Tax Structure, Growth and Welfare in the UK*

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April 21, 2008

Abstract

This paper studies the quantitative implications of changes in the composition of taxes for long-run growth and expected lifetime utility in the UK economy over 1970-2005. Our setup is a dynamic stochastic general equilibrium model incorporating a detailed fiscal policy structure, and where the engine of endogenous growth is human capital accumulation. The government’s spending instruments include public consumption, investment and education spending. On the revenue side, labour, capital and consumption taxes are employed. Our results suggest that if the goal of tax policy is to promote long-run growth by altering relative tax rates, then it should reduce labour taxes while simultaneously increasing capital or consumption taxes to make up for the loss in labour tax revenue. In contrast, a welfare promoting policy would be to cut capital taxes, while concurrently increasing labour or consumption taxes to make up for the loss in capital tax revenue.

Keywords: Fiscal policy, Economic growth, Welfare
JEL Classification: E62, O52.

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*We thank Guido Cozzi, Campbell Leith, Hamish Low, Klaus Walde and seminar participants at the University of Glasgow and the Royal Economic Society 2008 conference for comments. Any errors are ours.
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1 Introduction

The relationship between the government’s tax structure (distribution of revenue by type of tax) and the economy’s long-run growth rate has always received a great deal of theoretical and empirical attention (see e.g. Turnovsky (1995) at theoretical level and Auerbach (2006) at policy level). A robust finding of the literature, based on calibrated dynamic general equilibrium (DGE) models, is that the growth effects of tax reforms are likely to be small (see e.g. Lucas (1990) and Stokey and Rebelo (1995)). There has also been a lot of empirical work conducted on the link between growth rates and tax structure (see, e.g. Mendoza et al. (1997) and also Angelopoulos et al. (2007a) for a recent review of this literature). Broadly consistent with the findings of the quantitative DGE models, the preponderance of estimation evidence suggests that tax reform has either small or insignificant effects on growth.

In contrast to research on tax structure and growth, much less work has been undertaken on the welfare effects of altering the tax policy mix. Notable exceptions include the U.S. DGE studies by Lucas (1990), Cooley and Hansen (1992) and the dynamic stochastic general equilibrium (DSGE) study by McGrattan (1994). These papers conclude, in contrast to the growth studies discussed above, that the welfare effects of reforming the tax structure can be substantial.

The broad issue of tax reform and its economics consequences has recently moved back to centre stage in U.K. academic and policy circles (see e.g. the ongoing research for the Mirrlees Review, available at the IFS website). Despite this increased interest, we are not aware of any quantitative studies which attempt to assess the general equilibrium growth and welfare effects of the tax structure on the UK economy.

In light of the above, in this paper, we conduct tax policy analysis for the UK economy over 1970-2005 using a DSGE setup. Our model is a stochastic variant of Lucas’ (1990) well-known model in which human capital accumulation is the engine of endogenous growth. We extend Lucas’ model in four ways. First, by allowing for a more realistic disaggregation of government spending into its basic growth and utility enhancing activities, i.e. public investment, education and consumption. Second, by including a consumption tax\(^1\), which allows us to examine the growth and welfare trade-offs between income (capital and labor) and consumption taxes, as in e.g. Cooley and Hansen (1992)\(^2\). Third, by allowing for externalities from per

\(^1\)Note that taxes on goods and services constitute about 28.5% of total tax revenue in the UK, see, e.g. IMF Government Finance Statistics.

\(^2\)Cooley and Hansen (1992) work with a different (deterministic) model, which does
capita aggregate human capital (see e.g. Lucas (1988) and Tamura (1991)).
Fourth, by allowing for uncertainty when calculating welfare as in e.g. McGrattan (1994). Our welfare evaluations (where welfare is measured as expected lifetime utility of the representative household) follow recent developments in micro-founded DSGE models applied mainly in examining the welfare implications of different (monetary and fiscal) stabilization policies (see e.g. Schmitt-Grohé and Uribe (2004, 2007)). However these papers do not consider the welfare effects of the tax structure and also do not allow for (endogenous) growth.

In our analysis, we focus on two types of policy experiments. First, the case in which changes in one of the three distorting tax rates (capital, labour, consumption) are met by changes in lump-sum transfers/taxes. Working vis-a-vis the benchmark case in which a lump-sum policy instrument is the residually adjusted variable, helps us to identify and understand the effects of each distorting tax policy instrument. Second, the more interesting case in which changes in one of the three distorting tax rates are met by changes in another distorting tax rate. Working in this way, allows us to assess the effects of changing the composition of distorting tax rates.

The results of our analysis suggest that if the goal of tax policy is to promote long-run growth by replacing one distorting tax rate with another, then it should reduce labour taxes, while simultaneously increasing capital or consumption taxes to make up for the loss in labour tax revenue. Lucas (1990) also reports negative effects on the growth rate by increasing the labour tax, while decreasing the capital tax. This is because human capital is an engine of endogenous growth. However, both in Lucas (1990) and here, the growth effects of changes in the relative tax rates are small. For instance, if the tax rate on labour is reduced by about 10%, the net growth rate would increase to about 2.43% (when the capital tax increases) or to about 2.42% (when the consumption tax increases), from the data average of 2.41%. Our results are hence consistent with the findings in Stokey and Rebelo (1995), who compare the growth effects of the tax structure in different models and conclude that these effects are likely to be around the range suggested by Lucas (1990).

Our welfare evaluations (where welfare is measured as expected lifetime utility of the representative household) follow recent developments in micro-founded DSGE models applied mainly in examining the welfare implications of different (monetary and fiscal) stabilization policies (see e.g. Schmitt-Grohé and Uribe (2004, 2007)). However these papers do not consider the welfare effects of the tax structure and also do not allow for (endogenous) growth. Hence, they do not examine the growth effects of the tax structure. In addition, they focus (as in Lucas (1990)) in discrete tax reforms, i.e. in changes in the tax structure involving setting one tax rate to zero, whereas we focus on changes in the tax rates that are within the historical experience of the UK.

McGrattan (1994) also does not consider human capital and does not allow for endogenous growth. In addition, she focuses on capital and labour taxes and does not consider consumption taxes.
In contrast, if the goal of tax policy is to promote welfare, then policymakers should cut capital taxes, while concurrently increasing labour or consumption taxes to make up for the loss in capital tax revenue. For instance, the welfare gains of substituting the capital tax with the labour tax are about 1% of extra consumption in each time period for a 10% decrease in the capital tax. Again, the UK results are similar in magnitude to those reported for the USA by Lucas (1990). The welfare gains from decreasing the capital tax by 10% and increasing the consumption tax to make up for the loss in tax revenue is about 1.5% of extra consumption in each time period. There are also welfare gains from substituting the labour tax with the consumption tax, of the order of 0.25% for a 10% decrease in the labour tax.

The rest of the paper is organized as follows. Section 2 presents the theoretical model. Section 3 discusses the data, calibration and the long run solution. Section 4 contains the results and Section 5 concludes. Additionally, an Appendix presents information on the second-order welfare function.

2 Theoretical Model

In this section, building on Lucas’ (1990) model, we present and solve a DSGE model in which the engine of endogenous, long-term growth is human capital accumulation. To conduct our policy analysis, in comparison to the Lucas setup, we add: (a) Externalities generated by the average stock of human capital in the society. This can in turn justify public education expenditure. (b) A rather detailed spending-tax mix on the part of the government. Specifically, the government spends on education, infrastructure investment, public consumption and lump-sum transfers. On the other side of the budget, the government imposes taxes on capital income, labour income and private consumption spending. (c) We operate in a stochastic environment which allows us to account for the effects of uncertainty on welfare.

The general equilibrium solution of the model consists of a system of dynamic relations jointly specifying the paths of output, private consump-

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4We choose Lucas’ model because it is well known and its conclusions are rather robust to changes in parameter values (see Stokey and Rebele (1995)). Note that, in our model, long-term growth can also be generated by accumulation in public infrastructure capital. Nevertheless, since human capital is optimally chosen, while government policy is exogenously set, the engine of endogenous growth is essentially human capital accumulation.

