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Stimulating long-term growth and welfare in the U.S.

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Stimulating long-term growth and welfare in the U.S.*

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Abstract

We develop an endogenous growth model to quantify how permanent structural policy changes that enhance the fiscal policy mix, markets' functioning, and public institutions' quality affect long-term growth and welfare. The reforms include increased public investment, reduced market power through lower price markups for patents and intermediate goods, and an improved institutional framework that reduces rent-seeking. All reforms, except lower patent prices, lead to per-capita output and welfare gains along the transition and balanced growth paths. In contrast, a lower markup in the research sector hurts innovation, leading to lower growth over both paths and welfare losses along the transition.

Keywords: endogenous growth, structural policy, welfare
JEL Codes: H30, O41, O43

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1 Introduction

Over the past century and a half, real per capita GDP in the U.S. has been growing on trend at roughly 2%. Historical growth accounting suggests that the main factors contributing to this rate include higher capital per worker (see, e.g. Solow (1957)), more years of schooling (see, e.g. Barro and Lee (2015)), and better ways of using scarce social resources to reduce misallocation (see, e.g. Restuccia and Rogerson (2017)). Moreover, it is also widely recognised that, at least for countries like the U.S., the economy cannot achieve long-term sustained growth without technological progress in which research translates to innovative ideas and new and better products over time (see, e.g. Romer (1990), Jones (1995, 2016, 2022a), Aghion and Howitt (2005, 2009), Sala-i-Martin (2010), Barro (2013) and Fernald and Jones (2014)).[1]

Complementary to growth accounting, the literature has also explored the underlying forces that shape the evolution of the above growth factors. These typically include physical infrastructure, market efficiency, the education system, openness and the institutional framework within which firms, individuals and governments interact (see, e.g. Aghion and Howitt (2009), Acemoglu (2009) and Sala-i-Martin (2010)). In this paper, combining features from several growth models, we quantitatively assess how permanent structural policy changes, or reforms, can affect several of these underlying influences and, in turn, long-term growth and welfare.

Following the related literature, we adopt a relatively broad interpretation of policy and concentrate on policy reforms that enhance the fiscal policy mix, markets' functioning and public institutions' quality.[2] In particular, we investigate how growth and welfare over the transition and on the balanced growth path (BGP) are affected by (i) an increase in public investment spending financed by reducing transfer income; (ii) a reduction in the

[1] The decomposition of growth sources for the U.S., provided by the Bureau of Labor Statistics, illustrates the dominance of TFP; see bls.gov/productivity/tables/home.htm. For example, they report a per capita growth rate of 2.3% during 1948-2022, and, of this 2.3%, 0.9 percentage points (ppt) are due to higher capital per worker, 0.2 ppt to the composition of labour, including years of education, and 1.2 ppt to TFP. In another study, Jones (2022a) decomposes the 1.2 ppt due to TFP into contributions by research intensity (0.6), declining misallocation due to better opportunities to minorities (0.3) and population growth (0.3).

[2] See also, e.g. Prescott (2002), who broadly defines policy to include the regulatory and legal environment and fiscal policies. See also studies on the EU economies, like Pfeiffer *et al.* (2023) of the European Commission and Masuch *et al.* (2018) of the European Central Bank, that adopt a similarly broad perspective of structural policies. A review of the earlier literature on reforms, defined as "significant changes in a policy area", can be found in Drazen (2000, chapter 10).

market power of research firms that reduces the price of blueprints; (iii) a reduction in the market power of intermediate goods producers that results in lower prices for their products; (iv) an improved institutional framework which reduces rent-seeking related to the public budget; and (v) an increase in the overall population.[3] We examine permanent changes since we are interested in long-term outcomes rather than business cycles.

To implement these reforms, we build on the Romer-Jones model of long-term endogenous growth and thus distinguish between final good, intermediate goods and research firms, where the latter produce ideas or blueprints that enhance the total factor productivity (TFP) of the goods-producing firms in equilibrium (see, e.g. Jones (2019) for a review). Given its importance in producing objects and ideas, we add human capital accumulation to the household's problem as another potential driver of long-term endogenous growth.[4] Next, since government policies can be an essential factor in shaping the allocation of resources, we incorporate productivity-enhancing public capital/investment and public consumption into the government's setup. Finally, we add resource misallocation to our model by allowing firms to enter a rent-seeking competition related to government budget allocations. In this model, ideas and individual human capital growth rates drive long-term per capita output growth. In contrast, along the transition, in addition to these two drivers, per capita output growth is also affected by the accumulation of labour inputs, physical capital and public capital. We calibrate the model using U.S. data from as early as 1925.

1.1 Motivation for reforms

Our focus on the structural reforms outlined above is motivated as follows. First, permanent changes in the fiscal policy mix in favour of public investment spending that improves infrastructure continue to occupy the centre stage of policy agendas, especially after the COVID-19 pandemic crisis that has left little room for further expansionary demand-side policies. For gov-

[3] Although, strictly speaking, population growth is not a policy reform, we explain below why we include it.

[4] Even if the quantity of human capital (e.g. years of schooling) may not be the primary driver of long-term growth (see Jones (2022a)), its quality seems to be necessary. See, e.g. Hanushek and Kimko (2000) and Hanushek and Woessmann (2015), who provide evidence that, while time in school is insignificantly related to growth rates, acquired skills as measured by test scores in mathematics and science, are essential to long-term growth. As Hanushek and Woessmann (2015, p. 11) point out, "a given level of education can produce ... new ideas, making it possible for education to affect long-run growth rates even if no additional education is added to the economy". Barro and Lee (2015) provide similar evidence for the role of quality-adjusted educational attainment.

ernment investment policy in the U.S., see e.g. Leeper *et al.* (2010), Bouakez *et al.* (2017, 2020), Ramey (2020) and Malley and Philippopoulos (2023).

Second, there is a general belief, at least among policymakers, that moving to more competitive markets is necessary for a more efficient supply side. On the other hand, more demanding competition and the anticipation of lower returns may discourage frontier innovation in setups with long-term endogenous growth where imperfect competition is a crucial ingredient, as in Romer (1990) and Jones (1995). Hence, it is not surprising that, although there is strong evidence of an increase in various indices typically used to measure market power in the U.S. at least since the 1980s, their implications are mixed (see, e.g. the early reviews in Aghion and Griffith (2005) and Aghion and Howitt (2009, ch. 12), the recent papers by Bento (2020, 2021), as well as the literature on the U.S. economy as surveyed by e.g. Syverson (2019) and De Loecker *et al.* (2020)).[5]

Third, although institutional quality has many dimensions (see, e.g. Acemoglu *et al.* (2005) for a review on institutions and long-term growth), firms' engagement with the public sector to promote their private interests and profits at the cost of the general public has perennially been present in policy debates and academic research on misallocation and growth. See, e.g. the review of Restuccia and Rogerson (2017), who emphasise that an essential source of misallocation reflects discretionary provisions made by the government that favour specific firms.[6] Jones (2022a) also highlights the role that resource misallocation, in general, can play in the growth performance of the U.S. economy.

Finally, we include the effects of higher population growth not only because, ultimately, it is the main engine of long-term growth in the semi-endogenous growth literature (see, e.g. Jones (2019, 2022b) and Vollrath

[5]In this literature, measures of market power include price markups, market concentration, profitability and sales share. These measures show a persistent increase over time in the U.S. However, despite the concern of policymakers (see, e.g. the recent policy actions by the Biden administration to promote market competition), the macro implications are varied. For example, a higher concentration can lead to increased innovation and productivity (see Autor *et al.* (2020)), greater technological intensity and higher output growth (see Kwon *et al.* (2023)), and a more efficient aggregate environment (see Bighelli *et al.* (2023) for the European economy). On the other hand, according to Bento (2020, 2021), barriers to entry can decrease firm-level innovation and aggregate productivity. Moreover, it is not clear that developments in these measures automatically translate to more market power. For example, economies of scale and globalisation can also drive markups, concentration and profitability (see the reviews of Syverson (2019) and De Loecker *et al.* (2020), as well as the article on market power in The Economist, July 15th 2023, pp. 49-51).

[6]This can be in the form of privileged subsidies, tax treatments, government-created demand for a firm's product, etc. But it can also be select legislation and regulation that reduce competition and support prices.

(2020)) but also because population plays a changing role across different stages of growth (see, e.g. the review in Aghion and Howitt (2009, chapter 10)). Moreover, demographic developments can play a critical role in shaping the labour market's performance and the allocation of the workforce to various sectors (see, e.g. Boeri and van Ours (2013, ch. 9)).

1.2 Main results

Our main results are as follows. First, a permanent increase in public investment, financed by a cut in income transfers, stimulates the growth rates of ideas and human capital and, thereby, the per capita GDP growth rate along the transition and on the BGP. Social welfare also rises thanks to relatively substantial increases in per capita private and public consumption, which offset the fall in leisure as households find it optimal to work harder in a more productive economy that allows for higher wages. On the negative side, higher public spending implies a larger contestable prize, a slice of which rent-seeking firms fight for, and this means a misallocation of labour away from productive activities that do not allow the increase in public investment to have its otherwise complete beneficial effect. To give an indicative quantitative result, focusing, for instance, on the BGP, this kind of reform implies that a permanent increase of public investment as a share of GDP by one percentage point, other things equal, increases the growth rate of per capita GDP from a base of 2.08% to 2.14%. Although the growth rate change is small, recall that, in a growing economy, per capita GDP increases exponentially with its gross growth rate. For example, starting at $60,000, which is the value of 2022 per capita GDP in the U.S., after 40 and 100 years, per capita GDP on the BGP increases by roughly 3.6 and 31.8 thousand dollars, respectively, relative to the values implied using the base growth rate of 2.08%. Moreover, welfare gains on the BGP, measured typically in consumption equivalent units, will be 3.53% relative to the base.[7]

Second, a permanent reduction in the market power, and hence in the price of blueprints and the associated profits enjoyed by research firms, discourages innovation and the production of new ideas, hurting growth. On the other hand, it makes the blueprints used by other firms more accessible, stimulating growth. In our model, the former effect dominates, so, in general equilibrium, lower profits by research firms hurt growth and, hence, per capita private and public consumption. Social welfare may increase or decrease depending on whether the increase in leisure, as economic activity

[7] In addition to providing further details relating to the BGP, the results section below will report findings for growth rates, per capita magnitudes and welfare across different time horizons over the transition path.

has fallen, is stronger or weaker than the decrease in per capita consumption. Quantitatively, again focusing on the BGP, a permanent cut in the price of blueprints that translates to a fall in the profits of the research sector by 10%, *ceteris paribus*, will lower the growth rate of per capita GDP from 2.08% to 2.06% implying that per capita GDP will fall by about 0.7 and 5.6 thousand dollars after 40 and 100 years, respectively, relative to the base. On the other hand, welfare rises on the BGP by 0.88%, but this happens simply because of more leisure. Thus, moving to a more competitive market, when firms in this market produce patents, is counter-productive. This finding is consistent with the evidence provided by Autor *et al.* (2020), Kwon *et al.* (2023) and Bighelli *et al.* (2023).

Third, regulatory policies that reduce the market power of the intermediate goods sector lead to lower intermediate goods prices, lower profits, and higher growth. This outcome mainly happens because intermediate goods get cheaper, boosting the final good sector and, thus, GDP. Also, our results show that higher competition for intermediate goods enhances welfare, thanks to higher per capita private and public consumption, compensating households for less leisure. Moreover, employment goes up generally, except for jobs in the research sector, whose price markup is proportional to profits in the intermediate goods sector. Quantitatively, on the BGP, a permanent cut in the price of intermediate goods translates to a fall in intermediate goods firms' profits by 10%, *ceteris paribus*, increasing the per capita GDP growth rate from 2.08% to 2.11%. This implies that the BGP per capita GDP rises by approximately 1.8 and 15.2 thousand dollars after 40 and 100 years, respectively, relative to the base. There is also a considerable increase in welfare on the BGP by 9.6%.

Fourth, most of the aggregate effects of a permanent reduction in rent-seeking activities are qualitatively similar to those from better public infrastructure. Nevertheless, a decrease in rent-seeking has extra dividends. For example, it incentivises private firms to use their labour force productively rather than to use it for redistributive contests. Moreover, it allows the society to allocate its scarce resources to provide utility- and productivity-enhancing public goods and services rather than to augment individual profits and incomes. Quantitatively, on the BGP, an assumed permanent reduction in the fraction of time households allocate to rent-seeking services when at work from 1% to zero implies that the growth rate of per capita GDP rises from 2.08% to 2.13%, indicating that per capita GDP increases by approximately 3.1 and 26.6 thousand dollars after 40 and 100 years, respectively, relative to the base. Moreover, welfare gains on the BGP are 1.45%.

Finally, an increase in population allows all firms to increase their labour inputs, including the number of employees in the research sector, which boosts the growth rate of ideas and, hence, the per capita GDP growth rate. Although this looks similar to the prediction of the Jones semi-endogenous growth model in which, eventually, economic growth is driven only by population growth (see, e.g. Jones (2019, 2022b)), in our decentralised model, equilibrium labour for each sector is chosen optimally rather than as an exogenous fraction of the total population. Regarding welfare, it also rises as the gains from higher per capita private and public consumption are strengthened by more leisure time as households choose to increase the time allocated to both work and leisure at the expense of time allocated to education. Quantitatively, on the BGP, if population growth is assumed to rise permanently from 1.1% to 1.2%, other things equal, the growth rate of per capita GDP will increase from 2.08% to 2.14%, implying that per capita GDP increases by about 3.5 and 30.3 thousand dollars after 40 and 100 years, respectively. Furthermore, welfare gains on the BGP are 2.3%.

1.3 Contribution relative to previous work

Our research complements and adds to the literature and current policy discussion relating to the aggregate effects of fiscal policy over the business cycle (see, e.g. Leeper *et al.* (2010), Sims and Wolff (2018), Bouakez *et al.* (2020), Ramey (2020) and Malley and Philippopoulos (2023). We also contribute to the literature on structural reforms in the U.S. since we are the first study to quantify the effects of reducing market power and rent-seeking into a general equilibrium endogenous growth setup with the three distinct production sectors *a la* Romer-Jones. More specifically, regarding market power, our work enriches the literature on two-sector dynamic general equilibrium models with imperfect competition and an endogenous determination of the number of firms and hence product variety (see, e.g. Bilbiie *et al.* (2012, 2007), Etro and Colciago (2010) and Bento (2020, 2021)). Also, it complements the empirical studies of, e.g. Syverson (2019), De Loecker *et al.* (2022), Autor *et al.* (2020) and Kwon *et al.* (2023) on market power and its implications in the U.S. Finally, for quantitative studies of direct or indirect rent-seeking via lobbying in the U.S., see, e.g. Huneeus and Kim (2018) and Angelopoulos *et al.* (2021), but again not in a three-sector growing economy.

We organise the rest of the paper as follows. Section 2 sets out the model, Section 3 the calibration, Section 4 the quantitative analysis, and Section 5 the conclusions.

2 Model

Our decentralised model economy draws on the work of Lucas (1988), Romer (1990), Jones (1995, 2019, 2022a, 2022b), McGrattan and Schmitz (1999) and Gross and Klein (2022). Table 1 shows that the setup comprises firms, households and a government. We distinguish firms into final good, intermediate goods and research, as in the Romer-Jones setup. These firms are indexed respectively by $f = 1, 2, \ldots, N_{f,t}$, $i = 1, 2, \ldots, N_{i,t}$ and $b = 1, 2, \ldots, N_{b,t}$, and choose their various inputs optimally. Identical households are indexed by $h = 1, 2, ..., N_t$ and, in addition to consumption-saving decisions, they choose optimally the time allocated to leisure, work and education, where the latter augments their human capital, $h_{h,t}$.

Table 1: Economic Environment

	Final good firm
Output:	$y_{f,t} = A_f \left(\widetilde{k}_t^g\right)^{\phi} (l_{f,t}^w)^a \left(\sum_{i=1}^{N_{i,t}} x_{f,i,t}^{1-\alpha}\right)$
Profits:	$\pi_{f,t} = (1-\tau_t^f)[y_{f,t} - w_t(l_{f,t}^w + l_{f,t}^r) - p_{i,t}x_{i,t}] + \left(\frac{l_{f,t}^r}{L_t^r}\right)\kappa(G_t^c + G_t^i)$
	Intermediate goods firm
Output:	$x_{i,t} = A_i \left(\widetilde{k}_t^g\right)^{\phi} \left(N_{b,t} l_{i,t}^w\right)^{\alpha} k_{i,t}^{1-\alpha}$
Profits:	$\pi_{i,t} = (1-\tau_t^f)[p_{i,t}x_{i,t} - w_t(l_{i,t}^w + l_{i,t}^r) - q_t] - i_{i,t} + \left(\frac{l_{i,t}^r}{L_t^r}\right)\kappa(G_t^c + G_t^i)$
Private capital:	$k_{i,t+1} = (1-\delta)k_{i,t} + i_{i,t}$
	Research sector
Output:	$N_{b,t+1} = (1-\delta^{n_b}) N_{b,t} + M \left(\widetilde{k}_t^g\right)^{\phi} L_{b,t}^w H_t (N_{b,t})^{\mu}$
Profits:	$\Pi_{b,t} = (1-\tau_t^f)[q_t N_{b,t} - w_t(L_{b,t}^w + L_{b,t}^r)] + \left(\frac{L_{b,t}^r}{L_t^r}\right)\kappa(G_t^c + G_t^i)$
	Households
Utility:	$u = \frac{(c_{h,t} + \lambda\frac{(1-\kappa)G_t^c}{N_t})^{1-\sigma}}{1-\sigma} \left(1 - l_{h,t}^w - l_{h,t}^e\right)^{\psi(1-\sigma)}$
BGT constraint:	$(1+\tau_t^c) c_{h,t} + b_{h,t+1} = (1-\tau_t^y) \left(w_t h_{h,t} l_{h,t}^w + \pi_{h,t}\right) + \left(1 + r_t^b\right) b_{h,t} + g_t^t$
Human capital:	$h_{h,t+1} = (1-\delta^h) h_{h,t} + D \left(\widetilde{k}_t^g\right)^{\phi} \left(l_{h,t}^e h_{h,t}\right)^{\theta} \left(\frac{H_t}{N_t}\right)^{1-\theta}$
	Government
BGT constraint:	$G_t^c + G_t^i + G_t^t + (1 + r_t^b) B_t = B_{t+1} + T_t$
Public capital:	$K_{t+1}^g = (1-\delta^g) K_t^g + (1-\kappa) G_t^i$
	Aggregate economy & population
RES constraint:	$N_t c_{h,t} + N_{i,t} i_{i,t} + (1-\kappa)(G_t^c + G_t^i) = N_{f,t} y_{f,t}$
POP growth:	$\frac{N_{t+1}}{N_t} = 1 + \gamma^n$

On the policy side, the government has several fiscal policy instruments whose use shapes incentives, factor accumulation and, eventually, the drivers of macroeconomic growth. In particular, the taxes include those on firms' profits, τ_t^f, personal income, τ_t^y, and consumption, τ_t^c. On the spending side, we allow for public consumption, G_t^c, investment, G_t^i, and income transfers to households, G_t^t. Public investment spending augments public infrastructure capital, $\widetilde{k}_t^g$, where the latter enhances firms' productivity and households' human capital. Government consumption spending complements private consumption, $c_{h,t}$, in households' utility. Rent-seeking is defined here as the ability of firms to extract fiscal favours in the form of extra transfers that can augment their profits, $\pi_{j,t}$, where $j = f, i, b$. This results in resource misallocation at a social level since firms need to use a fraction of their labour for rent-seeking, $l_{j,t}^r$, instead of productive activities, $l_{j,t}^w$, and because they eventually extract a part of public spending, $\kappa(G_t^c + G_t^i)$, earmarked for productivity- and utility-enhancing public goods and services. Finally, it is helpful to recall that specific functional forms are required in an endogenously growing economy to allow for a stationary detrended transformation in equilibrium (see, e.g., Jones *et al.* 2005a, p. 5 and references therein).

