Rule of Thumb Consumers, Public Debt and Income Tax*

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Abstract

This paper analyzes a New Keynesian model with Rule-of-Thumb consumers (ROTC) as in Galí *et al.*(2007) and a fiscal policy which levies a proportional income tax. We find that, when the share of ROTC is above a specified threshold and differently from the usual Leeper (1991) result, the determinacy condition requires for both monetary and fiscal policy to be either active of passive. Furthermore we show that the introduction of a set of ROTC can reverse the traditional predictions of a change in government spending on the economy as a whole: under a reasonable parametrization of the model, an increase in government spending can lead, against the common Keynesian wisdom, to a decrease in total output. Finally we point out that with the introduction of a distortive fiscal policy and independently of the parametrization used, private consumption responds negatively to a positive government spending shock.

JEL *classification*: E32; E62; H30

Keywords: Rule-of-thumb-consumers, monetary-fiscal policy interactions, distortive taxation, public spending, private consumption.

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1 Introduction

In the last decade macroeconomic researchers have shown particular interest in the study of the relationship between public spending and private consumption. Due to the fact that government spending is one of the most important instruments of economic policy and private consumption is the main component of GDP, understanding the relationship between these two variables is an important point for economic theorists as well as for policy makers.

The original Keynesian theory pointed out that in the face of an increase in government spending (when financed with public debt), the current disposable income of private agents increases and therefore they consume more. This theoretical prediction had been reversed in the late seventies by the Real Business Cycle approach (RBC henceforth) in which private households, infinitely lived, maximize their lifetime utility. With this feature, when government spending increases and independently of how this is financed, private consumers' after tax net income decreases, and as a consequence, they consume less.[1] On the other hand, recent empirical work[2] seem to contradict the RBC paradigm, finding in the data a positive correlation between public and private consumption.

A recent theoretical work by Galí, Lopez-Salido and Valles (2007) (GLV henceforth) tries to tackle this economic puzzle, introducing in an otherwise standard New Keynesian framework (NK henceforth) with optimizing agents, sticky prices and monopolistic competition in good markets, a set of Rule-of-thumb consumers (ROTC henceforth) who are excluded from the financial markets and hence do not smooth consumption over time. The main result of their work is that if we allow for the number of ROTC to be large enough in the economy, and government expenditure to be partly financed with public debt, private consumption may increase in the face of a positive government spending shock.

Due to its reasonable tractability and its policy implications, the GLV approach has received increased interest in the literature. From this, one of the aspects most stressed is that the introduction of a set of ROTC in an otherwise standard NK model can drastically change the determinacy conditions of the model.[3] To this extent the main contribution can be found in Bilbiie(2006). He shows that in a NK model with no capital accumulation, a walrasian labour market and no fiscal policy, a high share of ROTC may require for determinacy, using Leeper's (1991) definition, a *passive* monetary policy (whereby nominal interest rate is adjusted such that real rate decreases in response to positive inflation). The basic intuition for this result is that when the monetary authority increases the interest rate, the system experiences a downward pressure on wages. This, combined with a sticky price environment implies an increase in profits which are held only by the optimizer consumers (OPTC henceforth). The increase in OPTC wealth generated by the increase in profits may generate an increase

[1] See, inter alia, Baxter and King(1992).
[2] See inter alia Fatas and Mihov(2004), Blanchard and Perotti(1999) and Schmitt-Ghroe and Uribe(2007).
[3] See inter alia GLV(2004), Di Bartolomeo and Rossi(2006) and Bilbie(2006).

in total demand putting, via the *Phillips curve,* upward pressure on prices. A monetary authority wishing to stabilize the price level may therefore need to cut the real interest rate in the face of an inflationary shock.

However all these works assume a neutral fiscal policy. Government spending changes are therefore financed by levying a lump-sum tax (i.e. changes in taxes do not have any consequences on the aggregate variables of the model). As pointed out by Favero *et al.*(2005), this assumption is at odds with the reality. Linnemann (2005) and Schmitt-Grhoe *et al.* (2006) show that the introduction of a more realistic distortive fiscal policy (i.e. proportional income tax), drastically changes the determinacy condition as well as the policy implications of the model. The main reason is that changes in the tax rate cause a change in the consumers *marginal rate of substitution* between consumption and leisure. This implies that tax rate adjustments have a direct feedback on the level of the aggregate variables.

The aim of this work is to extend the GLV analysis incorporating in a ROTC-NK model a distortive fiscal policy. From this point this paper makes two main contributions. The first is to check if the GLV results on the positive correlation between public and private consumption survive with a richer characterization of the fiscal policy. The second is to fully describe which are the monetary-fiscal mix requirements for determinacy within a NK model with ROTC and distortive taxation.

First of all, we show that the bifurcation point found by Bilbiie (2006), due to the non linear relationship between ROTC and the sensitivity of the demand side of the economy (i.e. dynamic IS), for monetary policy, can potentially be extended to fiscal policy. In particular, when the share of ROTC is above a specified threshold, an increase in government spending could, *ceteris paribus,* not only decrease private consumption but also total output.

Second, we find that an active monetary policy which respects the Taylor principle (whereby the nominal interest rate is adjusted such that the real rate increases in response to positive inflation) can lead to a unique equilibrium even with a high number of ROTC in the economy, as long as the fiscal regime abandons its passive role (whereby the tax rate increases in response to positive public debt shock), and adopts an active one.

Third, due to the presence of distortive taxation, and independently of the parametrization used, the positive correlation between private and public spending is no longer a feature of the model.

The remainder of the paper is as follow. Section 2 describes the theoretical model. Section 3 presents the determinacy analysis and the simulation results. Section 4 concludes.

2 The model

The totality of households is normalized to unity. Of this, a fraction $(1-\lambda)$ with $\lambda \leq 1$, behave in a traditional optimizing way. Hence they maximize their (infinite) lifetime utility, hold profits coming from the monopolistic nature of the goods market, and participate in perfect and complete financial markets. The remaining λ households are defined as in Gali *et al.*(2007). They care only for their current disposable income and they hold no financial assets nor any profit share. For these consumers all their wealth is represented by their wage and therefore they cannot smooth consumption over time.