5As is typical in the literature, the three types of government expenditure have distinct roles: public investment augments public infrastructure capital that can provide production externalities to firms; public education spending can enhance the productivity of households’ private education choices; public consumption goods and services can provide direct utility to households. See e.g. Turnovsky (1995) for a review book.
tion, private physical capital, the growth rate of human capital, the fractions of time allocated to work, education and leisure, public capital, and one residually determined policy instrument. To obtain these paths, we solve the second-order approximation of our model’s equilibrium conditions around the deterministic steady-state (see also e.g. Schmitt-Grohé and Uribe (2004)). In contrast to solutions which impose certainty equivalence, the solution of the second-order system allows us to take account of the uncertainty agents face when making decisions. More importantly, as pointed out by Woodford (2003), Schmitt-Grohé and Uribe (2004) and many others, the second-order approximation to the model’s policy function helps to avoid potential spurious welfare rankings which may arise under certainty equivalence. In other words, when we evaluate different policies and regimes, we will approximate both the equilibrium solution and welfare (defined as the conditional expectation of lifetime utility) to second-order (see e.g. Schmitt-Grohé and Uribe (2007)). This represents a departure from the earlier literature (see e.g. Lucas (1990) and Cooley and Hansen (1992)).

2.1 Households

The economy is populated by a large number of identical households indexed by the superscript $h$ and identical firms indexed by the superscript $f$, where $h, f = 1, 2, \ldots, N_t$. The population size, $N_t$, evolves at a constant rate $n \geq 1$, so that $N_{t+1} = nN_t$ where $N_0$ is given. Each household’s preferences are given by the following time-separable function:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C^h_t, l^h_t, G^c_t) \tag{1}$$

where $E_0$ is the conditional expectations operator; $C^h_t$ is private consumption of household $h$ at time $t$; $l^h_t$ is leisure of household $h$ at time $t$; $G^c_t$ is average (per household) public consumption goods and services at time $t$, and $0 < \beta < 1$ is the subjective rate of time preference. The instantaneous utility function, $U^h_t = U(C^h_t, l^h_t, G^c_t)$, is increasing in all its arguments, concave and satisfies the Inada conditions. Specifically, we use a Cobb-Douglas form in composite consumption and leisure:

$$U^h_t = \frac{[(C^h_t + \phi G^c_t)^\mu (l^h_t)^{1-\mu}]^{1-\sigma}}{1-\sigma} \tag{2}$$

where, $1/\sigma \ (\sigma > 1)$ is the intertemporal elasticity of substitution between consumption in adjacent periods, $0 < \mu < 1$ is the weight given to composite consumption, $(C^h_t + \phi G^c_t)$, relative to leisure, and $\phi$ is the weight given to
public consumption in composite consumption. The way we model composite consumption is as in e.g. Ashauer (1985) and Christiano and Eichenbaum (1992).

Each household $h$ saves in the form of investment, $I_{p;h}^{p,h}$, and receives interest income, $r_t K_{p;h}^{p,h}$, where $r_t$ is the return to private capital and $K_{p;h}^{p,h}$ is the beginning-of-period private capital stock (the superscript $p$ refers to private, as opposed to public, physical capital). Each household has one unit of time in each period $t$, which is allocated between leisure, $l_{h}^{h}$, work, $u_{h}^{h}$, and education, $e_{h}^{h}$, so that $l_{h}^{h} + u_{h}^{h} + e_{h}^{h} = 1$. A household with a stock of human capital, $H_{h}^{h}$ receives labour income, $w_t u_{h}^{h} H_{h}^{h}$, where $w_t$ is the wage rate and $u_{h}^{h} H_{h}^{h}$ is $h$’s effective labour. Finally, each household receives dividends paid by firms, $\Pi_{h}^{h}$, and an average (per household) lump-sum transfer/tax, $\bar{G}_{t}^{p}$. Accordingly, the budget constraint of each household is:

$$(1 + c_{t}^{c}) C_{h}^{h} + I_{t}^{p,h} = (1 - k_{t}) (r_{t} K_{p;h}^{p,h} + \Pi_{t}^{h}) + (1 - l_{t}) w_{t} u_{h}^{h} H_{h}^{h} + \bar{G}_{t}^{p}$$  \(3\)

where $0 \leq \tau_{c}^{c}, \tau_{t}^{k}, \tau_{l}^{l} < 1$ are respectively the tax rates on consumption, capital income and labour income.

Each household’s physical and human evolve according to:

$$K_{t+1}^{p,h} = (1 - \delta^{p}) K_{t}^{p,h} + I_{t}^{p,h}$$  \(4\)

and

$$H_{t+1}^{h} = (1 - \delta^{h}) H_{t}^{h} + (e_{h}^{h} H_{t}^{h})^{\theta_{1}} (\bar{H}_{t})^{1-\theta_{1}} \bar{B}_{t}$$  \(5\)

where, $0 \leq \delta^{p}, \delta^{h} \leq 1$ are constant depreciation rates on private physical and human capital respectively. The second expression on the r.h.s. of (5), consisting of three multiplicative terms, can be interpreted as the quantity of “new” human capital created at time period $t$. This expression is comprised of the following arguments: (i) $(e_{h}^{h} H_{t}^{h})$ is $h$’s effective human capital; (ii) $\bar{H}_{t}$ is the average (per household) human capital stock in the society; (iii) $\bar{B}_{t} = B (g_{t}^{f})^{\theta_{2}}$ represents human capital productivity, where $B > 0$ is a constant scale parameter and $g_{t}^{f}$ is average (per household) public education expenditure expressed in efficiency units (see below).\(^7\) The parameters $0 < \theta_{1} \leq 1, 0 \leq (1 - \theta_{1}) < 1$ and $0 \leq \theta_{2} < 1$ capture the productivity of private human capital.\(^6\)

\(^6\) The assumption that individual human capital accumulation is an increasing function of the per capita level of economy-wide human capital encapsulates the idea that the existing know-how of the economy provides an external positive effect. Equivalently it can be thought of as a learning-by-doing effect as discussed in Romer (1986). Examples of other papers which use the per capita level of aggregate human capital in either the goods or human capital production functions include Lucas (1988), Azariadis and Drazen, (1990), Tamura (1991) and Glomm and Ravikumar (1992).

\(^7\) The assumption that individual human capital accumulation depends on the per
human capital, aggregate human capital externality and public education spending respectively. 

Households act competitively by taking prices, policy variables and aggregate outcomes as given. Thus, each household $h$ chooses $\{C^h_t, u^h_t, e^h_t, l^h_t, I^h_t, K^p,h_{t+1}, H^h_{t+1}\}$ to maximize (1) subject to (3), (4), (5), the time constraint $l^h_t + u^h_t + e^h_t = 1$, and initial conditions for $K^p,0$ and $H^h_0$.

Substituting (4) into (3) for $I^h_t$ and using the time constraint for $l^h_t$, we derive the first-order conditions. Specifically, the static optimality conditions for consumption, $C^h_t$, work effort, $u^h_t$, and education, $e^h_t$ are (where $\Lambda^h_t$ and $\Psi^h_t$ are multipliers associated with (3) and (5) respectively):

$$\frac{\mu \left[ (C^h_t + \phi G^h_t)^{\mu} (l^h_t)^{1-\mu} \right]^{1-\sigma}}{(C^h_t + \phi G^h_t)} = \Lambda^h_t (1 + \tau^h_t)$$

$$\frac{(1-\mu) \left[ (C^h_t + \phi G^h_t)^{\mu} (l^h_t)^{1-\mu} \right]^{1-\sigma}}{(1 - u^h_t - e^h_t)} = \Lambda^h_t (1 + \tau^h_t) w_t H^h_t$$

and

$$\Psi^h_t = \frac{\Lambda^h_t (1 - \tau^h_t) w_t H^h_t}{B \theta_1 \left( e^h_t \right)^{\theta_1-1} \left( H^h_t \right)^{\theta_1} \left( \bar{H}^h_t \right)^{1-\theta_1} \left( g^e_t \right)^{\theta_2}}. \quad (8)$$

The Euler-equations for private physical capital, $K^p,h_{t+1}$, and human capital, $H^h_{t+1}$, are:

$$\Lambda^h_t = \beta E_t \left\{ \Lambda^h_{t+1} \left[ 1 - \delta^p + (1 - \tau^h_{t+1}) r_{t+1} \right] \right\} \quad (9)$$

and

$$\Psi^h_t = \beta E_t \left\{ \Lambda^h_{t+1} \left[ 1 - \tau^h_{t+1} \right] w_{t+1} H^h_{t+1} \right\} +$$

$$+ \beta E_t \left\{ \Psi^h_{t+1} \left[ 1 - \delta^h + B \theta_1 \left( e^h_{t+1} \right)^{\theta_1} \left( H^h_{t+1} \right)^{\theta_1-1} \left( \bar{H}^h_{t+1} \right)^{1-\theta_1} \left( g^e_{t+1} \right)^{\theta_2} \right] \right\}. \quad (10)$$

household public education share, $g^e_t$, is consistent with the goal of public education policy in practice, as well as with theoretical work (see e.g. Glomm and Ravikumar (1992) and Blankenau (2005)). Blankenau (2005, pp. 493-4) also has a good discussion of the effects of public education on students’ achievement. As he points out, assuming a positive effect is not uncontroversial, this is why public expenditures "are included with a parameter $\theta_2$ to gauge their relative importance in producing human capital". 