2.1 Firms

We start by building upon the Romer-Jones production model in which productivity growth arises from an expanding variety of intermediate inputs or machines built using an idea or a blueprint.[8] Identical final-good firms produce a single final good. These firms hire labour from households and rent a variety of differentiated intermediate inputs from intermediate-goods firms. The latter hire labour, invest in physical capital and, to operate, purchase blueprints or ideas from research firms. Research firms hire researchers to produce blueprints or ideas, and we assume that each research firm makes one blueprint. All firms can benefit from public infrastructure and are engaged in a Tullock-type rent-seeking competition. In equilibrium, the number of final good firms, $N_{f,t}$, will be assumed to be equal to the number of intermediate goods firms, $N_{i,t}$, and this number will be set equal to the number of research firms, $N_{b,t}$, where the latter is endogenously determined as in the Romer-Jones setup.

[8] For the base model, in addition to Romer (1990) and Jones (1995), see, e.g. Barro and Sala-i-Martin (2004, chapter 6), Acemoglu (2009, chapter 13) and Aghion and Howitt (2009, chapter 3).

2.1.1 Final-good sector

At each t, there are $f = 1, 2, ...N_{f,t}$ identical final-good producers. Each f produces $y_{f,t}$ using the technology:

$$y_{f,t} = A_{f,t}(l_{f,t}^w)^a \left(\sum_{i=1}^{N_{i,t}} x_{f,i,t}^{1-\alpha} \right), \tag{1}$$

where $l_{f,t}^w$ and $x_{f,i,t}$ are, respectively, the units of labour input and the amount of each intermediate input or machine of variety $i = 1, 2, ..., N_{i,t}$ used by firm f in production, and $0 < \alpha < 1$ is a technology parameter. This production function follows the literature cited above and implies that product varieties, and hence ideas, are labour-augmenting.[9] Further, note that

$$A_{f,t} \equiv A_f \left(\widetilde{k}_t^g \right)^{\phi}, \tag{2}$$

where $A_f > 0$ is a scale parameter; $\widetilde{k}_t^g$ is per firm productivity-enhancing public capital expressed in efficiency units; and the parameter $0 < \phi < 1$ measures the productivity of $\widetilde{k}_t^g$.[10]

In each period t, each firm f maximises its after-tax gross profit defined as:

$$\pi_{f,t} \equiv (1 - \tau_t^f) \left(y_{f,t} - w_t(l_{f,t}^w + l_{f,t}^r) - \sum_{i=1}^{N_{i,t}} p_{i,t} x_{f,i,t} \right) + \left(\frac{l_{f,t}^r}{L_t^r} \right) G_t^p, \tag{3}$$

where $p_{i,t}$ is the price of intermediate input of variety i relative to the single final good price or the *numeraire*; $l_{f,t}^r$ is the units of labour input used for rent-seeking activities by each final good firm f (e.g. legal and financial activities, lobbying, etc.), while L_t^r is the total amount of these inputs used by all firms in the economy;[11] G_t^p denotes the contestable pie (defined below); and $0 \leq \tau_t^f < 1$ is the corporate tax rate on firms' gross profits. Notice that $\frac{l_{f,t}^r}{L_t^r}$ is the classic rent-seeking technology or redistributive contest introduced by Tullock (1967) and used, for example, by Murphy *et al.* (1991), Esteban and Ray (2011) and Angelopoulos *et al.* (2021).

[9] This becomes more apparent below. In particular, in a symmetric equilibrium (where intermediate goods firms are alike *ex-post*, the number of final good firms, $N_{f,t}$, equals the number of intermediate goods firms, $N_{i,t}$, and this number is set equal to the endogenously determined number of research firms, $N_{b,t}$) we will have $y_{f,t} = A_{f,t}(l_{f,t}^w N_{b,t})^a (x_{i,t})^{1-a}$.

[10] Efficiency units imply congestion in the use of public capital (see, e.g. Lansing (1988) and Agénor (2011)). Thus, $\widetilde{k}_t^g \equiv \frac{K_t^g}{H_t N_{b,t}}$, where K_t^g is the total quantity while H_t and $N_{b,t}$ are respectively the aggregate stock of human capital and the total number of blueprints. As said, in equilibrium, $N_{f,t} = N_{i,t} = N_{b,t}$.

[11] Thus, $L_t^r \equiv \sum_{f=1}^{N_{f,t}} l_{f,t}^r + \sum_{i=1}^{N_{i,t}} l_{i,t}^r + \sum_{b=1}^{N_{b,t}} l_{b,t}^r$.

First-order conditions Final good firms act competitively. The first-order conditions for $l^w_{f,t}$, $l^r_{f,t}$ and $x_{f,i,t}$, giving the demand for the two types of labour and each intermediate input i, respectively, are:

$$w_t = \frac{\alpha y_{f,t}}{l^w_{f,t}}, \tag{4}$$

$$(1-\tau^f_t)w_t = \left(\frac{1}{L^r_t}\right) G^p_t, \tag{5}$$

$$p_{i,t} = \frac{(1-\alpha)\, y_{f,t}\, (x_{f,i,t})^{-\alpha}}{\sum_{i=1}^{N_{i,t}} x^{1-\alpha}_{f,i,t}}. \tag{6}$$

2.1.2 Intermediate-goods sector

At each t, there are $i = 1, 2, ...N_{i,t}$ intermediate-goods producers, one for each input of variety i. Each i produces $x_{i,t}$ using the technology:

$$x_{i,t} = A_{i,t}\left(N_{b,t} l^w_{i,t}\right)^{\alpha} k^{1-\alpha}_{i,t}, \tag{7}$$

where $l^w_{i,t}$ and $k_{i,t}$ are, respectively, the labour and capital inputs used by firm i in production, and $N_{b,t}$ is the labour-augmenting number of blueprints in the economy.[12] Further note, as in (2) above, that:

$$A_{i,t} \equiv A_i \left(\widetilde{k}^g_t\right)^{\phi}, \tag{8}$$

where $A_i > 0$ is a scale parameter; and $\widetilde{k}^g_t$ and $0 < \phi < 1$ are as defined above.[13]

The motion of private capital used by each firm i is:

$$k_{i,t+1} = (1-\delta)k_{i,t} + i_{i,t} \tag{9}$$

where $0 \le \delta \le 1$ is the depreciation rate.

[12] This functional form allows us to obtain a stationary equilibrium system where all quantities can grow at the same rate on the BGP (see Appendix C). It is also like the production function of the final good in equilibrium (see footnote 9 above). In Jones (1995), one unit of capital is transformed into a unit of output, $x_{i,t} = k_{i,t}$. Gross and Klein (2022) use a standard neoclassical production function of the form $x_{i,t} = A_{i,t}\left(l^w_{i,t}\right)^{\alpha} k^{1-\alpha}_{i,t}$ which then implies that different quantities need to grow at different rates on the BGP (see Appendix of their paper).

[13] We could assume that the productivity parameter of public capital, ϕ, varies across sectors (see Malley and Philippopoulos (2023)). Here, for simplicity, we use a common ϕ across sectors. We report, however, that using sector-specific values of ϕ does not affect our main results.

Each i purchases a blueprint to operate, which works like a fixed cost within each period. In other words, as in Gross and Klein (2022), we assume that patents or blueprints last for one period only (hence q_t enters the flow payoff in each period), which, in the calibration below, will correspond to 20 years.[14] Therefore, each i maximises the discounted value of its after-tax net cash flows, or its value, defined as:[15]

$$\sum_{t=0}^{\infty} \beta_{i,t}\pi_{i,t} \equiv \sum_{t=0}^{\infty} \beta_{i,t} \Big[(1-\tau_t^f)[p_{i,t}x_{i,t} - w_t(l_{i,t}^w + l_{i,t}^r) - q_t] - \\ - i_{i,t} + \left(\frac{l_{i,t}^r}{L_t^r}\right) G_t^p\Big], \tag{10}$$

where q_t is the price of the blueprint purchased from the research sector; $l_{i,t}^r$ is the labour input used for rent-seeking activities by each intermediate good firm i; and $\beta_{i,t}$ is the firm's time discount factor (defined below).

First-order conditions Each intermediate-goods firm i acts monopolistically in its product market by taking into account its product's demand function, equation (6).[16] The first-order conditions for $l_{i,t}^w$, $l_{i,t}^r$ and $k_{i,t+1}$, giving the demand for the two types of labour and physical capital, respectively, are:

$$w_t = \frac{(1-\alpha)^2 y_{f,t} (x_{i,t})^{-\alpha}}{\frac{1}{N_{f,t}} \sum_{i=1}^{N_{i,t}} x_{i,t}^{1-\alpha}} \frac{\alpha x_{i,t}}{l_{i,t}}, \tag{11}$$

$$(1-\tau_t^f) w_t = \left(\frac{1}{L_t^r}\right) G_t^p, \tag{12}$$

$$1 = \beta_{i,1} \left[1 - \delta + \frac{(1-\tau_{t+1}^f)(1-\alpha)^2 y_{f,t+1}(x_{i,t+1})^{-\alpha}}{\frac{1}{N_{f,t+1}} \sum_{i=1}^{N_{i,t+1}} x_{i,t+1}^{1-\alpha}} \frac{(1-\alpha)x_{i,t+1}}{k_{i,t+1}}\right]. \tag{13}$$

[14] In contrast, e.g. Romer (1990), they last forever. Also, notice that all blueprints trade at the same price, q_t (see also, e.g. Jones (1995)).

[15] That is, as in most of the literature, the firm's gross profit, $(p_{i,t}x_{i,t} - w_t l_{i,t} - q_t)$, includes all types of costs except new investment, $i_{i,t}$ (see, e.g. Altug and Labadie (1994, pp. 171-172) and Miao (2014, p. 363-364) for similar problems and details). Also, Sargent (1987, pp. 80-81) shows the relation between the firm's profit and net cash flow. In particular, the firm's value, defined as the PDV of its net cash flows, equals the initial capital stock plus the PDV of its profits.

[16] As said this is $p_{i,t} = \frac{(1-a)y_{f,t}(x_{f,i,t})^{-a}}{\sum_{i=1}^{N_{i,t}} x_{f,i,t}^{1-a}}$. Since $x_{f,i,t} = \frac{x_{i,t}}{N_{f,t}}$, this becomes $p_{i,t} = \frac{(1-a)y_{f,t}(x_{i,t})^{-a}}{\frac{1}{N_{f,t}}\sum_{i=1}^{N_{i,t}} x_{i,t}^{1-a}}$. Then, when maximizing, each firm i takes $y_{f,t}$ and aggregate variables as given.

To operate, an intermediate firm will purchase the blueprint only if the associated profit is non-negative at each t. Thus,

$$(1-\tau_t^f)[p_{i,t}x_{i,t} - w_t(l_{i,t}^w + l_{i,t}^r) - q_t] - i_{i,t} + \left(\frac{l_{i,t}^r}{L_t^r}\right) G_t^p \geq 0. \tag{14}$$

Without loss of generality, we rewrite (14) as:[17]

$$q_t \equiv \Gamma \left[\frac{(1-\tau_t^f)[p_{i,t}x_{i,t} - w_t(l_{i,t}^w + l_{i,t}^r)] - i_{i,t} + \left(\frac{l_{i,t}^r}{L_t^r}\right) G_t^p}{(1-\tau_t^f)} \right], \tag{15}$$

where we will calibrate the auxiliary parameter $0 < \Gamma \leq 1$ to give us an average profit rate as in the data.[18] In other words, the extra profits generated in the intermediate goods sector, thanks to imperfect substitutability between intermediate goods as in the standard Dixit-Stiglitz framework, can be shared with the research sector that sells the patent being necessary for the production of intermediate goods. The higher the value of $0 < \Gamma \leq 1$, the higher the price of blueprints and the larger the fraction of these profits that goes to the research sector.

2.1.3 Research sector

For simplicity, as in most of the literature, we work with a research sector rather than individual research firms. At each t, we assume that the total number of blueprints evolves according to:[19]

$$N_{b,t+1} = (1-\delta^{n_b}) N_{b,t} + M_t L_{b,t}^w H_t (N_{b,t})^{\mu}, \tag{16}$$

[17] That is, the inequality condition (14) is $q_t \leq \frac{(1-\tau_t^f)[p_{i,t}x_{i,t} - w_t(l_{i,t}^w + l_{i,t}^r)] - i_{i,t} + \left(\frac{l_{i,t}^r}{L_t^r}\right) G_t^p}{(1-\tau_t^f)}$, which is equivalent to (15) to the extent that $0 < \Gamma \leq 1$.

[18] When $\Gamma = 1$, the net cash flow of the intermediate goods firm becomes zero in equilibrium. This is also the upper boundary since if $\Gamma > 1$, intermediate goods firms make losses and hence close down. Note that the case in which $\Gamma = 1$ is similar to that in most papers by Jones. For example, in Jones (1995, p. 781), the research sector "sets the price of the blueprint to extract the PDV of the intermediate sector's monopoly profit". To the best of our knowledge, Jones focuses mainly on the balanced growth path when he studies decentralized economies of this type. Finally, note that this equation can be compared to equation (19) in Gross and Klein (2022).

[19] Thus, following most of the related literature, we treat the number of blueprints, and hence the number of firms, as continuous rather than discrete or an integer. See Barro and Sala-i-Martin (2004, p. 287) for justification of this assumption in this family of models. In any case, as in the literature, we will solve for growth rates and ratios rather than levels.

where $L^w_{b,t}$ is the units of labour input used by this sector for the production of research; H_t is the economy's total human capital stock; $0 \leq \delta^{n_b} \leq 1$ is the depreciation rate which, since blueprints last for one period will be set at 1 in the calibration section; and $\mu < 1$ is a technology parameter whose range of values, as argued by, e.g. Jones (2019, 2022a), captures the fact that "ideas are becoming harder to find".[20] Further note, as in (2) and (8) above, that:

$$M_t \equiv M \left(\widetilde{k}^g_t \right)^{\phi}, \tag{17}$$

where $M > 0$ is a scale parameter; and $\widetilde{k}^g_t$ and $0 < \phi < 1$ are as defined above.

The research sector maximises the discounted present value of its after-tax gross profits defined as:

$$\sum_{t=0}^{\infty} \beta_{b,t} \Pi_{b,t} \equiv \sum_{t=0}^{\infty} \beta_{b,t} \left[(1 - \tau^f_t)[q_t N_{b,t} - w_t(L^w_{b,t} + L^r_{b,t})] + \left(\frac{L^r_{b,t}}{L^r_t} \right) G^p_t \right], \tag{18}$$

where $L^r_{b,t}$ is the labour used by this sector for rent-seeking activities; and $\beta_{b,t} = \beta_{i,t}$ (defined below).

First-order condition Using the law of motion for $N_{b,t}$ in the above profit function, the first-order conditions for $L^w_{b,t}$ and $L^r_{b,t}$ or the demand for the two types of labour in this sector are respectively:

$$(1 - \tau^f_t) w_t = \frac{\beta_{b,1} \left(1 - \tau^f_{t+1}\right) q_{t+1} M_t L^w_{b,t} H_t \left(N_{b,t}\right)^{\mu}}{L_{b,t}}, \tag{19}$$

$$(1 - \tau^f_t) w_t = \left(\frac{1}{L^r_t} \right) G^p_t. \tag{20}$$

2.2 Households

Households consume, work and save in the form of government bonds. They also own the firms and so receive their dividends. In addition to time allocated to work and leisure, they allocate time to education, which augments their human capital.

There are $h = 1, 2, ..., N_t$ identical households. Each h maximises lifetime utility defined as:

$$\sum_{t=0}^{\infty} \beta^t u \left(c_{h,t}, 1 - l^w_{h,t} - l^e_{h,t}; g^c_t \right), \tag{21}$$

[20] For empirical evidence, see Bloom *et al.* (2020). For a theoretical generalization of semi-endogenous and fully endogenous growth models in this literature, see Cozzi (2023).

where $c_{h,t}$ denotes private consumption; $\left(1 - l_{h,t}^{w} - l_{h,t}^{e}\right)$ is the fraction of time allocated to leisure where $l_{h,t}^{w}$ and $l_{h,t}^{e}$ are the time-fractions allocated to work and education respectively; g_t^c is per capita utility-enhancing public goods provided by the government (defined below); and $0 < \beta < 1$ is households' time discount factor.[21] Following, e.g. Finn (1998) and Jones *et al.* (2005a, 2005b), we use the functional form:

$$u\left(c_{h,t}, 1 - l_{h,t}^{w} - l_{h,t}^{e}; g_t^c\right) \equiv \frac{\left(c_{h,t} + \lambda g_t^c\right)^{1-\sigma}}{1-\sigma}\left(1 - l_{h,t}^{w} - l_{h,t}^{e}\right)^{\psi(1-\sigma)}, \qquad (22)$$

where $\sigma > 0$ $(\neq 1)$; $\psi > 0$ and λ is a preference parameter so that if $\lambda > 0$ (resp. $\lambda < 0$), private consumption and per capita public consumption are substitutes (resp. complements). Substitutes (resp. complements) mean that the marginal utility of private consumption decreases (resp. increases) with public consumption.

The within-period budget constraint of each h is:

$$\left(1 + \tau_t^c\right) c_{h,t} + b_{h,t+1} = \left(1 - \tau_t^y\right)\left(w_t h_{h,t} l_{h,t}^{w} + \pi_{h,t}\right) + \left(1 + r_t^b\right) b_{h,t} + g_t^t, \quad (23)$$

where $b_{h,t+1}$ is one-period government bonds purchased at t; w_t is the wage rate; $h_{h,t}$ is h's human capital at the beginning of t; r_t^b is the return to bonds purchased at $t-1$; $\pi_{h,t}$ is dividends paid by firms to each household to each h; g_t^t is a transfer to each household from the government; and $0 \leq \tau_t^y$, $\tau_t^c < 1$ are tax rates on income and consumption.[22] We assume that interest income from bonds is untaxed.

Each h's stock of human capital evolves as:

$$h_{h,t+1} = (1 - \delta^h) h_{h,t} + D_t \left(l_{h,t}^{e} h_{h,t}\right)^{\theta} \left(\frac{H_t}{N_t}\right)^{1-\theta}, \qquad (24)$$

where $0 \leq \delta^h \leq 1$ is human capital's depreciation rate; $\frac{H_t}{N_t}$ is per capita human capital in the society working as a positive externality; and $0 < \theta < 1$ is a technology parameter. Further note, as in (2), (8) and (17) above, that

$$D_t = D\left(\widetilde{k}_t^g\right)^{\phi}, \qquad (25)$$

where $D > 0$ is a scale parameter; and $\widetilde{k}_t^g$ and $\phi \geq 0$ have been defined above.

[21] As Boppart and Krusell (2019) point out, growth theory should not abstract from endogenous labour supply (recall, by contrast, that most endogenous growth models assume an inelastic labour supply, typically set at 1). Boppart and Krusell (2019) also study the implications of various utility functions like this within a balanced-growth perspective.

