

Dynamic Option Adjusted Spread and the Value of Mortgage Backed Securities

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Abstract

We extend a reduced form model for pricing pass-through mortgage backed securities (MBS) and provide a novel hedging tool for investors in this market. To calculate the price of an MBS, traders use what is known as option-adjusted spread (OAS). The resulting OAS value represents the required basis points adjustment to reference curve discounting rates needed to match an observed market price. The OAS suffers from some drawbacks. For example, it remains constant until the maturity of the bond (thirty years in mortgage-backed securities), and does not incorporate interest rate volatility. We suggest instead what we call dynamic option adjusted spread (DOAS), which allows investors in the mortgage market to account for both prepayment risk and changes of the yield curve.

Keywords: Asset pricing, Mortgage Backed Securities, Term Structure.

JEL classification numbers: C23, G34

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1. Introduction

Mortgage Backed Securities (MBS) are securities collateralised by residential mortgage loans. The MBS market has grown to become the largest fixed income market in the United States. Probably one of the reasons of this enormous growth was the higher return paid by these securities and the perception that most of them carried a lower risk than other fixed income securities¹. However, although the market for MBS was very dynamic and many studies have been interested in pricing these financial instruments (see for example Longstaff, 2004 and Chen, 2004), there are still quite a few issues concerning the risk management of these securities.

Because of the borrowers' prepayment option in the underlying mortgage loans, mortgage-backed securities have characteristics similar to those of callable bonds. Unlike callable bonds for which the issuers' refinancing strategies are assumed to be close to optimal, mortgage borrowers may be slow to refinance when it would be financially favourable and sometimes prepay when it is financially unfavourable.

Investors in mortgage-backed securities hold long positions in non-callable bonds and short positions in call (prepayment) options. The non-callable bond is effectively a portfolio of zero coupon bonds, and the call option gives the borrower the right to prepay the mortgage at any time prior to the maturity of the loan. Therefore, the value of the MBS is the difference between the value of the non-callable bond and the value of the call (prepayment) option. In the market place, dealers generally price the mortgage by pricing these two components separately.

To evaluate the call option, the Option-Adjusted Spread methodology uses option pricing techniques. When the option component is quantified and taken away from the total yield spread, the yield to maturity of a non-benchmark bond can be compared to a risk-free of a benchmark security². Any model used to value a MBS should be able to value the non-callable component of a mortgage and the call option component. *Ceteris paribus*, given that interest rate and prepayment risks have been accounted for, and incorporated in the theoretical model, one would expect the theoretical price of an MBS to be equal to its market price. If these values are not equal, then market participants demand compensation for the unmodeled risks.

¹ The market turbulence of these months shows that it is not the case. However in this study we only consider pricing pass-through MBS and assume that these are guaranteed by the US Treasury. We leave credit-liquidity risk and the pricing of MBS on the agenda for future research.

² See the option adjusted spread application in this paper.

The difference in values might be due to unmodeled risks which are attributable to the structure and liquidity of the bond. One of these unmodeled risks is the forecast error associated with the prepayment model. For example, the actual prepayment may be faster or slower than what the model predicts. In this case, the OAS is the market price for the unmodeled risks. Because there is no agreement on how to model prepayments among mortgage holders and many different interest rate models exist, option-adjusted spread calculation suffers from the lack of a standard term.

The academic literature in this area has mainly focused on modelling OAS dynamics such that the embedded mortgage call option price can be estimated and consequently the mortgage priced (see for example, Dunn and Spatt (1986), Liu and Xu (1998), Schwartz and Torous (1992) amongst others). Although helping to clarify a number of issues concerning the pricing of MBS, these models are not used in practice since in many cases they are unable to fit the observed market prices. On the other hand academics and practitioners have instead opted for econometric models to estimate the parameters of interest to calibrate reduced form models and price MBS (see for example Chen, 2004)³. Therefore reduced form models seem to be the ideal way of pricing MBS. However, since most of these models are proprietary models their functional form is not known in the market. Furthermore as mentioned above, these models may be miss-specified since they assume a constant option adjusted spread over the lifetime of the mortgage.

