CHARACTERIZATION OF NON-SYMMETRIC FORMS OF COMPOSITE LAMINATES.

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PRESENTATION CONTENTS

Uncoupled Laminates

- Characterization of **F**ully **O**rthotropic angle-ply **L**aminates (**FOL**s) with up to 21 plies in terms of angle- and cross-ply sub-sequence symmetries.
- Important sub-sets of **F**ully **O**rthotropic **L**aminates (**FOL**s) include: **Q**uasi-**H**omogeneous **O**rthotropic **L**aminates (**QHOL**s), **E**xtensionally **I**sotropic **L**aminates (**EIL**s) and **F**ully **I**sotropic **L**aminates (**FIL**s).
- Dimensionless parameters are presented for each sequence from which the laminate properties are readily calculated.
- Expressions relating the dimensionless parameters to the well-known lamination parameters are given, together with graphical representations of feasible domains for use in design optimization.
- Rules for mixing **FOL**s to form laminates containing any number of plies are demonstrated.
- Examples are given for Tapered **FOL**s.

Coupled Laminates

- **E**xtensionally **A**nisotropic angle-ply **L**aminates (**EAL**s) with up to 21 plies have been derived to complement those for **FOL**s.
- **EAL**s are shown to be the predominant material characteristic for passive bend-twist coupling in wing box structures.

- Bending-Extension coupling: An overview of current and planned activities on laminate tailoring of the coupling response between in- and out-of-plane behaviour.

CHARACTERIZATION

Composite laminate materials are typically characterized in terms of their response to mechanical or thermal loading, which is generally associated with a description of the coupling behaviour, described by the **ABD** relation:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} \mathrm{A}_{11} & \mathrm{A}_{12} & \mathrm{A}_{16} \\ & \mathrm{A}_{22} & \mathrm{A}_{26} \\ \mathrm{Sym.} & & \mathrm{A}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} + \begin{bmatrix} \mathrm{B}_{11} & \mathrm{B}_{12} & \mathrm{B}_{16} \\ & \mathrm{B}_{22} & \mathrm{B}_{26} \\ \mathrm{Sym.} & & \mathrm{B}_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} \mathrm{B}_{11} & \mathrm{B}_{12} & \mathrm{B}_{16} \\ & \mathrm{B}_{22} & \mathrm{B}_{26} \\ \mathrm{Sym.} & & \mathrm{B}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} + \begin{bmatrix} \mathrm{D}_{11} & \mathrm{D}_{12} & \mathrm{D}_{16} \\ & \mathrm{D}_{22} & \mathrm{D}_{26} \\ \mathrm{Sym.} & & \mathrm{D}_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

Couplings generally exist between:

- in-plane (extension or membrane) and out-of-plane (bending or flexure) actions, when $\mathrm{B}_{ij} \neq 0$,
- in-plane shear and extension, when $\mathrm{A}_{16} = \mathrm{A}_{26} \neq 0$, and
- out-of-plane bending and twisting, when $\mathrm{D}_{16} = \mathrm{D}_{26} \neq 0$.

The Engineering Sciences Data Unit or ESDU offers a classification system[1] in which:

- Extensional (**A**), coupling (**B**) and bending (**D**) stiffness matrices are used together with an extended subscript notion to describe the form of the elements in each matrix.

The matrix sub-scripts are:

0 = All elements zero

F = All elements Finite

S = Specially orthotropic form

An additional subscript is also introduced here:

I = Isotropic form

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[1] Engineering Sciences Data Unit, "Stiffnesses of laminated plates", ESDU Item No. 94003, 1994.

For example, *balanced and symmetric stacking sequences*, which generally possess bend-twist coupling, are referred to by the designation:

$$\mathbf{A}_{\mathrm{S}}\mathbf{B}_{0}\mathbf{D}_{\mathrm{F}},$$

signifying that the elements of the extensional stiffness matrix (**A**) are **S**pecially orthotropic in nature, i.e.

$$A_{16} = A_{26} = 0, \tag{1}$$

the (bending-extension) coupling matrix (**B**) is null, whilst all elements of the bending stiffness matrix (**D**) are **F**inite.