[22] Thus, as in practice, we allow for double taxation.

First-order conditions The household's first-order conditions for $b_{h,t+1}$, $l^w_{h,t}$, $l^e_{h,t}$ and $h_{h,t+1}$ respectively are:

$$\frac{\left(1+\tau^c_{t+1}\right)(c_{h,t}+\lambda g^c_t)^{-\sigma}(1-l^w_{h,t}-l^e_{h,t})^{\psi(1-\sigma)}}{(1+\tau^c_t)(c_{h,t+1}+\lambda g^c_{t+1})^{-\sigma}(1-l^w_{h,t+1}-l^e_{h,t+1})^{\psi(1-\sigma)}} = \beta\left(1+r^b_{t+1}\right), \tag{26}$$

$$\begin{aligned}\psi(c_{h,t}+\lambda g^c_t)^{1-\sigma}(1-l^w_{h,t}-l^e_{h,t})^{\psi(1-\sigma)-1} = \\ = \frac{(c_{h,t}+\lambda g^c_t)^{-\sigma}}{(1+\tau^c_t)}(1-l^w_{h,t}-l^e_{h,t})^{\psi(1-\sigma)}(1-\tau^y_t)w_t h_{h,t},\end{aligned} \tag{27}$$

$$\begin{aligned}\psi(c_{h,t}+\lambda g^c_t)^{1-\sigma}(1-l^w_{h,t}-l^e_{h,t})^{\psi(1-\sigma)-1} = \\ = \mu_{h,t}\frac{\theta D_t(l^e_{h,t}h_{h,t})^{\theta}\left(\frac{H_t}{N_t}\right)^{1-\theta}}{l^e_{h,t}},\end{aligned} \tag{28}$$

$$\begin{aligned}\mu_{h,t} = \beta\frac{(c_{h,t+1}+\lambda g^c_{t+1})^{-\sigma}}{\left(1+\tau^c_{t+1}\right)}(1-l^w_{h,t+1}-l^e_{h,t+1})^{\psi(1-\sigma)}(1-\tau^y_{t+1})\times \\ \times w_{t+1}l^w_{h,t+1} + \beta\mu_{\times,t+1}\left[1-\delta^h+\frac{\theta D_{t+1}(l^e_{h,t+1}h_{h,t+1})^{\theta}\left(\frac{H_{t+1}}{N_{t+1}}\right)^{1-\theta}}{h_{h,t+1}}\right].\end{aligned} \tag{29}$$

i.e. the demand for bonds, hours at work, hours in education and supply of human capital.

2.3 Government

The within-period government budget constraint is (in total terms):

$$\begin{aligned}G^c_t + G^i_t + G^t_t + (1+r^b_t)B_t = B_{t+1} + N_t\tau^y_t(w_t h_{h,t}l^w_{h,t} + \pi_{h,t}) + \\ +N_t\tau^c_t c_{h,t} + N_{f,t}\tau^f_t[y_{f,t} - w_t(l^w_{f,t}+l^r_{i,t}) - p_{i,t}x_{i,t}] + N_{i,t}\tau^f_t[p_{i,t}x_{i,t} - \\ -w_t(l^w_{i,t}+l^r_{i,t}) - q_t] + N_{b,t}\tau^f_t[q_t - w_t(l^w_{b,t}+l^r_{b,t})],\end{aligned} \tag{30}$$

where G^c_t, G^i_t and G^t_t are, respectively, government spending earmarked for public consumption, investment and transfers to households. Given tax rates and public spending items, this constraint will determine the end-of-period bonds, B_{t+1}, residually.

We assume that the contestable pie is public spending on consumption and investment.[23] Thus,

$$G^p_t \equiv \kappa(G^c_t + G^i_t), \tag{31}$$

where $0 \leq \kappa < 1$ is the fraction of the contestable pie extracted. In other words, although the government earmarks $G^c_t + G^i_t$ for utility- and productivity-enhancing public goods, only a fraction of it, $0 < 1-\kappa \leq 1$,

[23] We could assume that the pie incorporates all types of public spending, including spending earmarked for household transfers. This is not important to our results.

is used for this purpose, because the rest, $0 \leq \kappa < 1$, is grabbed by rent-seeking firms as an extra fiscal transfer that augments their profits. Hence, the motion of public capital is:

$$K_{t+1}^g = (1 - \delta^g)K_t^g + (1 - \kappa)G_t^i, \tag{32}$$

where $0 \leq \delta^g \leq 1$ is the depreciation rate. Similarly to productivity-enhancing spending, per capita utility-enhancing public goods provided by the government are $g_t^c \equiv \frac{(1-\kappa)G_t^c}{N_t}$.

2.4 Exogenous variables

Regarding policy instruments, we assume constant tax rates and that public spending and debt are proportional to final output. Thus, lump-sum transfers, G_t^t, are the residual policy instrument that closes the government budget. In particular,

$$G_t^k = s_t^k N_{f,t} y_{f,t}, \tag{33}$$

$$B_t = \left(\frac{B}{N_f y_f} \right) N_{f,t} y_{f,t}, \tag{34}$$

where $s_t^k = s^k + \varepsilon_t^k$; $k \equiv c, i, t$ for different public spending items; $0 < s^k < 1$ are parameters; and ε_t^k denotes policy shocks.

Finally, population size evolves as:

$$\frac{N_{t+1}}{N_t} = 1 + \gamma^n, \tag{35}$$

where $\gamma^n \geq 0$ is a parameter.

2.5 Macroeconomic system

Collecting equations, our macroeconomic system, including market-clearing conditions, is presented in Appendix B. In this equilibrium system, we assume that intermediate goods firms are alike *ex-post* and, as said above, that the number of firms is the same across the three sectors and equal to the endogenously determined number of blueprints, $N_{b,t}$; that is, $N_{f,t} = N_{i,t} \equiv N_{b,t}$.[24]

[24] Our assumption about the endogenous determination of the number of firms is comparable to the one employed in two-sector models, e.g. Bilbiie *et al.* (2012, 2007) and Etro and Colciago (2010), where an entry decision achieves this endogeneity and hence product variety. It is also comparable to the model of Blanchard and Giavazzi (2003), where the same kind of endogeneity happens via the assumption that the degree of substitutability between intermediate goods is a function of the number of firms where the latter adjusts to make profits zero in the long run. But, as said above, this literature has no innovation and no research sector.

In this system, the exogenous motion of population, $\frac{N_{t+1}}{N_t} > 1$, implies that variables are not stationary. In addition, the motions of ideas and human capital, $N_{b,t+1}$ and $h_{h,t+1}$, can also cause non-stationarity to the extent that the solutions of the associated endogenous variables, in combination with parameter values, result in $\frac{N_{b,t+1}}{N_{b,t}} > 1$ and $\frac{h_{h,t+1}}{h_{h,t}} > 1$. Hence, we need to detrend the non-stationary variables by N_t, $N_{b,t}$ and $h_{h,t}$ and then solve the transformed equilibrium system. We report the latter in Appendix C.

Accordingly, if $\frac{N_{b,t+1}}{N_{b,t}} > 1$ and $\frac{h_{h,t+1}}{h_{h,t}} > 1$ in the long run, the model features long-run endogenous per capita growth. In this situation, the economy is on the BGP, and all per capita quantities grow at the same positive rate. In particular, as Appendix D shows, the BGP net growth rate of per capita GDP, γ^{y_f}, is the sum of human capital growth, γ^h, and ideas growth, γ^{n_b}. These BGP growth rates remain constant in response to temporary policy changes but can change in response to permanent changes. Appendix D also shows that ignoring human capital and fiscal policy implies, as in, e.g. Jones (2019), $\gamma_t^{y_f} = \gamma^h = \gamma^{n_b}$ on the BGP. In other words, in this special case, the long-run per capita GDP growth rate is driven by the creation of new ideas, which in turn is determined by population growth only.

3 Calibration

We start with an annual calibration of the structural and policy parameters and then convert the relevant coefficients to a 20-year calibration to reflect that patents expire after 20 years (see, e.g. Gross and Klein (2022)).[25] Given our interest in long-run growth, we use the most extended available time series to help approximate parameter means for the structural parameters and the most recent data for the policy parameters.

3.1 Structural parameters

In Table 2, the model's scale parameters A_f, A_i, and M are normalised to unity and population growth, n_a, and the depreciation rates, δ_a and δ_a^g, are based directly on the data. Note that we calculate average exponential population growth using the Federal Reserve Economic Data (FRED) database (1929-2022), and the mean depreciation rates for δ_a and δ_a^g using the Bureau of Economic Analysis (BEA) fixed asset accounts Tables 1.1 and 1.3 (1925-2021). Moreover, we chose β_a to target an annual return on bonds, r^b, of 4%.

[25] Note that annual parameters, which we convert to a 20-year basis, will be denoted with the subscript 'a'.

The parameters σ and δ_a^h are from Jones *et al.* (2005b)). Other parameter values following the literature include $\alpha = 0.64$ and $\theta = 0.5$ (see Jones *et al.* (2005b) and Angelopoulos *et al.* (2012)). Also, $\mu = -1$ is required to obtain a stationary solution. Finally, we set $\lambda = -1$ to reflect a one-for-one complementarity between private and public consumption[26] and Γ to target a profit share for intermediate and research firms of 10%.

Table 2: Structural Parameters

Coef.	Value	Definition
A_f	1.000	scale parameter in final good production
A_i	1.000	scale parameter in intermediate goods production
α	0.640	labour's share of output
β_a	0.990	time discount factor
δ_a	0.047	depreciation rate private capital
δ_a^g	0.040	depreciation rate public capital
δ_a^h	0.025	depreciation rate human capital
θ	0.500	h_h elasticity of new human capital
λ	-1.000	preference parameter in the utility function
μ	-1.000	technology parameter in blueprints production
M	1.000	scale parameter in blueprints production
γ_a^n	0.011	net population growth rate
σ	1.400	coefficient of relative risk aversion $(1/\sigma)$
Γ	0.5402	blueprint pricing function parameter

3.2 Policy parameters

The public consumption and investment shares reported in Table 3 are from the U.S. National Income and Product Accounts (NIPA) in 2022. Corporate taxes apply to final goods, intermediate goods and research firms. In contrast, labour income taxes apply to households. The values we use for these rates are those calculated by Malley and Philippopoulos (2023) and follow the methods set out in Jones (2002). The mean gross federal debt to GDP ratio in 2022 is from the FRED database. Finally, the value of the public productivity parameter, ϕ, is set at the lower end of the range reported in the literature (see, e.g. Malley and Philippopoulos (2023) and the review in Ramey (2020) for references to the literature).

[26] In contrast, $\lambda = 0$ implies that public consumption is a resource drain and $\lambda = 1$ that public consumption directly crowds out private consumption. See, e.g. Malley and Philippopoulos (2023) and references therein for further discussion of how the literature treats this parameter. Note that the value of λ does not materially affect our key results.

Table 3: Policy Parameters

Coef.	Value	Definition
s^c	0.141	public consumption share of final output
s^i	0.034	public investment share of final output
τ^f	0.259	corporate tax rate
τ^y	0.299	labour income tax rate
τ^c	0.069	consumption tax rate
ϕ	0.050	public capital elasticity
$\left(\frac{B}{y_f}\right)_a$	121.11	public debt share of final output

3.3 Calibration (20-year)

To translate the relevant parameters from the annual calibration in Tables 2 and 3 to a 20-year frequency requires the transformations reported in Table 4. Also, following the literature, the 20-year depreciation rate on blueprints is $\delta^{n_b} = 1$.

Table 4: 20-year conversion

$\beta = (\beta_a)^{20} = 0.818,$	$\delta = 1 - (1 - \delta_a)^{20} = 0.615,$
$\delta^g = 1 - (1 - \delta^g_a)^{20} = 0.561,$	$\delta^h = 1 - (1 - \delta^h_a)^{20} = 0.397,$
$\frac{B}{y_f} = \left(\frac{B}{y_f}\right)_a \left(\frac{1}{20}\right) = 6.056,$	$\gamma^n = (1 + \gamma^n_a)^{20} - 1 = 0.237.$

In Table 5, using the 20-year calibration, we solve for D (to target human capital growth, γ^h, on the BGP), ψ (to target work-time, l^w_h), and κ (to target rent-seeking time, $3l^r_t/\widetilde{\psi}_{b,t}$). In particular, an index of human capital per person (1950-2019) suggests that average exponential human capital growth, γ^h, over this period is roughly half a percentage point (see Federal Reserve Economic Data (FRED)).[27] The work-time target is 0.31, following Cooley and Prescott (1995) and Malley and Philippopoulos (2023)). We assume that the proportion of time households allocate to rent-seeking services at work is 1%.[28] This conservative value is close to the lowest rate typically employed by the quantitative rent-seeking literature. For instance, Angelopoulos *et al.* (2021) also use 1% for the U.S., while, in papers for the European economies,

[27] On a 20-year basis, the net growth rate of human capital is 0.1050.

[28] That is, from the market-clearing condition in the labour market, (C.19) in Appendix C, the fraction of time that households put in productive work is $(l^w_{f,t} + l^w_{i,t} + l^w_{b,t})/\widetilde{\psi}_{b,t}$, while the rest, $3l^r_t/\widetilde{\psi}_{b,t}$, goes to the provision of rent-seeking services so that here we set $3l^r_t/\widetilde{\psi}_{b,t} = 0.01$. This equation implies the calibrated value of κ we use in our base solutions.

the calibrated value of this fraction is about 5-10% (see, e.g. Angelopoulos *et al.* (2009) and Christou *et al.* (2021)). After solving the model, this 1% implies a value of κ around 0.12, suggesting that rent-seekers extract around 2.8% of GDP. Note that we deliberately use a low value of rent-seeking time to show that even such a slight distortion has important macroeconomic implications.

Table 5: Implied parameters 20-year calibration

Coef.	Value	Definition
D	2.1993	scale parameter in human capital production
κ	0.1187	rent extraction parameter
ψ	1.5269	utility function parameter

4 Quantitative analysis

In the following analysis, the initial BGP is defined as the solution of the model using the parameters and policy variables listed in Tables 2-5 above. In this initial equilibrium, the economy grows at a constant rate of about 2.1%, i.e. the long-run average growth rate in the U.S. annual data.

As discussed above, we shock the model by assuming five different permanent changes: (i) an increase in public investment spending by 1-ppt (i.e. from 3.4% in the data to 4.4%); (ii) a lower market power of research firms achieved by decreasing the non-competitive price at which they sell their blueprints to intermediate goods firms; in particular, we lower the parameter Γ as defined in equation (15) so that the profits of research firms fall by 10% relative to their base value (i.e. Γ changes from 0.5402 to 0.4892); (iii) a lower market power of intermediate goods firms achieved by decreasing the non-competitive price at which they sell their products to final good producers; specifically, we increase the parameter Ω as defined in Appendix F so that the net cash flow of intermediate goods firms falls by 10% from their base solution (i.e. Ω changes from 0 to 0.19); (iv) the elimination of rent-seeking by lowering the parameter κ (i.e. from $\kappa = 0.1187$ to $\kappa = 0$ or, equivalently, from 1% to 0% rent-seeking time, or 2.8% to 0% rent-seeking costs as a share of GDP); and finally (v) an increase in the exogenous growth rate of population from 1.1% to 1.2%.

We realise that our approach to policy reforms (ii), (iii) and (iv) is rather generic. More specifically, regarding (ii) and (iii), as in Syverson (2019, p. 25), we define market power as "the ability of the firm to influence the price at which it sells its product(s)", and changes in the (calibrated) parameters Γ and Ω help us to capture this. Papers that work similarly, in the sense that non-competitive prices, and hence market power, are directly affected by changes in parameters and exogenous policy instruments, include Blanchard and Giavazzi (2003), Eggertsson *et al.* (2014) and Pfeiffer *et al.* (2023), all three for the European economy. The same applies to (iv), where institutional quality or the fraction of socially beneficial public spending eventually grabbed by rent-seeking firms, is captured by changes in the (calibrated) parameter κ. Papers that work similarly include Murphy *et al.* (1991), Esteban and Ray (2011), Angelopoulos *et al.* (2009, 2021) and Christou *et al.* (2021).

Since the above changes are assumed to be permanent, the model converges to a new BGP. In each experiment reported below, we compare the initial and terminal BGPs and then analyse the transition dynamics. Finally, since the shock sizes considered below are arbitrary, Appendix G documents a range of outcomes for each reform. Throughout, we assume perfect foresight.

4.1 Higher public investment

In this experiment, fixing the public debt-to-GDP ratio and the remaining fiscal policy instruments as in the 2022 data, government transfers adjust to finance the assumed increase in public investment by 1 ppt.

4.1.1 Balanced growth path

Table 6 presents the results of some key variables on the base and shocked BGPs. We focus on variables that determine the economy's real growth rate and directly shape social welfare (see Appendices D and E, respectively, for details). As can be seen in this table, a permanent increase of public investment by one ppt enhances the long-term growth rates of both individual human capital, γ^h, and ideas, γ^{n_b}, and thereby the economy's long-term real growth rate (recall that on the BGP, the latter is simply the sum of γ^h and γ^{n_b}). Specifically, on the BGP, the growth rate of per capita GDP increases from 2.08% to 2.14%. This higher growth rate implies that if we start with a per capita GDP level of about $60,000 (i.e. the 2022 value in the U.S.), after, say, 40 and 100 years, per capita GDP would be about 3.6 and 31.8 thousand dollars higher respectively than under the base growth rate scenario of 2.08%.[29]

Regarding social welfare, productive public investment incentivises work and education, so leisure is lower on the new BGP. Nevertheless, welfare, χ_{bgp}, rises due to an increase in per capita private and public consumption, whose

[29] Since output levels are exponential functions of output growth rates, even small increases in the latter translate to substantial gains in levels. In their empirical growth study, Prichett *et al.* (2016) also show significant gains or losses in per capita income levels as a result of an initial growth episode, positive or negative, in various countries.

rise more than offsets the adverse effect of less leisure on households' welfare. We can understand the increase in per capita private and public consumption by the increase of their detrended counterparts, $\widetilde{c}_h$ and $\widetilde{g}^c$. Recall that $\widetilde{c}_{h,t} \equiv \frac{C_t}{N_t h_{h,t} N_{b,t}}$ and $\widetilde{g}_t^c \equiv \frac{G_t^c}{N_t h_{h,t} N_{b,t}}$ (see Appendix C). In other words, since $\widetilde{c}_h$ and $\widetilde{g}^c$ are higher on the new BGP, while, at the same time, the growth rates of ideas, $N_{b,t}$, and human capital, $h_{h,t}$, are also higher, implies that per capita consumption, $c_{h,t} \equiv \frac{C_t}{N_t} \equiv \widetilde{c}_{h,t} h_{h,t} N_{b,t}$, and utility-enhancing public services, $g_t^c \equiv \frac{G_t^c}{N_t} \equiv \widetilde{g}_t^c h_{h,t} N_{b,t}$, grow by more on the new BGP. Notice that the resulting welfare gain of about 3.53%, as typically measured by the permanent change in consumption equivalent units (see Appendix E), is substantial relative to gains implied by other temporary reforms in the literature (see, e.g. Malley and Philippopoulos (2023) for details).