This paper makes two important contributions to the literature on pricing pass-through MBS. Firstly, we propose the so called dynamic option adjusted spread (DOAS) which, as we shall explain in Section 5, accounts for volatility shifts in the interest rates term structure. Secondly, we show that the DOAS can also be used as a hedging tool by investors in this market.

The paper is organised as follows: we discuss the MBS model used in this study in Section 2, Section 3 discusses the interest rate model and its calibration, Section 4 presents a numerical example, Section 5 the dynamic option adjusted spread, Section 6 presents the empirical results finally Section 7 concludes.

³ Obviously practitioners have been able to develop reduced-form models since, generally, they also dispose of proprietary data needed to calibrate the model.

2. The Mortgage Backed Security Model

Consider the following probability space (Ω, F, P) and the process $p(t, D, C, z)$ adapted to the filtration F . The price process depends on the risk neutral vector of discounted bond prices $D(t)$ $0 < t < T$, with Q being the risk neutral probability measure. z is a state variable that will be defined shortly and $C(t)$ is the cash-flow paid by the mortgage at t . In this model z may represent the option adjusted spread used to match the theoretical and the market prices of the MBS.

Define the price process for a mortgage at time t as the expected value of the discounted future cash-flows:

$$p(t) = E^Q\left[\sum_{t=0}^T C(t)D(t)\right] \quad (1)$$

The main problem when using equation (1) to price a MBS is that the borrower can at each time consider a prepayment action. In the introduction we have already mentioned different ways of modelling the prepayment option when pricing MBSs. In this paper we shall follow Chen (2004) and implement a reduced form model⁴. In general, when pricing a MBS one has to generate the mortgage cash flows $C(t)$ using, for example, a reduced form model. Once the cash-flows have been generated, the value of the mortgage can be obtained by discounting the simulated cash flows and summing them up.

Using Monte Carlo to generate m paths for $C(t) | m$, we have that

$$E^Q(p) = \frac{1}{m} \sum_{i=0}^T \left[\sum_{m=1}^M C(t)D(t) \right], \lim_{m \rightarrow \infty} C(t) | m \rightarrow C, \text{ and therefore the simulated price,}$$

say p^* , will converge to the true price p . Using Equation (1) one can also estimate the option adjusted spread z . Suppose that p is the observed market price of the mortgage. As we do when we want to obtain implied volatilities for plain vanilla options, we can compute z using a root finding method to solve:

⁴ Refer to the Appendix for a description of the model.

$$p(t_0, C, D, z) = p \quad (2)$$

3. The Term Structure Model

To solve Equation (2) one has to simulate the term structure of interest rates out of the maturity of the mortgage. We extend the above model by using a two factor Heath, Jarrow, and Morton (1992) model (HJM). The HJM model belongs to a class of models, and therefore one needs to specify the initial forward rates and volatilities to specify the model itself. Below we explain the way we have dealt with this problem.

The HJM attempts to construct a model of the term structure of interest rates that is consistent with the observed term structure. The state variable in this model is the forward rate in time t for instantaneous borrowing at $T > t$, $F(t, T)$. In differential form the model can be written as:

$$dF(t, T) = m(t, T)dt + \sum_{k=1}^N \sigma_k(t, T)dW_k(t) \quad \text{for } 0 \leq t \leq T \quad (3)$$

Or also in integral form

$$F(t, T) = F(0, T) + \int_0^t m(v, T)dv + \int_0^t \sum_{k=1}^N \sigma_k(v, T)dW_k(v) \quad (4)$$