Alternatively, *unbalanced stacking sequences*, with coupling between in-plane shear and extension only, would be referred to by the designation:

$$\mathbf{A}_{\mathrm{F}}\mathbf{B}_{0}\mathbf{D}_{\mathrm{S}},$$

signifying that all elements of the extensional stiffness matrix (**A**) are **F**inite, the (bending-extension) coupling matrix (**B**) is null, whilst the elements of the bending stiffness matrix (**D**) are **S**pecially orthotropic in nature, i.e.

$$D_{16} = D_{26} = 0, \tag{2}$$

Fully **O**rthotropic angle-ply **L**aminates or **FOL**s therefore have the designation:

$$\mathbf{A}_{\mathrm{S}}\mathbf{B}_{0}\mathbf{D}_{\mathrm{S}}$$

Extensionally **I**sotropic **L**aminates or **EIL**s, have the designation:

$$\mathbf{A}_{\mathrm{I}}\mathbf{B}_{0}\mathbf{D}_{\mathrm{S}}$$

Fully **I**sotropic **L**aminates or **FIL**s, with the designation:

$$\mathbf{A}_{\mathrm{I}}\mathbf{B}_{0}\mathbf{D}_{\mathrm{I}}.$$

EILs and **FIL**s represent sub-sets of **FOL**s:

matrices simplify further in **EIL**s, in which the designation $\mathbf{A}_{\mathrm{S}}$ is replaced with $\mathbf{A}_{\mathrm{I}}$ to indicate that:

$$\mathrm{A}_{11} = \mathrm{A}_{22} \tag{3}$$

and

$$\mathrm{A}_{66} = (\mathrm{A}_{11} - \mathrm{A}_{12})/2 \tag{4}$$

and further still in **FIL**s, in which the designation $\mathbf{D}_{\mathrm{S}}$ is replaced with $\mathbf{D}_{\mathrm{I}}$ to indicate that:

$$\mathrm{D}_{\mathrm{ij}} = \mathrm{A}_{\mathrm{ij}}\, H^2/12 \tag{5}$$

where H is the laminate thickness, corresponding to the total number of plies (n) of thickness (t).

Quasi-**H**omogeneous **O**rthotropic **L**aminates (**QHOL**s) are **FOL**s that satisfy Eq. (5).

ARRANGEMENT AND FORM OF STACKING SEQUENCE DATA

The resulting sequences are characterized by sub-sequence symmetries using a **double prefix** notation: the **1st character** relates to the form of the **angle-ply sub-sequence** and the **2nd character** to the **cross-ply sub-sequence**.

The double prefix contains combinations of the following characters:

A to indicate **Anti-symmetric form**;

N for **Non-symmetric**; and

S for **Symmetric**.

Additionally, for cross-ply sub-sequence only,

C is used to indicate **Cross-symmetric form**.

The form (and number) of the stacking sequences with up to 21 plies can be summarized as:

AC (210), ***AN*** (14,532), ***AS*** (21,609),

SC (12), ***SN*** (192), ***SS*** (1,029),

$_{+}NS_{+}$ (220), $_{+}NS_{-}$ (296),

$_{+}NN_{+}$ (5,498), $_{+}NN_{-}$ (15,188) and $_{+}NN_{\circ}$ = $_{+}NN_{\bullet}$ (10,041).

NON-DIMENSIONAL PARAMETERS

An extract from a Table of stacking sequences:

Ref.	Sequence	n	$n_\pm$	$n_\circ$	$n_\bullet$	ζ	$\zeta_\pm$	$\zeta_\circ$	$\zeta_\bullet$
AS 1	+ − − ● + + −	7	6	0	1	343	342	0	1
AS 2	+ − − ○ + + −	7	6	1	0	343	342	1	0
AS 3	+ − − + − + + −	8	8	0	0	512	512	0	0
AS 4	+ − ● − ● + ● + −	9	6	0	3	729	630	0	99
⋮		⋮							
AS 9	+ − − + ○ − + + −	9	8	1	0	729	728	1	0
AS 10	+ ● − − − + + + ● −	10	8	0	2	1000	704	0	296