2.1 Optimizers (OPTC) $(1-\lambda)$

The (lifetime) OPTC utility function has a standard form and it simply includes consumption and labour

$$U_t = E_0 \sum_{t=0}^{+\infty} \beta^t u\left(C_t^o, N_t^o\right) \tag{1}$$

Where $u\left(.,.\right)$ represents instantaneous utility. We assume, in line with most of the literature, that $\frac{du}{dC^o} > 0$ and $\frac{du}{dN^o} < 0$. The shape of u is

$$u\left(C_t^o, N_t^o\right) = \log C_t^o - \theta \frac{\left(N_t^o\right)^{1+\varphi}}{1+\varphi} \tag{2}$$

Where $\beta \in (0,1)$ is the discount factor, C^o is the level of consumption of the OPTC, N^o is the OPTC labour supply, $\theta > 0$ indicates how leisure is valued relative to consumption. The parameter $\varphi > 0$ is the inverse of the Frisch elasticity of labour supply and represents the risk aversion to variations in leisure.

The OPTC flow budget constraint is

$$\int_0^1 P_t\left(j\right) C_t^o\left(j\right) dj + R_t^{-1} \frac{B_{t+1}}{1-\lambda} + \frac{E_t\left(Q_{t,t+1} V_{t+1}\right)}{1-\lambda} = \left(W_t N_t^o + \frac{D_t}{1-\lambda}\right)\left(1-\tau_t\right) + \frac{B_t}{1-\lambda} + \frac{V_t}{1-\lambda} - S^o \tag{3}$$

Where $P_t\left(j\right)$ is the price level of the variety of good j, W_t is the nominal wage, D_t are the nominal profits coming from the monopolistic competitive structure of the goods market,B_{t+1} is the nominal payoff of the one period risk-less bond purchased at time t, R_t is the gross nominal return on bonds purchased in period t, $Q_{t,t+1}$ is the stochastic discount factor for one period ahead payoff and V_t is nominal payoff of a state-contingent asset portfolio. The government is assumed to pay a level of public spending, G_t and the service of debt, levying a proportional income tax, τ_t. S^o is a steady state transfer such that at SS the two types of agents consume and supply the same amount of labour.

The expenditure minimization problem implies that households, creating their consumption basket, exploit any relative price differences present in the economy. This, combined with the CES Dixit-Stiglitz aggregator, results in a demand function for any single good that is downward sloping.

$$C_t^o(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} C_t^o$$

Where the price index is found by

$$P_t = \left(\int_0^1 P_t(j)^{1-\varepsilon}\, dj\right)^{\frac{1}{1-\varepsilon}}$$

at the optimum we have

$$\int_0^1 P_t(j)\, C_t^o(j)\, dj = P_t C_t^o$$

Where the parameter ε represents the elasticity of substitution among goods and it is a measure of the market power held by each firm.

The first order condition for consumption is

$$\beta\left(\frac{C_t}{C_{t+1}}\right)\left(\frac{P_t}{P_{t+1}}\right) = Q_{t,t+1}$$

Taking conditional expectation on both sides and rearranging gives

$$\beta R_t E_t\left[\left(\frac{C_t}{C_{t+1}}\right)\left(\frac{P_t}{P_{t+1}}\right)\right] = 1 \tag{4}$$

Where $R_t = \frac{1}{E_t(Q_{t,t+1})}$ is implied by the non arbitrage condition. This expression is the familiar *Euler equation* for consumption. It describes the attitude to smooth consumption over time once the opportunity cost implied by the real interest rate has been taken into account. The first order condition with respect to labour states that the *marginal rate of substitution* between labour and consumption must be equal to the after tax real wage

$$\theta\left(N_t^o\right)^{\varphi} C_t^o = \frac{W_t}{P_t}\left(1 - \tau_t\right)$$

From the last expression one can see that taxation distorts the leisure-consumption choice. Any change in the tax rate has a direct effect on real wage and therefore on the marginal rate of substitution between consumption and labour.

2.2 Rule of Thumb Consumers(ROTC) (λ)

The ROTC utility function is represented by a single period expression. In particular, following Galí *et al.(*2007*)*, it is assumed that the shape of the instantaneous utility is the same for the two types of consumers. Therefore

$$U_t = \log C_t^r - \theta\frac{\left(N_t^r\right)^{1+\varphi}}{1+\varphi} \tag{5}$$

As stressed above the ROTC do not participate in the financial markets and do not hold any profit, therefore their budget constraint can be expressed as follows

$$\int_0^1 P_t(j) C_t^r(j) = W_t N_t^r (1-\tau_t) - S^r \tag{6}$$

Where $C^r(j)$ and N^r are level of consumption of each product and labour supply of the ROTC. Furthermore, it is assumed that as the OPTC, they exploit any relative price difference in creating their consumption basket. Hence, at the optimum

$$P_t C_t^r = \int_0^1 P_t(j) C_t^r(j)\, dj \tag{7}$$

On the consumption side the ROTC are forced to consume all their income in each period, therefore consumption can be easily inferred by combining (6) with (7). The FOC for the optimal supply of labour implies

$$\theta (N_t^r)^{\varphi} C_t^r = \frac{W_t}{P_t}(1-\tau_t) \tag{8}$$

The last two expressions state the ROTC "*hand to mouth*" attitude towards consumption. This means that they consume in every period all their wealth which, as previously stated, is equal to their after tax labour income. The optimal supply of labour takes the same analytical form as that of the OPTC.