Here $F(0, T)$ is the fixed initial forward rate curve, $m(t, T)$ is the instantaneous forward rate drift, $\sigma_k(t, T)$ is the instantaneous volatility process of the forward rate curve, and $W_k(t)$ is a standard Brownian motion process. The model above is very general and encompasses all the short rate models such as, for example, the Hull and White (1993) model. The drift process is specified as:

$$m(t, T) = \sum_{k=1}^N \sigma_k(t, T) \int_t^T \sigma_k(t, s)ds \quad (5)$$

The hardest problem when using the HJM approach to simulate $F(t, T)$ is that the model is specified in terms of instantaneous forward rates and the latter are not

observable. To overcome the problem we use the following deterministic specification for the volatilities, and the Musiela parameterization:

$$\sigma_k(t, T) = \bar{\sigma}_k(t, T - t)$$

That means that our model belongs to the Gaussian class of models and maturity is specified as time to maturity. Therefore if we set $\tau = T - t$, it follows that:

$$d\bar{F}(t, \tau) = \bar{m}(t, \tau)dt + \bar{\sigma}(t, \tau)dW(t) \quad (6)$$

With the drift specified as:

$$\bar{m}(t, \tau) = \bar{\sigma}(t, \tau) \int_0^\tau \bar{\sigma}(t, s) ds + \frac{\partial}{\partial \tau} \bar{F}(t, \tau) \quad (7)$$

We use the above parameterisation when simulating the forward rates. The spot rate $r(t)$ used to discount the cash flows can be determined from (6) as follows:

$$r(t) \equiv \lim_{\tau \rightarrow t} \bar{F}(t, \tau)$$

To use the two factor model above, one has to specify the initial forward rates and volatilities. In practical applications of our model we use Bloomberg to obtain the forward rates necessary to initiate the process. Also from Bloomberg one can obtain implied volatilities on interest rate caps necessary for the calibration of the model. Two volatilities are used in this case. The first is set fixed for all the maturities and equal to the implied volatility of a thirty year interest rate cap option. The second refers to implied volatilities of interest rates caps with maturities between 1 and 30 years. An Euler discretization scheme, with 360 time steps and 5000 simulations, is used.

4. Numerical Example

In this section we provide a preliminary numerical example and describe how the OAS is computed. The MBS price can be obtained by simulating the mortgage's cash flows (i.e. $C(t)$) over the lifetime of the mortgage using a prepayment model. The prepayment model used in this study is described in the Appendix. Furthermore one also needs to simulate the term structure dynamics over the relevant horizon. We use the model described in Section 3. Finally equation (1) gives us the bond's price. The simulated MBS price with the respective standard error in bracket is 102.1786 (0.063124).

The value of the mortgage is equal to 102.1786%. Suppose the size of the underlying mortgage pool is \$1,000,000.00, the price of a mortgage-backed security issued from the underlying pool will be \$1,021,786.00. For simplicity we assume that the observed market price is 100% of the par value. Since all the elements of equation (1) are known and the market price of the mortgage (or a similar one) can be observed, one can now compute, using equation (2) and a root finding method, the option adjusted spread. The option adjusted spread in this example is 46 basis points.

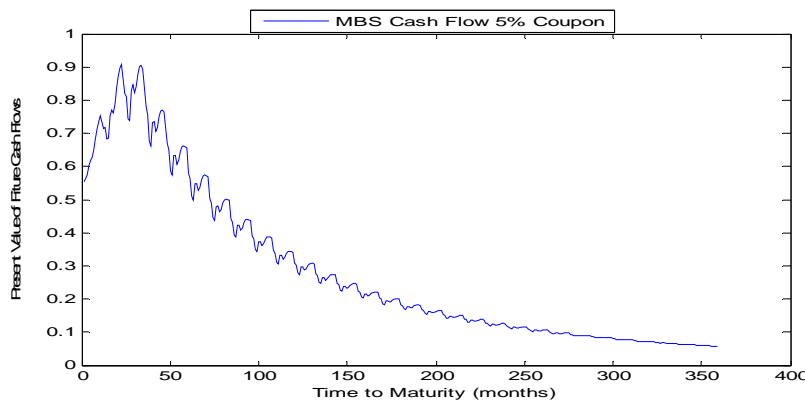


Figure 1: MBS Cash Flow

Figure 1 shows the simulated paths of the monthly cash flows of the mortgage. As the bond approaches maturity the value of the prepayment option decreases and consequently the mortgage cash flow becomes less uncertain.