The parameter restrictions in (5) imply increasing returns to scale (IRS) at the social level. Lucas (1988) and Benhabib and Perli (1994) are examples of other studies which employ the IRS assumption in either or both the physical and human capital production functions. Finally, note that following e.g. Lucas (1988), we assume that human capital is basically the only private input in human capital accumulation.
2.2 Firms

To produce its homogenous final product, \( Y^f_t \), each firm \( f \) chooses private physical capital, \( K^{p,f}_t \), and effective labour, \( u^f_t H^f_t \), and also takes advantage of public infrastructure. The production function of each firm \( f \) is:

\[
Y^f_t = \tilde{A}_t \left( K^{p,f}_t \right)^{\alpha_1} \left( u^f_t H^f_t \right)^{1-\alpha_1}
\]  

(11)

where \( \tilde{A}_t \equiv A_t \left( k^f_t \right)^{\alpha_2} \) represents total factor productivity, where \( A_t \) is an exogenous stochastic process whose motion is defined below, and \( k^f_t \) is average (per firm) public infrastructure capital expressed in efficiency units (see below). The parameters \( 0 < \alpha_1 < 1, 0 < (1 - \alpha_1) < 1 \) and \( 0 \leq \alpha_2 < 1 \) capture the productivity of private capital, private labour and public infrastructure respectively.\(^9\)

Firms act competitively by taking prices, policy variables and aggregate outcomes as given. Accordingly, subject to (11), each firm \( f \) chooses \( K^{p,f}_t \) and \( u^f_t H^f_t \) to maximize a series of static profit functions:

\[
\Pi^f_t = Y^f_t - r_t K^{p,f}_t - w^f_t u^f_t H^f_t.
\]  

(12)

The resulting familiar first-order conditions are:

\[
\frac{(1 - \alpha_1) Y^f_t}{u^f_t H^f_t} = w_t
\]  

(13)

\[
\frac{\alpha_1 Y^f_t}{K^{p,f}_t} = r_t.
\]  

(14)

2.3 Government budget constraint

Total expenditure on public consumption, \( G^c_t \), public infrastructure investment, \( G^i_t \), public education, \( G^e_t \), and lump-sum transfers/taxes, \( G^o_t \), are financed by total tax revenue from capital income, labour income and consumption spending. Thus,

\[
G^c_t + G^i_t + G^e_t + G^o_t = \tau^k_t \sum_{h=1}^{N_t} \left( r_t K^{p,h}_t + \Pi^h_t \right) + \tau^l_t \sum_{h=1}^{N_t} w^h_t u^h_t H^h_t + \tau^c_t \sum_{h=1}^{N_t} C^h_t
\]  

(15)

where only six of the seven \( (G^c_t, G^i_t, G^e_t, G^o_t, \tau^k_t, \tau^l_t, \tau^c_t) \) policy instruments can be exogenously set with the seventh residually determined (see below).\(^9\)

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\(^9\)The parameter restrictions in (11) imply increasing returns to scale (IRS) at the social level. See e.g. Baxter and King (1993) for the same production function.
use a balanced budget. Ignoring public debt is not critical here since changes in lump-sum taxes/ transfers are equivalent to debt financing (see e.g. Baxter and King (1993)).

Also note that public capital \((K^g_t)\) evolves according to:

\[ K^g_{t+1} = (1 - \delta^g) K^g_t + G^i_t \]  

where, \(0 \leq \delta^g \leq 1\) is a constant depreciation rate on public capital.

### 2.4 Stationary decentralized competitive equilibrium

Given the paths of six of the seven policy instruments and initial conditions for the state variables, \((K^p_0, H_0, K^g_0)\), a decentralized competitive equilibrium (DCE) is defined to be a sequence of allocations \(\{C_t, u_t, e_t, K^p_{t+1}, H_{t+1}, K^g_{t+1}\}_{t=0}^\infty\), prices \(\{r_t, w_t\}_{t=0}^\infty\) and one policy instrument, such that (i) households maximize utility; (ii) firms maximize profits; (iii) all markets clear; and (iv) the government budget constraint is satisfied in each time period. Market clearing values will be denoted without the superscripts \(h, f\).

Since the model allows for long-term growth, we transform variables to make them stationary. We first define per capita quantities for any variable \(X\) as \(\overline{X}_t \equiv X_t / N_t\), where \(X_t \equiv (Y_t, C_t, K^p_t, K^g_t, H_t, G^i_t, G^r_t, G^p_t)\). We next express these quantities as shares of per capita human capital, e.g. \(x_t \equiv \overline{X}_t / \overline{H}_t\), and define the gross human capital growth rate as \(\gamma_t \equiv \overline{H}_{t+1} / \overline{H}_t\).

Using this notation, substituting out prices \(\{r_t, w_t\}_{t=0}^\infty\) and substituting for \(\Lambda_t\) and \(\Lambda_{t+1}\) in (8) and (10) respectively, we obtain the following per capita stationary DCE:

\[ y_t = A_t (k^p_t)^{\alpha_1} (u_t)^{(1-\alpha_1)} (k^g_t)^{\alpha_2} \]  

\[ n \gamma_t k^p_{t+1} - (1 - \delta^p) k^p_t + c_t + g^c_c + g^c_t + g^c_t = y_t \]  

\[ n \gamma_t = 1 - \delta^h + (e_t)^{\theta_1} B (g^c_t)^{\theta_2} \]  

\[ (1 + \tau^c_t) \lambda_t = \mu (e_t + \phi g^c_t)^{\mu(1-\sigma)-1} (1 - u_t - e_t)^{(1-\mu)(1-\sigma)} \]  

\[ n \gamma_t k^g_{t+1} - (1 - \delta^g) k^g_t = g^r_t \]  

\[ \lambda_t = \beta (\gamma_t)^{\nu(1-\sigma)-1} E_t \left\{ \lambda_{t+1} \left[ 1 - \delta^p + \frac{(1 - \tau^g_{t+1})^{\alpha_1} y_{t+1}}{k^p_{t+1}} \right] \right\} \]  

\[ (1-\mu) u_t (e_t + \phi g^c_t)^{\mu(1-\sigma)} (1-u_t-e_t)^{(1-\mu)(1-\sigma)-1} = \lambda_t (1-\alpha_1) (1 - \tau^c_t) y_t \]  

\[ \psi_t = \frac{\mu (e_t + \phi g^c_t)^{\mu(1-\sigma)-1} (1 - u_t - e_t)^{(1-\mu)(1-\sigma)} (1 - \alpha_1) (1 - \tau^r_t) y_t}{(1 + \tau^c_t) u_t \theta_1 (e_t)^{\theta_1-1} B (g^c_t)^{\theta_2}} \]
\[ \psi_t = \beta (\gamma_t)^{\mu(1-\sigma)-1} E_t \left\{ \lambda_{t+1} (1 - \alpha_1) (1 - \tau^{l+1}_t) y_{t+1} \right\} + \beta (\gamma_t)^{\mu(1-\sigma)-1} E_t \left\{ \psi_{t+1} \left[ 1 - \delta^h + \theta_1 (e_{t+1})^{\theta_1} B \left( g_{t+1}^e \right)^{\theta_2} \right] \right\} \]
\[ g^c_t + g^l_t + g^e_t + g^o_t = \tau^k_t \alpha_1 y_t + \tau^l_t (1 - \alpha_1) y_t + \tau^e_t e_t \]

where \( \lambda_t \) and \( \psi_t \) are the transformed shadow prices associated with (3) and (5) respectively in the household’s problem.\(^{10}\)

Therefore, the stationary DCE is summarized by the above system of ten equations in the paths of \( \gamma_t, y_t, c_t, u_t, e_t, k^p_{t+1}, k^g_{t+1}, \lambda_t, \psi_t, \) and one residually determined policy instrument. This equilibrium is given the assumed policy regime and the paths of exogenous stochastic variables which are defined below.