2.3 Aggregation rules and market clearing condition

The aggregate expressions for consumption and labour are simply the weighted average of the single consumer type variables. Therefore aggregate consumption follows

$$C_t = \lambda C_t^r + (1-\lambda) C_t^o \tag{9}$$

and aggregate labour

$$N_t = \lambda N_t^r + (1-\lambda) N_t^o \tag{10}$$

In the absence of capital accumulation, everything produced must be consumed in the same period. Furthermore each product can be purchased by the private sector(C_t) or by the government (G_t)

$$Y_t(j) = C_t(j) + G_t(j) \tag{11}$$

In aggregate

$$Y_t = C_t + G_t \tag{12}$$

2.4 Firms

In this economy, firms are assumed to possess an identical production technology. This production function is linear in labour and can be written as

$$Y(j) = N(j) \tag{13}$$

Furthermore, it is worth noting that each firm faces the following demand function

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} Y_t \tag{14}$$

where

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{15}$$

Following the NK literature it is assumed that prices are sticky. For the sake of simplicity, we model this feature of the economy following the Calvo contracts (1983). In each period there is a (randomly selected) set of firms, let's say $(1-\alpha)$, who reset their price optimally, while the remaining α keep their prices fixed. When a firm is allowed to reset its prices, it takes into account the expected future stream of profits discounted for the probability to not reset its prices. In particular the maximization problem of a price setter can be written as

$$\max_{P_t^*(j)} E_t \sum_{i=0}^{+\infty} \alpha^i Q_{i,t+i} \left(\left(\frac{P_t^*(j)}{P_{t+i}}\right) Y_{t+i}(j) - MC_{t+i} Y_{t+i}(j)\right) \tag{16}$$

where the FOC with respect to $P_t^*(j)$

$$P_t^*(j) = \left(\frac{\varepsilon}{\varepsilon-1}\right) \frac{E_t \sum_{i=0}^{+\infty} \alpha^i \beta^i \left(\frac{C_t^o}{C_{t+i}^o}\right)\left(\frac{P_t}{P_{t+i}}\right)\left(MC_{t+i}\left(P_{t+i}\right)^{\varepsilon} Y_{t+i}\right)}{E_t \sum_{i=0}^{+\infty} \alpha^i \beta^i \left(\frac{C_t^o}{C_{t+i}^o}\right)\left(\frac{P_t}{P_{t+i}}\right)\left(P_{t+i}\right)^{\varepsilon} P_{t+i}^{-1} Y_{t+i}} \tag{17}$$

while the price level follows

$$P_t^{(1-\varepsilon)} = \left[(1-\alpha) P_t^{*(1-\varepsilon)} + \alpha P_{t-1}^{(1-\varepsilon)}\right] \tag{18}$$

2.5 The Goverment

The government uses income tax revenues, $P_t \tau_t Y_t$ to finance a stream of public spending, $P_t G_t$[4], and the service of public debt. Therefore the government budget constraint can be expressed as

$$R_t^{-1} B_{t+1} = B_t - P_t \tau_t Y_t + P_t G_t \tag{19}$$

[4] As the private sector, the government exploit any price differences in the market to form its consumption basket G_t. This jointly with a CES aggregator gives the following downward sloping demand function for each single public spending good. $G_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} G_t$

where $P_tG_t - \tau_tP_tY_t$ is the primary budget. The government budget constraint can be expressed in real terms as

$$R_t^{-1}\frac{B_{t+1}}{P_t} = \frac{B_t}{P_{t-1}}\frac{P_{t-1}}{P_t} - \tau_tY_t + G_t \tag{20}$$

2.6 Monetary Policy

Monetary policy fixes the nominal interest rate, R_t, in every period. Its only aim is price stability. Following Clarida *et al.*(2000), it is assumed that the monetary authority responds only to current inflation. The monetary policy rule can therefore be expressed as follow

$$R_t = R + \phi\pi_t \tag{21}$$

Where $R = \frac{1}{\beta}$ is the steady state interest rate and ϕ is the policy parameter that identifies the response of the interest rate to the inflation rate.

2.7 Fiscal Policy

For the fiscal policy we assume a government revenues rule of the type

$$Y_t\tau_t = \delta_0 + \delta_1\frac{\tau}{B}(B_t - B) + \delta_2\frac{\tau}{Y}(Y_t - Y) \tag{22}$$

where $\delta_0 = (1-\beta)B + G$ and δ_1 and δ_2 are policy parameters identifying the relative weight given to debt stabilization and output stabilization. This fiscal rule has the characteristic of being SS neutral (at steady state the fiscal rule collapses to $\tau = \frac{(1-\beta)B}{Y} + \frac{G}{Y}$ which is equal to $\tau = \delta_0/Y$) and it permits the analysis, following Leeper (1991), from the active-passive policy perspective.

2.8 Log Linearization

This section presents a log-linearized version of the model around the non stochastic steady state (SS).[5] Henceforth, all the upper hat variables identify the variable percentage deviation from its SS value (i.e.$\widehat{X}_t = \log\left(\frac{X_t}{X}\right)$). While $\pi_t = \log P_t - \log P_{t-1}$ identifies the inflation rate.

The log linearization of the OPTC Euler equation and optimal supply of labour are

$$\widehat{C}_t^o = E_t\widehat{C}_{t+1}^o - \left(\widehat{R}_t - E_t\pi_{t+1}\right) \tag{23}$$

$$\widehat{C}_t^o + \varphi\widehat{N}_t^o = \widehat{W}_t - \widehat{P}_t - \frac{\tau}{1-\tau}\widehat{\tau}_t \tag{24}$$

[5] Detailed description of the SS in Appendix A

while the ROTC consumption and labour follow

$$\widehat{C}_t^r = \widehat{W}_t - \widehat{P}_t + \widehat{N}_t^r - \frac{\tau}{1-\tau}\widehat{\tau}_t \tag{25}$$

$$\varphi\widehat{N}_t^r + \widehat{C}_t^r = \widehat{W}_t - \widehat{P}_t - \frac{\tau}{1-\tau}\widehat{\tau}_t \tag{26}$$

Log linearizing (17) and (18) around a zero inflation SS yields to the traditional New Keynesian Phillips Curve (NKPC)

$$\pi_t = \beta E_t \pi_{t+1} + \kappa(\widehat{MC}_t - \widehat{P}_t) \tag{27}$$

Where $\kappa = \frac{(1-\alpha)(1-\beta\alpha)}{\alpha}$. The log linearization of the aggregation rules for consumption and labour yield

$$\widehat{C}_t = \lambda\widehat{C}_t^r + (1-\lambda)\,\widehat{C}_t^o \tag{28}$$

$$\widehat{N}_t = \lambda\widehat{N}_t^r + (1-\lambda)\,\widehat{N}_t^o \tag{29}$$

while the market clearing condition follows

$$\widehat{Y}_t = \gamma_c\widehat{C}_t + (1-\gamma_c)\,\widehat{G}_t \tag{30}$$

where $\gamma_c = \frac{C}{Y}$. Furthermore from the production function (13)

$$\widehat{Y}_t = \widehat{N}_t \tag{31}$$

The log linearization of the monetary and fiscal rule yields

$$\widehat{R}_t = \phi\pi_t \tag{32}$$

$$\widehat{\tau}_t = \delta_1\widehat{B}_t + (\delta_2 - 1)\widehat{Y}_t$$

.Finally, a log linearization of the government budget constraint can be written as

$$\widehat{B}_{t+1} = \widehat{R}_t + \frac{1}{\beta}\left(\widehat{B}_t - \pi_t + \frac{1-\gamma_c}{B}\widehat{G}_t - \frac{\tau Y}{B}\left(\widehat{Y}_t + \widehat{\tau}_t\right)\right) \tag{33}$$