5. Dynamic Option Adjusted Spread

The option-adjusted spread (OAS) can be viewed as a measure of the yield spread. It is constant over the benchmark curve chosen for the valuation process. The reason why this spread is referred to as option-adjusted is because the cash flows of the underlying security are adjusted to reflect the embedded option. Most market participants find it more convenient to think about yield spread than price differences. One issue with the option spread is that it assumes the yield spread to stay unchanged over the maturity of the bond. Therefore, if future interest rates become volatile, the OAS remains unchanged. Clearly in this situation a prepayment model, using an option adjusted spread approach, is miss-specified. Furthermore this implies that traders will have to compute it and re-calibrate their models frequently. This may carry an additional cost in terms of time necessary for the re-calibration. In this section we propose a modification of the OAS that we call Dynamic Option Adjusted Spread (DOAS). The DOAS allows one to capture prepayment risks as well as changes in the yield curve. Furthermore, a potential investor holding a mortgage can also use it as a hedging tool.

From an investor point of view the DOAS can be viewed as an investment⁵. The value of this portfolio can be positive or negative depending on the spread adjustment. A bond having a positive OAS has a positive portfolio value. On the other hand, a bond with a negative OAS will have a negative portfolio value.⁶

To compute the dynamic option adjusted spread, we use the following procedure. Simulate the bond's cash-flows, at each t , over the lifetime of the mortgage. Compute the option adjusted spread (i.e z) and use it to adjust the cash-flows of the bond at each t . We have in this way the adjusted cash-flows. The difference, at each t , between a plain vanilla bond cash flow (C_p) and the mortgage cash flow, is the dynamic option adjusted spread in t . The summation of these up to t_0 is the portfolio value

$$PV_0 = E^Q \sum_{t=0}^T [(C_p) - (C)]_t \quad (8)$$

⁵ We call this investment a portfolio value (PV).

⁶ OAS can be negative when the mortgage coupon is low but interest rate volatility is relatively high. In this case investors in this market might not be very concerned with the MBS optionality, at least not in the short run.

Equation (8) describes the way the portfolio value is computed. Therefore the portfolio value is just the difference between a non-callable bond and a callable bond. It might be worth noticing that, by buying a MBS and investing in the above portfolio, the investor has indeed created a synthetic non-callable bond but with the difference that he is also hedging against interest rate risk⁷.

Figure 2 below shows the conditional prepayment rate (CPR) function, the refinancing incentive (RI) and the portfolio value (PV). At the beginning of the mortgage there is a positive spread (i.e. the difference between the value of the portfolio and the cash flow of the mortgage). The difference would compensate the investor if the option is exercised by the borrower. The spread is particularly relevant in the first one hundred months which, in general, corresponds to the time when the prepayment risk is higher. As the prepayment risk becomes less accentuate, the spread decreases.

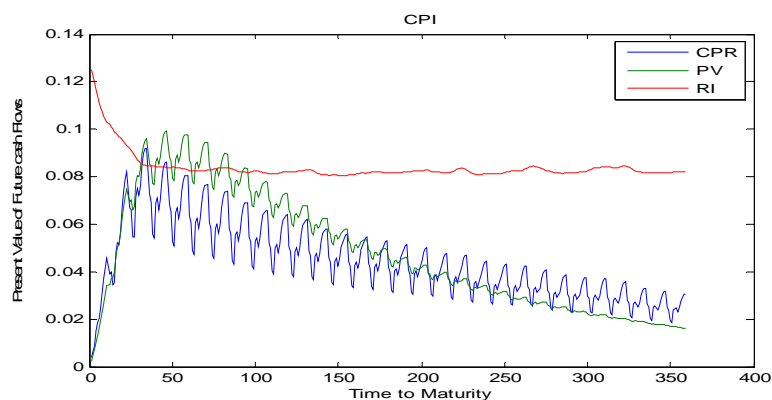


Figure 2: Prepayment Model

⁷ In effect the idea that there is a positive relationship between option adjusted spread and prices of non-callable securities (in this cases Treasury securities) was first reported in Brown (1999). He also suggests, in line with our model, that the option adjusted spread is a noisy measure.

