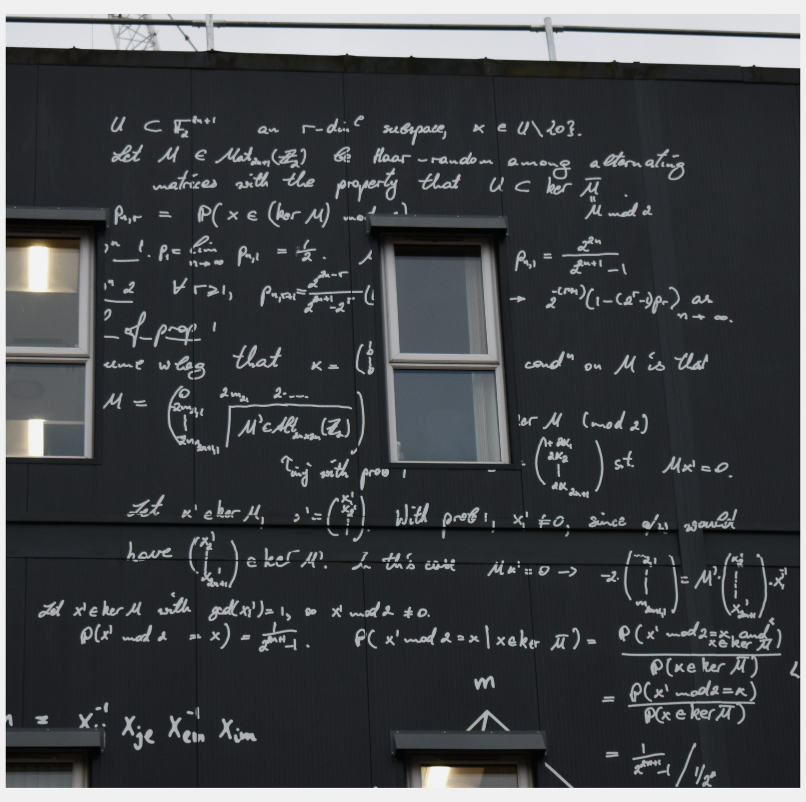
Yang-Baxter Equation & Quantum Groups



A visual language for quantum groups.

What it means: The Yang–Baxter equation is an important equation in mathematical physics, describing two equivalent ways of factorising a single three-particle scattering event into a sequence of two-particle scatterings. Finding solutions to the Yang–Baxter equation is a major area of research, with solutions providing examples for many models, from quantum spin chains to superconducting systems. It is closely related to the braid relation in knot theory and the representation theory of quantum groups which describe the symmetries of quantum systems. The diagram on the building is a way of encoding the relations of such a quantum group.

Diophantine Equations to Random Matrices

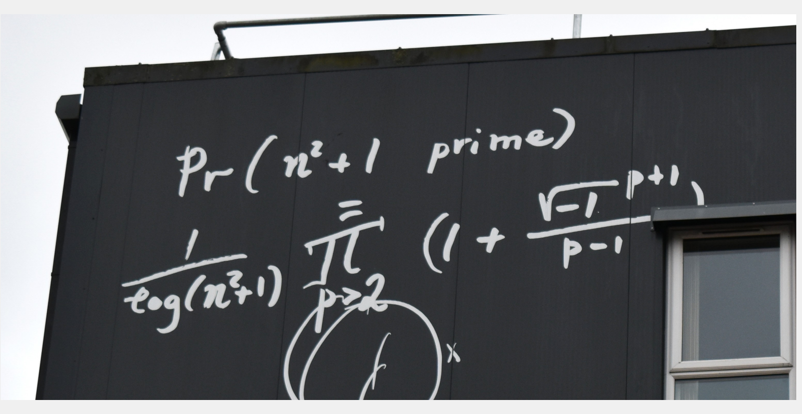


Integer solutions to equations.

Let $U \subset E_k^n$ be an r-dim subspace...