Table 6: Higher public investment

Determinants of welfare & CCS		Base	Shock
final output:	$\widetilde{y}_f$	0.0427	0.0438
private consumption:	$\widetilde{c}_h$	0.0353	0.0359
public consumption:	$\widetilde{g}^c$	0.0053	0.0054
work time:	l_h^w	0.3100	0.3127
education time:	l_h^e	0.1053	0.1055
leisure time:	$1 - l_h^w - l_h^e$	0.5847	0.5818
annual human capital growth:	γ^h	0.0050	0.0053
annual ideas growth:	γ^{n_b}	0.0158	0.0161
CCS_{bgp}:	χ_{bgp}		3.5303

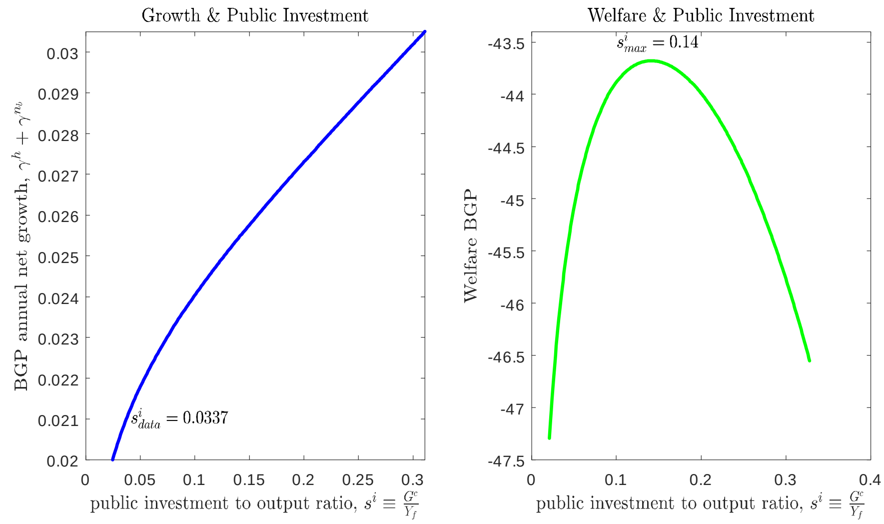
Figure 1, which plots public investment as a share of GDP on the horizontal axis and the economy's long-term real growth rate and social welfare on the vertical axes, confirms the above logic. The effect on the growth rate is monotonically positive, which is unsurprising since public investment augments public capital, with the latter enhancing productivity for the three different types of firms in our model. Moreover, as mentioned above, we assume that lump-sum transfer changes finance this extra public spending.[30]

In contrast, there is a trade-off regarding social welfare. This is because higher growth, and hence a constantly increasing per capita consumption, comes at the cost of less leisure. It is interesting to notice that the maximum GDP share of public investment is around 14% in this experiment, which is

[30] Allowing transfers to adjust to accommodate the exogenous change in fiscal policy is the usual assumption in the literature. In contrast, see Malley and Philippopoulos (2023) for the implications of alternative distorting public financing instruments in a model without endogenous long-term growth.

over three times higher than this share in the 2022 data. Although historically, this share was around 6.5%-7% in the 1960s and almost 20% during WWII, the finding that it is currently underprovided is consistent with results reported by Malley and Philippopoulos (2023) and Ramey (2020). Naturally, the maximum share would be significantly lower if distorting tax instruments were used to finance the increase in public investment (see, e.g. Malley and Philippopoulos (2023), although in a non-growing economy). Nonetheless our analysis shows that, even when financed by cuts in lump-sum transfers, public investment is not a free lunch since lower transfers lead to more work, less leisure and lower welfare. Also, lower transfers would cease to be lump-sum and possibly worsen inequality in models with household heterogeneity and inequality. Thus, our quantitative normative results are indicative only.

Figure 1: Growth, Welfare and Public Investment



4.1.2 Transition dynamics

Figures 2a and 2b present the economy's behaviour over the transition from the initial to the new BGP. The former figure shows the paths of the main variables that capture macroeconomic outcomes and determine social welfare. At the same time, the latter figure focuses on the paths of production inputs that drive the magnitudes included in 2a. The dotted blue line shows the value of a variable at the initial BGP, and the solid green line its value as the economy transitions to the new BGP. The mauve dotted line in the graph for final output shows the sum of the growth rates of ideas and individual human capital so that the difference between this and the green line is due to other variables that endogenously change along the transition to the new BGP (see Appendix equation D.2 for details).

Figure 2a includes three subplots illustrating the time paths of three growth rates: ideas, individual human capital and per capita GDP. It also consists of two subplots for the paths of the detrended, stationary values of per capita private and public consumption (as defined in Appendix C). There are another three subplots for the paths of leisure hours, as well as the paths of the non-stationary values of per capita private and public consumption, which are the three variables that enter the household's utility function before transformations (see equation (21)).[31] Finally, the last subplot shows the consumption subsidy (CCS) time path, where a positive value indicates a welfare gain vis-a-vis the initial BGP.

Figure 2a: Public Investment Shock (Growth & Welfare)

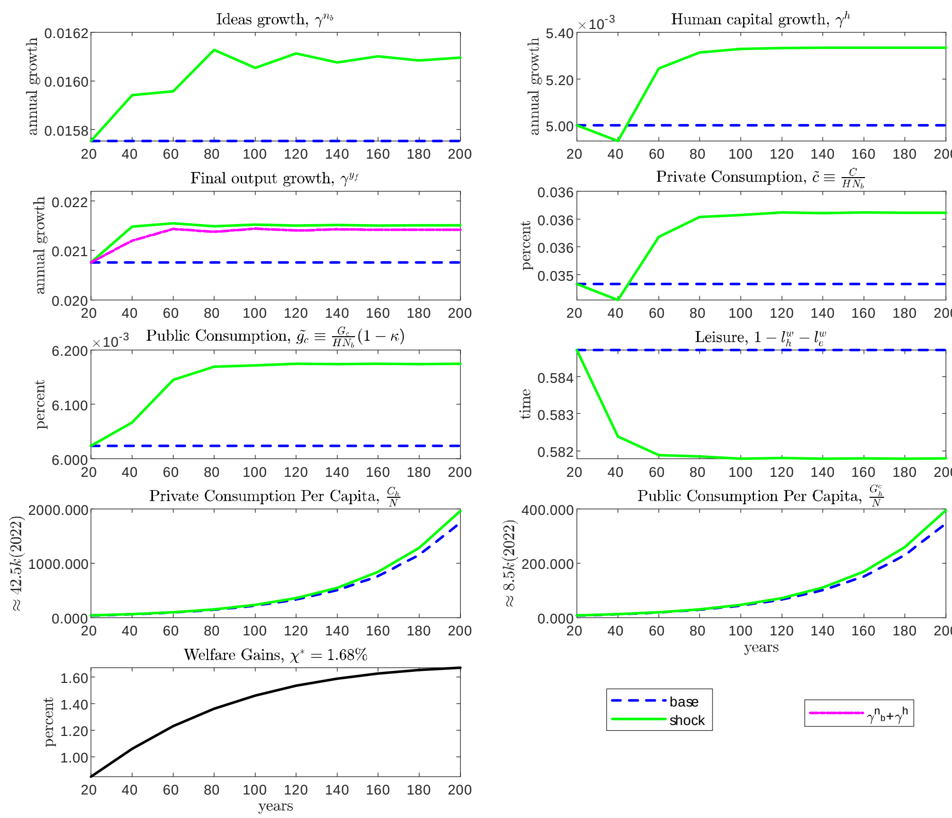
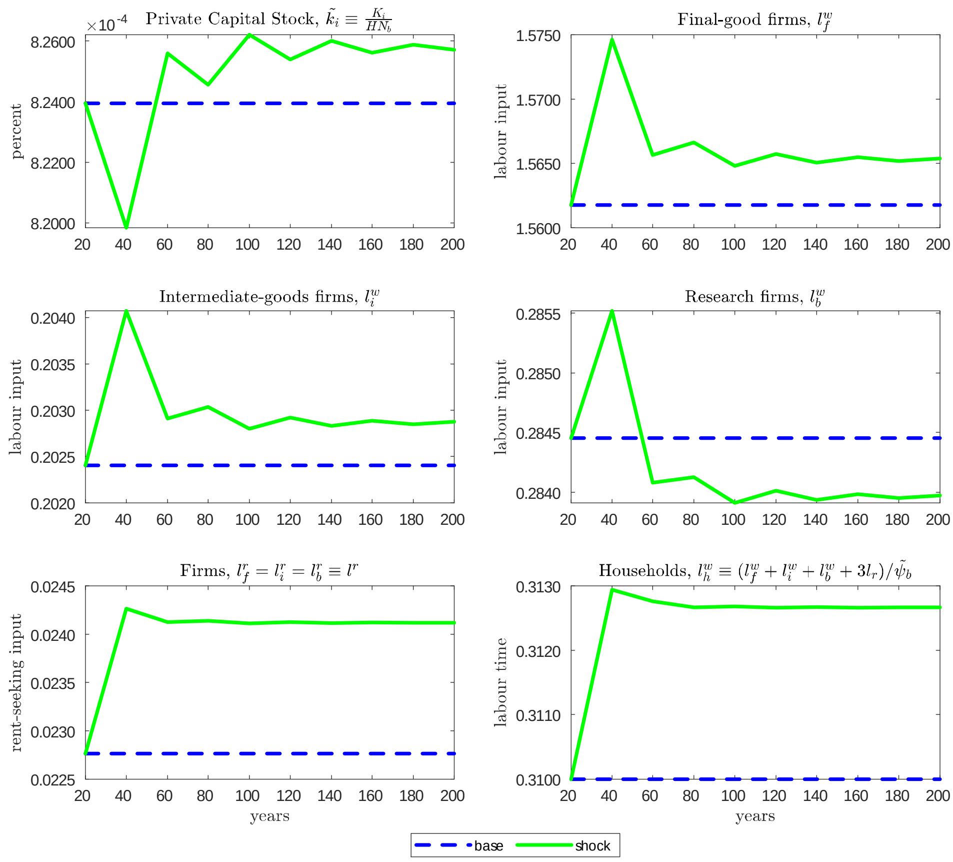


Figure 2b includes six subplots for the path of the detrended physical capital stock, the paths of the labour input used for productive activities by

[31] We calculate per capita, private and public consumption paths using U.S. data from 2022 data as initial values (see Appendix D for details).

final, intermediate and research firms, the labour input used for rent-seeking activities used by each firm (this is common across firms) and finally the path of households' time at work. Note that the Figures for the remaining experiments in this section will follow the same format as Figures 2a,b.

Figure 2b: Public Investment Shock (Inputs)



The message from Figure 2a is similar to that from Table 6, where we compared the initial to the new BGP. That is, an increase in public investment is growth-enhancing (see the three graphs for the growth rates with a more marked increase in the growth rate of ideas), which benefits private and public per capita consumption. The CCS subplot reveals that the consumption increases more than offset the fall in leisure time, so there are welfare gains along the entire transition path to the new BGP.

Figure 2b reveals the factors that drive economic growth over time. Private physical capital increases (after a drop in the short term). The workforce used by final good and intermediate goods firms for productive activities, $l_{f,t}^w$

and $l_{i,t}^{w}$, also increase relative to the base. The same happens to that used by research firms, $l_{b,t}^{w}$, in the short term (by contrast, in the medium and long run, the increase in human capital shown in Figure 2a allows research firms to reduce their labour input used for the production of ideas). But, at the same time, all firms find it optimal to also increase their workforce used for rent-seeking activities, l_t^r. This happens because the increase in public investment spending leads to a direct increase in the contestable prize that firms compete for, and this triggers an increase in the units of labour input employed for rent-seeking activities. In other words, the detrimental consequences of rent-seeking from state coffers (here, in terms of labour misallocation away from productive uses) intensify as public spending rises, weakening the beneficial effects that an increase in public investment could have had on the macro-economy. For a similar result in a quantitative business cycle model for the U.S. (see, e.g. Angelopoulos *et al.* (2021)). The increase in the demand for productive labour from the side of final and intermediate goods firms, as well as the increase in the demand for labour for rent-seeking activities by all firms, dominates, so households' hours at work, $l_{h,t}^{w}$, rise too.

4.2 Less market power to research firms

We next study the effects of exogenously reducing the blueprint's non-competitive price so that the research firm's profit falls by 10% from its initial BGP value. As said above, we achieve this by lowering the $0 < \Gamma \leq 1$ parameter in the blueprint pricing function (15) from 0.5402 to 0.4892. A lower price, generally, makes blueprints more accessible to other firms, and this can stimulate growth, but, on the other hand, it discourages the production of new ideas, thus hurting other firms and the economy's growth.

4.2.1 Balanced growth path

Inspection of the results in Table 7 reveals that a cut in research firms' profits leads to a reduction in the growth rates of ideas and individual human capital and, eventually, a reduction in the economy's growth rate. This happens because the incentive to produce ideas is now weaker. The detrended final output and private consumption values are higher in the new BGP. However, this is only because the denominator (in particular, the stocks of ideas and human capital) is lower and not because the per capita magnitudes in the numerator are higher. Per capita GDP and per capita private and public consumption are all lower in the new BGP since the constant rate at which all per capita quantities can grow is the sum of the growth rates of ideas and human capital, and this sum is lower in the new BGP as just said.

Nevertheless welfare is higher on the new BGP. This occurs simply because the losses from lower per capita private and public consumption are more than offset by more leisure hours in the long-run equilibrium. Leisure hours rise because equilibrium labour has decreased due to less production.

Table 7: Lowering profits of research firms

Determinants of welfare & CCS		Base	Shock
final output:	$\widetilde{y}_f$	0.0427	0.0430
private consumption:	$\widetilde{c}_h$	0.0353	0.0356
public consumption:	$\widetilde{g}^c$	0.0053	0.0053
work time:	l_h^w	0.3100	0.3077
education time:	l_h^e	0.1053	0.1046
leisure time:	$1 - l_h^w - l_h^e$	0.5847	0.5877
annual human capital growth:	γ^h	0.0050	0.0049
annual ideas growth:	γ^{n_b}	0.0158	0.0157
CCS_{bgp}:	χ_{bgp}		0.8750

In terms of magnitudes, on the BGP, the growth rate of per capita GDP falls from the base rate of 2.08% to 2.06%, which again is not negligible in terms of the level of per capita GDP for the reasons discussed above. This lower growth rate implies, again starting with a per capita GDP level of $60,000, that after 40 and 100 years, per capita GDP would be about 0.7 and 5.6 thousand dollars lower, respectively, than in the base.

Figure 3 plots Γ against the economy's growth rate and social welfare on the BGP. Recall that the higher the Γ, the higher the research sector's pricing power and profits. Γ's effect on growth and welfare is monotonic but

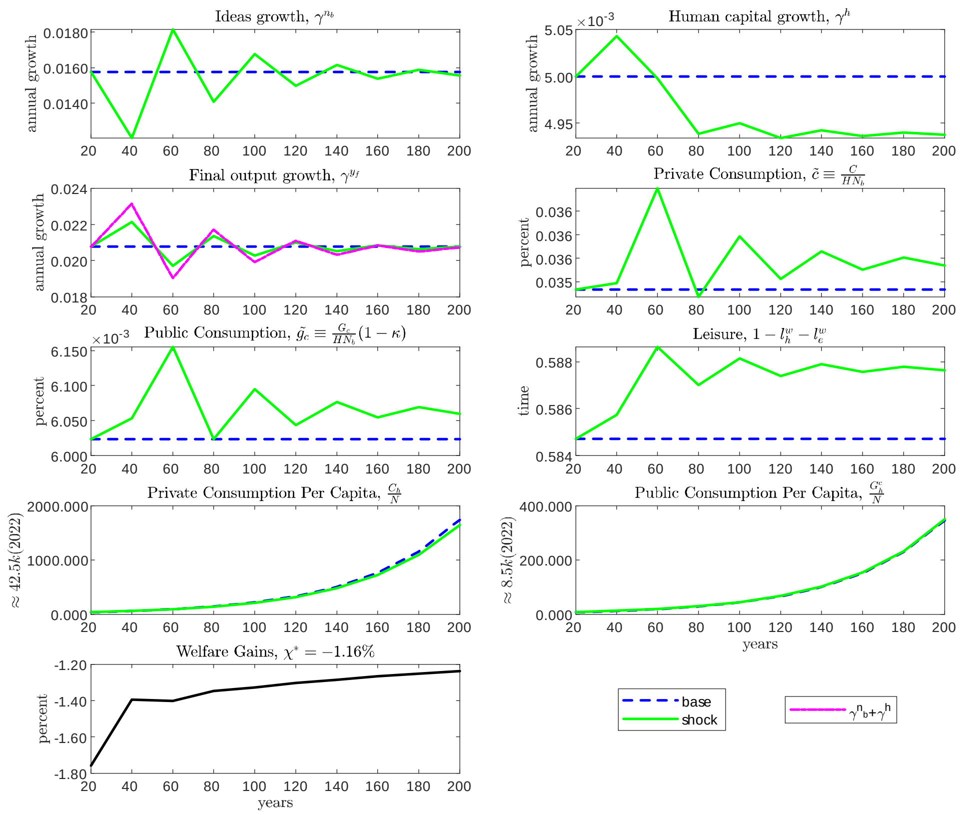
Figure 3: Growth, Welfare and Profit of Research Firms

in opposite directions. As Γ rises, the incentive to generate ideas strengthens, driving long-term economic growth. On the other hand, as we switch to a more productive economy, people work more, reducing leisure, which, in our parameterization, dominates these welfare comparisons. Note that if we increase the parameter Γ to its upper boundary (unity), which means that the research firms extract all profits made by intermediate goods firms as explained in equation (15), then the profits enjoyed by the research sector increase by nearly 100% as a share of GDP relative to the base (in particular, they rise from 1.43% to 2.78% of GDP).

4.2.2 Transition dynamics

We start by comparing transition results to results for the BGP and postpone the discussion of some variables' damped oscillations observed in Figures 4a-4b to the end of this subsection. Most transition results in Figure 4a are

Figure 4a: Profits Shock Research Firms (Growth & Welfare)



qualitatively similar to those in Table 7 for the BGP. The only difference from the BGP is that now, along the transition, welfare is lower over time (see the negative value of the consumption gain). This happens because the drop in per capita private consumption is more pronounced along the transition than on the BGP, and this cost is now more substantial than the gain from more leisure time. But the key message is the same as on the BGP: a reduction in $0 < \Gamma \leq 1$ hurts economic growth and macroeconomic performance over time. Reversing the argument, in our setup calibrated to the U.S. economy, an increase in Γ, and thus extra returns from research, are associated with higher growth both on the BGP and along the transition. This result generalises the predictions of the Romer-Jones model. It is also consistent with studies that provide evidence that market power is associated positively with innovation (see, e.g. Autor *et al.* (2020), Kwon *et al.* (2023) and Bighelli *et al.* (2023), as well as the review paper of Syverson (2019)).