5.1 Numerical Example

Using the same approach as explained in Section 4 and using equation (8) we can also compute the portfolio value (with standard error in bracket). The portfolio value is, in this case, 2.07006% (0.00289) of the par value.

The DOAS in our example is 2.07006% par value. If we assume that the pool size of the mortgage is \$1,000,000.00, the portfolio value will be \$ 20,700.60. The investor can buy this option to hedge interest rate risk. The next section further clarifies this.

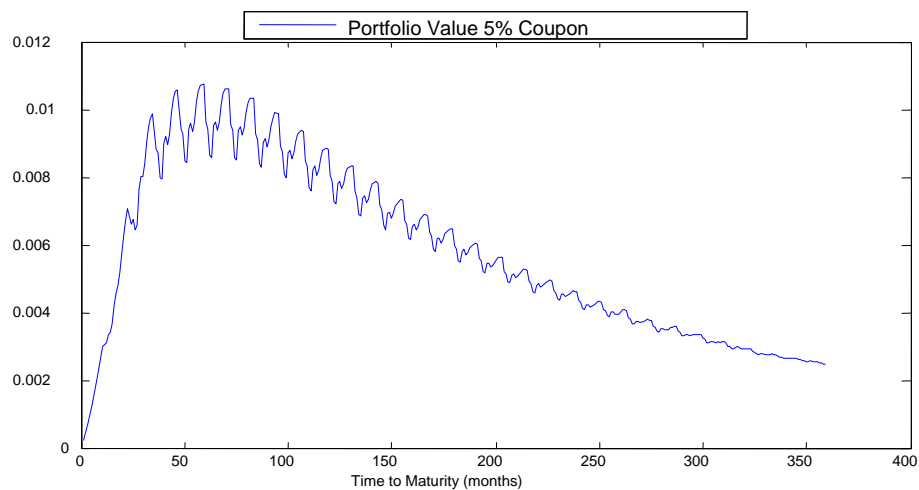


Figure 3: Portfolio Value

5.2 Numerical Example

The investor can use the portfolio described above as a hedging instrument against prepayment risk in general and changes of the yield curve. The examples below show exactly this.

Example1: 5% Coupon rate:

Investor A buys at time t_0 a 30-year mortgage-backed security with the price of the MBS being 100% of the face value. The investor receives Treasury rate plus 46 basis point (OAS). We assume the pool size to be \$1,000,000.

Another investor, say, Investor *B* buys at time t_0 the same mortgage and a DOAS option. The DOAS option is 2.07006% of the par value. Therefore the value of this investment will be 102.07%.

Suppose at time t_1 the interest rate volatility increases from 13bp to 26bp. What is the impact of this increase on the MBS price and the investor's portfolio?

At time t_1 , the price of the mortgage drops to 99.8534 % or \$ 998,534.00. Therefore that implies a \$1,466 loss on the mortgage for Investor *A*.

On the other hand, the value of the investment for the Investor *B*, is given by:

$$\begin{aligned} \text{Pay-off} &= \text{bond value at time } t_1 - \text{bond value at time } t_0 \\ &+ (\text{portfolio value at time } t_1 - \text{portfolio value at time } t_0) \end{aligned}$$

$$\text{Pay-off} = 99.8534 - 100 + (2.08289 - 2.07006) = -0.1337 \text{ or } \$1,337$$

Example2: 6% coupon rate:

We now show another example choosing a coupon rate that is above the initial interest rate used in the simulation. Under this scenario the prepayment risk is more relevant than in the previous example. Investor *A* buys at time t_0 the mortgage and receives interests plus 227.70 basis points. Investor *B* buys the same mortgage but also invests into a DOAS option whose price is 9.9080% for a total of 109.908%.

