Guidance on structural reliability analysis of marine structures

Chapter 1 Introduction and summary

1.1 Overview of structure reliability
Structural reliability design is a more advanced design method than the deterministic traditional design. In traditional design methods, design parameters are used in deterministic values without uncertainties. In reality, these values are not unique values but rather have probability distributions that reflect many uncertainties. For marine structures, the following uncertainties are obvious: fluctuations of loads, variability of material properties and uncertainties in analysis models, etc. These all result in a probability of failure of marine structures, so that the structure does not perform as intended. A presentation using a probabilistic model is possible and is more suitable. Structural reliability design considers all these uncertainties, including the uncertainties in the process of design, manufacture, and servicing. Until now, reliability design has not been used extensively for practical design. One of the reasons is that an ideal frequency interpretation of the estimated failure probability would require a large population of the particular type of structure, in conjunction with perfect analysis methods and full knowledge of the governing uncertainties. This will practically never be fulfilled. Another reason is that it is a new method, and if not well understood, structural reliability theory can be misused.
In order to do the reliability design, fulfilment of a required target reliability is necessary in order to ensure a sufficient safety level in design. Such a requirement can either be derived by a utility optimisation at the decision analysis stage, or by requiring that the safety level resulting from the design by the reliability analysis shall be the same as the safety level resulting from the current deterministic design practice.

1.2 General steps of a structural reliability analysis
In order to carry out reliability design of a structure, the first task is to do an analysis of its structural reliability. In general, when calculating structural reliability, the following procedure is suggested.
1) Establish a target reliability, i.e. reliability safety index, or a decision model.
2) Identify all possible and significant failure modes of the structure or operation under consideration.
3) Formulate failure criteria and establish a relevant failure limit state function for each of mode of failure.
4) Choose and identify stochastic variables and parameters for each failure mode of the structure or operation under consideration.
5) Calculate the reliability or failure probability of the structure of each failure mode of the structure or operation under consideration.
6) Assess the structure’s reliability against the given reliability target; whether the calculated reliability is sufficient or not and modify the concept if necessary.
7) Evaluate the results of the reliability analysis with respect to parametric sensitivity considerations.
8) Document these outcomes to inform the structure’s design

The following sections of this report will illustrate the above steps in detail.

1.3 Uncertainties of marine structures

1.3.1 Uncertainties

The load acting on a marine structure is very difficult to determine exactly, because of the many uncertainties. Marine structures are taken here to include both floating vessels and structures and fixed structures at sea, which experience the sea surface in its infinitely variable and irregular nature. A completely exact mathematical representation of it as functions of time, wind speed, wind direction, current, etc is not possible.

The strength of apparently identical marine structures will not be, in general, identical, because of the limitations on control of properties of steel and other materials used in structures and because of limitations in the manufacture of their components etc. In addition, the uncertainties associated with residual stresses arising from welding etc, may affect the strength of the structure.

Additional uncertainties in the strength will arise due to the uncertainties associated with the assumptions and the methods of analysis used to calculate strength. Further uncertainties are associated with the possible numerical errors in the process of analysis. These errors can accumulate in one direction or possibly trend to cancel each other. Whatever the case may be, the above uncertainties have to be reflected in any reliability or failure analysis.

These limitations and uncertainties indicate that a certain variability in loads and strength or hull capacity about mean values will result.

1.3.2 Types of uncertainties of marine structures

The uncertainties of marine structures can conveniently be divided into two types: objective uncertainties and subjective uncertainties. Objective uncertainties include the following uncertainties:
1) Loads acting on the marine structure.
2) Properties of materials of marine structures and components
3) Calculation models:
In addition to the above the objective uncertainties, the following subjective uncertainties may cause variabilities in the strength or capacity of the marine structure.

1) The degree of effectiveness of plating due to shear lag effects.
2) The Navier hypothesis of plane section remaining plane and perpendicular to the neutral axis.
3) The presence of small holes and the cutouts that may exist in the deck plating.
4) The residual strength after buckling and the effects of initial deformation on the buckling loads.
5) Human errors.

The above uncertainties associated with marine structures and corresponding physical representation in an analysis have various sources that may be grouped as follows:

- Physical uncertainties: such as yield stresses of materials
- Measurement uncertainties: such as geometry or dimension of plating
- Statistical uncertainties: such as types of distribution of stochastic variables
- Model uncertainties: such as correction coefficients of ultimate strength of the marine structure, etc
- Other uncertainties: such as human errors and the transitions between the quoted different uncertainties types that may exist.

1.4 General typical failure modes

In general, structure failure modes can be divided into the following modes:

1) Yielding
2) Buckling
3) Fracture or rupture
4) Deformation
5) Fatigue
6) Wear
7) Corrosion and erosion
8) Flooding, foundering, capsizing

In this paper only yielding and buckling failure modes of marine structures, are covered in detail. The yielding and buckling failure modes are more important than other failure modes in the initial stage of marine structure design and use. The failure criteria under ultimate loads are to be formulated in accordance with the current state-of-the-art within the particular technical field.