Figure 4b: Profits Shock Research Firms (Inputs)

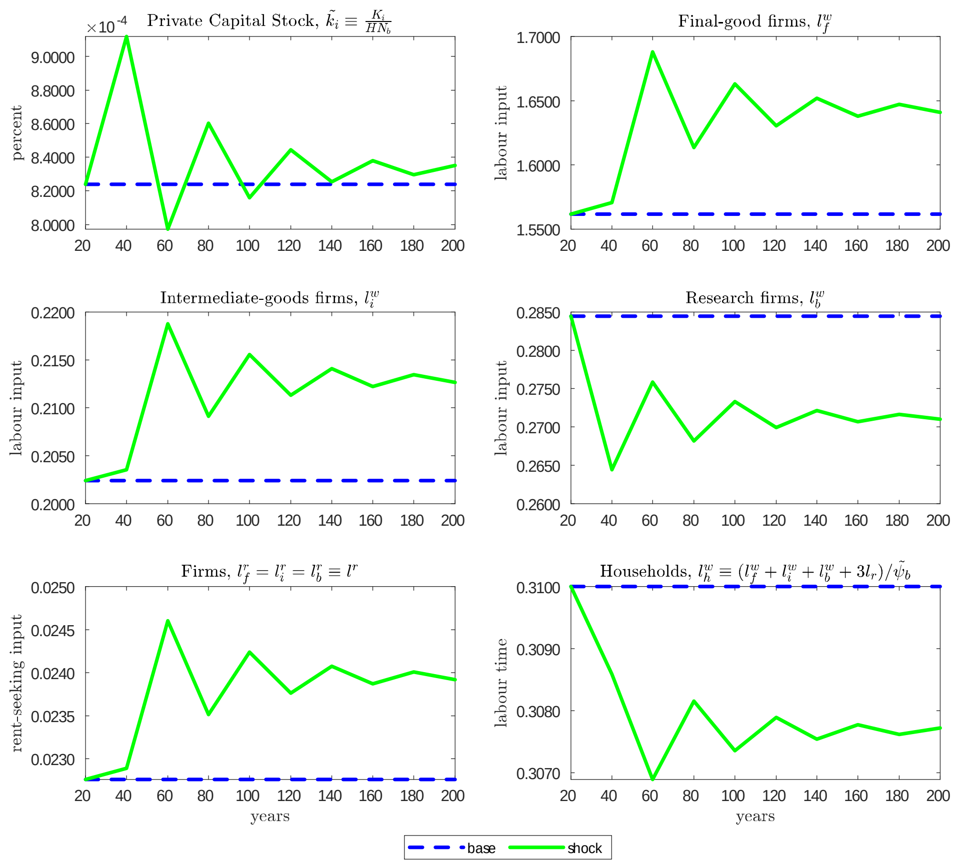


Figure 4b reveals an intuitive reallocation of labour across sectors caused by the assumed cut in the price markup realised initially by research firms. In particular, there is a marked fall in $l^w_{b,t}$ as research firms now sell at a lower price, so they reduce their output, while $l^w_{f,t}$ and $l^w_{i,t}$ rise as the cost of blueprints used by intermediate goods firms, and in turn the cost of intermediate goods used by final good firms, both fall. Notice that the drop in the demand for labour by research firms, $l^w_{b,t}$, dominates any other developments, so that $l^w_{h,t}$ falls, and leisure rises, on the side of households. Also, note the increase in the labour input used for rent-seeking activities, l^r_t. This rise is probably explained by the significant fall in $l^w_{b,t}$, which releases workers from productive effort to rent-seeking activities.

Finally, observe the damped oscillations of ideas and GDP growth rates around the base in Figure 4a. This differs from the previous experiment of increasing public investment shown in Figure 2a above, where convergence to the new BGP was much smoother. Typically, oscillations of this kind have to do with negative values of eigenvalues (see, e.g. Azariadis (1993, chapter 2)). We report that this is the case: we get real eigenvalues in all experiments, but there is a bigger number of negative ones in the case of changes in Γ than in the case of changes in public investment studied in the previous subsection (and, we shall see below, the same will apply in the next two experiments with changes in Ω and κ). Intuitively, this happens because there are direct tradeoffs across sectors in these three cases (especially for changes in Γ and Ω). For example, when Γ falls, this isn't good for the profits of the research sector but is beneficial for the other two sectors for the reasons discussed above. By contrast, a change in public investment exerts similar effects on all sectors and agents. We believe this cross-sectoral tradeoff drives the sawtooth paths in Figure 4a and, in turn, Figure 4b.

4.3 Less market power to intermediate-goods firms

We next study the effects of exogenously reducing the non-competitive price at which intermediate goods firms sell their products. The price reduction will be such that the intermediate goods firms' net cash flow falls by 10% from its initial BGP value. As said at the beginning of this section, this is achieved by increasing the parameter $0 \leq \Omega \leq \alpha$ as defined in Appendix F, from its base value of 0 to 0.19.[32] Lower prices leading to lower markups and profits for intermediate goods firms hurt these firms, but, on the other hand,

[32] That is, increases in the parameter Ω away from 0 amount to higher substitutability among intermediate products (see also, e.g. Blanchard and Giavazzi (2003) and Eggertsson *et al.* (2014)).

they can benefit the final good firms that need to purchase intermediate goods.

4.3.1 Balanced growth path

The results in Table 8 reveal that a cut in intermediate goods firms' profits leads to an increase in the growth rates of ideas and individual human capital and, eventually, an increase in the economy's growth rate. This happens mainly because intermediate goods are cheaper (we report that p_i, in equation (C.3) in Appendix C, falls from 2.68 to 2.30), and this boosts the final good sector and, thus, the GDP. The detrended values of final output, private

Table 8: Lowering profits of intermediate-goods firms

Determinants of welfare & CCS		Base	Shock
final output:	$\widetilde{y}_f$	0.0427	0.0468
private consumption:	$\widetilde{c}_h$	0.0353	0.0385
public consumption:	$\widetilde{g}^c$	0.0053	0.0058
work time:	l_h^w	0.3100	0.3128
education time:	l_h^e	0.1053	0.1059
leisure time:	$1 - l_h^w - l_h^e$	0.5847	0.5813
annual human capital growth:	γ^h	0.0050	0.0052
annual ideas growth:	γ^{n_b}	0.0158	0.0159
CCS$_{bgp}$:	χ_{bgp}		9.6024

consumption and public consumption ($\widetilde{y}_f$, $\widetilde{c}_h$ and $\widetilde{g}^c$) are all higher in the new BGP, which, in combination with the increase of the denominator, in particular, the higher stocks of ideas and human capital, implies that per capita private and public consumption are all higher in the new BGP. Leisure time has decreased, but this is more than compensated by higher per capita private and public consumption. Looking at the numbers, the growth rate of per capita GDP increases from the base rate of 2.08% to 2.11%. In terms of per capita values, after 40 and 100 years, per capita GDP would be about 1.8 and 15.2 thousand dollars higher than the base.

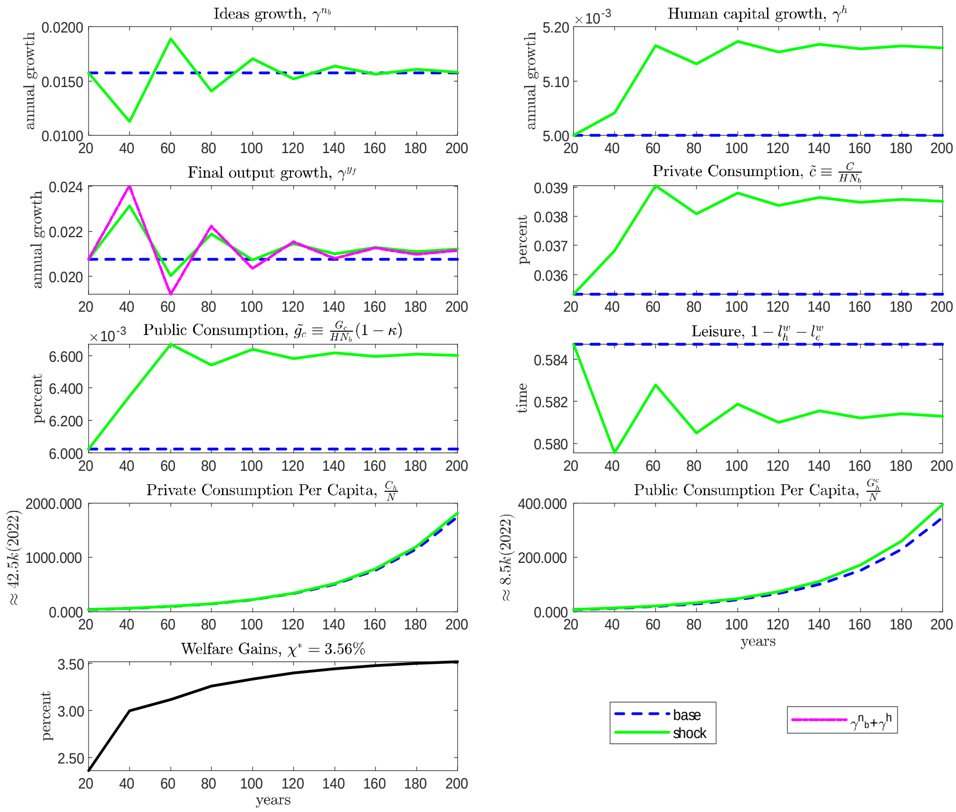
Figure 5 plots Ω against the economy's growth rate and social welfare on the BGP. As Ω rises, i.e. as the intermediate goods market becomes more competitive, growth and welfare increase. Recall, however, that the highest possible value of Ω can take is $\Omega = \alpha = 0.64$, corresponding to perfect competition for the firms in this market. In this polar case, the net cash flow of intermediate goods firms as a share of GDP falls from 8.57% when $\Omega = 0$ to 1.71% when $\Omega = 0.64$.

Figure 5: Growth, Welfare and Profits of Intermediate Goods Firms

4.3.2 Transition dynamics

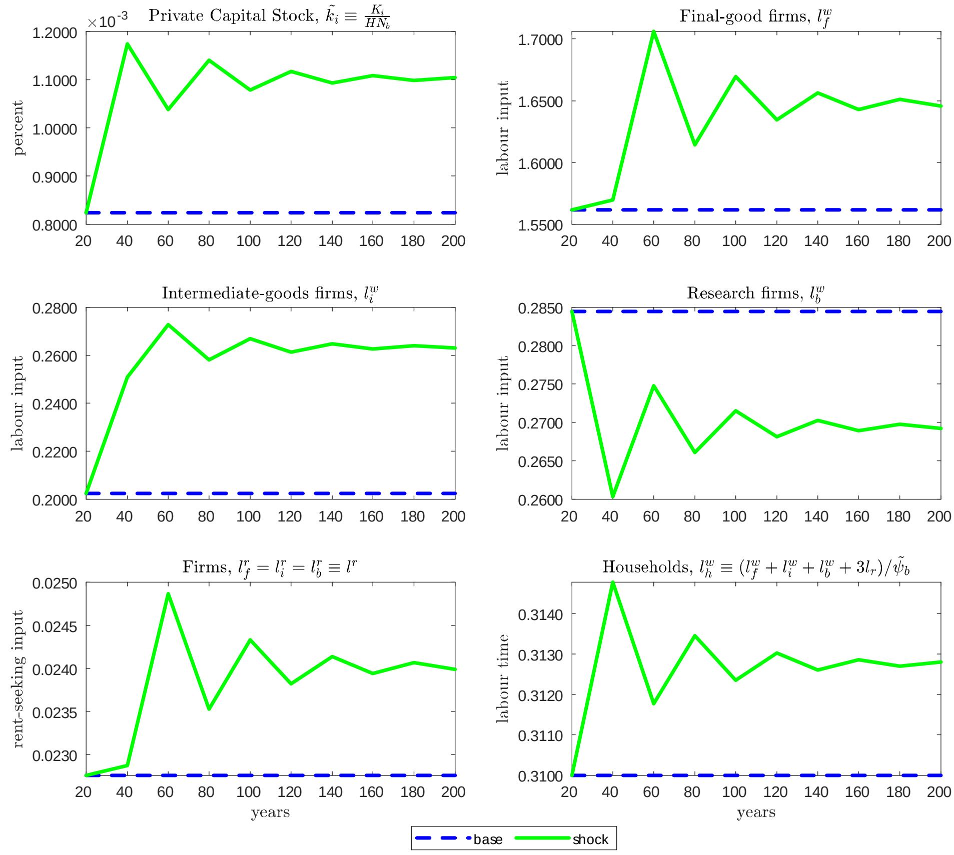
Comparing results on the BGP to results along the transition, the message from Figure 6a is similar to that from Table 8. Namely, per capita private and public consumption rise over time, and this explains the rise in welfare despite the loss from less leisure time. Figure 6b shows what happens to productive inputs. As the intermediate goods market becomes more competitive, demand for inputs, $k_{i,t}$ and $l^w_{i,t}$, rises in this market (imperfect competition is typically related to under-investment and under-employment; see, e.g. Guo and Lansing (1999)) and this crowds-in the units of labour input used by final good firms, $l^w_{f,t}$, who purchase the intermediate goods. On the other hand, since the price of blueprints is proportional to intermediate goods profits (see equation (15)), research firms are hurt and so reduce their demand for labour, $l^w_{b,t}$. Also, notice that the units of labour input firms use for rent-seeking, l^r_t, also rise as the rise in GDP implies a larger contestable prize that increases firms' appetite for rent-seeking extraction. The increase in the demand for productive labour from the side of intermediate and final goods firms, as well as the increase in the demand for labour for rent-seeking activities by all firms, dominates, so households' hours at work, $l^w_{h,t}$, rise too.

Figure 6a: Profits Shock Intermediate Goods Firms (Growth & Welfare)



Finally, the sawtooth paths of ideas and GDP growth rates around the base, as shown in Figure 6a, can be explained by the number of negative eigenvalues discussed at the end of the previous subsection 4.2. Intuitively, a cut in the price markup of intermediate goods is bad news for intermediate goods and research firms but good news for the final good firms.

Figure 6b: Profits Shock Intermediate Goods Firms (Inputs)



4.4 Eliminating rent-seeking

We now examine what happens when we eliminate rent-seeking activities from the side of firms. We capture this by resolving the model when $\kappa = 0$, which implies that rent-seeking time falls from 1% to 0%.

4.4.1 Balanced growth path

The results in Table 9 are qualitatively similar to Table 6. In other words, eliminating rent-seeking allows for a more efficient allocation of resources, which is beneficial for accumulating ideas and human capital and, hence, for the economy's growth. The latter can support higher per capita private and public consumption, whose increase can explain the rise in social welfare despite the decrease in leisure time as people find it optimal to work and study slightly more. Quantitatively, on the BGP, the per capita GDP growth rate rises from the base rate of 2.08% to 2.13%. Notice also the increase in welfare despite the decrease in leisure time. In terms of per capita values, after 40 and 100 years, per capita GDP would be about 3.1 and 26.6 thousand dollars higher, respectively, than in the base.

Table 9: Eliminating rent-seeking

Determinants of welfare & CCS		Base	Shock
final output:	$\widetilde{y}_f$	0.0427	0.0452
private consumption:	$\widetilde{c}_h$	0.0353	0.0365
public consumption:	$\widetilde{g}^c$	0.0053	0.0056
work time:	l_h^w	0.3100	0.3140
education time:	l_h^e	0.1053	0.1061
leisure time:	$1 - l_h^w - l_h^e$	0.5847	0.5798
annual human capital growth:	γ^h	0.0050	0.0053
annual ideas growth:	γ^{n_b}	0.0158	0.0160
CCS_{bgp}:	χ_{bgp}		1.4543

Figure 7: Growth, Welfare and Rent Seeking

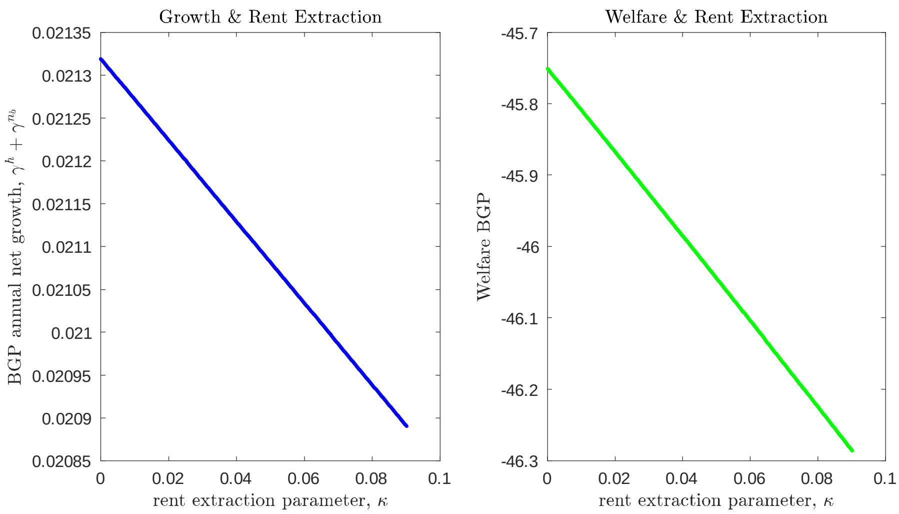
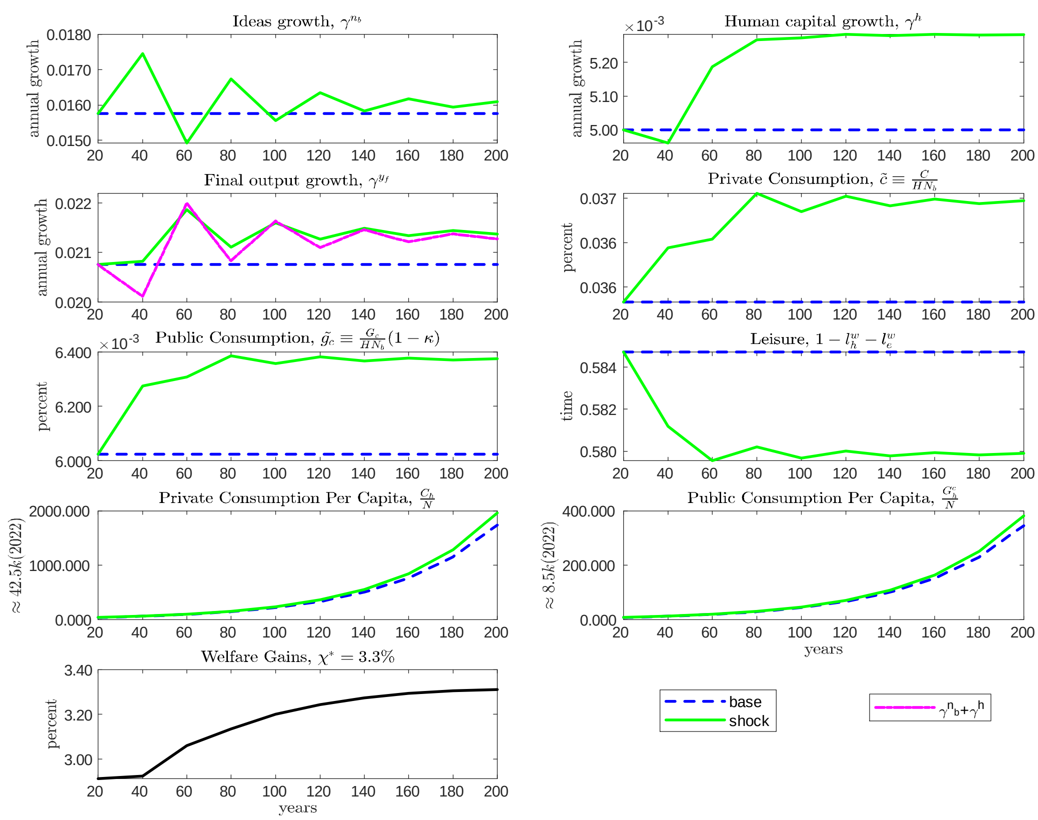


Figure 7 plots the degree of rent-seeking against the economy's long-run growth rate and social welfare. The effect on both is monotonically negative, confirming that the worse the institutional quality (or the higher κ), the worse the aggregate macroeconomic performance. Recall that here, we work with a representative household model. Hence, rent-seeking activities can be considered a negative-sum game in a macro equilibrium.