\section{2.5 Alternative tax structures}

Regarding policy instruments, we first express each government spending item, which has already been written as share of \( H_t \), as a share of output. That is, we define:
\[ g^j_t \equiv \tilde{g}^j_t y_t \]

where \([j = c, i, e, o]\) and \( \tilde{g}^j_t \equiv G^j_t / Y_t \).

We thus have seven stationary policy instruments, \((g^c_t, g^i_t, g^e_t, g^o_t, \tau^k_t, \tau^l_t, \tau^e_t)\), out of which only six can be set exogenously. Given that we wish to examine the implications of changes in the composition of taxes, we first examine the case in which changes in each of the distorting tax rates \((\tau^c, \tau^l, \tau^k)\), in turn, is met by changes in lump-sum transfers/taxes, \( \tilde{g}^o \), holding the spending instrument rates at their data averages. This experiment helps us identify the general equilibrium effects of each distorting tax policy instrument relative to a non-distorting base. We next examine the case in which changes in each of the three distorting tax rates, in turn, is met by opposite changes in each of the remaining distorting tax rates. As in the first experiment, the spending instrument rates are held at their data averages. Further details are provided at the beginning of Section 4 below.

\section{2.6 Process for technology}

We next specify the evolution of exogenous stochastic variables. Given the above assumptions, only total factor productivity, \( A_t \), is stochastic among

\(^{10}\)Note that \( \lambda_t = \Lambda_t / H_t^{\mu(1-\sigma)-1} \) and \( \psi_t = \Psi_t / H_t^{\mu(1-\sigma)-1} \) where \( h \)-superscripts are omitted in a symmetric equilibrium.
the exogenous variables. Following usual practice in the RBC literature, we assume that $A_t$ follows an AR(1) process:

$$A_t = A\left(1-\rho^a\right)A_{t-1}^{\rho^a}e^{\varepsilon_t^a}$$  \hspace{1cm} (19)

where $A > 0$ is a constant, $0 < \rho^a < 1$ is the autoregressive parameter and $e^{\varepsilon_t^a} \sim iid(0, \sigma_a^2)$ are the random shocks to productivity.

3 Data, Calibration and Steady-State

3.1 Data

The model’s structural parameters relating to preferences, production and physical and human capital accumulation are next calibrated using annual data for the United Kingdom from 1970-2005. The data are obtained from the OECD, IMF, ECFIN and the Office for National Statistics (ONS). The OECD databases include: (i) Main Economic Indicators (MEI); (ii) Economic Outlook (EO); (iii) International Sectoral Database (ISDB); and (iv) OECD Statistics. The IMF data is from the International Financial Statistics (IFS) database. Effective tax rates are obtained from the ECFIN Effective Average Tax Base (see Martinez-Mongay, 2000). Public spending on education data are obtained from ONS. As our aim is to use the model to evaluate long-run growth and welfare around the steady-state, it is important that the calibrated parameters imply a sensible long-run solution. This provides the criterion for choosing those parameters we cannot retrieve from the data or previous empirical studies, especially the exponents in the production function for human capital.

3.2 Calibration

The numeric values for the model’s parameters are reported in Table 1. To calibrate the model, we work as follows. We set the value of $(1 - \alpha_1)$ equal to labour’s share in income (i.e. 0.601) using the ISDB dataset. Given labour’s share, capital’s share, $\alpha_1$, is then determined residually. Following e.g. Baxter and King (1993), we set $a_2$ equal to the public investment share in GDP (i.e. 0.011), as obtained from the EO database. The population gross growth rate $n$ is calculated using IFS data to be 1.003.

The discount rate, $1/\beta$ is equal to 1 plus the \textit{ex-post} real interest rate, where we use the ex-post real interest rate from MEI. This implies a value 0.976 for $\beta$. Following Kydland (1995, ch. 5, p. 134), we set $\mu$, the weight given to composite consumption relative to leisure in the utility function,
equal to the average value of work versus leisure time, which is obtained using data on hours worked from the EO database.\textsuperscript{11} Given the lack of relevant data, we follow the study by Baier and Glomm (2001), and set the relative weight of public consumption services in composite consumption at $\phi = 0.1$. We also use a value for the intertemporal elasticity of consumption $(1/\sigma)$ that is common in the DSGE literature (i.e. $\sigma = 2$).

### Table 1: Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A &gt; 0$</td>
<td>0.234</td>
<td>technological progress in goods production</td>
</tr>
<tr>
<td>$B &gt; 0$</td>
<td>0.204</td>
<td>technological progress in human capital production</td>
</tr>
<tr>
<td>$0 &lt; \alpha_1 &lt; 1$</td>
<td>0.399</td>
<td>productivity of private capital</td>
</tr>
<tr>
<td>$0 &lt; 1 - \alpha_1 &lt; 1$</td>
<td>0.601</td>
<td>productivity of effective labour</td>
</tr>
<tr>
<td>$0 &lt; \alpha_2 &lt; 1$</td>
<td>0.011</td>
<td>productivity of public capital</td>
</tr>
<tr>
<td>$0 &lt; \beta &lt; 1$</td>
<td>0.976</td>
<td>rate of time preference</td>
</tr>
<tr>
<td>$0 \leq \phi \leq 1$</td>
<td>0.100</td>
<td>public consumption weight in composite consumption</td>
</tr>
<tr>
<td>$0 &lt; \mu &lt; 1$</td>
<td>0.347</td>
<td>composite consumption weight in utility</td>
</tr>
<tr>
<td>$\sigma &gt; 1$</td>
<td>2.000</td>
<td>$1/\sigma$ is the intertemporal elasticity of consumption</td>
</tr>
<tr>
<td>$n \geq 1$</td>
<td>1.003</td>
<td>population growth</td>
</tr>
<tr>
<td>$0 \leq \delta^p \leq 1$</td>
<td>0.050</td>
<td>depreciation rate on private capital</td>
</tr>
<tr>
<td>$0 \leq \delta^g \leq 1$</td>
<td>0.050</td>
<td>depreciation rate on private capital</td>
</tr>
<tr>
<td>$0 \leq \delta^h \leq 1$</td>
<td>0.018</td>
<td>depreciation rate on public capital</td>
</tr>
<tr>
<td>$0 &lt; \tilde{g}^e &lt; 1$</td>
<td>0.049</td>
<td>public education spending share of output</td>
</tr>
<tr>
<td>$0 &lt; \tilde{g}^i &lt; 1$</td>
<td>0.011</td>
<td>public investment spending share of output</td>
</tr>
<tr>
<td>$0 &lt; \tilde{g}^c &lt; 1$</td>
<td>0.223</td>
<td>public consumption spending share of output</td>
</tr>
<tr>
<td>$0 &lt; \tilde{g}^o &lt; 1$</td>
<td>0.169</td>
<td>other public spending share of output</td>
</tr>
<tr>
<td>$0 &lt; \tau^l &lt; 1$</td>
<td>0.265</td>
<td>labour tax rate</td>
</tr>
<tr>
<td>$0 &lt; \tau^k &lt; 1$</td>
<td>0.470</td>
<td>capital tax rate</td>
</tr>
<tr>
<td>$0 &lt; \tau^c &lt; 1$</td>
<td>0.185</td>
<td>consumption tax rate</td>
</tr>
<tr>
<td>$0 \leq \theta_1 \leq 1$</td>
<td>0.510</td>
<td>productivity of household human capital</td>
</tr>
<tr>
<td>$0 \leq 1 - \theta_1 \leq 1$</td>
<td>0.490</td>
<td>productivity of aggregate human capital</td>
</tr>
<tr>
<td>$0 \leq \theta_2 \leq 1$</td>
<td>0.050</td>
<td>productivity of public education spending</td>
</tr>
<tr>
<td>$0 &lt; \rho^e &lt; 1$</td>
<td>0.920</td>
<td>AR(1) parameter technology</td>
</tr>
<tr>
<td>$\sigma_a &gt; 0$</td>
<td>0.030</td>
<td>std. dev. of technology innovations</td>
</tr>
</tbody>
</table>

Regarding the fiscal policy parameters, we employ data averages. For example, we use data from ONS to obtain the mean of public education spending share of output.