1.5 Concepts, definitions and notations
This section not only includes concepts, definitions and notations but also the relative calculation methods, explanations and examples. References are mainly SSC-351, SSC-368, SSC-398, and DNV Classification notes No.30.6 etc.

**1.5.1 Concepts and definitions**

Asymptotic distributions of the extreme value:

The extreme value distribution for a random process with defined probability characteristics for the outcome (e.g., a Gaussian random process) is a function of time, or equivalently, the number of peaks within the time. As time or number of peaks increase, the distribution of the extreme value shifts to the right. The asymptotic distribution corresponds to an infinite length of time or number of peaks. The asymptotic form of the extreme value distribution depends largely on the tail behaviour of the initial distribution of outcomes of the random process. Gumbel showed that the asymptotic distribution takes one of three forms: a double exponential form, an exponential form and an exponential form with an upper bound.

Average of highest 1/m-th value:

This is the average value of the highest 1/m\(^{th}\) peaks in a random process. For a random process whose peaks are Rayleigh distributed,

- Average of 1/3 highest values = \(2 \sqrt{m_0}\)
- Average of 1/10 highest values = \(2.55 \sqrt{m_0}\)
- Average of 1/1000 highest values = \(3.85 \sqrt{m_0}\)

where \(m_0\) is the mean square value of the process. The multipliers shown are for amplitudes rather than heights (double amplitudes). The average of 1/3 highest value is also called the Significant Value. These multipliers may be used for waves and wave bending moments and may err slightly on the conservative side.

Basic variables:

A set of variables entering the limit-state equation, including variables for model uncertainties in the limit-state function itself.

Bounded random variables:

Bounded random variables mean the bounds on the random variables are not \(-\infty\) and \(\infty\). For example, material yield strength is always a positive quantity, and its lower bound is zero. An upper bound on a load is sometimes used, resulting in a truncated probability density function.

Characteristic values:

A normal value to characterise the magnitude of a stochastic variable. The characteristic value is to be defined as a fractile of the probability distribution.

Characteristic load:

Reference value of a load to be used in determination of load effects.

Characteristic load effect:
Effect of a single characteristic load or combination of characteristic loads.

Characteristic resistance:
The normal capacity that may be used for determination of design resistance of a structure or structural element. The characteristic value of resistance is more easily based on a defined percentile of the test results, or lower bound values.

Characteristic strength:
The nominal value of material to be used for determination of design strength or of characteristic resistance. The characteristic strength is to be determined with the same confidence levels as that for the characteristic resistance.

Check point:
The Design Point, the most likely failure point, refers to Design Point.

Code calibration:
This is the process of selecting a target reliability level and a corresponding set of partial safety factors for use in a probability based design code. Reliability analyses of comparable past experience (existing structures, and systematic structural designs to traditional codes) are useful in the code calibration process.

Code optimisation:
This is the process of selecting partial safety factors for use in probabilistically based safety check equations, in such a manner that the scatter in the reliability of structures built to the code is minimized, and centred around the target value.

Conditional probability and Bayes theorem:
A conditional probability is denoted \( P[A|B] \) when A is one event and B is another event upon which the outcome of event A depends. An example of a conditional probability is a probability of structural failure calculated for a given sea state. The actual lifetime probability of failure will be different if all the sea states are considered. Bayes’ Theorem applies to conditional events. By Bayes’ Theorem, the probability that event A occurs conditioned on the probability that event B has already occurred is given by

\[
P(A|B) = \frac{P(A \cap B)}{P(B)}
\]

where A and B are the event domains and \( A \cap B \) is their intersection, i.e., the outcome space that contains both A and B at the same time (mutual occurrence)

Confidence level:
Probability, in percent, that a given observation is not the result of sampling error. A confidence level of 75 percent means that there is a probability of 25 percent that sampling error (chance fluctuations of sampling and testing) could have produced the result.

Dependent random variables:
Two random variables that are not independent random variables. This means that their joint density function is not the product of the marginal densities. The outcome of any one of the random variables is dependent on the outcome of the
other, i.e., there is a correlation between the realization of one random variable and realizations of the other. For $x$ dependent on $y$, the following is true:

$$f(x|y) = \frac{f_{x,y}(x|y)}{f_y(y)}$$

where $f(x|y)$ is the conditional density, $f_y(y)$ is a marginal density, and $f_{x,y}(x|y)$ is the joint density evaluated with $x$ given $y$.

Design life:
The time period from commencement of construction until condemnation or re-qualification/re-certification of a structure.

Design point:
The most probable outcome of the basic variables when failure occurs, i.e. the point on the limit-state surface with the highest probability density.

Design value:
Value to be used in the deterministic design procedure, i.e., characteristic value modified by the material or load coefficient.