4.4.2 Transition dynamics

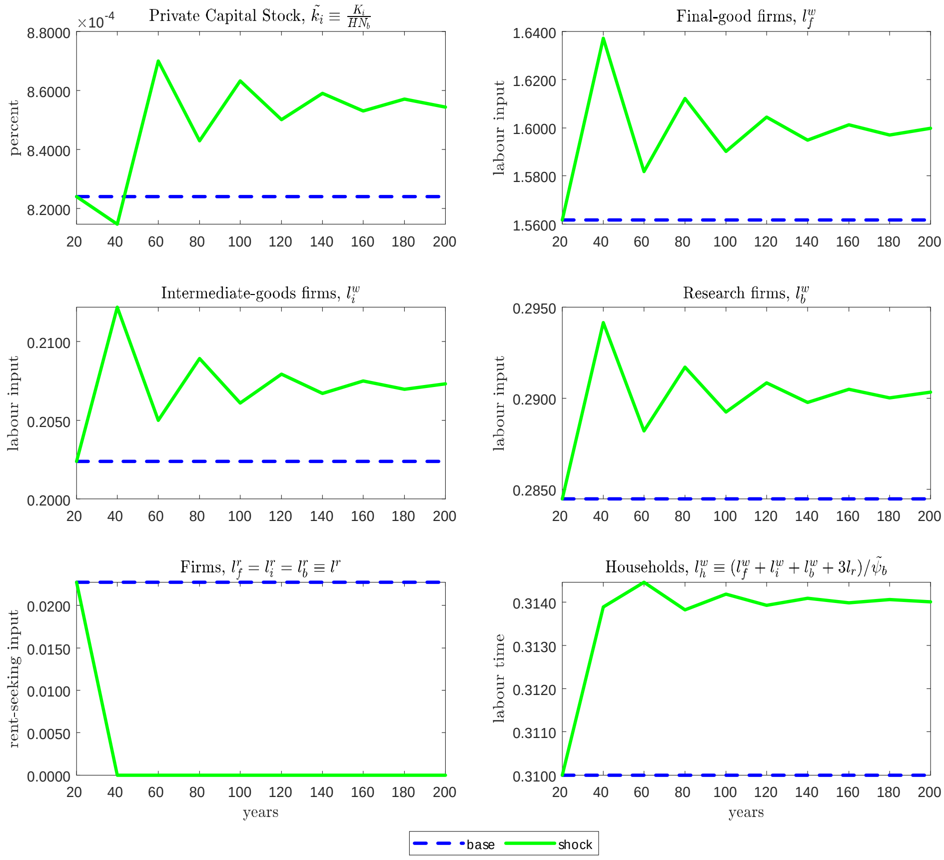
Figures 8a-8b present the economy's behaviour over the transition to a BGP without rent-seeking. The graphs and their messages in Figures 8a-8b are similar to those in Figures 2a-2b since both an increase in public investment spending and an improvement in institutional quality are growth-enhancing.

Figure 8a: Rent Seeking Shock (Growth & Welfare)



Nevertheless, there are also differences. We note two of them. First, and more importantly, there are reallocation differences from Figure 2b. In Figure 8b, an improvement in institutional quality stimulates all three labour inputs used for productive activities, $l^w_{f,t}$, $l^w_{i,t}$ and $l^w_{b,t}$. Also, it reduces the labour input used for rent-seeking, l^r_t. This is symmetrically opposite from the effects of an increase in public investment spending in Figure 2b, which implied an increase in the contestable prize and, hence, an increase in rent-seeking at the cost of less productive use of the labour force. That is, there is a double dividend from better institutions. They incentivise self-interested private firms to use their labour force productively rather than use it for redistributive contests and, at the same time, allow for allocating scarce social resources to provide utility- and productivity-enhancing public goods and services rather than to augment individual incomes and profits, which is again as in most of the literature (see, e.g. Murphy *et al.* (1991), Esteban and Ray (2011), Acemoglu and Robinson (2019), as well as the computable macro models in Angelopoulos *et al.* (2009, 2011, 2021)). Second, in Figure 8a, as in Figures 4a and 6a, the growth rates of ideas and GDP feature sawtooth paths due to the number of negative eigenvalues discussed above. Intuitively, a lower κ enhances overall efficiency but, at the same time, it also reduces rent-seeking firms' profits.

Figure 8b: Rent Seeking Shock (Inputs)



4.5 Higher population growth

We now study what happens when population growth increases from 1.1% in the data to 1.2%. Jones (2019, 2022a, 2022b) argued that a larger population means more researchers, more ideas, and higher growth. On the other hand, a larger population size may reduce per capita output and welfare. Also, in our model, an increase in the supply of researchers will translate to the production of more ideas and, hence, higher growth only if firms find it profitable to increase their output.

4.5.1 Balanced growth path

Table 10 shows that an increase in population considerably boosts the growth rate of ideas, γ^{n_b}, from 1.58% to 1.66%. Then, the higher growth rate of

Table 10: Higher population growth

Determinants of welfare & CCS		Base	Shock
final output:	$\widetilde{y}_f$	0.0427	0.0425
private consumption:	$\widetilde{c}_h$	0.0353	0.0351
public consumption:	$\widetilde{g}^c$	0.0053	0.0053
work time:	l_h^w	0.3100	0.3108
education time:	l_h^e	0.1053	0.1041
leisure time:	$1 - l_h^w - l_h^e$	0.5847	0.5851
annual human capital growth:	γ^h	0.0050	0.0048
annual ideas growth:	γ^{n_b}	0.0158	0.0166
CCS_{bgp}:	χ_{bgp}		2.2998

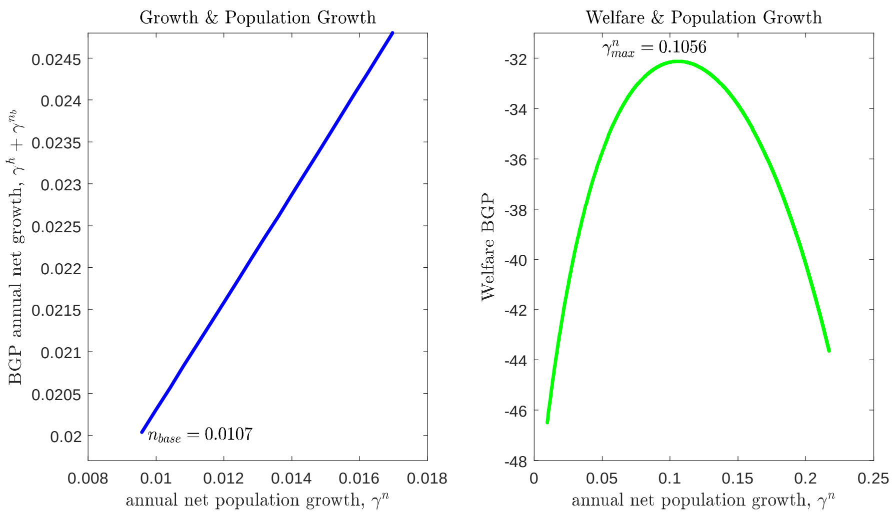
ideas allows per capita GDP, and in turn, per capita private and public consumption, to grow on the new BGP. As said above, this is as in Romer-Jones literature but in a setup where labour demand (and supply) for each sector is chosen optimally rather than being determined as an exogenous fraction of the total population, as in most related literature.[33] Regarding magnitudes, on the BGP, the per capita GDP growth rate rises from the base rate of 2.08% to 2.14%. In terms of per capita values, after 40 and 100 years, per capita GDP would be about 3.5 and 30.3 thousand dollars higher than in the base. Notice also the increase in welfare on the BGP by around 2.30% as a result of both higher per capita consumption and more leisure time, as

[33] Notice that growth and leisure time move in the same direction on the BGP. Boppart and Krusell (2019) also search for setups that allow for decreasing work hours in a growing economy (in their model, there is no education time, so when leisure time rises, work time falls).

households find it optimal to devote more time to work and leisure at the expense of time to education.

Figure 9 plots the population growth rate, γ^n, against the economy's long-run growth rate and social welfare. The effect of γ^n on the economy's growth rate is monotonically increasing because a higher population growth increases the growth rate of ideas. On the other hand, as we switch to a more productive economy, people work more, reducing leisure, which results in a welfare trade-off. In our model and given our parameterization, the "optimal" γ^n is around 10%. Again, this quantitative normative result is indicative since a larger population has richer implications (positive and negative) than merely an increase in the supply of workers in general and researchers in particular.

Figure 9: Growth, Welfare and Population Growth



4.5.2 Transition dynamics

Figures 10a and 10b show variables along the transition to the new BGP as the population grows more than in the base. The logic of results along the transition path in Figure 10a is the same as that on the BGP, i.e., the apparent increase in the production of ideas stimulates the growth of per capita quantities over time, which implies an increasing welfare gain where leisure time reinforces the latter. Figure 10b also reveals that $l^w_{f,t}$, $l^w_{i,t}$ and $l^w_{b,t}$, as well as l^r_t, all rise too. Note that the increase in l^r_t is caused by the larger contestable prize as the economy grows. In other words, a bigger population size accommodates an increase in all types of labour inputs chosen by firms.

Figure 10a: Population Shock (Growth & Welfare)

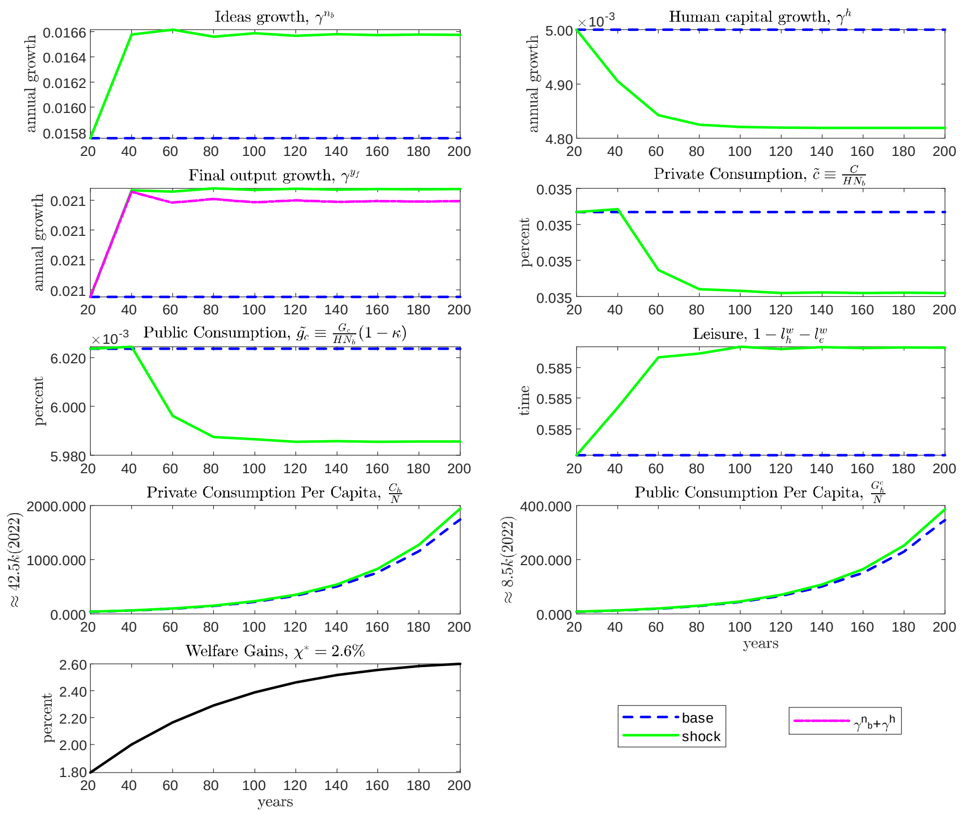
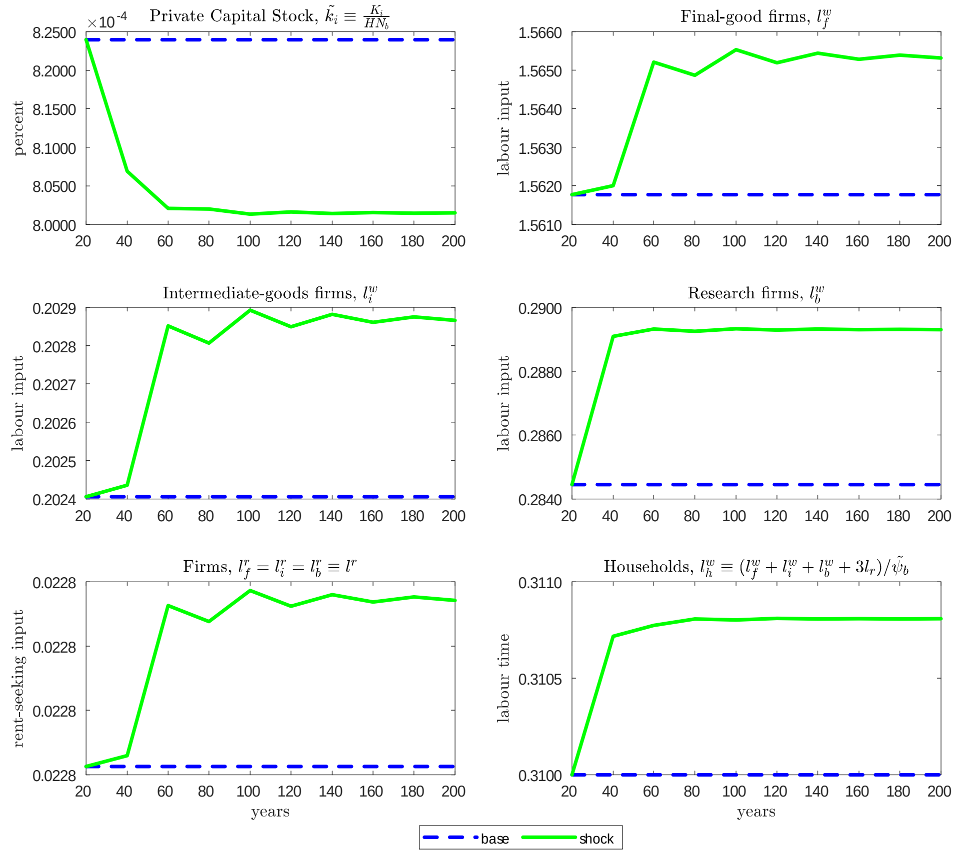


Figure 10b: Population Shock (Inputs)



5 Conclusions, caveats and extensions

Building upon the celebrated Romer-Jones model, we quantified the implications of various structural reforms that shape the accumulation of some critical factors that account for long-term economic growth. Departing from an initial equilibrium carefully calibrated to U.S. data, our structural reforms included an increase in public investment spending financed by lower income transfers, a reduction in the market power of research firms in the form of lower prices for patents, a reduction in the market power of intermediate goods firms in the form of lower price markups for their products, and an improvement in the institutional framework as reflected in a reduction in rent-seeking activities by firms. Given the importance of population growth in the Romer-Jones literature, we also examined the implications of a larger population, which provides a quantitative context for the other experiments. In all cases, we assumed small permanent changes relative to the data or the calibrated parameters.

Our results showed that these changes generally lead to significant per capita output and welfare gains, both on the BGP and along the transition path. The exception is patent prices because technology and innovation are the main drivers of growth in an economy like the U.S., so the anticipation of higher returns, at least up to a point, is necessary to encourage innovation and drive long-term endogenous growth. Considering this result alongside the beneficial effect of lower market power in the intermediate goods sector, the lesson is that one-size-fits-all competition policies across sectors are not a good idea. This implication is consistent with Syverson's (2019, pp. 36-37) discussion, who argues that it is essential to consider the "sector-specific mechanisms" to understand the implications of rising market power. Our results further suggest that for higher public investment to deliver its maximum benefit, rent-seeking from state coffers should be controlled by combining policy reforms.

It is also worth recalling that the reforms we have considered concentrated on improving efficiency without increasing taxes or the public debt burden as a share of GDP. Nonetheless, tough political decisions still need to be made. In particular, altering the fiscal mix in favour of public investment requires agreeing on and eliminating waste relating to transfer spending. Enforcing anti-competition legislation and reducing rent-seeking demands the political will to take on powerful special interests. Finally, harnessing the potential benefits of a higher population needs a less partisan debate on immigration policy.

We can improve our work in several directions. First, although we have taken a step in the right direction, we have treated the degree of market power in different sectors and the economy's institutional quality as given. Although this is a relatively common approach in the literature, as Blanchard and Giavazzi (2003, p. 885) point out, this is, admittedly, done in a reduced-form fashion. Thus, it would be interesting to go deeper and identify their determinants, particularly the channels through which regulatory and fiscal policy instruments in the hands of policymakers can affect them.[34] Second, as is usually the case in studies on structural reforms, a natural question to ask is, "Why don't we observe socially beneficial reforms in practice?". If we leave aside answers like ignorance and irrationality, a response is that re-

[34]Regarding the degree of market power, such instruments can include floor and ceiling prices, quantity restrictions and licenses, barriers to entry, the design of patent systems and R&D incentives, mergers and acquisitions policy, and more traditional tax-spending instruments.

forms have distributional effects so that special interests may dominate. This consideration would mean the model needs to be augmented by household heterogeneity. We leave these extensions for future research.

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Supplementary Material

Appendix A: Data Sources

Table A.1: Data Sources

NIPA Accounts	
Table 1.1.5	Gross Domestic Product (1929-2022)
Table 3.1	Government Current Receipts and Expenditures (1929-2022)
Table 3.17	Selected Government Current & Capital Expenditures by Function (1959-2021)
Fixed Asset Accounts	
Table 1.1	Current-Cost Net Stock of Fixed Assets & Consumer Durable Goods (1925-2021)
Table 1.3	Current-Cost Depreciation of Fixed Assets & Consumer Durable Goods (1925-2021)
Table 1.5	Investment in Fixed Assets & Consumer Durable Goods (1925-2021)
Table 7.1	Current-Cost Net Stock of Government Fixed Assets (1925-2021)
Table 7.3	Current-Cost Depreciation of Government Fixed Assets (1925-2021)
Table 7.5	Investment in Government Fixed Assets (1925-2021)

Appendix B: Macroeconomic system

In our solutions, we assume the number of firms is the same across the three sectors and equal to the endogenously determined number of blueprints or ideas. Thus,

$$N_{f,t} = N_{i,t} \equiv N_{b,t}.$$

Then, imposing *ex-post* symmetricity within each type of firm, we have the following system:

Final good sector

$$y_{f,t} = A_{f,t} \left(N_{b,t} l^w_{f,t}\right)^{\alpha} x^{1-\alpha}_{i,t}, \tag{B.1}$$

$$\pi_{f,t} \equiv (1-\tau^f_t)[y_{f,t} - w_t(l^w_{f,t} + l^r_{f,t}) - p_{i,t}x_{i,t}] + \frac{G^p_t}{3N_{b,t}}, \tag{B.2}$$

$$w_t = \frac{\alpha y_{f,t}}{l^w_{f,t}}, \tag{B.3}$$

$$l^r_{f,t} = \frac{1}{(1-\tau^f_t)w_t} \frac{G^p_t}{3N_{b,t}}, \tag{B.4}$$

$$p_{i,t} = \frac{(1-\alpha)\, y_{f,t}}{x_{i,t}}, \tag{B.5}$$

where $A_{f,t} = A_f \left(\widetilde{k}_t^g\right)^\phi$, $\widetilde{k}_t^g \equiv \frac{N_{b,t}k_t^g}{N_t h_{h,t} N_{b,t}} = \frac{K_t^g}{H_t N_{b,t}}$, and $G_t^p \equiv \kappa(G_t^c + G_t^i)$.