CALCULATION OF MEMBRANE AND BENDING STIFFNESS TERMS

The calculation procedure for the elements (A_{ij} and D_{ij}) of the extensional (**A**) and bending (**D**) stiffness matrices, using the dimensionless parameters provided in the Tables, are as follows:

$$A_{ij} = \{n_\pm/2 \times Q'_{ij+} + n_\pm/2 \times Q'_{ij-} + n_\circ Q'_{ij\circ} + n_\bullet Q'_{ij\bullet}\} \times t \tag{6}$$

$$D_{ij} = \{\zeta_\pm/2 \times Q'_{ij+} + \zeta_\pm/2 \times Q'_{ij-} + \zeta_\circ Q'_{ij\circ} + \zeta_\bullet Q'_{ij\bullet}\} \times t^3/12 \tag{7}$$

LAMINATION PARAMETERS

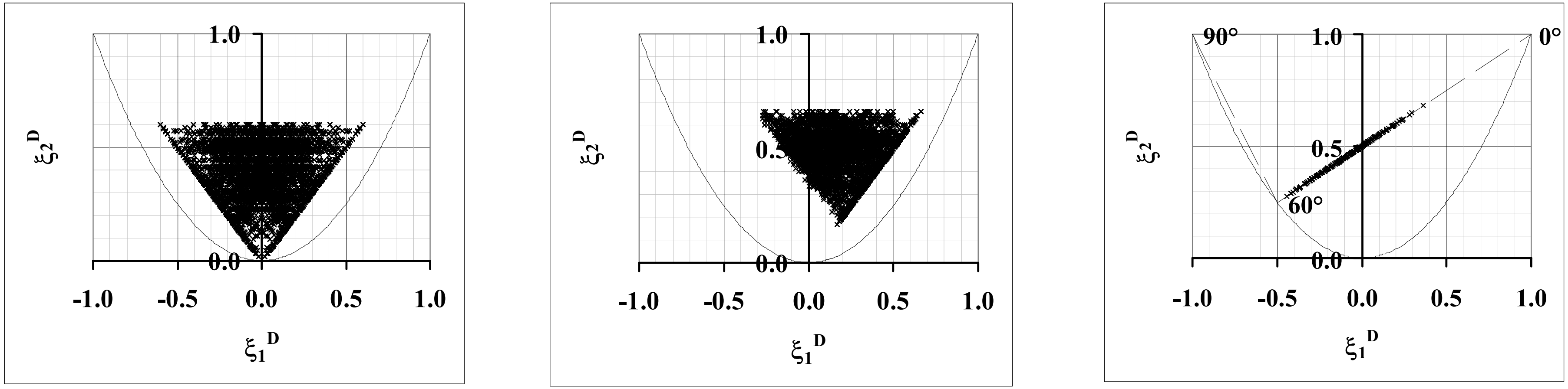
It is beneficial, for design optimization, to express the stiffness properties in terms of lamination parameters, which can be conveniently presented in graphical form.

The flexural stiffness terms are fully defined by two linear design variables:

$$\xi_1^{D} = \xi_9 = \{\zeta_{\pm}\mathrm{Cos}(2\theta_{\pm}) + \zeta_{\circ}\mathrm{Cos}(2\theta_{\circ}) + \zeta_{\bullet}\mathrm{Cos}(2\theta_{\bullet})\}/\zeta \qquad (8)$$

$$\xi_2^{D} = \xi_{10} = \{\zeta_{\pm}\mathrm{Cos}^2(2\theta_{\pm}) + \zeta_{\circ}\mathrm{Cos}^2(2\theta_{\circ}) + \zeta_{\bullet}\mathrm{Cos}^2(2\theta_{\bullet})\}/\zeta$$

Optimized lamination parameters may then be matched against a laminate stacking sequence.



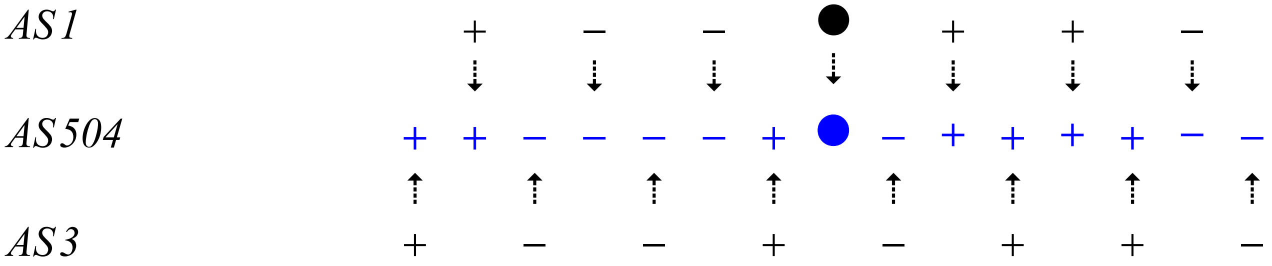
(a) $_{+}NN_{-}$ (15,188) (b) $_{+}NN_{\circ}$ (10,041) (c) **EIL**s