What it means: For how many integers d does the equation $d\,y^2 = x^4 + 1$ have solutions in integers x, y? For example if $d = 2$, then $x = y = 1$ is a solution because $2 \times (1)^2 = (1)^4 + 1$, while if $d = 3$, it turns out that there are no solutions. As we go through the integers, how often do we encounter an integer d for which the equation has solutions? The answer turns out to be surprisingly subtle. The maths on this part of the building works out a somewhat strange connection between this problem and random matrices, not just for this equation but for infinitely many others of a similar kind.

The Probability of Primes

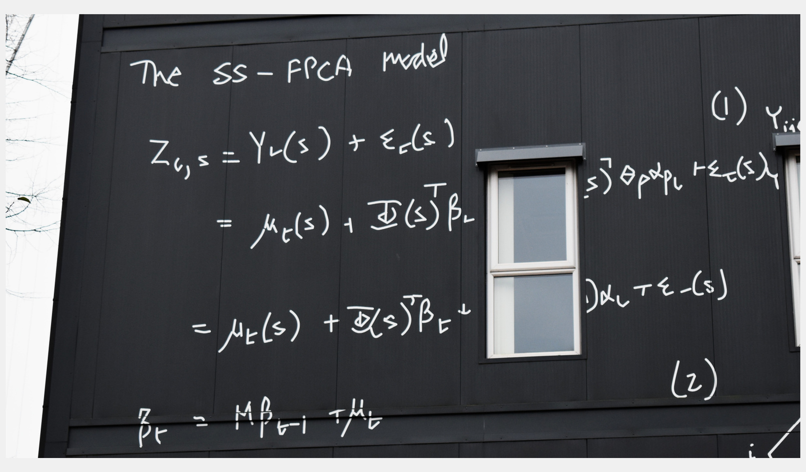


Exploring the probability that $n^2 + 1$ is prime.

$$\Pr(n^2+1 \text{ prime}) = \frac{1}{\log(n^2+1)} \prod_{p>2} \left(1 - \frac{\sqrt{-1}^{\,p+1}}{p-1}\right)$$

What it means: Gauss discovered that an integer of size x is prime with probability inversely proportional to the natural logarithm of x. But what is the probability that the first integer after a perfect square is prime? Examples like 5, 17, 101, and 90001 show that such numbers can be prime. Computational data suggest that this probability is about 1.3 times higher than Gauss's prediction. Bateman and Horn explained the factor 1.3 as the product of densities over all primes shown in the picture. However, as yet there is no rigorous proof for this idea.

Functional Principal Component Analysis

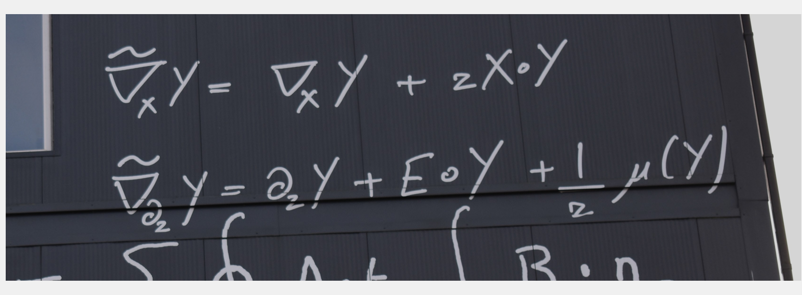


Analysing data in the form of curves or functions.

$$Z_{t,s} = \mu_t(s) + \Phi(s)^\top \beta_t + \Phi(s)^\top \Theta \alpha_t + \varepsilon_t(s)$$

What it means: Not all data are simple numbers. Sometimes, data points are entire functions, like a person's growth curve over time, brainwave signals, or daily temperature profiles. Functional Data Analysis provides tools to understand this type of data. In this state-space version of Functional Principal Component Analysis for spatiotemporal data, the equation shown breaks down the observed dynamic process $Z_{t,s}$ at time point t and location s into a mean function $\mu_t(s)$ plus combinations of spatial basis functions $\Phi(s)$. The dynamic component β_t can evolve over time and follows an autoregressive model: $\beta_t = M\beta_{t-1} + u_t$, so the model captures temporal dependence as well as the main functional patterns in the data. It's a powerful way to find patterns in complex, high-dimensional datasets.