2.9 Equilibrium

This section presents the equilibrium of the model. Further analysis is simplified by rewriting the model as a function of aggregate variables only. First, combining (25) with (26), we obtain

$$\widehat{N}_t^r = 0 \tag{34}$$

and

$$\widehat{C}_t^r = \widehat{W}_t - \widehat{P}_t - \frac{\tau}{1-\tau}\widehat{\tau}_t \tag{35}$$

From the last two expressions one can see that the introduction of distortive taxation is completely internalized in the ROTC consumption, while their labour supply remains constant at the SS level[6]. Therefore changes in the tax rate over the business cycle do not have any effect on the ROTC labour supply.

Combining the last expression with the optimal labour supply of the OPTC yields

$$\widehat{C}_t^o + \varphi\widehat{N}_t^o = \widehat{C}_t^r \tag{36}$$

Furthermore, combining (29) with (34) it is possible to rewrite the total supply of labour as

$$\widehat{N}_t = (1-\lambda)\,\widehat{N}_t^o \tag{37}$$

Therefore aggregate labour fluctuations are just a function of changes in OPTC labour supply. Moreover, plugging these results into the equation for total consumption yields

$$\widehat{C}_t = \lambda\left[\widehat{C}_t^o + \frac{\varphi}{1-\lambda}\widehat{N}_t\right] + (1-\lambda)\,\widehat{C}_t^o$$

Simplifying gives

$$\widehat{C}_t = \widehat{C}_t^o + \varphi\frac{\lambda}{1-\lambda}\widehat{N}_t \tag{38}$$

From the latter we can rewrite the Euler equation in terms of aggregate consumption as

$$\widehat{C}_t = E_t\left(\widehat{C}_{t+1}\right) - \left(\widehat{R}_t - E_t\pi_{t+1}\right) - \varphi\frac{\lambda}{1-\lambda}E_t\Delta\widehat{N}_{t+1} \tag{39}$$

As in Galí et al(2007) we find it useful to iterate forward the last expression such that

$$\widehat{C}_t = \varphi\frac{\lambda}{1-\lambda}\widehat{N}_t - E_t\sum_{i=0}^{+\infty}\left(\widehat{R}_{t+i} - \pi_{t+i+1}\right) \tag{40}$$

From the last expression it is easy to see that an increase in public spending has the potential to increase private consumption. The intuition is straightforward. An increase in $\widehat{G}_t$, given any path of future interest rate and inflation, through the market clearing condition, generates an increase in total output, and therefore in the demand of labour. The increase in labour demand has a direct and positive effect on the real wage. An increase in the real wage boosts ROTC consumption and therefore potentially total consumption. From the formula it is clear that the multiplier effect on consumption is greater, the higher the share of ROTC. However, something is missing from this apparently strong result. The problem lies in the fact that $\widehat{N}_t$ is not truly exogenous to $\widehat{C}_t$.

[6] For the ROTC the substitution effect on the labour supply is equal to the income effect.

On the contrary, total labour demand and therefore total output are functions of private consumption as well as of public spending. To have the full picture it is necessary to substitute in (40) the market clearing condition and the production function. The resulting equation can be expressed as follows

$$\widehat{C}_t = \varphi\frac{\lambda}{1-\lambda}\left(\gamma_c\widehat{C}_t + (1-\gamma_c)\,\widehat{G}_t\right) - E_t\sum_{i=0}^{+\infty}\left(\widehat{R}_{t+i} - \pi_{t+i+1}\right)$$

Solving for consumption

$$\widehat{C}_t = \left(1-\varphi\frac{\lambda}{1-\lambda}\gamma_c\right)^{-1}(1-\gamma_c)\,\widehat{G}_t - \left(1-\varphi\frac{\lambda}{1-\lambda}\gamma_c\right)^{-1}E_t\sum_{i=0}^{+\infty}\left(\widehat{R}_{t+i} - \pi_{t+i+1}\right) \tag{41}$$

It is clear from the last equation the precise multiplier of public spending on private consumption is represented by the term $\left(1-\varphi\frac{\lambda}{1-\lambda}\gamma_c\right)^{-1}$. The multiplier depends directly and in a non linear way on λ. In particular a necessary condition to have $\frac{d\widehat{C}}{d\widehat{G}} > 0$ requires

$$\lambda < (1+\varphi\gamma_c)^{-1} \tag{42}$$

A few points are worth stressing. First, for high level of λ or high values of φ (inelastic labour supply) an increase in public spending decreases private consumption. The intuition is as follows: when there is an increase in government spending, there is an increase in labour demand and as a consequence an increase in real wage. This generates an increase in ROTC consumption. At the same time higher real wages imply lower profits, which are held only by the OPTC. For each unit of decrease in profit, the OPTC's receive $\frac{1}{1-\lambda} > 1$ less profits. When λ is above the threshold, the negative effect on profit overcompensates the boost in ROTC consumption, generating a negative relationship between public and private spending. This is not all. This negative correlation can have drastic consequences on aggregate demand. Expressing (41) in terms of aggregate output yields

$$\widehat{Y}_t = \left(\frac{1}{\gamma_c}-\varphi\frac{\lambda}{1-\lambda}\right)^{-1}\frac{(1-\gamma_c)}{\gamma_c}\widehat{G}_t - \left(\frac{1}{\gamma_c}-\varphi\frac{\lambda}{1-\lambda}\right)^{-1}E_t\sum_{i=0}^{+\infty}\left(\widehat{R}_{t+i} - \pi_{t+i+1}\right) \tag{43}$$

From the latter, one can see that when λ is above the threshold there is a negative correlation which affects not just private consumption but the entire aggregate demand. Therefore, and in contrast with most of the Dynamic Stochastic General Equilibrium literature, the presence of ROTC could generate a situation where $\frac{d\widehat{Y}_t}{d\widehat{G}_t} < 0$. The initial increase in labour demand generates a contemporaneous and expected decrease in OPTC income which is greater than the boost in ROTC consumption and than the public spending increase put together. The initial increase in labour demand is overturned by the decrease in OPTC wealth. This will have a contractionary effect on the labour demand which is greater than the one generated by an increase in $\widehat{G}_t$. Moreover, it is interesting to note that this result is completely independent of the type of fiscal policy present in the economy.