Deterministic process:
If an experiment is performed many times under identical conditions and the records obtained are always alike, the process is said to be deterministic.

Ergodic hypothesis:
This states that a single sample function is quite typical of all other sample functions representing realization of a random process. Therefore we can estimate the various statistics of interest by averaging over time, using the one realization rather than averaging over an ensemble of realizations. An ergodic random process is necessarily stationary. A stationary random process is not necessarily ergodic.

Expected load history:
Expected load history for a specified time period, taking into account the number of load cycles and the resulting load levels for each cycle.

Expected maximum value:
The expected value (average) of the maximum peak (e.g., wave bending moment) in a sequence of $N$ peaks of a zero mean Gaussian random process was determined by Cartwright and Longuet-Higgins, and is approximated by

$$E[\max(z_1, z_2, \ldots, z_n)] = \left[2 \ln\left(\sqrt{1-e^2}N\right)\right]^{0.5} + C \left[2 \ln\left(\sqrt{1-e^2}N\right)\right]^{0.5}$$

where Euler's constant $C = 0.5772$. Here, $m_0$ is the area under the power spectral density, i.e., the mean square value of the process. It should be noted that the most probable extreme value (i.e., the mode) is given by the above equation, but with the second term on the right hand side deleted.

Expected value: i.e. mean value:
First order statistical moment of the probability distribution function for the considered variable.

**Extreme value:**

The extreme value of a random process is the largest value over a period of time. Each realization of the random process will have an extreme value. Thus there is also a distribution of extreme values, i.e., the extreme value is a random variable that has its own special distribution, mean value, variance, etc.

**Failure:**

An event causing an undesirable condition considered as failure (e.g. loss of component or system function, or deterioration of functional capacity to such an extent that the safety of the unit, personnel or environment is significantly reduced). A failure mode refers to a particular physical mechanism by which a structure or a part of it fails.

**Fatigue limit state:**

The fatigue limit state is associated with the damaging effect of repeated loading that may lead to a loss of specified function or to ultimate collapse. Fatigue limit state capacity for structural details is typically defined using S-N curves, while the demand is defined in terms of the lifetime stress range versus number of cycles histogram.

**First-order reliability index:**

The approximation to the reliability index resulting from use of a first-order reliability method.

**First-order reliability method (FORM):**

A reliability method in which the limit-state surface, in the standardised normal space, is approximated by a linear surface at the Design Point. The essential steps in this method of reliability analysis for the determination of the probability of failure are:

a) The basic correlated random variables $X$ defining the limit state function $G(X)$, with prescribed probability distributions, are transformed to a set of independent standard normal variables $U$.

b) The limit state surface $g(U)$ in the standard normal space is approximated by its tangent hyperplane at the point of the limit surface closet to the origin. This point has the highest probability density, and is called the Design Point or the Most Probable Failure Point.

c) The probability content within the linearized failure domain is found as an estimate of the actual failure probability. The FORM probability of failure is

$$P_f = \Phi(-\beta)$$

where $\beta$ is the reliability index, which is also the distance of the Design Point from the origin in the $u$ space. The FORM reliability index is invariant for mechanically different limit state functions representing the same failure event.

**Hasofer-Lind safety index:**
In the history of structural reliability theory, there have been several definitions of the safety index. Some fell from favour because of a problem known as lack of invariance. By this, it is meant that mechanically different limit state functions representing the same physical failure mode resulted in different values of the safety index. The Ha sofer-Lind index does not suffer from the lack of invariance problem.

Independent random variables:
Two random variables $x$ and $y$ are independent if their joint density function is equal to the product of their individual densities

$$f(x, y) = f(x)f(y)$$

where $f(x, y)$ is the joint density function and $f(x), f(y)$ are the individual density functions. Also $f(x), f(y)$ are called marginal density functions. The outcomes of independent random variables occur without any reference to one another. Normally in reliability analysis, strength and load are considered independent random variables.

Inherent uncertainties:
This kind of uncertainty is inherent to the variable, and cannot be reduced even if additional information is provided. This is a term that, in many cases, may involve the same sources as ‘objective’ uncertainties. Examples are the inherent variability of wave heights, extreme wave bending moment or the variability in yield strength.

Level (in structural reliability):
Level refers to the extent of information about the structural reliability problem that is provided and used.

Level I methods:
Level I methods are deterministic reliability methods that use only one ‘characteristic’ value to describe each uncertain variable. Level I methods correspond to standard deterministic design methods.

Level II methods:
Level II methods are reliability methods that use two values to describe each uncertain variable, i.e., its mean and variance, supplemented with a measure of correlation between the variables, usually the covariance. Reliability index methods, such as the first-order second-moment method, are examples of Level II methods.

Level III methods:
Level III methods are reliability methods that use the joint probability distribution of all the uncertain variables to describe each uncertain variable. Level III methods include numerical integration, approximate analytical methods, such as first- and second-order reliability methods, and simulation methods.