\textsuperscript{11}To obtain this we divide total hours worked by total hours available for work or leisure, following e.g. Ho and Jorgenson (2001). For example, they assume that there are 14 hours available for work or leisure on a daily basis with the remaining 10 hours accounted for by physiological needs.
spending as a share of GDP, which gives $g^e = 0.049$. EO data give $g^i = 0.011$ for the government investment spending share and $g^c = 0.272$ for the government consumption spending share. To avoid double counting in the government budget constraint, we have to adjust this ratio since government education spending is recorded as government consumption spending in the national accounts. Thus we set $g^c = 0.272 - 0.049 = 0.223$. We also use the effective average tax rates from the ECFIN paper by Martinez-Mongay (2000) for 1970-2000, to obtain the average tax rates as $\tau^k = 0.47$, $\tau^l = 0.265$ and $\tau^c = 0.185$. It is interesting to point out that, in the UK, labour tax revenue is lower than in most of EU countries, while revenue from taxes on capital is higher than the EU-27 average and consumption taxes are below the EU-27 average. The budget constraint then implies $g^o = 0.169$. It is important to point out that, given the above data averages, all tax regimes defined above imply the same long-run solution.

To accurately gauge the persistence of TFP shocks, we estimate the AR(1) relation given by (19) using TFP data from the ONS. The estimated values for $\rho$ is 0.92 and is significant at less than the 1% level of significance. The standard deviation of this process, $\sigma$, is 0.03.

A monetary valued measure for human capital is needed to calibrate the scale parameters $A$ and $B$. In particular, model consistent values for $A$ and $B$ can be obtained by solving equations (5) and (11) using data averages, long-run values and the other calibrated parameters. As we require data for the endogenous variables as shares of human capital, it is important to obtain a monetary-valued measure of human capital that is comparable to monetary valued quantities such as consumption, income, physical capital and government spending. To our knowledge, such a measure does not exist for the UK. In our related work for the USA (see Angelopoulos et al. (2007b)), we used data on human and physical capital from Jorgenson and Fraumeni (1989, 1992a,b). These measures are reported in billions of constant 1982

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12 These are the effective average tax rates KITN, LITR and CITR respectively in Martinez-Mongay (op cit.).
13 See e.g. the Eurostat statistical book, edition 2007, on "Taxation trends in the European Union" for a rich presentation of taxation in all 27 member countries of the EU.
14 For this exercise, we obtain a model consistent $y$. In particular, $y$ is found from equation (17b), using EO and ONS data. As a dataset for the share of time individuals spend on education as opposed to work or leisure is not available, we acquire a proxy for $e$ to calibrate $B$. This is achieved by assuming that agents spend on average 16 years on education over the 62 minus 6 years available on average for work or education, so that agents allocate 28.6% of their non-leisure time to education as opposed to work. Non-leisure time is found using data on hours worked, obtained from EO.
15 Generally, empirical studies use measures of school enrolment ratios or years of schooling as general proxies of labor quality or human capital. However, in our setup, these
dollars for 1949-1984. The basic idea used in the construction of this dataset is that the output of the education sector is considered as investment in human capital.\footnote{\textsuperscript{16}In this context, Jorgenson and Fraumeni (1992a) note: “investment in human beings, like investment in tangible form of capital such as buildings and industrial equipment, generates a stream of future benefits. Education is regarded as an investment in human capital, since benefits accrue to an educated individual over a lifetime of activities”. Jorgenson and Fraumeni (1989) also note that “in order to construct comparable measures of investment in human and nonhuman capital, we define human capital in terms of lifetime labor incomes for all individuals in the US population. Lifetime labor incomes correspond to asset values for investment goods used in accounting for physical or nonhuman capital”.}

Unfortunately, such data are not available for the UK. We use, however, the Jorgenson and Fraumeni (1989, 1992a,b) dataset, by setting the depreciation rate for human capital to the value they calculate, so that $\delta^h = 0.0178$. We also assume that the depreciation rate for physical capital is at 5%, the level calculated by Jorgenson and Fraumeni, and also set the same depreciation rate for public capital. Note that the depreciation rates matter for the long-run value of the investment share in GDP, but have little effect on near steady-state dynamics in this class of model (see e.g. King and Rebelo, 1999, p. 954).

To construct a measure of human capital, we use education spending data from OECD Statistics, following Harbenger’s (1978) approach\footnote{Conolly (2004) provides an example of the Harbenger method which is essentially a variant of the perpetual inventory method. For example, to initialize the capital series, education spending in the first period is divided by the net growth rate of education spending and by the assumed depreciation rate of human capital. For the other periods, human capital is equal to the un-depreciated human capital of the previous period plus new spending on education.\footnote{We used Harbenger’s method on NIPA accounts data on education spending in USA and compared the resulting human capital series to the human capital measure given by Jorgenson and Fraumeni. We found that Harbenger’s method underestimates the average value of human capital, so we used the degree of proportionality between the two series in USA to re-scale Harbenger’s human capital series for the UK.}}. This data is then re-scaled by a factor that we find in the U.S. data\footnote{We used Harbenger’s method on NIPA accounts data on education spending in USA and compared the resulting human capital series to the human capital measure given by Jorgenson and Fraumeni. We found that Harbenger’s method underestimates the average value of human capital, so we used the degree of proportionality between the two series in USA to re-scale Harbenger’s human capital series for the UK.}. We then use this measure to construct the data averages needed for the calibration of $A$ and $B$.

Given the functions for the calibration of $A$ and $B$, we calibrate $\theta_1$ and $\theta_2$ so that we obtain an economically meaningful and data-consistent long-run solution. In particular, to ensure that the balanced growth rate is equal to the data average of 1.024, we set a value of $(1 - \theta_1) = 0.49$ for the human capital externality (which implies that the productivity of individual human capital is $\theta_1 = 0.51$), and a value of $\theta_2 = 0.05$ for the productivity of public proxies are measures of the input to the production function of human capital (time spent on education) and not of the output of this activity, new human capital.
education spending. The value of the gross growth rate of 1.024 is the growth rate of labour productivity for the UK, calculated using ISDB data.

Regarding the calibrated values of $\theta_1$ and $\theta_2$, it is important to report the following. For higher externalities, the growth rate becomes too low. This happens because, with very high externalities, there are free riding problems in the creation of human capital. On the other hand, for low externalities, the implied share of time allocated to education ($e$) in the long-run increases and also the growth rate increases to unrealistic values. The same movements are observed with changes of $\theta_2$, although to a lesser extent. By contrast, our calibrated values $(1 - \theta_1) = 0.49$ and $\theta_2 = 0.05$ guarantee a growth rate consistent with the data average growth of labour augmenting technology of 2.4%. Finally, note that the calibrated value for $\theta_2$ is also approximately equal to the government education spending share in GDP.

### 3.3 Steady-state solution

The steady-state solution implied by this calibration is reported in Table 2. As can be seen, the long-run solution for all variables is close to the data average, or, in the case of the capital shares in GDP, the implied long-run value using data on private and public investment.

Regarding the allocation of time, there are no data that decompose total time in work, education and leisure. However, survey data on hours worked from EO, indicate that agents who work, allocate about 65.3% of their time to leisure. Hence, the model’s long-run solution is compatible with the assumption that agents who educate themselves make the same effort/leisure choice, so that total leisure time in the labour force is 67.5%. In addition, the model solution indicates that the breakdown of non-leisure time to work and education effort is roughly two thirds to work and one third to education. This is again consistent with the average percentage of the years agents allocate to education over the total number of years available for work and
education (i.e. $16/(62 - 6)$).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Data Average</th>
<th>Model Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>1.024</td>
<td>1.024</td>
</tr>
<tr>
<td>$l$</td>
<td>N.A.</td>
<td>0.675</td>
</tr>
<tr>
<td>$e$</td>
<td>N.A.</td>
<td>0.098</td>
</tr>
<tr>
<td>$u$</td>
<td>N.A.</td>
<td>0.227</td>
</tr>
<tr>
<td>$c/y$</td>
<td>0.606</td>
<td>0.567</td>
</tr>
<tr>
<td>$i/y$</td>
<td>0.110</td>
<td>0.150</td>
</tr>
<tr>
<td>$k^p/y$</td>
<td>1.433</td>
<td>1.948</td>
</tr>
<tr>
<td>$k^g/y$</td>
<td>0.142</td>
<td>0.142</td>
</tr>
</tbody>
</table>

To sum up, this model economy for the UK is consistent with externalities in private human accumulation and productive public education expenditure. Lucas (1988) supports a value of human capital externality of 0.4, but (since his externality is modeled as a direct argument in the goods production function) its effect on output produced is much higher relative to our calibrated externality. The associated value of the productivity of public education expenditure, $a_2 = 0.05$, and the value of the productivity of public capital, $a_2 = 0.011$, are also within the range assumed in the related literature (see e.g. Blankenau (2005) p. 501).