The second important point regards the effect of an increase in the real interest rate on the economy as a whole. As pointed out by Bilbiie (2006), when λ is above the threshold described above, an increase in the real interest rate can potentially generate an increase in aggregate demand. As a consequence, in order for monetary policy to achieve price stabilization it may have a passive policy rule[7]. The reason is as follows. First of all, an increase in the real rate makes OPTC's current consumption decrease through the Euler equation. As a consequence, there is a downward shift in labour demand and a reduction in real wages. ROTC disposable income is reduced and therefore $\widehat{C}_t^r$ decreases. Nevertheless, a fall in real wages generates an increase in profits. As stressed above, a unit of increase in profit generates $\frac{1}{1-\lambda}$ increase in OPTC wealth. When the share of ROTC is high enough (or equally labour supply is inelastic), this effect on profits may overturn the reduction in aggregate demand generated through the Euler equation. Furthermore, an increase in the interest rate causes a greater upward shift in the demand for bonds than in a situation with no ROTC. This is because in this framework public debt is net wealth (i.e. both consumers pay the service of public debt but only the OPTC hold bonds). Again, for each unit of increase in real interest rate the OPTC consumers receive $R_t \frac{1}{1-\lambda}$ units of wealth, thus increasing the possibility of an expansion in aggregate demand.

On the supply side, using the market clearing condition and the definition of real marginal cost, we can express the New Keynesian Phillips Curve ($NKPC$) in terms of aggregate variables as follows

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \left(\left(\frac{1}{\gamma_c} + \varphi \right) \widehat{Y}_t - \frac{(1-\gamma_c)}{\gamma_c} \widehat{G}_t + \frac{\tau}{1-\tau} \widehat{\tau}_t \right) \tag{44}$$

3 The model in action

This section analyzes the determinacy conditions and the calibration results of the model.

First of all it is important to have a full description of the policy mix, monetary and fiscal, that guarantees a unique equilibrium when λ, the share of ROTC consumers, is above and below the threshold introduced in the previous section. As stressed by Bilbiie (2006), for λ above the threshold, the system needs a passive monetary policy in order to have a unique equilibrium, whilst, the opposite is true when λ is smaller than the threshold. This important result is here integrated with a *distortive* fiscal policy.

The second important goal of this section is to check, using a similar parametrization as in Galí *et al.*(2007) augmented with distortive taxation, the behavior of private consumption in the face of a government spending shock. This point is not trivial. With distortive fiscal policy a change in the tax rate has direct feed back on the endogenous variables of the model. In particular, as discussed above, an increase in the tax rate generates, *ceteris paribus*, a decrease in total output and in private consumption. For this reason, when the Ricardian equivalence does not hold, the government budget constraint cannot be separated from the rest of model as long

[7] We define passive monetary policy as in Leeper (1991). Passive monetary policy is such that it responds to the inflation rate by increasing the nominal interest rate by less than one. In the context of this model, a passive rule implies $\phi < 1$.

as the timing of how the public sector decides to finance its spending has direct consequences on the endogenous variables of the model.

3.1 Calibration

This section discusses the parameters values chosen for the determinacy analysis and the calibration of the model. We assume the elasticity of substitution among goods, ε, is equal to 6. This implies a SS real wage is equal to 0.83 and a SS markup of 20%, which is in line with the literature. The discount factor β has been fixed at 0.99. As a consequence, the gross annual interest rate is 3%. θ, the parameter of relative disutility of labour to consumption, has been chosen to obtain an average SS labour supply of 1/3. The SS ratio between private consumption and total output, γ_c, is 0.75. This value implies a SS ratio G/Y of 0.25 which is in line with the level of public consumption of most of the industrialized countries. The ratio between the SS annual stock of public debt and the annual GDP is fixed at 0.6, which is the average of the ratio of public debt to GDP of most industrialized countries. This in turn means that the SS level of the tax rate is equal to 30% and $\gamma_b = B/Y$ being equal to 2.4. As most of the NK literature, we assume that prices remain unchanged on average for one year. Therefore α, the parameter ruling the degree of price stickiness, is fixed at 0.75. The stochastic component of the model is represented by a government spending shock. In particular we assume that government spending follows an exogenous and stationary AR(1) process of the type $\widehat{G}_t = \rho\widehat{G}_{t-1} + \zeta_t$ with $\zeta \sim^a i.i.d.N\,(0,1)$ and $\rho = 0.9$. The determinacy and consequently the calibration exercise has been studied with different values of δ_2, the fiscal policy parameter of the output gap. A value of $\delta_2 = 0$ implies a policy rule very similar to the one studied by Leeper (1991), and describes a situation in which the total government revenues do not respond to output fluctuations. To study a countercyclical fiscal policy in terms of output, δ_2 has been fixed at 2 . The model has been solved with two pairs of λ and φ. One of these ($\lambda = 0.3$, $\varphi = 1$) guarantees for the model to be consistent with the Keynesian logic, whilst the other ($\lambda = 0.5$, $\varphi = 3$), reverses the Keynesian logic. Similarly, in order to describe the active-passive policy mix, the determinacy condition is analyzed for a broad range of policy parameters, ϕ and δ_1.