Suppose that at time t_1 the interest rates volatility increases, as before, from 13bp to 26bp. What is the impact of this increase on the bond price, and the investor's portfolio? At time t_1 the price of the mortgage drops to 99.9825 % or \$ 999,825.00. The loss for the Investor *A* is therefore \$ 175.00. As a consequence of the increase in interest rate volatility the value of the DOAS option increases to 9.9275%. The pay-off for the Investor *B* is therefore given by:

$$\begin{aligned} \text{Pay-off} &= \text{bond value at time } t_1 - \text{bond value at time } t_0 \\ &+ (\text{portfolio value at time } t_1 - \text{portfolio value at time } t_0) \end{aligned}$$

$$\text{Pay-off} = 99.9825 - 100 + (9.9275 - 9.9080) = 0.0020 \% \text{ or } \$20.00$$

6. Empirical Results

In this section we use the model described in Sections 2, 3 and 5 to price mortgage backed securities with different coupon rates⁸. Table 1 shows the MBS prices and the option adjusted spread. As expected the price of the mortgage increases as the coupon rate increases⁹. This is because of the refinancing incentive for the borrower when the coupon rate is above the market interest rate.

Table 1: Mortgage-Backed Security Values and Dynamic Option Adjusted Spreads

Coupon Rate %	5.00	5.50	6.00	6.50	7.00
MBS Price	102.17	106.28	110.26	114.21	117.58
SE	0.06312	0.06204	0.07211	0.06606	0.05265
OAS bp	46.18	135.66	227.70	321.05	412.33
DOAS %	2.0700	6.0034	9.9080	13.7460	17.0849
SE	0.00289	0.00837	0.01223	0.01690	0.02427

Note: SE are standard errors obtained by 100 trials. OAS bp is the option adjusted spread in basis point.

The highest price is reached when the coupon is 7% and it is 117.58. Such a high premium clearly cannot be explained by par plus a number of refinancing points¹⁰. These high prices are consistent with what generally is observed in the market where mortgage prices can easily reach these levels (see also Longstaff, 2004, for a discussion on this issue).

Conditionally on the interest rate level used in our simulation, we note that higher coupon rates will increase the incentive for the borrower to repay the mortgage and this clearly will affect the spread that an eventual investor would require as a

⁸ Note, we are unable to challenge our model against markets prices. In fact data needed to calibrate our model are generally proprietary data and we were not able to obtain these information. However as we shall mention, our empirical results are in line with what the literature would predict and our MBS prices can, in certain cases, reach the values observed in the market place.

⁹ Note that the initial rate used for the simulation is 5%.

¹⁰ This is generally the conclusion reached by many practitioners and academic models.

compensation for the prepayment option. In fact our model suggests a spread over the Treasury curve of more than 400bp when a 7% coupon is considered. We have also computed standard errors from the simulation by using 100 independent trials of the model in section 2. These empirical results are in line with theoretical and empirical studies in this area (see for example Gabaix et al., 2007).

At the bottom of Table 1, we report the simulated dynamic options adjusted values. As we see, given the interest rate level used in the simulation, the value of the option increases as the coupon increases. This is consistent with a higher prepayment risk implicit with higher coupons. As mentioned an investor can buy this option, and pay a higher price for the mortgage, to hedge the prepayment risk and changes in the slope of the yield curve

Conclusions

Mortgage Backed Securities are assets collateralised by a pool of mortgages and allow investors to gain higher rates of return (with a relatively lower risk) compared to other fixed income instruments. Given the importance of these securities¹¹ in the last decade there has been a proliferation of models trying to explain the optimal prepayment behaviour of the borrower. Their main problem is that they cannot always explain, within a rational analytical framework, how borrowers decide to refinance their loans. Therefore, some of these approaches have tried to model the prepayment option as an endogenous problem (see Stanton and Wallace, 1998 amongst others) but MBS prices obtained by using these frameworks cannot generally match market prices.