## 4 Results: Growth and Welfare

Using the solution of the second-order approximation of the model around its deterministic steady-state, we now examine the effects on the long-run growth rate and expected lifetime utility from changes in the composition of taxes focusing on the policy scenarios introduced in subsection 2.5.\(^{19}\)

After defining long-run growth and welfare in subsection 4.1, subsection 4.2 studies the effects of increases in one of the distorting tax rates ($\tau^c, \tau^l, \tau^k$), which is met by a decrease in the share of lump-sum taxes (or equivalently an increase in lump-sum taxes/transfers), $\tilde{g}^o$. Note that in each experiment, we only change one of the distorting tax rates at a time, allowing $\tilde{g}^o$ to adjust to keep the government budget constraint satisfied.

Subsection 4.3 then analyses the effects of replacing one distorting tax rate with another. In particular, we study increases in $\tau^c$ accompanied by decreases in $\tau^l$; increases in $\tau^k$ accommodated by decreases in $\tau^c$; and increases in $\tau^l$ met by decreases in $\tau^k$. Again, we exogenously alter only one

\(^{19}\)We use the Matlab functions made available by Schmitt-Grohé and Uribe (2004), to solve and simulate the second-order approximation of the model.
rate at a time. Finally note that in all policy experiments, we keep the GDP share of government spending fixed at its data average.\footnote{Here we are referring to the spending activities of the government that make direct use of real resources and thus enter the economy’s resource identity in equation (17b) above (this is what Tanzi and Schuknecht (2000) also call "real government spending"; see also Atkinson and Stiglitz (1980, p. 16).}

### 4.1 Measures of long-run growth and expected lifetime utility

Long-run growth is simply measured by the balanced growth rate, $\gamma$, which is the common constant rate at which all quantities grow in the long-run of our economy. Welfare is defined as the conditional expectation of the discounted sum of lifetime utility (see eq. (1)). To this end, we first undertake a second-order approximation of the within-period utility function (see eq. (2)) around the non-stochastic steady-state (see subsection 3.3) and then take the discounted "infinite" sum of approximate within-period utility functions (see Appendices 6.2-6.4). We undertake this for varying levels of the tax rates using the solution(s) of the second-order approximation to the stationary equilibrium as given by equations (17a – 19). Note that in comparison to the related literature (see e.g. Schmitt-Grohé and Uribe (2007)), we work with an endogenous growth model.

### 4.2 Replacing a distorting tax rate with lump-sum taxes

We start with a comparison of distorting tax rates ($\tau^c, \tau^l, \tau^k$), when the lump sum instrument, $\tilde{g}^o$, adjusts to satisfy the government budget constraint.

#### 4.2.1 Growth effects

First, consider the effects of different tax rates on long-run growth. In Figure 1, subplot (1,1), where (1,1) refers to row and column numbers respectively, we derive the consumption tax-growth rate relationship. Subplots (1,2) and (1,3) derive the labour tax-growth rate and capital tax-growth rate relationship respectively.

[Figure 1 about here]

As can be seen, increases in any of the distorting tax rates, holding government spending other than transfers and other distorting tax rates constant, decreases the growth rate. This is as expected given the distortion of individual choices. It is worth noting that increases in the labour tax rate hurt
growth more than increases in the other two tax rates. To better understand these effects, Figure 2 plots the effects of the three distorting tax rates on the main endogenous variables, education time, $e$, work time, $u$, private consumption, $c$, output, $y$ and investment, $i$. There are three lines in each sub-plot in Figure 2. The continuous lines with stars represent the effects of changes in the consumption tax, $\tau^c$; the dashed lines the effects of changes in the capital tax, $\tau^k$; and finally the continuous lines the effects of changes in the labour tax, $\tau^l$.

In general, Figure 2 suggests that increases in any of the three distorting tax rates have adverse effects on all endogenous variables. Reference to output, consumption and investment in Figure 2 shows that the tax rate on capital has the largest adverse effects followed by the labour and consumption tax rates respectively. On the other hand, when examining labour market variables, the negative effects of the labour tax rate are higher than the capital tax rate. This follows since increases in the labour tax create important and direct disincentives for both work and education effort. In particular, the labour tax reduces the return to labour and hence work effort. Moreover, expected future returns to human capital also fall, thus, leading to a fall in human capital investment and education effort. This latter result explains why the net growth effect of labour versus capital taxes is larger, as human capital growth is mainly determined by private education effort. Also, since consumption taxes reduce education effort by more than capital taxes, the growth rate is hurt more by consumption than by capital taxes.

To summarize, if the goal of tax policy is to minimize the negative effects of taxation on long-run growth, and if it is possible for the government to have a lump-sum tax policy instrument at its disposal, the above results suggest that higher capital taxes rates are the least harmful, followed by consumption and labour taxes respectively. In other words, the largest marginal contribution to increasing growth would come from cutting labour taxes.

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21 Obviously, there are also income effects in the labour markets, due to the changes in tax rates. As all tax rates decrease income, agents want to work more to make up for the loss in income. However, for all three tax rates, the substitution effects from lower returns to work and education time dominate and hence work and education time are reduced in all cases.

22 The reason that consumption taxes decrease work and education effort time is that as consumption becomes more expensive, agents tend to substitute consumption for leisure. The reason that capital taxes decrease work and education effort is that these taxes reduce the return to the labour input (in addition to decreasing the return to the capital input) in the production function, as human and physical capital are complements in the production process.
4.2.2 Welfare effects

We next consider welfare as defined above. The welfare curves for changes in \( \tau^c \), \( \tau^l \) and \( \tau^k \) respectively, being met by changes in \( \tilde{g}^o \) are shown in subplots (2,1), (2,2) and (2,3) in Figure 1. Again, increases in any of the distorting tax rates results in decreases in welfare. In our model, welfare depends on the time-paths of the equilibrium solutions for composite consumption, growth and leisure time (see the calculations in the Appendix).23 As we see in Figures 1 and 2, higher tax rates imply less consumption, less growth and more leisure time (as both work and education time fall with higher tax rates). The first two effects dominate so that welfare falls as distorting tax rates rise.

It is interesting to note that welfare falls more under higher capital tax rates, compared to higher labour and consumption tax rates. This is despite the fact that the fall in the growth rate was larger under higher labour and consumption tax rates (see above subsection). The welfare result pertaining to capital taxes is due to the large negative effect that these taxes have on consumption, and also on the fact that leisure is not increased as much by higher capital taxes as by higher labour and consumption taxes (see Figure 2). Comparing labour and consumption taxes, we see that the adverse effect of the former is larger although any differences are small. Again, labour taxes have larger negative effects on consumption, but since they also have larger positive effects on leisure, the quantitative welfare differences between these two tax rates are small.

To formally compare the quantitative welfare effects of different distorting tax rates, we follow e.g. Lucas (1990), Cooley and Hansen (1992) and Schmitt-Grohé and Uribe (2007), and compute the welfare gains, or losses, associated with alternative tax mixes by computing the percentage change in private consumption that the individual would require so as to be equally well off between two policy regimes. This is defined as ”\( \xi \)” (see Appendix 6.5 for the derivation of \( \xi \) in our model). Subplots (3,1), (3,2) and (3,3) show, respectively, the welfare gain/loss from consumption over labour taxes, consumption over capital taxes and labour over capital taxes - for a range of values of \( \tilde{g}^o \).

In accordance with our previous discussion, we see first that there are welfare gains from higher consumption tax rates as opposed to either higher labour or capital tax rates, as well as from higher labour as opposed to higher capital tax rates.

23Consumption is made up of private and public consumption. However, as we keep public consumption constant across our experiments, this does not affect the differences in welfare caused by different tax rates, which is our aim in this study.
capital tax rates\textsuperscript{24}. On the contrary, and for symmetrically opposite reasons to those explained above, there are welfare losses from decreasing consumption as opposed to decreasing either labour or capital tax rates, and from decreasing labour as opposed to decreasing capital tax rates. The second observation is that the welfare effects seem to be large in magnitude, especially when the comparison is with respect to capital taxes. The additional gains from higher consumption or labour tax rates, instead of capital tax rates, amount to 2\% of consumption for each additional percentage point of $\bar{g}^o$.

To summarize, if the goal of tax policy is to minimize the detrimental effects of taxation on expected lifetime utility, the above results indicate that higher consumption tax rates are least harmful followed by labour and capital taxes respectively. In other words, the largest marginal contribution to welfare would come from cutting capital taxes. This is also checked below.