Table 1 : Baseline calibration

Parameter	Value	Description
β	0.99	Discount factor
λ	0.5	Share of ROTC
δ_2	$2; 0$	Fiscal parameter on the output gap
ε	6	Elasticity of substitution among goods
γ_c	0.75	Steady state share of C/Y
θ	3	Relative weight of disutility of labour
φ	1	Frisch inverse elasticity of labour supply
α	0.75	Degree of price stickiness
γ_b	2.4	Annual ratio public debt to GDP
ρ	0.9	Persistence of the public spending shock

3.2 Results

Figure (1) shows the combination of λ and φ that guarantee the validity of the Taylor principle and the positive correlation, *ceteris paribus*, between public spending and total output. As previously stated we refer to this area as the Keynesian side of the economy which is consistent with low values of λ and high values of φ (i.e. low share of ROTC and inelastic labour supply) or with high values of λ and low values of φ (high share of ROTC and elastic labour supply).

Within this side of the economy, the determinacy of equilibrium requires, as shown by Leeper (1991), a particular mix of monetary and fiscal policy. In particular a monetary policy wishing to stabilize inflation (active monetary policy) must be complemented by a fiscal policy which adjusts tax revenues in order to stabilize public debt (passive fiscal policy). On the other side if the fiscal authority does not stabilize the stock of public debt (active fiscal policy), determinacy requires monetary policy to forsake price stabilization (passive monetary policy). In the context of this model, an active monetary policy requires for ϕ to be greater than one (equivalently called Taylor principle), whilst a passive fiscal policy needs δ_1 to be positive.

Figure (2) shows, given a combination of λ and φ that leaves the economy Keynesian ($\lambda = 0.3$, $\varphi = 1$), the mix of monetary and fiscal policy parameters that leads to a unique equilibrium.[8] As stressed before, the active passive policy mix required for equilibrium is as in Leeper (1991). The reason is now well known. Let us consider a Keynesian economy facing a government spending shock. At the time of the shock aggregate output increases.[9] This leads, through the NKPC, to upward pressure on inflation. The active interest rule results in monetary policy increasing the nominal rate by more than the inflation rate. This increase on one hand makes today consumption more expansive, causing a downward shift on the aggregate demand, and on the other hand

[8] Appendix A conteins the details of the determinacy analysis.

[9] Note that at the moment nothing is said about the nature of the fiscal policy.

raises the demand for bonds. The overall result is a stabilization of price level and an increase in the cost of serving public debt. In order to repay this cost the fiscal authority must raise tax revenues (passive fiscal policy).

Figure (3) reports the impulse response function (IRF) of the variables of the model in the face of a government spending shock within the Keynesian side of the economy, with an active monetary policy ($\phi = 1.2$) and a passive fiscal policy($\delta_1 = 0.2$). The policy parameter on output gap δ_2 takes here the value of 0 that corresponds to a neutral, in terms of output, fiscal rule and the value of 2 which corresponds to a countercyclical, always in terms of output, fiscal policy.

At the time of the shock there is an expansion in aggregate demand, generated by an increase in total output. Through the NKPC, this leads to an upward pressure on prices resulting in positive inflation rate. The monetary authority, following an active rule, raises the nominal interest rate more than the inflation rate.

On the government side, the public spending shock worsens the government budget constraint, which is therefore balanced with the emission of public debt. The increase in interest rate, increasing the real return on bonds, makes the OPTC willing to purchase all the public debt assets present on the market, while the (passive) fiscal authority repays the cost of this service, raising the (distortive) income tax rate.

Due to the distortive nature of the fiscal policy, any increase in the tax rate generates a decrease in disposable income and therefore in total output. At this point it is convenient to make a distinction between the two types of consumer. The ROTC's supply labour as they always were at SS[10]. Therefore any changes in disposable real wage are internalized by their consumption. As a consequence, an increase in tax rate (a decrease in disposable income) translates to a direct decrease in ROTC consumption.

For the OPTC the situation is more complicated. They maximize the stream of their lifetime utility. Hence, when a government spending shock occurs, and independently on how this is financed, their expected after tax disposable income decreases and therefore they consume less. However, due to the tendency of the government to balance its budget with public debt, which now, purchased only by OPTC, is net wealth (both consumers pay the service of debt but only the OPTC hold public debt assets), the decrease in OPTC consumption, caused by the present and expected future raise in the tax rate, is smaller than in a situation with no ROTC.

Similarly, an increase in the interest rate makes OPTC present consumption more expansive. At the same time it raises the return on bond and potentially, through a downward pressure on real wages, increases the profit rate. Again the overall reduction in OPTC present consumption determined by a higher real interest rate is smaller than a situation with no ROTC.

The weaker impact of monetary and fiscal instruments on this economy may help to explain the high persistence of the variables of this model (i.e. the smaller the impact of the policy, the slower is the convergence to the SS value). The system takes in fact approximately 50 quarters, more than 12 years, to return to the steady state level.

As one can see from Figure (3), the difference between a fiscal policy which is neutral ($\delta_2 = 0$) or countercyclical

[10] See note 6.

($\delta_2 = 2$) is almost irrelevant. The joint effect of the need to balance the government budget constraint with the increase in the real interest rate and the net wealth characteristic of public debt causes, when $\widehat{G}_t$ increases, a greater response in bonds than in output. As a consequence (passive fiscal policy), and independently of δ_2, the tax rate has to increase, generating a similar effect on the dynamics of the model under the two different specifications of the fiscal rule.

Figures (4) and (5) show the monetary-fiscal policy mix that guarantee a unique equilibrium in the *non-Keynesian* side of the economy. In particular, in figure (4) the fiscal rule has a neutral behavior in terms of output ($\delta_2 = 0$), while the second figure sketches the equilibrium condition on the policy parameters with a countercyclical fiscal policy ($\delta_2 = 2$). In this side of the economy, the particular mix that guarantees equilibrium requires for both policies to be either active or to be passive. The intuition for this result is as follows: when the combination of λ and φ is such that the Keynesian logic is inverted, a monetary policy wishing to stabilize the price level needs to adopt a passive rule. *Ceteris paribus*, such a policy implies a potential decrease in the real interest rate. This would push upward OPTC present consumption and therefore the demand for bonds, increasing the stock of public debt. In order to stabilize the system a sound fiscal policy needs to raise taxation (passive fiscal policy). Alternatively, if the fiscal authority does not stabilize the stock of debt (active policy) the equilibrium condition needs for the monetary authority to increase the real rate (active policy). Doing so, given the positive effect of interest rates on the return of bonds and on profits, total output and consequently inflation would increase, deflating the cost of public debt. As one can see from Figure (4), a countercyclical fiscal policy is consistent with the active-active policy mix, while it reduces the determinacy area in the passive-passive policy mix. In particular, given a passive monetary policy, determinacy requires a very mild or a very strong passive fiscal policy.