A selection of feasible domains of lamination parameters, for: (a), (b) **FOL**s, including form and number of sequences represented and; (c) the **EIL** sub-set, for π/3 isotropy, with up to 21 plies. The **FIL** sub-set, with π/3 isotropy, all correspond to the point (0, 0.5).

MIXING RULES FOR COMBINING STACKING SEQUENCES

One example: Interlacing of laminate stacking sequences

A more intuitive method for combining laminates is now demonstrated, involving the interlacing of individual plies of one laminate with those of another. In this way two laminates may be combined: one with *n* plies, the other with *n* + 1 plies, e.g. sequences *AS 1* and *AS 3* give rise to sequence *AS 504*:



Interlacing is generally not applicable for combining sequences with equal numbers of plies, although exceptions to this rule have been identified.

FULLY ORTHOTROPIC TAPERED LAMINATES

The following series provides the laminate number, together with the details of the particular ply to be dropped, i.e. ply orientation (+/–/●/○) and corresponding ply number:

NN 3979(●$_{11}$) ➡ *NN 1796*(●$_{10}$) ➡ *NN 560*(○$_{11}$) ➡ *NN 153*(○$_{10}$) ➡ *NN 21*(○$_{9}$) ➡ *NN 2*

Hence terminating the 11th ply of sequence *NN 3979*, corresponding to a ● ply, gives sequence *NN 1796*. Terminating the 10th ply of sequence *NN 1796*, which is also a ● ply, gives sequence *NN 560*, and so on.

+/–/●/○/○/●/–/+/○/●/●/○/○/–/●/+/+/●/–/○

+/–/●/○/○/●/–/+/○/●/○/○/–/●/+/+/●/–/○

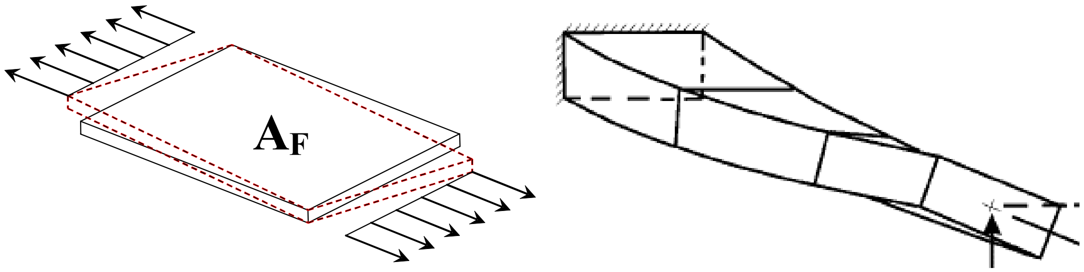
+/–/●/○/○/●/–/+/○/○/○/–/●/+/+/●/–/○

+/–/●/○/○/●/–/+/○/○/–/●/+/+/●/–/○

+/–/●/○/○/●/–/+/○/–/●/+/+/●/–/○

+/–/●/○/○/●/–/+/–/●/+/+/●/–/○

EXTENSIONALLY ANISOTROPIC LAMINATES



(a) (b)

Illustration of (a) coupling (A_F) between extension and shear, producing (b) bend-twist deformation in aircraft wing-box structures.

This study compares bend-twist response in wing-box structures for both $\mathbf{A}_\mathrm{F}\mathbf{B}_0\mathbf{D}_\mathrm{S}$ and $\mathbf{A}_\mathrm{F}\mathbf{B}_0\mathbf{D}_\mathrm{F}$ laminate skins, in which laminates were carefully chosen to isolate the effects of bend-twist coupling response; achieved through identical stiffness terms, with the exception of D_{16} and D_{26}, which are zero in the $\mathbf{A}_\mathrm{F}\mathbf{B}_0\mathbf{D}_\mathrm{S}$ laminate.