If on the one hand various different models have been proposed in the literature to price MBS, on the other hand there has been very little work on hedging and risk management of these securities. In this paper we have tried to fill this gap. We extend a reduced form model to price MBS and propose a novel approach to managing interest rates risk. Firstly we pointed out that most reduced form models, by relying on the option adjusted spread, might be miss-specified. Secondly, we suggested what we call the dynamic option adjusted spread. We show that an investor in the MBS market, by taking a long position on an option (DOAS), can hedge out interest rate risk. The DOAS is simply the difference between the cash flows of a non-callable bond and a callable bond over the maturity of the mortgage. The concept of

¹¹ And the subsequent turmoil in the financial markets caused by the collapse of these securities.

DOAS can be easily extended to other fixed income securities such as callable bonds and a variety of exotic swaps.

References

- Brace, A., D., Gatarek, and M., Musiela, 1997, "The Market Model of Interest Rate Dynamics", *mathematical Finance*, 7, 127-155.
- Brown, D. 1999, "The Determinants of Expected Returns on Mortgage-backed Securities: An Empirical Analysis of Option-adjusted Spreads", *Journal of Fixed Income* 9, 8-18.
- Chen Jian, 2004, "Simulation Based Pricing of Mortgage Backed Securities, Proceeding of the 2004 Winter Simulation Conference.
- Dunn, Kenneth B., and Chester S., Spatt, 1986, "The Effect of Refinancing Costs and Market Imperfections on the Optimal Call Strategy and the Pricing of Debt Constracs", Working Paper, Carnegie-Mellon University.
- Gabaix X., Arvind K., and O., Vigneron, 2007, "Limits of Arbitrage: Theory and Evidence from the Mortgage Backed Securities Market", *Journal of Finance*, Vol. 62, 557-595.
- Heath, D., R., A., Jarrow, and A., Morton, 1992, "Bond Pricing and the Term Structure of Interest Rates: A New Methodology for Contingent Claims Valuation", *Econometrica* 60 (1), 77-105.
- Hull, J., and A., White, 1993, "One Factor Interest rate Models and the Valuation of Interest Rate Derivative Securities", *Journal of Financial and Quantitative Analysis* (28), 235-254.
- Liu Jian Guao and Eugene Xu, 1998, "Pricing of Mortgage Backed Securities with Option Adjusted Spread", *Managerial Finance*, vol. 24, No. 9/10.
- Longstaff, F., 2004, "Borrower Credit and the Valuation of Mortgage Backed Securities", UCLA Anderson School, Working Papers in Finance.
- Schwartz, Eduardo, S., and Walter N., Torous, 1992, "Prepayment, Default, and the Valuation of Mortgage Pass-Through Securities", *Journal of Business*, 65, 221-239.
- Stanton, Richard and N., Wallace, 1998, "Mortgage Choice: What`s the Point?", *Real Estate Economics*, 26, 173-205.

Appendix 1

The model assumes that four factors (i.e. refinancing incentive, burnout, seasoning, and seasonality) explain 95% of the variation in prepayment rates. These factors are then combined into one model to project prepayments:

$$CPR_t = RI_t \times AGE_t \times MM_t \times BM_t$$

where, RI_t represents the refinancing incentive; AGE_t represents the seasoning multiplier; MM_t represents the monthly multiplier; BM_t represents the burnout multiplier.

Therefore, the prepayment model is:

$$CPR_t = RI_t \times AGE_t \times MM_t \times BM_t$$

where:

$$RI_t = 0.28 + 0.14 \tan^{-1}[-8.571 + 430(WAC - r_{10}(t))]$$

$$AGE_t = \min\left(1, \frac{t}{30}\right)$$

$$BM_t = 0.3 + 0.7 \left(\frac{B_{t-1}}{B_0}\right)$$

MM_t takes the following values, which start from January and end in December: (0.94, 0.76, 0.74, 0.95, 0.98, 0.92, 0.98, 1.1, 1.18, 1.22, 1.23, 0.98), r_{10} is 10-year Treasury rate, and WAC is the weighed average coupon rate.