Level IV methods:
Level IV methods are reliability methods that compare a structural prospect with a reference prospect according to the principles of engineering economic analysis under uncertainty.

Limit state:
A state only within which the structure can satisfy the requirement. For structures, categories of limit-states are:
ULS = Ultimate limit-state.
FLS = Fatigue limit-state.
PLS = Progressive collapse limit-state.
SLS = Serviceability limit-state.

Limit state exceedence probability:
Limit state exceedence probability is the probability of reaching or exceeding a specified limit state that is determined from

$$P_f = \int_{G(X) \leq 0} f_X(X) dX$$

where $f_X(X)$ is the joint probability density function of the basic variable vector $X$. $G(X)$ is limit state function.

Limit state function:
This is a function, often denoted $G(X)$ where $X$ is a vector of basic variables, that characterizes the safety margin in a given mode of failure. This function has negative values when the structure component fails, and has positive values when the structure component is safe.

Limit state surface:
The surface separating the region of failure states from the region of safe states in the space spanned by the basic variables.

Load:
Any action causing stress or strain in the structure. Load categories are:
P = Permanent loads.
L = Live loads, or variable functional loads.
D = Deformation loads.
E = Environment loads.
A = Accidental loads.

Load coefficient:
Partial safety coefficient by which the characteristic load or load effect is multiplied to obtained the design load or load effect.

Load effect:
Effect of a single load or combination of loads on the structure, such as stress, deformation, displacement, motion, etc.

Material coefficient:
Partial safety coefficient by which the characteristic strength is divided to obtain the design strength.
Mean square value:
The mean square value of a random variable $X$ is defined by
\[ E(x^2) = \int_{-\infty}^{\infty} x^2 f(x) dx \]
and its root-mean-square or rms value is simply $rms = \sqrt{E(x^2)}$

Mean value: i.e. expected value:
For a given probability density function $f(x)$ relating to a random variable $X$, the mean or average value $\mu$ is given by
\[ \mu = E(x) = \int_{-\infty}^{\infty} x f(x) dx \]
where $E(x)$ denotes the expected value of X.

Median value:
The median value of $X$, denoted $\tilde{x}$, is defined from the cumulative distribution function $F(x)$ as $\tilde{x} = F^{-1}(0.5)$, i.e., it is a value of $X$ corresponding to a cumulative distribution function of 0.5. This implies that, on the average, $\frac{1}{2}$ the outcomes of the random variable will lie below $\tilde{x}$ and $\frac{1}{2}$ above it.

Mode value:
The mode of a random variable $X$ is the value of $X$ corresponding to the peak of the probability density for the random variable. The mode is also called the most probable of the random variable.

Model uncertainties:
Model uncertainties are the inherent uncertainties of the selected calculation models (load model, strength model, function model for the structure, etc.). These uncertainties arise because of errors in how the prediction models attempt to represent reality. They can be reduced with additional information. Model uncertainties are typically estimated based on comparing the analysis procedure with experimental data, or in some case using professional judgement or other indirect information such as the non-occurrence of cracks in relation to expectation. Some sources of model uncertainties are described under ‘subjective uncertainties’. The largest model uncertainty in marine structures usually relates to loads such as slamming loads. Strength prediction techniques (e.g., for buckling strength) also have their own model uncertainties. This type of uncertainty is usually quantified in terms of a basis (i.e., actual value to predicted value ratio) and a coefficient of variation.

Most probable extreme value:
This is the value of the random variable corresponding to the peak of the extreme value density function, i.e., the mode. Thus, the most probable extreme wave bending moment is the mode value of the extreme bending moment density function, i.e., the value of the moment at the peak of the density function.

Narrow band process:
This is a random process whose time realizations are such that there is one peak between every up-crossing and down-crossing of the mean level. Process ‘cycles’
are thus discernible. The power spectral density function of the process realization has a central tendency, i.e., it is clustered about a central frequency. The peaks of a zero mean narrow band Gaussian random process have the Rayleigh distribution function given by

\[ f_p(a) = \frac{a}{m_0^2} e^{-\frac{a^2}{2m_0^2}} \quad a \geq 0 \]

where \( m_0 \) is the mean square value of the process, also equal to the area under the energy spectrum for the process. Records of waves and wave bending moments over a short period of time (3 hours) are usually considered to be narrow-band processes.

Nominal values:
In this report, they are taken as equivalent to characteristic values (refer to Characteristic values).

Objective uncertainties:
These are uncertainties associated with random variables for which statistical data can be collected and examined. They can be quantified by a mean, a coefficient of variation, and a form of the probability distribution function derived from available statistical information. The variability in the yield strength of steel is an example.

Omission sensitivity factor:
A factor giving the change in the reliability index when a random variable is replaced by a fixed value.