Results reveal negligible difference in the static response, suggesting that bend-twist coupling of the individual laminate skins is not only potentially detrimental to static instability, but that it is unnecessary in terms of the magnitude of the wing box bend-twist response.

BENDING-EXTENSION COUPLING

It is clear from these recent studies[2,3] that the extensional (**A**) and bending (**D**) stiffness matrices possess one of two forms: either specially orthotropic ($\mathbf{A}_S$ or $\mathbf{D}_S$) or fully coupled ($\mathbf{A}_F$ or $\mathbf{D}_F$).

The isotropic form of these matrices, where $\mathbf{A}_I$ (and/or $\mathbf{D}_I$) replaces $\mathbf{A}_S$ (and/or $\mathbf{D}_S$), are excluded from this characterisation because they have been shown[4] to represent subsets of the specially orthotropic form.

By contrast, the coupling (**B**) stiffness matrix has several complex forms, whence $\mathbf{B}_0$ in all the preceding laminate descriptions must now be replaced with one of the alternative designations given in the following Table….

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[2] York, C. B. "On composite laminates with extensional-anisotropy," 49th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conf., AIAA-2008-1752, Schaumburg, Illinois, April 2008.

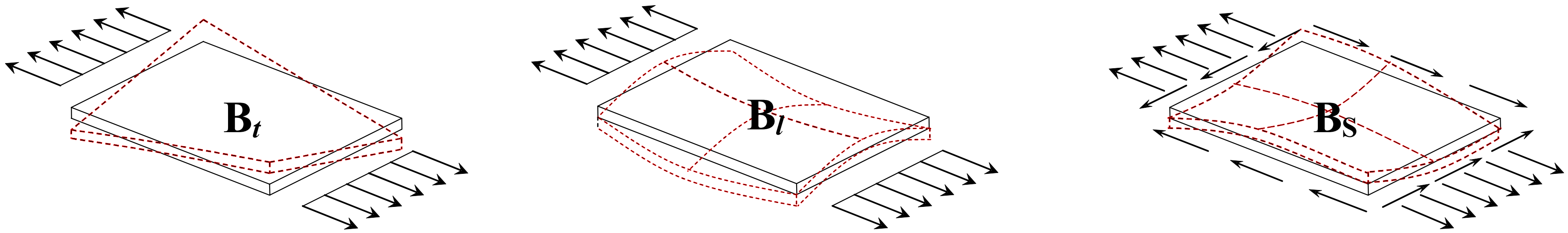
[3] York, C. B. "Characterization and ply mixing rules for non-symmetric forms of fully orthotropic laminates," Proc. 48th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conf., AIAA-2007-2083, Honolulu, Hawaii, 2007.

[4] York, C. B. "Stacking sequences for extensionally isotropic, fully isotropic and quasi-homogeneous orthotropic laminates," 49th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conf., AIAA-2008-1940, Schaumburg, Illinois, April 2008.

Descriptions of coupling behaviour nomenclature

Designation and coupling description	Laminate form	Matrix form
$\mathbf{B}_l$ Bend-stretch.	Unbalanced cross-ply sub-sequence.	$B_{11} = B_{22} \neq 0$
$\mathbf{B}_t$ Bend-shear.	Unbalanced angle-ply sub-sequence.	$B_{16} = B_{26} \neq 0$
$\mathbf{B}_{lt}$ Bend-stretch and bend-shear.	Unbalanced angle- and cross-ply sub-sequences.	$B_{12} = B_{66} = 0$
$\mathbf{B}_\mathrm{S}$ Anticlastic ($B_{12} \neq 0$) bend-stretch and twist-shear ($B_{66} \neq 0$).	Balanced laminate modified by orthotropic layer on one outer surface.	$B_{16} = B_{26} = 0$
$\mathbf{B}_\mathrm{F}$ Fully coupled.	Un-balanced non-symmetric laminates.	$B_{ij} \neq 0$

The following figure reveals the duality of bending-extension coupling.



(a) *(b)* *(c)*

The descriptions of coupling behaviour relate an applied (bending and/or twisting) *moment* resultant and the associated extensional (and/or shearing) strains, whereas the illustrations relate an applied (axial and/or shear) *force* resultant and the associated curvatures (and/or twist-curvature), i.e. between stretching and: (a) twisting or (b) bending, and between (c) shearing and twisting, and stretching and bending.