4.3 Replacing one distorting tax rate with another one

In this subsection, we assume away the possibility of changes in lump-sum taxes/transfers, and instead evaluate the growth and welfare effects of replacing one distorting tax rate with another. The effects on long-run growth and welfare are presented in Figure 3, while Figures 4 and 5 present effects on other endogenous variables.

4.3.1 Growth effects

The effects on the growth rate from increases in the consumption tax rate, when the labour tax rate is reduced, are shown in Figure 3, subplot (1,1). Subplots (1,2) and (1,3) show respectively the growth effects from increases in the capital tax rate when the consumption tax rate is reduced, and from increases in the labour tax rate when the capital tax rate is reduced. In all experiments, all the other policy rate components in the government budget constraint remain unchanged.

Note first that increases in labour taxes \textit{vis-a-vis} both capital and consumption taxes hurt the growth rate (see subplots (1,3) and (1,2) respectively). In addition, capital taxes hurt the growth rate more than consumption taxes, but the effects in this case are trivial (see subplot (1,2)).

\[\text{[Figure 3 about here]}\]

In Figure 4, we present the effects on other endogenous variables from changes in the two income tax rates ($\tau^l$ and $\tau^k$), when accommodated by

\textsuperscript{24}Note that the data average of the residually determined $\bar{g}^o$ share is 16.9\%.
changes in the consumption tax rate, $\tau^c$. There are two lines in each sub-plot in Figure 4. The continuous lines present the effects of changes in the labour tax, $\tau^l$, met by changes in $\tau^c$, while the lines with stars present the effects of changes in the capital tax, $\tau^k$, met by changes in $\tau^c$. This figure helps us to better understand the effects of income (labour and capital) tax rates vis-a-vis the consumption tax rate. As can be seen, substituting a reduced income tax rate for a higher consumption tax rate has positive effects on output, consumption and investment, which are larger when it is the capital income tax rate that is reduced. Regarding the labour market, lower labour income tax rates result in increases in both work and education effort time as the returns to work and education get higher, while lower capital income tax rates result in a decrease in work and education effort time. In this case, it is the income effects that dominate (recall that income is increased when the capital tax rate is decreased) so that agents can afford to enjoy more leisure time. These different effects on education time explain why growth is reduced when labour taxes rise relative to consumption taxes, and why growth is increased when capital taxes rise relative to consumption taxes.

In Figure 5, we present the effects on endogenous variables from switching from labour tax rates, $\tau^l$, to capital tax rates, $\tau^k$. As can be seen, increases in $\tau^l$, met by decreases in $\tau^k$, boost output, consumption and investment. On the other hand, work and education effort time are reduced when $\tau^l$ rises. In this case, both the substitution (higher labour tax) and income effects work in the same direction in the labour market. Hence, the fall in the growth rate when $\tau^l$ rises and $\tau^k$ falls.

To summarize, if the goal of tax policy is to promote long-run growth by altering the distorting tax rates, our results suggest that labour taxes should be reduced, while simultaneously increasing capital or consumption taxes to make up for the loss in labour tax revenue. However, the gains in terms of growth from such changes in the tax structure are found to be very small, in accordance with the results in Lucas (1990) and Stokey and Rebelo (1995). In particular, if the tax rate on labour was reduced to 20% (starting from the data average of 26.5%), the net growth rate would increase to 2.46% (when the capital tax increases) or to about 2.44% (when the consumption tax increases), from the data average of 2.41%.

### 4.3.2 Welfare effects

The welfare curves are shown below their respective growth curves in Figure 3. As can be inferred from subplots (2,1) and (2,2), higher consumption tax
rates, met by reduced tax rates on labour or capital are welfare improving. Compared to labour taxes, higher consumption taxes imply higher consumption (see Figure 4) and growth. They also imply less leisure, but the first two effects dominate so that welfare rises as the ratio of consumption to labour tax rates goes up. Compared to capital taxes, higher consumption taxes again imply higher consumption and leisure. They also imply less growth but, since this adverse effect is very small, the first two effects dominate and welfare rises substantially as the ratio of consumption to capital taxes rises. Finally, in subplot (2,3), a higher labour to capital tax rate ratio, although bad for growth, is good for welfare. As can be seen in Figure 5, the reason is that as labour tax rates rise and capital tax rates fall, both consumption and leisure rise and this increases welfare.

Finally, we turn to a quantitative assessment of how much these tax switches can benefit the economy. We calculate the welfare gains/losses of increasing the ratio of consumption to labour, capital to consumption, and labour to capital tax rates. The results are presented in subplots (3,1) to (3,3) respectively in Figure 3. In each case, the gains/losses are calculated with respect to the mean value of the respective tax ratios in the data. In each of the subplots, then, point zero on the vertical axis represents the current combination of tax rates in the UK.

The results suggest important welfare gains from re-allocating the tax burden away from capital to labour income and even bigger ones by shifting it to consumption spending. For instance, if the capital tax is decreased from its data average of 47% to 43%, welfare gains of about 1% of consumption can be obtained permanently if the loss in tax revenue is met by an increase in the labour tax. Whereas welfare gains of about 1.5% of consumption can be obtained permanently if the loss in tax revenue is by an increase in the consumption tax. Smaller welfare benefits are realized from substituting the labour tax with a consumption tax. For instance, welfare gains of about 0.2% of consumption can be made in this case if the labour tax is decreased from its data average of 26.5% to 23%. Note, these examples involve small changes in the tax rates (of about 10% change in the tax rates) that are within the historical (recent) experience for the UK. Quantitatively, the welfare effects reported here are similar to the effects reported in Lucas (1990), Cooley and Hansen (1992) and McGrattan (1994).

To summarize, if the goal of tax policy is to promote welfare by altering the distorting tax rates, our results suggest that capital taxes should be re-

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25We report that the KITN effective average tax rate from ECFIN is between 40.5 and 52.2 for the period 1980-2000. LITR is between 24.5 and 28.5 for the same period, while CITR is between 17.3 and 20.5.
duced, while simultaneously increasing labour or consumption taxes to make up for the loss in capital tax revenue. Our findings further suggest that it is also welfare improving, but less so, to reduce labour taxes, while increasing consumption taxes.

5 Conclusions

In this paper, we studied the quantitative implications of changes in the composition of taxes for long-run growth and expected lifetime utility in the UK economy from 1970-2005. We employed a DSGE setup incorporating a detailed fiscal policy structure and where the engine of endogenous growth was human capital accumulation. The model was based on Lucas’s (1990) model, which we extended by allowing for: (i) a more realistic disaggregation of government spending into its basic growth and utility promoting activities; (ii) a consumption tax; (iii) externalities from economy-wide human capital; and (iv) uncertainty in the model economy.

Our results suggest that the growth effects of tax reforms are likely to be small, whereas the welfare effects can be substantial. In particular, our results suggest that if the goal of tax policy is to promote long-run growth by altering relative tax rates, then it should reduce labour taxes while simultaneously increasing capital or consumption taxes to make up for the loss in labour tax revenue. In contrast, welfare promoting policy would be to cut capital taxes while concurrently increasing labour or consumption taxes to make up for the loss in capital tax revenue. The findings in this paper are similar to those obtained in similar studies for the U.S. and they also appear consistent with the important normative result that capital should not be taxed in the medium and long run (see e.g. Chamley (1986) and Lucas (1990)).

References


6 Appendix

6.1 Instantaneous utility

Using the notation set out in the paper, first consider the per capita representation of the instantaneous utility function given by (2):

$$ U_t = \frac{[(X_t)^\mu (l_t)^{1-\mu}]^{1-\sigma}}{1-\sigma} \quad (A.1) $$

where $X_t \equiv \bar{C}_t + \phi G^c_t$ denotes per capita composite consumption. Using our notation for stationary variables:

$$ \bar{U}_t = \frac{[(x_t \bar{H}_t)^\mu (l_t)^{1-\mu}]^{1-\sigma}}{1-\sigma} \quad (A.2) $$
where \( x_t \equiv \left( \frac{X_t}{H_t} \right) \), is stationary composite consumption and \( H_t \) is the beginning-of-period human capital stock. Since \( \gamma_t \equiv \Pi_{t+1}/\Pi_t \), we have for \( t \geq 1 \):

\[
\Pi_t = \Pi_0 \left( \prod_{s=0}^{t-1} \gamma_s \right) \quad (A.3)
\]

where \( \Pi_0 \) is given from initial conditions.