The Shape of Space and Quantum Fields

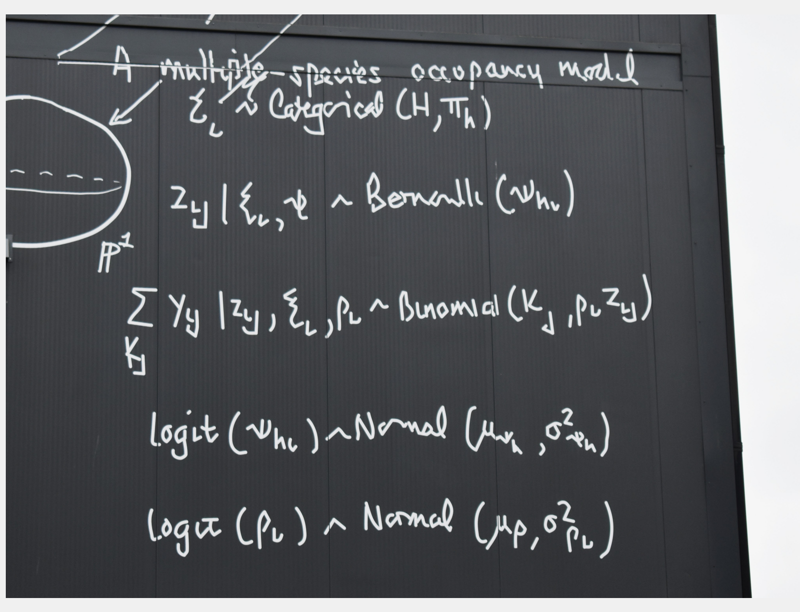


Drawing curves on curved surfaces.

$$\tilde{\nabla}_Y Y = \nabla_Y Y + \frac{1}{2}\mu(Y)$$

What it means: How many straight lines can you draw through two points? Work on generalisation of this simple problem to arbitrary curves and number of points got stuck until, by merging ideas from quantum field theory, the solution was found, and this solution opened a whole new area of research. To understand the shape of an object - put a quantum field theory on it, and study that. The equation describes the basic object in this theory - a Frobenius manifold - which encodes information about shape into the observables of a quantum field theory.

Environmental and Ecological Statistics



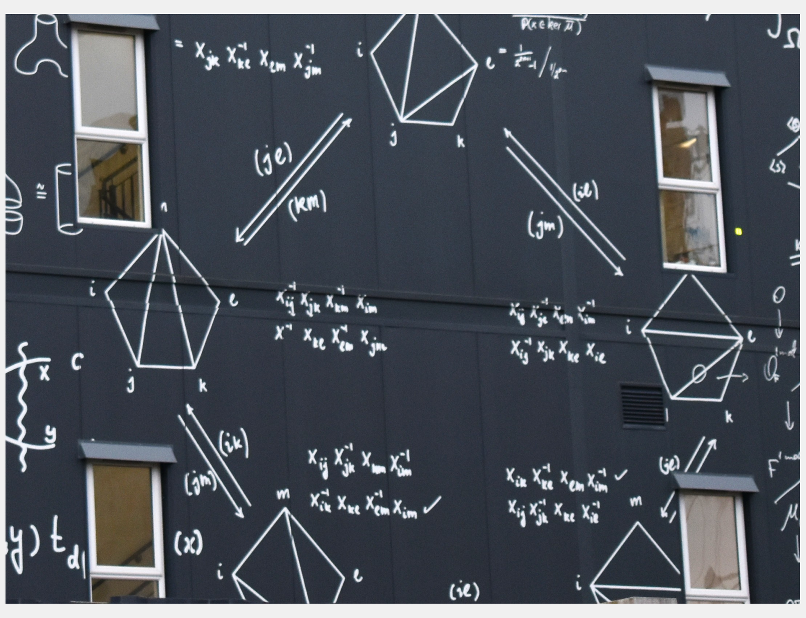
Estimating species presence from surveys with imperfect detection.