Order statistics:
The distribution of the largest peak (e.g., largest wave bending moment) in a sequence of N peaks of a random process can be determined using order statistics, assuming that the peaks are independent and identically distributed. The cumulative distribution function of the largest peak is given by

\[ F_{Z_N}(Z) = P[\max(z_1, z_2, \ldots, z_n) \leq z] = [F_z(z, \varepsilon)]^N \]

where \( F_z(z, \varepsilon) \) is the initial cumulative distribution function of the peaks and \( \varepsilon \) is the spectral bandwidth parameter. The corresponding probability density function is given by differentiating the cumulative distribution function:

\[ f_{Z_N}(z) = N[F_z(z, \varepsilon)]^{N-1} \cdot f_z(z, \varepsilon) \]

where \( f_z(z, \varepsilon) \) is the initial PDF of the peaks.

Parallel system:
In a parallel system, all links along the failure path must fail for the structure to fail. An example is a multi-component redundant structure such as a fixed offshore platform, in which a failure path is the failure of a group of members which leads to system collapse. The failure event resulting from one failure path can be modelled by a parallel system. Since there typically are many different failure paths, each represented by a parallel system, and since failure can occur in
any one of the failure paths, the entire system can be modelled as a giant series system with parallel subsystems, each representing a failure path.

Parametric sensitivity factors:

A factor giving the change in the reliability index by a unit change in a distribution parameter or a fixed value.

Partial coefficient:

Coefficient by which the characteristic value of a variable is modified to give the design value (i.e. a load or material coefficient.)

Partial safety factor format:

A safety check equation in a partial safety factor format employs multiple safety factors, which may address uncertainties in component loads, resistance, and also failure consequences, non-coincidence of peak loads from different sources, etc. Because there is more than one safety factor employed, the format is more efficient in that factors of safety are placed in a manner more commensurate with individual demands and uncertainties. Also, the partial safety factors are usually obtained using Level II reliability methods, consistent with a required target reliability level. A sample partial safety factor format is that recommended in the Load and Resistance Factor (LRFD) version of APIRP-2A. This is given by

$$L_{+++}\Phi_{i} \geq \gamma_{D}D + \gamma_{L}L + \gamma_{W}W + \cdots$$

where

- \( R_i \) = nominal strength or resistance of component i
- \( \Phi_{R_i} \) = partial strength factor for component i
- D = nominal gravity or dead load effect
- \( \gamma_{D} \) = load factor for dead load
- L = nominal live load effect
- \( \gamma_{L} \) = load factor for live load
- W = nominal environmental effect with prescribed return period
- \( \gamma_{W} \) = load factor for environmental load

Each resistance factor \( \Phi_{R_i} \) is calculated as a product of two factors, one representing strength uncertainty, and the other taking into account the consequence of failure of the component and the structural system. The load factors \( \gamma \) are also calculated as a product of two factors, one representing uncertainty in load intensity, and the other uncertainty in the related analysis procedures. A partial safety factor format is a Level I reliability based format if the safety factors employed are obtained from reliability analysis with a prescribed target reliability.

Percentile:

Percentile values of a random variable X are those values corresponding to specified values of the cumulative distribution function F(x). A 50 percentile value thus corresponds to x such that F(x)=0.5. This particular percentile is also the median value of the random variable. A 95 percentile value is a value such that
F(x)=0.95, i.e., only 5% of the outcomes of the random variable are expected to lie above it.

Probability of failure:
Although actually speaking, this should refer to the probability that the structure catastrophically fails, the term is generally and widely used as a substitute for limit state exceedence probability, i.e., the probability that the demand exceeds the capacity in any given limit state (including exceedence of deflection and elastic buckling stress etc.). It is an integral of probability density of the limit-state variables over the failure region of the space spanned by the basic variables.

Rackwitz-Fiessler Transformation:
In calculating the safety index, it is necessary to include information related to the form of the distribution of the basic variables. The tail of the distribution of the random variables is usually the location where most of the contribution to the probability of failure comes from. In the Rackwitz-Fiessler transformation, an equivalent normal distribution is fitted to the tail of the non-normal distribution at the most likely failure point (Design Point). The method requires that the cumulative distributions and the probability density function of both the actual distribution and the normal distribution be equal at the Design Point.

Random (stochastic) process:
Stochastic function of explicit time. Loads are often modelled as stochastic variables that are functions both of time and location. When such functions are time dependent they are called stochastic processes. When they are functions of location, they are called stochastic fields.

Random (stochastic) variable:
A variable characterizing a set of items where every member has a chance of occurring, as described by the probability density function.

Redundancy:
The ability of a component or system to maintain or restore its function when one failure has occurred. For instance, redundancy can be achieved by installation of more units or elements to restrain the loads, or by alternative means for performing a function.

Reliability:
Ability of a component or a system to perform its required function without failure during a specified time interval. This is found by probability density integrated over the safe states in the space spanned by the basic variables. This is the complement of the probability of failure $P_f$, i.e., reliability is the reliability of survival, given by $1-P_f$.