Figure (6) presents the IRF when both policies are active ($\phi = 1.2$, $\delta_1 = -0.2$). Let us first analyze a neutral fiscal policy in terms of output ($\delta_2 = 0$). At the time of the shock, $\widehat{G}_t$ increases and therefore, given the negative multiplier of public spending on output, $\widehat{Y}_t$ decreases. Through the NKPC, inflation decreases. Monetary policy cuts the interest rate more than the inflation rate, generating a decrease in the real interest rate. This causes a decrease in the demand for bond. The (active) fiscal policy raises the tax rate putting further downward pressure on total output. As a result, wages decrease, generating a reduction in labour income. The reduction in ROTC's wealth is directly transferred to their consumption level. This is not all. Due to the presence of sticky prices (and simultaneously perfectly flexible wages), wages decrease more than prices, generating a potential increase in profits. Furthermore, the decrease in the real rate makes OPTC present consumption cheaper. From the simulation, it appears that these two effects overcome the reduction of wealth, given by a higher tax rate, suffered by the OPTC, pushing their consumption upward. Nevertheless, as one can see, the increase in OPTC consumption is smaller, in absolute value, than the decrease in ROTC consumption. As a result, aggregate consumption decreases.

In this context, and differently from the Keynesian side of the economy, a countercyclical fiscal rule ($\delta_2 = 2$) changes in a non trivial way the impact on the model of a government spending shock. The initial decrease in total output generated by the increase in $\widehat{G}_t$, is more than compensated by a countercyclical fiscal policy. As a result, total output increases over the SS level. This, *ceteris paribus,* generates an upward pressure on the inflation rate and therefore, given an active monetary policy, on the real rate. Wages increase more than prices generating an increase in the present after-tax labour income and a decrease in profits. As a consequence, there is a boost in ROTC consumption. The reduction in profit rate causes a decrease in wealth of the OPTC which is greater than the increase in after-tax labour income, and in the increased opportunity cost of purchasing bonds. Moreover, OPTC present consumption, given an higher interest rate, is now more expansive. As a result, OPTC consume less and purchase fewer bonds.

Furthermore, in this context a countercyclical fiscal policy seems, from the simulation of the model, to have a fundamental stabilization role in the economy. In fact, as is clear from Figure (6), with such a fiscal policy the deviation of the variables from their SS values is much smaller than with a neutral fiscal rule.

Figure (7) shows the IRF in the case where both policies are passive. In order to have a combination of policy parameters that leads to determinacy with both specifications of fiscal rule (i.e. $\delta_2 = 0$ and $\delta_2 = 2$), ϕ, the parameter of monetary policy has been fixed at 0.8 while $\delta_1 = 0.1$. When a government spending shock hits the system, aggregate output falls, leading to downward pressure on prices and consequently to a negative inflation rate. Given a passive monetary rule, nominal interest rates fall less than inflation generating an increase in real rate. This makes the OPTC demand more bonds. Hence public debt increases. The *passive* fiscal authority raises the tax rate, generating a further decrease in total output. The combined effect of a lower labour demand and a higher tax rate makes after-tax disposable income drop. As a consequence, ROTC consumption decreases. Due to sticky prices, wages decrease more than inflation, generating an increase in profits and therefore an increase in wealth for the OPTC. However, from the simulation it appears, that the increase in wealth generated by a higher interest rate (bond channel) and higher profits is dominated by the decrease in present and future *labour income wealth,* due to the present and expected increases in the tax rate and by the intertemporal allocation (higher interest rate). Therefore OPTC consumption decreases. However, as one can see from Figure (7), these opposite economic effects make OPTC consumption return to its SS level quicker (around 10 quarters) than ROTC consumption (around 50 quarters).

The difference between a neutral and a countercyclical fiscal policy is very small. This is due to the greater impact (and therefore a bigger tax increase) of the public spending shock on debt than on output.

Colciago (2007) has pointed out that the results of Galí *et al.*(2007) were dependent on an *ad hoc* calibration of φ (0.2), the parameter of risk aversion in leisure. In particular, the author points out that the positive correlation between public and private consumption does not hold for higher values of φ.In this exercise we show that, even using the same parametrization as in Gali' *et al.*(2007)[11] ,within a model with distortive fiscal policy the correlation between public and private consumption is negative. The results are displayed in Figure(8). The combination of λ and φ $(0.5, 0.2)$ guarantees the economy to be in the Keynesian side. Hence, as in Figure (3) monetary policy is active while fiscal policy is passive. The mechanism of the model is similar to the one

[11] Detailed explanation of the calibration used in Gali' et al. can be found in Appendix B.

discussed for Figure(3). The main intuition is that independently of φ, the distortive nature of fiscal policy imposes a change in the marginal rate of substitution between leisure and consumption, and, as stressed above, these changes are completely internalized in ROTC consumption. In particular for each increase in public expenditure and a passive fiscal policy, the tax rate must somehow increase, generating a decrease in ROTC consumption. This, combined with the traditional RBC prediction of a negative correlation between OPTC consumption and public spending, causes a decrease in overall consumption.

4 Conclusions

In this paper we add to the traditional closed economy New Keynesian model with sticky prices and monopolistic competition in the goods market, a set of Rule of Thumb Consumers as in Galí *et al.*(2007), distortive income taxation and steady state public debt. We show that the introduction of ROTC generates a non linear sensitivity of aggregate demand not only, as described by Bilbiie (2006), to the interest rate, but also to government spending. In particular when the share of ROTC is above a threshold, an increase in public spending can potentially generate a decrease in total output.

Moreover, we show that when there is a negative correlation between government spending and total output an active monetary policy is consistent with a unique determinate equilibrium as long as the fiscal authority changes from passive to active. This can be interpreted as an answer to a recent empirical work by Favero & Monacelli (2005) where they show that since the late seventies an active monetary policy has been mixed with an active fiscal one.

Ultimately we find that the introduction of a non neutral fiscal instrument with a share of ROTC generates, for all the simulation experiments we run, a negative correlation between public spending and private consumption. These results challenge researchers to find a robust theoretical model which would better explain the positive correlation found in the data between public spending and private consumption.