$z_{ij} \mid \xi_i, \psi \sim$ Bernoulli(ψ_{ij})

$Y_{ij} \mid z_{ij}, \xi_i, p_i \sim$ Binomial$(K_j, p_i z_{ij})$

What it means: How do we know if a rare animal lives in a forest? Just because we don't see it doesn't mean it isn't there. This statistical model uses a hierarchical structure: an indicator z_{ij} describes whether species i is present at a site j, and the survey data Y_{ij} models detections of species i at site j over K_j sampling occasions. The last part adds a further layer by modelling the probability parameters ψ_{ij} and p_i on the logit scale with Normal distributions. This is crucial for conservation and environmental science, allowing researchers to make more accurate assessments of biodiversity from imperfect survey data.

Triangulations and Cluster Algebras



The triangulations of a pentagon.

$$\sum X_{ij}^k X_{jk}^l X_{ki}^m \ldots$$

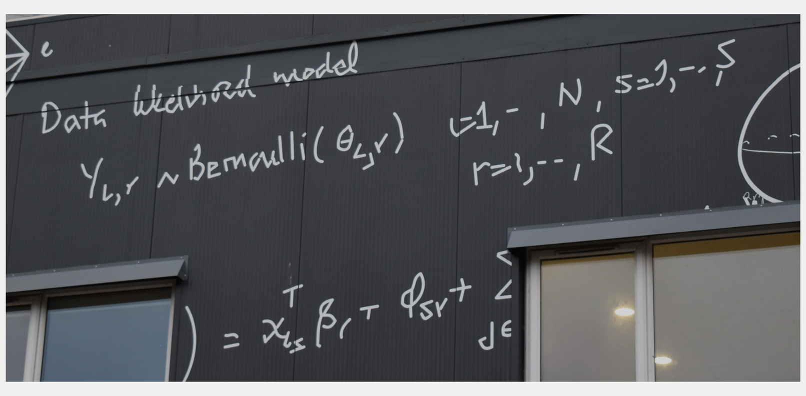
What it means: Shown are the five possible triangulations of a pentagon. For each quadrilateral in a triangulation, one obtains the Ptolemy relation for the sides and diagonals. The associated cluster algebra is an abstract mathematical object, called a ring, where all these Ptolemy relations hold at the same time. Here each triangulation forms a "cluster", an initial seed from which all others can be obtained by flipping the diagonals in a quadrilateral. The theory of cluster algebras is connected to many areas of mathematics, including Lie theory, combinatorics, and representation theory, and has applications in physics.

The Art of Tessellation

Repeated patterns that cover a surface perfectly.

What it means: Mathematical tiling theory examines collections of shapes that cover Euclidean space completely, without gaps or overlaps. Some of these collections are *aperiodic*, meaning that any tiling they form lacks translational symmetry, so an infinite copy of the pattern will align with the original in exactly one position only. The tilings on our floors have this property and does so using just a single prototile. The lines you see are matching rules, which must continue seamlessly from one tile to the next along with a missing colour rule. Recently, the hat tile achieved this same property using a single shape without relying on matching rules, a remarkable breakthrough.

Bayesian Hierarchical Models



A statistical model with multiple layers of probability.

$Y_{i,r} \sim$ Bernoulli$(\theta_{i,r})$

logit$(\theta_{i,r}) \sim$ Normal(μ_r, σ_r^2)

What it means: This "data likelihood model" is a sophisticated Bayesian hierarchical model. It models data at multiple levels. The first line says the observed data $(Y_{i,r})$ is like a simple coin flip, but the crucial part is that the probability $(\theta_{i,r})$ of that coin flip is not fixed but is itself drawn from another probability distribution. This allows statisticians to model variation across different groups and share information between them, leading to more robust and accurate conclusions, especially with limited data.

Contact Information & Credits

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www.gla.ac.uk/schools/mathematicsstatistics

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