Reliability index:
Fractile of the normalised normal probability function corresponding to the reliability.

Resistance:
Capacity of a structure or part of a structure to resist load effects.

Root mean square (rms):
The mean square value of a random variable X is defined by
\[ E(x^2) = \int_{-\infty}^{\infty} x^2 f(x) \, dx \]
and its root mean square or rms value is simply \( \text{rms} = \sqrt{E(x^2)} \)

Safety check equation:
In structural design, the performance of the structure is checked using safety check equations. In the working stress approach for fixed offshore platforms, as embodied in API RP-2A Recommended Practice, for example, the maximum or yield strength is divided by a safety factor to obtain an allowable stress. Designs are then limited so that the maximum calculated stress under extreme operating loads does not exceed the allowable value. This example safety check is of the form
\[ \frac{R}{SF} \geq D + L + W + \text{other load effects} \]
where \( R \) = nominal component strength
SF = safety factor
D = nominal gravity loads on components
L = nominal live load effects on components
W = nominal environmental load effects on components
Nominal loads are all combined with factors of one, and constant safety factors 1.67 and 1.25 are used for operating and extreme loadings. There are typically many safety check equations to be satisfied in a design, each of which addresses a different failure mode or design concern.

Safety margin:
This is the difference between capacity and demand, or strength and load. Either mean or characteristic values may be used to determine the safety margin.

Safety index:
The safety index is a number that is inversely related to the probability of failure. The safety index \( \beta \) and the probability of failure are related by
\[ P_f = \Phi(-\beta) \]
where \( \Phi \) is the standard normal distribution function. A safety index of 2.3 translates roughly to a probability of failure of 0.01, 3.1 to 0.001, 3.7 to 0.0001. A safety index of zero corresponds to a probability of failure of 0.5.

Second-order reliability method (SORM):
SORM is a reliability method in which the limit-state surface at the standardised normal space is approximated by a quadratic surface at the Design Point. In SORM, the essential steps are similar to FORM, except that the limit state surface in the standard normal ‘u’ space is approximated by a second order approximation such as a hyper-paraboloid fitted with its apex at the Design Point. The failure
domain probability content within the second order approximation is then estimated. For hyper-paraboloids, the probability content can be ‘exactly’ estimated.

Series system:
A series system is one that is composed of links connected in series, such that failure of any one or more of these links constitute a failure of the system, i.e., ‘weakest link’ system. In the case of the primary behaviour of a ship hull, for example, occurrence of any one of a number of modes of failure will constitute failure of the hull. The multiple failure modes can then be modelled as a series system.

Serviceability:
A condition in which a structure is considered to perform its design function satisfactorily.

Serviceability limit state:
The serviceability limit states are associated with constrains on the marine structure in terms of functional requirements such as the maximum deflection of a member or critical buckling loads that cause elastic buckling of plating.

Sensitivity:
Sensitivity is defined to be the measure of the susceptibility of reliability to changes in the input parameters.

Specified value:
Minimum, or maximum, value during the period considered. This value may take into account operational requirements, limitations and measures taken such that the required safety level is obtained.

Standard normal space:
A space of normally-distributed random variables with zero mean values, unit standard deviations and zero correlation coefficients.

Stationary random process:
A random process is stationary if the probability density function of its outcomes does not depend on time, i.e., the same probability density function is obtained for an ensemble of realizations of the random process at any given time as compared with any other time.

Structural system modelling:
The behaviour of a structure that can fail in more than one mode of failure is modelled for structural reliability evaluation purposes using structured representations of system behaviour. Series, parallel or general system representations are usual. A general system representation may take the form of a cut set (parallel subsystems connected in series) representation or a link set (series subsystems connected in parallel) representation. Failure tree representations are also possible.

Subjective uncertainties:
These are uncertainties associated with the lack of information and knowledge. They are typically quantified on the basis of the engineer’s prior experience and judgement. Examples of these include assumptions in the analysis, error in the design model, and empirical formula. The following subjective uncertainties contribute to strength variability:

a) Effectiveness of plating, e.g., due to shear lag
b) Use of Navier hypothesis in calculating hull girder response
c) Initial deformation and residual stress effects.

Uncertainty importance factors:
Factors giving the fraction of variance caused by variance in each basic variable.

Ultimate limit state:
The ultimate limit state considers structural performance or safety margin under extreme (typical lifetime maximum) loads. The ultimate limit state can be further divided into two modes of failure:

a. Failure due to spread of plastic deformation, e.g., as predicted for beams by plastic limit analysis. The initial yield moment for a beam can also be classified under this category.

b. Failure due to instability or buckling, e.g., of panel longitudinal stiffeners in the flexural and tripping modes, or the overall ‘grillage’ buckling of a gross panel consisting of longitudinal and transverse stiffeners.

1.5.2 Notations and Concepts

PDF (Probability Density Function) $f(x)$: The probability that a continuous random variable $x$ shall appear in the interval $[x, x+dx]$ is $f(x)dx$.