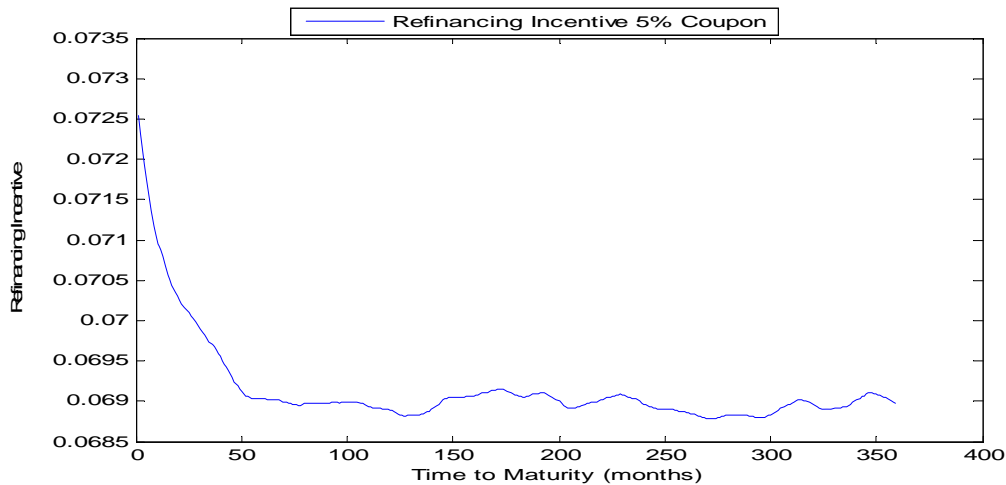


Figure 4: Refinancing Incentive (5% coupon)

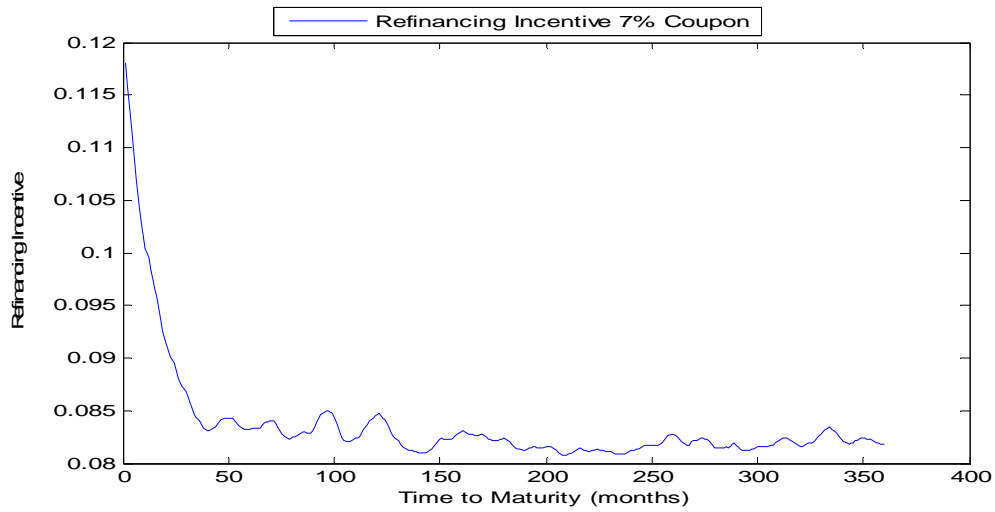


Figure 5: Refinancing Incentive (7% coupon)

Figure 4 and 5 above show the refinancing incentive function for 5% and 7% coupon rates. Borrowers have a higher incentive to exercise the prepayment option and refinance the mortgage when the coupon rate is higher than interest rates. This is shown in Figure 5.