The alternative forms of the coupling matrix (**B**) are contained in the laminate designations below, which include those previously identified elsewhere [5,6,7] together with others yet to be discovered.

Table 1 – Laminate designations ranging from fully uncoupled ($\mathbf{A}_S\mathbf{B}_0\mathbf{D}_S$) to fully coupled ($\mathbf{A}_F\mathbf{B}_F\mathbf{D}_F$) behaviour.

$\mathbf{A}_S\mathbf{B}_0\mathbf{D}_S^{(a)}$	$\mathbf{A}_S\mathbf{B}_0\mathbf{D}_F^{(b)}$	$\mathbf{A}_F\mathbf{B}_0\mathbf{D}_F^{(c)}$	$\mathbf{A}_F\mathbf{B}_0\mathbf{D}_S^{(\dagger)}$
$\mathbf{A}_S\mathbf{B}_l\mathbf{D}_S^{(d)}$	$\mathbf{A}_S\mathbf{B}_l\mathbf{D}_F$	$\mathbf{A}_F\mathbf{B}_l\mathbf{D}_F$	$\mathbf{A}_F\mathbf{B}_l\mathbf{D}_S$
$\mathbf{A}_S\mathbf{B}_t\mathbf{D}_S^{(e)}$	$\mathbf{A}_S\mathbf{B}_t\mathbf{D}_F$	$\mathbf{A}_F\mathbf{B}_t\mathbf{D}_F^{(j)}$	$\mathbf{A}_F\mathbf{B}_t\mathbf{D}_S$
$\mathbf{A}_S\mathbf{B}_{lt}\mathbf{D}_S^{(f)}$	$\mathbf{A}_S\mathbf{B}_{lt}\mathbf{D}_F$	$\mathbf{A}_F\mathbf{B}_{lt}\mathbf{D}_F$	$\mathbf{A}_F\mathbf{B}_{lt}\mathbf{D}_S$
$\mathbf{A}_S\mathbf{B}_S\mathbf{D}_S^{(g)}$	$\mathbf{A}_S\mathbf{B}_S\mathbf{D}_F$	$\mathbf{A}_F\mathbf{B}_S\mathbf{D}_F$	$\mathbf{A}_F\mathbf{B}_S\mathbf{D}_S$
$\mathbf{A}_S\mathbf{B}_F\mathbf{D}_S^{(\ddagger)}$	$\mathbf{A}_S\mathbf{B}_F\mathbf{D}_F^{(h)}$	$\mathbf{A}_F\mathbf{B}_F\mathbf{D}_F^{(k)}$	$\mathbf{A}_F\mathbf{B}_F\mathbf{D}_S$

Superscript notation, including all designations and stacking sequences identified in ESDU Data Item 94003:

(a) See Data Item 82013 for definitive list (≤ 21 plies)
(b) $[30/-30/-30/30]_T \equiv [30/-30]_S$
(c) $[30/30]_T$, $[30/0/30]_T$
(d) $[0/90]_T$, $[0/0/90/0]_T$, $[90/0/90/0]_T$
(e) $[45/-45]_T$, $[30/-30/30/-30]_T$, $[30/30/-30/-30]_T$
(f) $[0/45/-45/90]_T$
(g) $[0/30/-30/-30/0/30/30/-30]_T$
(h) $[0/45/-45]_T$
(j) $[30/-30/-30]_T$
(k) $[30/-30/0/-30]_T$
(†) Ref. 6 contains the definitive list (≤ 21 plies)
(‡) Ref. 7 reports 296 solutions for $\mathbf{A}_I\mathbf{B}_F\mathbf{D}_I$ 18-ply laminates

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[5] Engineering Sciences Data Unit, “Stiffnesses of laminated plates”, ESDU Item No. 94003, 1994.

[6] York, C. B. “On composite laminates with extensional-anisotropy,” 49th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conf., AIAA-2008-1752, Schaumburg, Illinois, April 2008.

[7] Vannucci, P. and Verchery, G., “A new method for generating fully isotropic laminates,” Comp. Struct. 58, 2002, pp. 75-82.

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