CPF (Cumulative Probability Function) $F(x)$: This is the probability that the random variable $x$ is less than or equal to given value $x$, i.e.,

$$F(x) = \int_{-\infty}^{x} f(t)dt$$

EPF (Exceedence Probability Function) $Q(X)$: This is the probability that a random variable $X$ exceeds a specified value $x$, and is given in terms of the probability distribution function as $1 - F(x)$, i.e. complemental probability function, since

$$Q(x) = 1 - F(x) = \int_{x}^{\infty} f(t)dt$$

Statistical moment of order $k$ about zero, $M_k$:

$$M_k = M_k(x) = \int_{-\infty}^{\infty} t^k f(t)dt$$

Expected value, $E(x)$:

$$E(x) = M_1 = M_1(x)$$

Central moment of order $n$, $m_n$:

$$m_n = \int (x - E(x))^n f(x)dx$$

Variance $\sigma^2$, standard deviation $\sigma$: The variance of the random variable $X$ is defined by
\[ \sigma^2 = E(x - \mu_x)^2 = \int_{-\infty}^{\infty} (x - \mu_x)^2 f(x) \, dx = E(x^2) - \mu^2 \]
i.e., \( \sigma^2 = m_2 - M_1^2 \)

The standard deviation of the random variable is \( \sigma \).

COV (Coefficient Of variation): \( COV = \frac{\sigma}{E(x)} = \left[ \frac{M_2 - M_1^2}{M_1^2} \right]^{0.5} = \frac{\sqrt{m_2}}{m_1} \)

Skewness coefficient, \( \lambda_3 = \frac{m_3}{\sigma^3} \)

Coefficient of kurtosis, \( \lambda_4 = \frac{m_4}{\sigma^4} \)

Joint (simultaneous) probability (density) distribution (function), \( f(X) \), i.e.,
\[ f(x_1, x_2, x_3, \ldots) : \text{The probability that any random variable } x_i \text{ in a set shall appear within an interval } [x_i, x_i + dx_i] \text{ at the same time as any other variable } x_i \text{ in the same set appears within an interval } [x_i, x_i + dx_i], \text{ is } f(X)dx_1dx_2 \cdots dx_n. \]

Joint moment of order \( i,j, \ldots \) about zero, \( M(X)_{i,j,\ldots} : M(X)_{i,j,\ldots} = \iiint x_1^i x_2^j \cdots f(X) \, dX \)

Covariance between \( x \) and \( y \), \( R_{x,y} \) : The covariance of two random variables, \( x \) and \( y \), is defined as
\[ R_{x,y} = E[(x - E(x))(y - E(y))] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x - \mu_x)(y - \mu_y) f(x, y) \, dx \, dy \]
\[ R_{x,y} = M(x, y)_{1,1} - E(x)E(y) \]

Correlation coefficient of \( x \) and \( y \), \( \rho_{x,y} \) : \( \rho_{x,y} = \frac{R_{x,y}}{\sigma_x \sigma_y} \)

The following list defines the main symbols appearing in this report.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L )</td>
<td>Ship length</td>
</tr>
<tr>
<td>( X )</td>
<td>Random variable</td>
</tr>
<tr>
<td>( \mu_x )</td>
<td>Mean of ( X )</td>
</tr>
<tr>
<td>( \sigma_x )</td>
<td>Standard deviation of ( X )</td>
</tr>
<tr>
<td>( \omega )</td>
<td>Wave frequency</td>
</tr>
<tr>
<td>( H(\omega) )</td>
<td>Frequency response function</td>
</tr>
<tr>
<td>( S_X(\omega) )</td>
<td>Wave spectrum</td>
</tr>
<tr>
<td>( S_y(\omega) )</td>
<td>Response spectrum</td>
</tr>
<tr>
<td>( N, n )</td>
<td>Number of records or encounters</td>
</tr>
<tr>
<td>( Z_u )</td>
<td>Random variable representing extreme amplitude of total bending moment in ( n )-records</td>
</tr>
</tbody>
</table>
\( P_f \) = Probability of failure
\( G(\cdot), g(\cdot) \) = Limit state function or performance function
\( \beta \) = Reliability index
\( \Phi(\cdot), \Phi(\cdot) \) = Standard normal probability density and distribution functions

Note: other Concepts, definitions and notations are defined where used first.

1.6 Benefits and drawbacks of using probability-based design method

Use of probabilistic methods in design can provide several benefits and some unique features. Among those are (SSC-368):

1) Explicit consideration and evaluation of uncertainties associated with the design variables.
2) Inclusion of all available relevant information in the design process.
3) Provide a framework of sensitivity measures.
4) Provide means for decomposition of global safety of a structure into partial safety factors associated with the individual design variables.
5) Provide means for achieving uniformity of safety within a given class of structures (or specified non-uniformity).
6) Minimum ambiguity when updating design criteria.
7) Provide means to weigh variables in terms of their significance.
8) Provide rationale for data gathering.
9) Provide guidance in novel design.
10) Provide the potential to reduce weight without loss of reliability, or improve reliability without increasing weight. The methods can identify and correct overly and unduly conservative designs.