Substituting (A3) into (A2) gives

\[
\bar{U}_t = \left\{ \left( H_0 x_t \left( \prod_{s=0}^{t-1} \gamma_s \right) \right) \right\}^{\mu} \frac{(l_t)^{1-\mu}}{1-\sigma} \quad \text{for } t \geq 1 \quad (A.4a)
\]

\[
\bar{U}_0 = \left[ \left( H_0 x_0 \right)^{\mu} (l_0)^{1-\mu} \right]^{1-\sigma} \quad \text{for } t = 0. \quad (A.4b)
\]

### 6.2 Steady-state utility

We next define the long-run as the state without stochastic shocks which implies that stationary variables are constant. Using (A4a, b), utility at the steady-state can be written as

\[
\bar{U}_t^* = \left[ \left( H_0 x_t \gamma_t \right)^{\mu} (l_t)^{1-\mu} \right]^{1-\sigma} \quad (A.5)
\]

where the * superscript denotes steady-state per capita utility. In the steady-state, non-stationary \( X_t \) grows at the constant rate \( \gamma \), which in turn implies for \( \sigma, \gamma > 1 \) that the growth of \( \bar{U}_t^* \) is constant and less than unity.

### 6.3 Second-order approximation of within period utility

Define for simplicity a variable \( z_t \equiv x_t \left( \prod_{s=0}^{t-1} \gamma_s \right) \), so that the second-order approximation of the within-period utility function in (A4) around its long-run is:

\[
\bar{U}_t^* \approx \bar{U}_t + [U_t z_t] \hat{z}_t + [U_t l_t] \hat{l}_t + \frac{1}{2} [U_t z_t + U_{zz} z_t^2] \hat{z}_t^2 + \frac{1}{2} [U_t l_t + U_{ll} l_t^2] \hat{l}_t^2 + [U_{zl} z_t l_t] \hat{z}_t \hat{l}_t \quad (A.6)
\]
where the partial derivatives in (A6), evaluated at the steady-state, are:

\[
\begin{align*}
U_z &= \mu \left( (\Pi_{0z})^\mu (t)^{1-\mu} \right)^{1-\sigma} \\
U_t &= (1-\mu) \left( \Pi_{0t} \right)^{1-\mu} \\
U_{zz} &= \mu [\mu (1-\sigma) - 1] \left( (\Pi_{0z})^\mu (t)^{1-\mu} \right)^{1-\sigma} \\
U_{tt} &= (1-\mu) \left( (1-\mu) (1-\sigma) - 1 \right) \left( (\Pi_{0z})^\mu (t)^{1-\mu} \right)^{1-\sigma} \\
U_{zt} &= \mu \left( (1-\mu) (1-\sigma) \right) \left( (\Pi_{0z})^\mu (t)^{1-\mu} \right)^{1-\sigma}
\end{align*}
\]

the second-order approximations for \( \hat{z}_t \) and \( \hat{t}_t \) are:

\[
\begin{align*}
\hat{z}_t &= \hat{x}_t + \sum_{s=0}^{t-1} \hat{\gamma}_s, \quad \text{where} \\
\hat{x}_t &\approx \left( \frac{c}{c+\phi y} \right) \hat{c}_t + \left( \frac{\phi y}{c+\phi y} \right) \hat{g}_t^c + \frac{1}{2} \left[ \frac{c}{c+\phi y} - \left( \frac{c}{c+\phi y} \right)^2 \right] (\hat{c}_t)^2 \\
&\quad + \frac{1}{2} \left[ \frac{\phi y}{c+\phi y} - \left( \frac{\phi y}{c+\phi y} \right)^2 \right] (\hat{g}_t)^2 - \left( \frac{\phi y}{(c+\phi y)^2} \right) \hat{c}_t \hat{g}_t^c
\end{align*}
\]

and

\[
\begin{align*}
\hat{t}_t &\approx -\left( \frac{a}{1-u-\epsilon} \right) \hat{u}_t - \left( \frac{e}{1-u-\epsilon} \right) \hat{e}_t - \frac{1}{2} \left[ \frac{u}{1-u-\epsilon} + \left( \frac{u}{1-u-\epsilon} \right)^2 \right] (\hat{u}_t)^2 \\
&\quad - \frac{1}{2} \left[ \frac{e}{1-u-\epsilon} + \left( \frac{e}{1-u-\epsilon} \right)^2 \right] (\hat{e}_t)^2 - \left( \frac{ue}{(1-u-\epsilon)^2} \right) \hat{u}_t \hat{e}_t;
\end{align*}
\]

and finally the second-order approximations for \( (\hat{z}_t)^2 \) and \( (\hat{t}_t)^2 \) are:

\[
\begin{align*}
(\hat{z}_t)^2 &\approx (\hat{x}_t)^2 + \sum_{s=0}^{t-1} \hat{\gamma}_s^2 + 2 \hat{x}_t \sum_{s=0}^{t-1} \hat{\gamma}_s \\
\hat{x}_t &\approx \left( \frac{c}{c+\phi y} \right) (\hat{c}_t)^2 + \left( \frac{\phi y}{c+\phi y} \right) (\hat{g}_t)^2 + 2 \left( \frac{\phi y}{(c+\phi y)^2} \right) \hat{c}_t \hat{g}_t^c \\
2 \hat{x}_t \sum_{s=0}^{t-1} \hat{\gamma}_s &\approx 2 \left( \frac{c}{c+\phi y} \right) \hat{c}_t \sum_{s=0}^{t-1} \hat{\gamma}_s + 2 \left( \frac{\phi y}{c+\phi y} \right) \hat{g}_t \sum_{s=0}^{t-1} \hat{\gamma}_s \\
(\hat{t}_t)^2 &\approx \left( \frac{a}{1-u-\epsilon} \right)^2 (\hat{u}_t)^2 + \left( \frac{e}{1-u-\epsilon} \right)^2 (\hat{e}_t)^2 + 2 \left( \frac{ue}{(1-u-\epsilon)^2} \right) \hat{u}_t \hat{e}_t.
\end{align*}
\]

The above gives the second-order approximation, \( \hat{U}_t^s \), for any \( t \geq 1 \). Therefore at \( t = 0 \), the expression for \( \hat{U}_t^0 \) is the same except that \( z_0 \equiv x_0 \).

### 6.4 Second-order approximation of lifetime utility

Finally, expected lifetime utility, \( V_t \), is given by the expected discounted sum of \( U_0^s \) and \( U_t^s \), i.e.:

\[
V_t \approx \hat{U}_0^s + E_0 \sum_{t=1}^{\infty} \beta^t \hat{U}_t^s
\]

(A.7)
In the simulations, $T = 300$ years and the sample average for $V$ is calculated using 1000 simulations.\footnote{Note that studies that use quarterly data, usually work with 1000 quarters, which implies 250 years.}

### 6.5 Welfare comparisons

Let $\bar{X}_t^A$ denote the contingent plan for per capita composite consumption associated with tax structure $A$, and $\bar{X}_t^B$ the contingent plan for per capita composite consumption associated with structure $B$. We can then, following e.g. Lucas (1990), define $\xi$ as the constant fraction of regime $B$‘s consumption process that a household would be willing to give up to be as well off under $A$ as under $B$. Hence, we write:

$$V_t^A = (1 - \xi)^{\mu(1-\sigma)}V_t^B$$

(A.8)

Solving for $\xi$, we obtain:

$$\ln(1 - \xi) = \frac{1}{\mu(1 - \sigma)} \times \ln \left( \frac{V_t^A}{V_t^B} \right)$$

$$\Rightarrow \quad \xi \approx \frac{1}{\mu(1 - \sigma)} \times \ln \left( \frac{V_t^A}{V_t^B} \right)$$

(A.9)

where, $V_t^B$ and $V_t^A$ are calculated by using the second-order approximation of welfare as defined in (A7) above and averaged over 1000 simulations.
Figure 1: Growth and welfare effects of changes in tax rates (relative to transfers)
Figure 2: Effects of changes in tax rates accommodated by transfers

- Tax rates on transfers
- Output (across tax rates) on transfers
- Education time (across tax rates) on transfers
- Work time (across tax rates) on transfers
- Consumption (across tax rates) on transfers
- Investment (across tax rates) on transfers
Figure 3: Growth and welfare effects of changes in relative tax rates
Figure 4: Effects of changes in income tax rates accommodated by consumption tax

- **Income taxes on consumption tax**
- **Output (across income taxes) on consumption tax**
- **Education time (across income taxes) on consumption tax**
- **Work time (across income taxes) on consumption tax**
- **Consumption (across income taxes) on consumption tax**
- **Investment (across income taxes) on consumption tax**
Figure 5: Effects of changes in capital tax accommodated by labour tax