Intermediate goods sector

$$x_{i,t} = A_{i,t}\left(N_{b,t} l_{i,t}^w\right)^\alpha k_{i,t}^{1-\alpha}, \tag{B.6}$$

$$\pi_{i,t} \equiv (1-\tau_t^f)[p_{i,t}x_{i,t} - w_t(l_{i,t}^w + l_{i,t}^r) - q_t] - i_{i,t} + \frac{G_t^p}{3N_{b,t}}, \tag{B.7}$$

$$w_t = \frac{(1-\alpha)^2 y_{f,t}}{x_{i,t}} \frac{\alpha x_{i,t}}{l_{i,t}^w}, \tag{B.8}$$

$$l_{i,t}^r = \frac{1}{(1-\tau_t^f)w_t} \frac{G_t^p}{3N_{b,t}}, \tag{B.9}$$

$$1 = \beta_{i,1}\left[1 - \delta + \frac{(1-\tau_{t+1}^f)(1-\alpha)^2 y_{f,t+1}}{x_{i,t+1}} \frac{(1-\alpha)x_{i,t+1}}{k_{i,t+1}}\right], \tag{B.10}$$

$$q_t \equiv \Gamma\left[\frac{(1-\tau_t^f)[p_{i,t}x_{i,t} - w_t(l_{i,t}^w + l_{i,t}^r)] - i_{i,t} + \left(\frac{l_{i,t}^r}{L_t^r}\right) G_t^p}{(1-\tau_t^f)}\right], \tag{B.11}$$

where $\beta_{i,1} \equiv \beta \frac{\left(1+\tau_{t+1}^c\right)(c_{h,t}+\lambda g_t^c)^{-\sigma}(1-l_{h,t}^w - l_{h,t}^e)^{\psi(1-\sigma)}}{(1+\tau_t^c)(c_{h,t+1}+\lambda g_{t+1}^c)^{-\sigma}(1-l_{h,t+1}^w - l_{h,t+1}^e)^{\psi(1-\sigma)}}$; $G_t^p \equiv \kappa(G_t^c + G_t^i)$; and $A_{i,t} \equiv A_i \left(\widetilde{k}_t^g\right)^\phi$.

Research sector

$$N_{b,t+1} = (1-\delta^{n_b})N_{b,t} + M_t N_{b,t} l_{b,t}^w N_t h_{h,t} (N_{b,t})^\mu, \tag{B.12}$$

$$\pi_{b,t} \equiv (1-\tau_t^f)[q_t - w_t(l_{b,t}^w + l_{b,t}^r)] + \frac{G_t^p}{3N_{b,t}}, \tag{B.13}$$

$$(1-\tau_t^f)w_t = \frac{\beta_{b,1}\left(1-\tau_{t+1}^f\right)q_{t+1}M_t\left(N_{b,t}l_{b,t}^w N_t h_{h,t}\right)\left(N_{b,t}\right)^\mu}{N_{b,t}l_{b,t}}, \tag{B.14}$$

$$l_{b,t}^r = \frac{1}{(1-\tau_t^f)w_t} \frac{G_t^p}{3N_{b,t}}, \tag{B.15}$$

where $M_t \equiv M\left(\widetilde{k}_t^b\right)^\phi$; $L_{b,t} = N_{b,t}l_{b,t}$; $H_t = N_t h_{h,t}$; $\Pi_{b,t} = N_{b,t}\pi_{b,t}$; $G_t^p \equiv \kappa(G_t^c + G_t^i)$; and $\beta_{b,1} \equiv \beta_{i,t}$.

Household and resource constraint

$$h_{h,t+1} = (1-\delta^h)h_{h,t} + D_t\left(l_{h,t}^e h_{h,t}\right)^\theta (h_{h,t})^{1-\theta}, \tag{B.16}$$

$$\frac{\left(1+\tau_{t+1}^{c}\right)(c_{h,t}+\lambda g_{t}^{c})^{-\sigma}(1-l_{h,t}^{w}-l_{h,t}^{e})^{\psi(1-\sigma)}}{(1+\tau_{t}^{c})(c_{h,t+1}+\lambda g_{t+1}^{c})^{-\sigma}(1-l_{h,t+1}^{w}-l_{h,t+1}^{e})^{\psi(1-\sigma)}}=\beta\left(1+r_{t+1}^{b}\right), \tag{B.17}$$

$$\begin{aligned}&\psi(c_{h,t}+\lambda g_{t}^{c})^{1-\sigma}(1-l_{h,t}^{w}-l_{h,t}^{e})^{\psi(1-\sigma)-1}=\\&=\frac{(c_{h,t}+\lambda g_{t}^{c})^{-\sigma}}{(1+\tau_{t}^{c})}(1-l_{h,t}^{w}-l_{h,t}^{e})^{\psi(1-\sigma)}(1-\tau_{t}^{y})w_{t}h_{h,t},\end{aligned} \tag{B.18}$$

$$\begin{aligned}&\psi(c_{h,t}+\lambda g_{t}^{c})^{1-\sigma}(1-l_{h,t}^{w}-l_{h,t}^{e})^{\psi(1-\sigma)-1}=\\&=\mu_{h,t}\frac{\theta D_{t}(l_{h,t}^{e}h_{h,t})^{\theta}\left(h_{h,t}\right)^{1-\theta}}{l_{h,t}^{e}},\end{aligned} \tag{B.19}$$

$$\begin{aligned}&\mu_{h,t}=\beta\frac{(c_{h,t+1}+\lambda g_{t+1}^{c})^{-\sigma}}{\left(1+\tau_{t+1}^{c}\right)}(1-l_{h,t+1}^{w}-l_{h,t+1}^{e})^{\psi(1-\sigma)}(1-\tau_{t+1}^{y})\times\\&\times w_{t+1}l_{h,t+1}^{w}+\beta\mu_{\times,t+1}\left[1-\delta^{h}+\frac{\theta D_{t+1}(l_{h,t+1}^{e}h_{h,t+1})^{\theta}\left(h_{h,t+1}\right)^{1-\theta}}{h_{h,t+1}}\right],\end{aligned} \tag{B.20}$$

$$N_{t}c_{h,t}+N_{i,t}i_{i,t}+(1-\kappa)(G_{t}^{c}+G_{t}^{i})=N_{f,t}y_{f,t}, \tag{B.21}$$

where $D_t = D(\widetilde{k}_t^g)^{\phi}$; $G_t^k = N_t g_t^k$; $k \equiv c, i, t$; and $g_t^c \equiv \frac{(1-\kappa)G_t^c}{N_t}$. Notice that we use the economy's resource constraint instead of the household's budget constraint in the equilibrium system (the latter will be satisfied residually).

Government budget constraint and public capital

$$\begin{aligned}&G_{t}^{c}+G_{t}^{i}+G_{t}^{t}+(1+r_{t}^{b})N_{t}b_{h,t}=\\&=N_{t}b_{h,t+1}+N_{t}\tau_{t}^{y}(w_{t}h_{h,t}l_{h,t}^{w}+\pi_{h,t})+N_{t}\tau_{t}^{c}c_{h,t}+\\&+N_{f,t}\tau_{t}^{f}[y_{f,t}-w_{t}(l_{f,t}^{w}+l_{f,t}^{r})-p_{i,t}x_{i,t}]+\\&+N_{i,t}\tau_{t}^{f}[p_{i,t}x_{i,t}-w_{t}(l_{i,t}^{w}+l_{i,t}^{r})-q_{t}]+\\&+N_{b,t}\tau_{t}^{f}[q_{t}-w_{t}(l_{b,t}^{w}+l_{b,t}^{r})],\end{aligned} \tag{B.22}$$

where $B_{t+1} = N_t b_{h,t+1}$; and $B_t = N_t b_{h,t}$.

Public capital

$$K_{t+1}^{g}=(1-\delta^{g})K_{t}^{g}+(1-\kappa)G_{t}^{i}. \tag{B.23}$$

Market-clearing: labour market

$$N_{b,t}(l_{f,t}^{w}+l_{f,t}^{r}+l_{i,t}^{w}+l_{i,t}^{r}+l_{b,t}^{w}+l_{b,t}^{r})=N_{b,t}(l_{f,t}+l_{i,t}+l_{b,t}+3l_{t}^{r})=N_{t}h_{h,t}l_{h,t}^{w}. \tag{B.24}$$

Market-clearing: dividend market

$$N_{f,t}\pi_{f,t}+N_{i,t}\pi_{i,t}+N_{b,t}\pi_{b,t}=N_{t}\pi_{h,t}. \tag{B.25}$$

Equations and unknowns

We, therefore, have 25 equations in the paths of 25 unknowns: $y_{f,t}$, $\pi_{f,t}$, $l^w_{f,t}$, $l^r_{f,t}$. $p_{i,t}$, $x_{i,t}$ $\pi_{i,t}$, $l^w_{i,t}$, $l^r_{i,t}$, $k_{i,t+1}$, q_t, $N_{b,t+1}$, $\pi_{b,t}$, $l^w_{b,t}$, $l^r_{b,t}$, $h_{h,t+1}$, $b_{h,t+1}$, $l^w_{h,t}$, $l^e_{h,t}$, $\mu_{h,t}$, $c_{h,t}$, r^b_t, K^g_{t+1}, w_t, $\pi_{h,t}$. This is given the exogenous variables defined in the main text and Appendix A. Note that here, we include the end-of-period public debt, $b_{h,t+1}$, in the list of endogenous variables; if, however, we set the public debt to GDP as in data, then one of the other fiscal instruments takes its place as an endogenous variable.

Appendix C: Stationary macroeconomic system

Since population, N_t, individual human capital, $h_{h,t}$, and ideas, $N_{b,t}$, can generate long-term endogenous growth, we need to transform the above system to make it stationary. In other words, we express non-stationary variables as ratios of these three growing quantities. In particular, we define $\widetilde{y}_{f,t} \equiv \frac{N_{f,t}y_{f,t}}{N_t h_{h,t} N_{b,t}} = \frac{Y_t}{H_t N_{b,t}}$, $\widetilde{x}_{i,t} \equiv \frac{N_{i,t}x_{i,t}}{N_t h_{h,t} N_{b,t}} = \frac{X_t}{H_t N_{b,t}}$, $\widetilde{k}_{i,t} \equiv \frac{N_{i,t}k_{i,t}}{N_t h_{h,t} N_{b,t}} = \frac{K_t}{H_t N_{b,t}}$, $\widetilde{i}_{i,t} \equiv \frac{N_{i,t}i_{i,t}}{N_t h_{h,t} N_{b,t}} = \frac{I_t}{H_t N_{b,t}}$, $\widetilde{c}_{h,t} \equiv \frac{N_t c_{h,t}}{N_t h_{h,t} N_{b,t}} = \frac{C_t}{H_t N_{b,t}}$, $\widetilde{b}_{h,t} \equiv \frac{N_t b_{h,t}}{N_t h_{h,t} N_{b,t}} = \frac{B_t}{H_t N_{b,t}}$, $\widetilde{k}^g_t \equiv \frac{N_{b,t}k^g_t}{N_t h_{h,t} N_{b,t}} = \frac{K^g_t}{H_t N_{b,t}}$, where $H_t = N_t h_{h,t}$ is total human capital. We also redefine the prices $\widetilde{w}_t \equiv \frac{w_t}{H_t}$ and $\widetilde{q}_t \equiv \frac{q_t}{H_t}$, the human capital multiplier $\widetilde{\mu}_{h,t} = \frac{\mu_{h,t}(h_{h,t})^\sigma}{(N_{b,t})^{1-\sigma}}$, and we add the auxiliary variable $\widetilde{\psi}_{b,t} \equiv \frac{H_t}{N_{b,t}}$ (see also Jones (2022b)). Further, note that $l^w_{f,t}$, $l^r_{f,t}$, $l^w_{i,t}$, $l^r_{i,t}$, $l^w_{b,t}$, $l^r_{b,t}$, $l^w_{h,t}$, $l^e_{h,t}$, $p_{i,t}$, r^b_t are ratios so they are not transformed. Finally, recall that $N_{f,t} = N_{i,t} \equiv N_{b,t}$ and that $\mu = -1$. Also, $l^r_{f,t} = l^r_{i,t} = l^r_{b,t} \equiv l^r_t$.

Then, we have the following stationary macroeconomic system.

Final good sector

$$\widetilde{y}_{f,t} = A_f \left(\frac{l^w_{f,t}}{\widetilde{\psi}_{b,t}} \right)^\alpha (\widetilde{x}_{i,t})^{1-\alpha} \left(\widetilde{k}^g_t \right)^\phi , \tag{C.1}$$

$$\widetilde{w}_t = \frac{\alpha \widetilde{y}_{f,t}}{l^w_{f,t}}, \tag{C.2}$$

$$p_{i,t} = \frac{(1-\alpha)\widetilde{y}_{f,t}}{\widetilde{x}_{i,t}}, \tag{C.3}$$

$$l^r_t = \frac{\kappa \left(s^c_t + s^i_t \right) \widetilde{y}_{f,t}}{3(1-\tau^f_t)\widetilde{w}_t}. \tag{C.4}$$

Intermediate goods sector

$$\widetilde{x}_{i,t} = A_i \left(\frac{l^w_{i,t}}{\widetilde{\psi}_{b,t}} \right)^{\alpha} (\widetilde{k}_{i,t})^{1-\alpha} \left(\widetilde{k}^g_t \right)^{\phi}, \tag{C.5}$$

$$\widetilde{k}_{i,t+1} \left(1+\gamma^n\right) \left(1+\gamma^h_t\right) = (1-\delta)\widetilde{k}_{i,t} + \widetilde{i}_{i,t}, \tag{C.6}$$

$$\widetilde{w}_t = \frac{(1-a)^2 a \widetilde{y}_{f,t}}{l^w_{i,t}}, \tag{C.7}$$

$$1 = \beta_{i,1} \left[1 - \delta + \frac{(1-\tau^f_{t+1})(1-\alpha)^3 \widetilde{y}_{f,t+1}}{\widetilde{k}_{i,t+1}} \right], \tag{C.8}$$

$$\widetilde{q}_t \equiv \Gamma \frac{(1-\tau^f_t)[p_{i,t}\widetilde{x}_{i,t} - \widetilde{w}_t(l^w_{i,t} + l^r_t)] - \widetilde{i}_{i,t} + \frac{\kappa\left(s^c_t + s^i_t\right)\widetilde{y}_{f,t}}{3}}{(1-\tau^f_t)}. \tag{C.9}$$

Research sector

$$(1-\tau^f_t)\widetilde{w}_t = \beta_{b,1}(1-\tau^f_{t+1}) \left(1+\gamma^n\right) \left(1+\gamma^h_t\right) \widetilde{q}_{t+1} \widetilde{\psi}_{b,t} M \left(\widetilde{k}^g_t \right)^{\phi}, \tag{C.10}$$

$$\frac{\widetilde{\psi}_{b,t+1}}{\widetilde{\psi}_{b,t}} = \frac{\left(1+\gamma^n\right)\left(1+\gamma^h_t\right)}{1+\gamma^{n_b}_t}. \tag{C.11}$$

Household and the resource constraint

$$\begin{aligned} &\frac{(1+\tau^c_{t+1})(\widetilde{c}_{h,t} + \lambda\widetilde{g}^c_t)^{-\sigma}(1-l^w_{h,t} - l^e_{h,t})^{\psi(1-\sigma)}}{(1+\tau^c_t)(\widetilde{c}_{h,t+1} + \lambda\widetilde{g}^c_{t+1})^{-\sigma}(1-l^w_{h,t+1} - l^e_{h,t+1})^{\psi(1-\sigma)}} = \\ &= \beta \left[\left(1+\gamma^h_t\right) \left(1+\gamma^{n_b}_t\right) \right]^{-\sigma} \left(1 + r^b_{t+1}\right), \end{aligned} \tag{C.12}$$

$$\frac{\psi}{(1-l^w_{h,t} - l^e_{h,t})} = \frac{(1-\tau^y_t)\widetilde{\psi}_{b,t}\widetilde{w}_t}{(1+\tau^c_t)(\widetilde{c}_{h,t} + \lambda\widetilde{g}^c_t)}, \tag{C.13}$$

$$\begin{aligned} \widetilde{\mu}_{h,t} &= \beta \left(1+\gamma^h_t\right)^{-\sigma} \left(1+\gamma^{n_b}_t\right)^{1-\sigma} \widetilde{\psi}_{b,t+1} \frac{(\widetilde{c}_{h,t+1} + \lambda\widetilde{g}^c_{t+1})^{-\sigma}}{(1+\tau^c_{t+1})} \times \\ &\times (1-l^w_{h,t+1} - l^e_{h,t+1})^{\psi(1-\sigma)} (1-\tau^y_{t+1}) \widetilde{w}_{t+1} l^w_{h,t+1} + \\ &+ \beta\widetilde{\mu}_{h,t+1} \left(1+\gamma^h_t\right)^{-\sigma} \left(1+\gamma^{n_b}_t\right)^{1-\sigma} [1-\delta^h + \theta(l^e_{h,t+1})^{\theta} D(\widetilde{k}^g_{t+1})^{\phi}], \end{aligned} \tag{C.14}$$

$$\begin{aligned} &\psi(\widetilde{c}_{h,t} + \lambda\widetilde{g}^c_t)^{1-\sigma}(1-l^w_{h,t} - l^e_{h,t})^{\psi(1-\sigma)-1} = \\ &= \widetilde{\mu}_{h,t}\theta(l^e_{h,t})^{\theta-1} D \left(\widetilde{k}^g_t\right)^{\phi}, \end{aligned} \tag{C.15}$$

$$\widetilde{c}_{h,t} + \widetilde{i}_{i,t} + (1-\kappa)(s^c_t + s^i_t)\widetilde{y}_{f,t} = \widetilde{y}_{f,t}. \tag{C.16}$$

Government budget constraint

$$\begin{aligned}
&\left(s_t^c + s_t^i + s_t^t\right)\widetilde{y}_{f,t} + (1+r_t^b)\widetilde{b}_{h,t} = \\
&= \left(1+\gamma_t^h\right)\left(1+\gamma_t^{n_b}\right)\widetilde{b}_{h,t+1} + \tau_t^y \widetilde{w}_t (l_{f,t}^w + l_{i,t}^w + l_{b,t}^w + 3l_t^r) + \\
&+ \tau_t^y \left[\widetilde{y}_{f,t} - \widetilde{w}_t (l_{f,t}^w + l_{i,t}^w + l_{b,t}^w + 3l_t^r) - \widetilde{i}_{i,t} + \kappa\left(s_t^c + s_t^i\right)\widetilde{y}_{f,t}\right] + \\
&+ \tau_t^f (1-\tau_t^y)\left(\widetilde{y}_{f,t} - \widetilde{w}_t (l_{f,t}^w + l_{i,t}^w + l_{b,t}^w + 3l_t^r)\right) + \tau_t^c \widetilde{c}_{h,t}.
\end{aligned} \tag{C.17}$$

Motion of public capital:

$$\widetilde{k}_{t+1}^g (1+\gamma^n)\left(1+\gamma_t^h\right)(1+\gamma_t^{n_b}) = (1-\delta^g)\widetilde{k}_t^g + (1-\kappa) s_t^i \widetilde{y}_{f,t}. \tag{C.18}$$

Market-clearing condition in the labour market

$$(l_{f,t}^w + l_{i,t}^w + l_{b,t}^w + 3l_t^r) = \widetilde{\psi}_{b,t} l_{h,t}^w. \tag{C.19}$$

Drivers of long-run endogenous growth

$$1+\gamma_t^h = 1 - \delta^h + (l_{h,t}^e)^\theta D(\widetilde{k}_t^g)^\phi, \tag{C.20}$$

$$1+\gamma_t^{n_b} = 1 - \delta^{n_b} + l_{b,t}^w \widetilde{\psi}_{b,t} M(\widetilde{k}_t^g)^\phi. \tag{C.21}$$

In the above we use:

$$\begin{aligned}
&\tfrac{N_{t+1}}{N_t} \equiv 1+\gamma^n; \quad \tfrac{h_{h,t+1}}{h_{h,t}} \equiv 1+\gamma_t^h; \quad \tfrac{N_{b,t+1}}{N_{b,t}} \equiv 1+\gamma_t^{n_b}; \quad \widetilde{g}_t^c \equiv (1-\kappa) s_t^c \widetilde{y}_{f,t}; \\
&\widetilde{b}_{h,t+1} = \tfrac{B_h}{Y_f}\widetilde{y}_{f,t}; \quad \beta_{i,1} = \beta_{b,1}; \quad s_t^k = \overline{s}^k + \varepsilon_t^k; \quad \text{and} \\
&\beta_{i,1} \equiv \beta \tfrac{(1+\tau_t^c)(\widetilde{c}_{h,t+1} + \lambda \widetilde{g}_{t+1}^c)^{-\sigma}(1-l_{h,t+1}^w - l_{h,t+1}^e)^{\psi(1-\sigma)}}{(1+\tau_{t+1}^c)(\widetilde{c}_{h,t} + \lambda \widetilde{g}_t^c)^{-\sigma}(1-l_{h,t}^w - l_{h,t}^e)^{\psi(1-\sigma)}} \left[\left(1+\gamma_t^h\right)(1+\gamma_t^{n_b})\right]^{-\sigma},
\end{aligned}$$

where for the spending shares $k \equiv c, i, t$.