In addition to the above benefits, reliability technology lends itself for certain uses for which it is more suitable than traditional design methods. In reference SSC-368, Wirsching lists some of its uses, which include:

1) To compare alternative designs, particularly in the early stages when several competing design concepts are considered.
2) To perform failure analysis of a component or a system.
3) To develop a strategy for design and maintenance of ageing structures (e.g., corrosion, fatigue), and to determine inspection intervals.
4) To execute ‘economic value analysis’ or ‘risk based economics’ to produce a design with minimum life cycle costs.
5) To develop a strategy for design, warranties, spare parts requirements.
6) In general, as a design tool to manage uncertainty in engineering problems.

Use and implementation of probabilistic methods are not without problems. Some of the drawbacks are:

1) Use of reliability analysis in safety and design processes requires more information on the environment, loads and the properties and characteristics of the structure than typical deterministic analyses. Often some information is not
available or may require considerable time and effort to collect. Time and schedule restrictions on design are usually limiting factors on the use of such methods.

2) Application of probabilistic and reliability methods usually require some familiarity with the basic concepts of probability, reliability and statistics. Practitioners and designers are gaining such familiarity through seminars, symposia and special courses. Educational institutions are also requiring more probability and statistics courses to be taken by students at the graduate and undergraduate levels. This, however, is a slow process that will take some time in order to produce the necessary ‘infrastructure’ for a routine use of reliability and probabilistic methods in design.

3) On a more technical side, reliability analysis did not deliver what it initially promised, that is, a true measure of the reliability of a structure by a ‘true and actual’ probability of failure. Instead what it delivered is ‘notional probabilities’ of failure and safety indices, which are most useful only as comparative measures. Only notional values are delivered because of the many assumptions and approximations made in the analysis producing such probabilities and indices. These approximations, deficiencies and assumptions are, however, made not only in probability-based design, but also in traditional design. Approximations are made in the determination of loads using hydrodynamic theory and in structural analysis and response to the applied loads. When all such assumptions and deficiencies are removed from the design analysis, the resulting probabilities of failure will approach the ‘true’ probabilities.

1.7 Role of reliability analysis in a general probability design procedure

This report focuses on reliability analysis. What about the role of reliability analysis in a general probabilistic design procedure? In order to define the role of reliability analysis in a general probabilistic procedure for the design of marine structures, Figure 1.1 is introduced.

Starting with a configuration of the marine structure and using random ocean waves as input, the wave loads acting on the structure can be determined (refer to Figure 1.1). Generally, for primary design analysis the most important loads are the large ones. Extrapolation procedures are usually used to determine the characteristics of these large loads. In the case of ocean-going vessels, for example, this is done either through the determination of a long-term distribution of the wave loads or through the evaluation of an extreme load distribution that may occur in a specific storm condition. In general, wave loads acting on an ocean-going vessel include low frequency loads due to the motion of the vessel in waves as a rigid body. They also include higher frequency loads
Combined extreme loads

Reliability analysis: Safety indices & probability of failure

Comparison with acceptable Safety indices & probability of failure

Acceptable?

START

Random ocean waves

Configuration of the structure

Wave loads on the structure

Other loads on the structure

Extreme wave loads

Combined extreme loads

Structural response to combined extreme loads

Structural capability or strength

Figure 1.1 Probabilistic analysis of marine structure
Due to slamming and spring which can be determined by considering the ship as a flexible body. In principal, these loads should be combined stochastically to determine the total wave load. Referring back to Figure 1.1, other loads besides wave loads occur on a marine structure. These loads may be important in magnitude, though usually less random in nature (except possibly for wind loads on offshore structures). For example, in the case of ocean-going vessels, these loads consist mainly of stillwater loads and thermal loads. Following Figure 1.1, the response of the marine structure to the total combined loads is determined and compared with the resistance or capacity of the structure. This comparison may be conducted through one of several reliability methods. Based on these methods, safety indices or probabilities of failure are estimated and compared with acceptable ones. A new cycle may be necessary if the estimated indices are below the acceptable ones.

1.8 Conclusions
In this chapter, first, the structural reliability design is reviewed and the general procedure of structural reliability analysis is presented. Then, an overview of uncertainties is given, including objective uncertainties and subjective uncertainties. In general, structure failure modes include yielding, buckling, fracture or rupture, deformation, fatigue, wear, corrosion, erosion, flooding, foundering, capsizing, and so on. Third, this chapter describes the associated reliability concepts, definitions and notations in detail. For most of them, detailed explanations are given. Fourth, benefits and drawbacks of using probability-based design method are listed. Finally, role of reliability analysis in a general probability design procedure is represented.