Therefore, we have 21 equations in the paths of 21 unknowns: $\widetilde{y}_{f,t}$, $l_{f,t}^w$, $p_{i,t}$, $\widetilde{x}_{i,t}$, $\widetilde{k}_{i,t+1}$, $l_{i,t}^w$, $\widetilde{i}_t$, $\widetilde{q}_t$, $1+\gamma_t^{n_b}$, $l_{b,t}^w$, $\widetilde{\psi}_{b,t}$, l_t^r, $\widetilde{c}_{h,t}$, $l_{h,t}^w$, $l_{h,t}^e$, $1+\gamma_t^h$, $\widetilde{w}_t$, r_t^b, $\widetilde{b}_{h,t+1}$, $\widetilde{\mu}_{h,t}$, $\widetilde{k}_{t+1}^g$. Note that here, relative to Appendix B, we have substituted out $\pi_{f,t}$, $\pi_{i,t}$, $\pi_{b,t}$ and $\pi_{h,t}$. Also note that, as in Appendix B, we include $\widetilde{b}_{h,t+1}$ in the list of endogenous variables. Finally, note that from those 21 unknowns, $\widetilde{k}_i$, ψ_b, r^b, $\widetilde{b}_h$, $\widetilde{k}^g$ are state-like variables.

Appendix D: Per capita levels and growth rates

To calculate any per capita quantity, x_t, over the transition, we start with the definition $\widetilde{x}_t \equiv \frac{N_{f,t} x_t}{N_t h_{h,t} N_{b,t}} \equiv \frac{X_t}{N_t h_{h,t} N_{b,t}}$ which can be rewritten in per capita terms as $x_t \equiv \frac{X_t}{N_t} \equiv \widetilde{x}_t h_{h,t} N_{b,t}$. Thus, along the transition path for $t \geq 1$:

$$x_t = \left(\frac{\widetilde{x}_t}{\widetilde{x}_{t-1}} \right) \left(\frac{h_{h,t}}{h_{h,t-1}} \right) \left(\frac{N_{b,t}}{N_{b,t-1}} \right) x_{t-1}, \tag{D.1}$$

where the initial value, x_0, is given and $\frac{h_{h,t}}{h_{h,t-1}} \equiv 1 + \gamma^h_{t-1}$ and $\frac{N_{b,t}}{N_{b,t-1}} \equiv 1 + \gamma^{n_b}_{t-1}$ have been defined above. We use U.S. data from 2022 as starting values for the analysis reported in the main text. Per capita growth rates in turn are simply $\gamma^x_t = \frac{x_t}{x_{t-1}} - 1$.

We can also find per capita final output growth along the transition and on the BGP analytically. What follows helps to contextualise per capita GDP growth on the BGP in our model with that in Jones (2019). We start by repeating equation (C.1) here for convenience:

$$\widetilde{y}_{f,t} = A_f \left(\frac{l^w_{f,t}}{\widetilde{\psi}_{b,t}} \right)^{\alpha} (\widetilde{x}_{i,t})^{1-\alpha} \left(\widetilde{k}^g_t \right)^{\phi},$$

so that, since $\widetilde{y}_{f,t} \equiv \frac{N_{f,t} y_{f,t}}{N_t h_{h,t} N_{b,t}} \equiv \frac{Y_{f,t}}{N_t h_{h,t} N_{b,t}}$, per capita GDP is:

$$\frac{Y_{f,t}}{N_t} = A_f \left(\frac{l^w_{f,t}}{\widetilde{\psi}_{b,t}} \right)^{\alpha} (\widetilde{x}_{i,t})^{1-\alpha} \left(\widetilde{k}^g_t \right)^{\phi} N_{b,t} h_{h,t}. \tag{D.2}$$

Note that we can compare this to the Jones' baseline model. For example, equation (D.2) is like equation (18) in Jones (2019) if we ignore intermediate goods, public capital and human capital and if we notice that $\frac{l^w_{f,t}}{\widetilde{\psi}_{b,t}} = \frac{l^w_{f,t} N_{b,t}}{N_t h_{h,t}}$ here is like $(1-s)$ in Jones (2019). This follows in the sense that $\frac{l^w_{f,t}}{\widetilde{\psi}_{b,t}}$ is the fraction of the population that works in the production of the final good as is $(1-s)$.

Taking logs and differentiating (D.2) with respect to time, the growth rate of per capita final output between t and $t-1$ is:

$$\gamma^{y_f}_t \simeq \alpha \gamma^{l^w_f}_t - \alpha \gamma^{\widetilde{\psi}_b}_t + (1-\alpha) \gamma^{\widetilde{x}_i}_t + \phi_f \gamma^{\widetilde{k}^g}_t + \gamma^h_t + \gamma^{n_b}_t, \tag{D.3}$$

where

$$\gamma_t^{y_f} \equiv \frac{\frac{Y_{f,t}}{N_t} - \frac{Y_{f,t-1}}{N_{t-1}}}{\frac{Y_{f,t-1}}{N_{t-1}}}, \quad \gamma_t^{l_f^w} \equiv \frac{l_{f,t}^w - l_{f,t-1}^w}{l_{f,t-1}^w}, \quad \gamma_t^{\widetilde{\psi}_b} \equiv \frac{\widetilde{\psi}_{b,t} - \widetilde{\psi}_{b,t-1}}{\widetilde{\psi}_{b,t-1}},$$

$$\gamma_t^{\widetilde{x}_i} \equiv \frac{\widetilde{x}_{i,t} - \widetilde{x}_{i,t-1}}{\widetilde{x}_{i,t-1}}, \quad \gamma_t^{\widetilde{k}^g} \equiv \frac{\widetilde{k}_t^g - \widetilde{k}_{t-1}^g}{\widetilde{k}_{t-1}^g},$$

$$\gamma_t^h \equiv \frac{h_{h,t} - h_{h,t-1}}{h_{h,t-1}} = -\delta^h + (l_{h,t-1}^e)^\theta D(\widetilde{k}_{t-1}^g)^\phi,$$

$$\gamma_t^{n_b} \equiv \frac{N_{b,t} - N_{b,t-1}}{N_{b,t-1}} = -\delta^{n_b} + l_{b,t-1}\widetilde{\psi}_{b,t-1} M(\widetilde{k}_{t-1}^g)^\phi.$$

Along the BGP, stationary variables do not change so that the long-run endogenous growth rate reduces to (we can now omit time subscripts):

$$\gamma_t^{y_f} = \gamma_t^h + \gamma_t^{n_b}, \tag{D.4}$$

where:

$$\gamma_t^h = -\delta^h + (l_h^e)^\theta D(\widetilde{k}^g)^\phi, \tag{D.5}$$

$$\gamma_t^{n_b} = -\delta^{n_b} + l_b \widetilde{\psi}_b M \left(\widetilde{k}^g\right)^\phi. \tag{D.6}$$

Note that we can again compare with Jones (2019). For example, if first, we drop human capital growth, γ_h, given by (D.5), and next drop public capital, $M\left(\widetilde{k}^g\right)^\phi$, human capital $h_{h,t}$ and set $\delta^{n_b} = 0$ in (D.6), we have since $\widetilde{\psi}_{b,t} \equiv \frac{N_t h_{h,t}}{N_{b,t}}$:

$$\gamma_t^{n_b} = l_b \frac{N_t}{N_{b,t}}. \tag{D.7}$$

Thus, taking logs and totally differentiating (D.7) with respect to time gives $\gamma_t^{n_b} = \gamma^n$, which is like equation (21) in Jones (2019). In this case, the long-run per capita GDP growth rate is driven by the creation of new ideas and the latter by population growth.

Appendix E: Welfare

Recall that households' period utility function is:

$$u_t = \frac{(c_{h,t} + \lambda g_t^c)^{1-\sigma}(1 - l_{h,t}^w - l_{h,t}^e)^{\psi(1-\sigma)}}{1-\sigma}, \tag{E.1}$$

which we rewrite as:

$$u_t = \frac{\left(\frac{N_t c_{h,t}}{N_t h_{h,t} N_{b,t}} + \lambda \frac{G_t^c}{N_t h_{h,t} N_{b,t}}\right)^{1-\sigma} \left(h_{h,t} N_{b,t}\right)^{1-\sigma} (1 - l_{h,t}^w - l_{h,t}^e)^{\psi(1-\sigma)}}{1-\sigma} = \tag{E.2}$$

$$= \frac{\left(\widetilde{c}_{h,t} + \lambda \widetilde{g}_t^c\right)^{1-\sigma} \left(h_{h,t} N_{b,t}\right)^{1-\sigma} (1 - l_{h,t}^w - l_{h,t}^e)^{\psi(1-\sigma)}}{1-\sigma},$$

where notice that $\widetilde{g}_t^c = (1-\kappa)s_t^c\widetilde{y}_{f,t}$.

Moreover, we have for $h_{h,t}$:

$$\begin{aligned} h_{h,t} &\equiv (1+\gamma_{t-1}^h)h_{h,t-1} = \\ &= (1+\gamma_0^h)(1+\gamma_1^h)...(1+\gamma_{t-1}^h)h_{h,0} = \\ &= \prod_{j=0}^{t-1}(1+\gamma_j^h)h_{h,0}, \end{aligned} \tag{E3}$$

where $h_{h,0}$ is the initial value of the individual human capital stock.

Similarly we have for $N_{b,t}$:

$$\begin{aligned} N_{b,t} &\equiv (1+\gamma_{t-1}^{n_b})N_{b,t-1} = \\ &= (1+\gamma_0^{n_b})(1+\gamma_1^{n_b})...(1+\gamma_{t-1}^{n_b})N_{b,0} = \\ &= \prod_{j=0}^{t-1}(1+\gamma_j^{n_b})N_{b,0}, \end{aligned} \tag{E.4}$$

where $N_{b,0}$ is the initial value of the stock of ideas.

Hence, we have for discounted lifetime utility or welfare:

$$\begin{aligned} U \equiv \sum_{t=0}^{\infty}\beta^t \Bigg[&\frac{\left(h_{h,0}N_{b,0}\right)^{1-\sigma}\left(\widetilde{c}_{h,t}+\lambda\widetilde{g}_t^c\right)^{1-\sigma}(1-l_{h,t}^w-l_{h,t}^e)^{\psi(1-\sigma)}}{1-\sigma}\times \\ &\times\frac{\left(\prod_{j=0}^{t-1}(1+\gamma_j^h)\right)^{1-\sigma}\left(\prod_{j=0}^{t-1}(1+\gamma_j^{n_b})\right)^{1-\sigma}}{1-\sigma}\Bigg], \end{aligned} \tag{E.5}$$

where $h_{h,0}$ and $N_{b,0}$ are given by initial conditions. This sum will be bounded to the extent that $\beta(1+\gamma_t^h)^{1-\sigma}(1+\gamma_t^{n_b})^{1-\sigma} < 1$ at least after a point in time.

By definition, U is also the household's value function at the beginning of the time horizon. Thus, to compare two regimes, we denote discounted lifetime utilities as U^S and U^R, and then calculate the constant consumption subsidy, χ, that would make the household indifferent between them solves $U^R = (1+\chi)^{1-\sigma}U^S$, i.e. $\chi = \left(\frac{U^R}{U^S}\right)^{\frac{1}{1-\sigma}} - 1$. Thus, if $\chi > 0$, regime R is superior, and vice versa.

We next consider welfare along the BGP, on which $\widetilde{c}_{h,t}$, $l_{h,t}^w$, $l_{h,t}^e$ remain constant. At the same time, individual human capital, ideas and population grow at constant rates so that, in this case, welfare simplifies to (we now omit time subscripts since all variables included here are constant over time):

$$U^{BGP} = \frac{\left(h_{h,0}N_{b,0}\right)^{1-\sigma}\left(\widetilde{c}_h+\lambda\widetilde{g}^c\right)^{1-\sigma}(1-l_h^w-l_h^e)^{\psi(1-\sigma)}}{1-\sigma}\times \\ \times\sum_{t=0}^{\infty}\left[\beta\left(1+\gamma^h\right)^{1-\sigma}\left(1+\gamma^{n_b}\right)^{1-\sigma}\right]^t, \tag{E.6}$$

or

$$U^{BGP} = \frac{(h_{h,0}N_{b,0})^{1-\sigma}(\widetilde{c}_h+\lambda\widetilde{g}^c)^{1-\sigma}\left(1-l_h^w-l_h^e\right)^{\psi(1-\sigma)}}{(1-\sigma)\left[1-\beta\left(1+\gamma^h\right)^{1-\sigma}\left(1+\gamma^{n_b}\right)^{1-\sigma}\right]}. \tag{E.6$'$}$$

Appendix F: Intermediate goods firms' market power

To quantify the market power enjoyed by intermediate goods producers, consider, for instance, their labour demand function in (B.8), rewritten here for convenience:

$$w_t = \frac{(1-\alpha)^2 y_{f,t}}{x_{i,t}}\frac{\alpha x_{i,t}}{l_{i,t}^w}. \tag{F.1}$$

In contrast, if these firms take the price of their product as given (meaning that they act competitively), and if we use (B.5) *ex-post*, (F.1) becomes:

$$w_t = \frac{(1-\alpha)y_{f,t}}{x_{i,t}}\frac{\alpha x_{i,t}}{l_{i,t}^w}. \tag{F.2}$$

Thus, in general, we can write:

$$w_t = \frac{(1-\alpha)^2 y_{f,t}}{(1-\Omega)x_{i,t}}\frac{\alpha x_{i,t}}{l_{i,t}^w} = \frac{(1-\alpha)^2 a y_{f,t}}{(1-\Omega)l_{i,t}^w}, \tag{F.3}$$

or in stationary form (see equation C.7):

$$\widetilde{w}_t = \frac{(1-a)^2 a\widetilde{y}_{f,t}}{(1-\Omega)l_{i,t}^w}, \tag{F.4}$$

where $0 \leq \Omega \leq \alpha$. The same arguments apply to the optimality condition for capital. Thus, we rewrite equation (C.8) as:

$$1 = \beta_{i,1}\left[1-\delta+\frac{(1-\tau_{t+1}^f)(1-\alpha)^3\widetilde{y}_{f,t+1}}{(1-\Omega)\widetilde{k}_{i,t+1}}\right]. \tag{F.5}$$

Therefore, in the base case, when intermediate goods producers act as monopolists, $\Omega = 0$. In contrast, price-taking is when $\Omega = \alpha = 0.64$. In our numerical exercise in the body of the paper, we increase Ω from its base value of 0 to 0.19 to generate a fall in intermediate profits of 10%.

The above can also be expressed in terms of prices and markups, as in most of the literature on imperfect competition. Recall from Appendix B that the price of the intermediate good is given by (B.5), which is repeated here for convenience:

$$p_{i,t} = \frac{(1-\alpha)\, y_{f,t}}{x_{i,t}},$$

so that (F.1), which is the case with monopolistic power, implies:

$$p_{i,t} = \frac{w_t}{(1-a)\frac{\alpha x_{i,t}}{l^w_{i,t}}}, \tag{F.6}$$

while (F.2), which is the case with price taking, implies:

$$p_{i,t} = \frac{w_t}{\frac{\alpha x_{i,t}}{l^w_{i,t}}}, \tag{F.7}$$

so that, since $(1-a) < 1$, the price is higher with market power, other things equal.

Thus, in general, we can write as above:

$$p_{i,t} = \frac{(1-\Omega)w_t}{(1-a)\frac{\alpha x_{i,t}}{l^w_{i,t}}}, \tag{F.8}$$

or in stationary form:

$$p_{i,t} = \frac{(1-\Omega)\widetilde{w}_t}{(1-a)\frac{\alpha \widetilde{x}_{i,t}}{l^w_{i,t}}}, \tag{F.9}$$

where if $\Omega = 0$, we are in a regime of market power, while if $\Omega = a$, there is perfect competition. In other words, the parameter Ω can be considered a measure of market power in price setting (markup) for intermediate goods firms. The lower it is, the smaller the substitutability of intermediate products and, hence, the more power intermediate goods firms have in price setting.

Appendix G: Reform ranges

Table G.1: Reform ranges

$\Delta\gamma_a^{y_f}$	χ_{bgp}	χ_{lt}	$\Delta\gamma_a^{y_f}$	χ_{bgp}	χ_{lt}	$\Delta\gamma_a^{y_f}$	χ_{bgp}	χ_{lt}
				Public Investment				
	1-ppt ↑			2-ppt ↑			3-ppt ↑	
0.067	3.530	1.678	0.124	6.195	2.847	0.175	8.257	3.660
				Lower Profits Research Firms				
	10% ↓			15% ↓			20% ↓	
-0.012	0.875	-1.158	-0.019	1.318	-1.825	-0.025	1.766	-2.561
				Lower Profits Intermediate Goods Firms				
	10% ↓			15% ↓			20% ↓	
0.033	9.602	3.561	0.048	13.940	4.748	0.063	18.021	5.576
				Eliminating Rent-Seeking				
	1% time			2% time			3% time	
0.057	1.454	3.271	0.120	3.591	7.270	0.194	6.804	12.320
				Population Growth				
	0.1-ppt ↑			0.2-ppt ↑			0.3-ppt ↑	
0.064	2.230	2.616	0.128	4.612	5.272	0.193	6.935	7.970

Note: $\Delta\gamma_a^{y_f} = \gamma_a^{y_f}(shock) * 100 - \gamma_a^{y_f}(base) * 100$.