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Appendix A

This section describes the steady state of the model. A few points are worth stressing. First of all, we impose, through a transfer, that the two agents have the same level of consumption and supply the same level of labour at SS. Hence the heterogeneity between the two consumers is only along the business cycle. Price are normalized to unity and we fix $\frac{G}{Y} = 1 - \gamma_c$. The OPTC budget constraint is

$$C^o = \left(WN^o + \frac{D}{1-\lambda} \right) (1-\tau) + \left(1 - R^{-1}\right) \frac{B}{1-\lambda} + S^o \tag{45}$$

Where S^o is the OPTC transfer. The SS ROTC budget constraint is

$$C^r = (WN^r)(1-\tau) + S^r \tag{46}$$

where S^r is the ROTC transfer. Furthermore we need to impose

$$(1-\lambda) S^o + \lambda S^r = 0 \tag{47}$$

From the SS Euler equation it is possible to find the SS interest rate

$$\frac{1}{\beta} = R$$

While the SS profits follow

$$D = (1-W) Y \tag{48}$$

Homogeneity requires

$$C^o = \left(WN^o + \frac{D}{1-\lambda} \right) (1-\tau) + (1-\beta) \frac{B}{1-\lambda} + S^o = C^r = WN^r (1-\tau) + S^r \tag{49}$$

Therefore

$$S^o = -\frac{\lambda}{1-\lambda} \left(D(1-\tau) + (1-\beta) B \right) \tag{50}$$

and

$$S^r = -\frac{(1-\lambda)}{\lambda} S^o \tag{51}$$

The SS government budget constraint can be written as

$$\tau Y = (1-\beta) B + G \tag{52}$$

Given that $\frac{C}{Y} = \gamma_c$,$\frac{G}{Y} = (1-\gamma_c)$ and that $\frac{B}{Y} = \gamma_b$ we can rewrite the last equation as

$$\tau = (1-\beta)\,\gamma_b + (1-\gamma_c) \tag{53}$$

Combining the fact that at SS $Y = N$ with the SS optimal labour supply it yields

$$\theta\gamma_c\,(N)^{\varphi+1} = W\,(1-\tau) \tag{54}$$

After rearranging, the latter yields the SS level of labour supply

$$Y = N = \left(\frac{W\,(1-\tau)}{\theta\gamma_c}\right)^{\frac{1}{\varphi+1}} \tag{55}$$

Consequently

$$\begin{aligned} G &= (1-\gamma_c)\,Y \\ C &= \gamma_c Y \end{aligned}$$

These equations give us to have a full description of the SS variables. A few points are worth mentioning. First of all the SS level of output is inefficient. This is due to the monopolistic competition nature of the good markets, to the distortive nature of fiscal policy taxation and to an (inefficiently) high level of public expenditure. From (55) the negative correlation between tax rate and output is clear. An increase in public spending or *ceteris paribus* an increase in public debt requires a higher level of tax rate (53) and this, in turn, lowers the level of output.

Appendix B

For determinacy purposes the dynamic system can be represented as follow

$$\pi_t = \beta E_t \pi_{t+1} + \kappa\left(\left(\frac{1}{\gamma_c} + \varphi\right)\widehat{Y}_t + \frac{\tau}{1-\tau}\tau_t\right) \tag{56}$$

$$\tau_t = \delta_1 \widehat{B}_t + (\delta_2 - 1)\widehat{Y}_t \tag{57}$$

$$\widehat{Y}_t = E_t \widehat{Y}_{t+1} - \left(\frac{1}{\gamma_c} - \varphi\frac{\lambda}{1-\lambda}\right)^{-1}\left(\widehat{R}_t - E_t\pi_{t+1}\right) \tag{58}$$

$$\widehat{R}_t = \phi\pi_t \tag{59}$$

$$\widehat{B}_{t+1} = \widehat{R}_t + \frac{1}{\beta}\left(\widehat{B}_t - \pi_t - \frac{\tau Y}{B}\left(\widehat{Y}_t + \tau_t\right)\right) \tag{60}$$

As one can notice, public spending has been omitted from these sets of equations. The reason is that $\widehat{G}_t$ follow a stationary AR(1) process and therefore does not affect the determinacy conditions.

Substituing (59) into (58) and (57) into (56) and (60), one obtains a system of three equations.

$$\pi_t = \beta E_t \pi_{t+1} + \kappa(\left(\frac{1}{\gamma_c} + \varphi + \frac{\tau}{1-\tau}\delta_2^*\right)\widehat{Y}_t + \frac{\tau}{1-\tau}\delta_1 \widehat{B}_t)$$

$$\widehat{B}_{t+1} = \phi\pi_t + \frac{1}{\beta}\left(\left(1 - \frac{\tau Y}{B}\delta_1\right)\widehat{B}_t - \pi_t - \frac{\tau Y}{B}\left(1 + (\delta_2 - 1)\right)\widehat{Y}_t\right)$$

$$\widehat{Y}_t = E_t \widehat{Y}_{t+1} - \left(\frac{1}{\gamma_c} - \varphi\frac{\lambda}{1-\lambda}\right)^{-1}(\phi\pi_t - E_t\pi_{t+1})$$

In matrix form

$$\begin{pmatrix} \beta & 0 & 0 \\ \left(\frac{1}{\gamma_c} - \varphi\frac{\lambda}{1-\lambda}\right)^{-1} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} E_t\pi_{t+1} \\ E_t\widehat{Y}_{t+1} \\ \widehat{B}_{t+1} \end{pmatrix} = \begin{pmatrix} 1 & -\kappa\left(\frac{1}{\gamma_c} + \varphi + \frac{\tau}{1-\tau}(\delta_2 - 1)\right) & -\frac{\tau}{1-\tau}\delta_1 \\ \left(\frac{1}{\gamma_c} - \varphi\frac{\lambda}{1-\lambda}\right)^{-1}\phi & 1 & 0 \\ \phi - \frac{1}{\beta} & -\frac{1}{\beta}\left(1 + (\delta_2 - 1)\right) & \frac{1}{\beta}\left(1 - \frac{\tau Y}{B}\delta_1\right) \end{pmatrix}\begin{pmatrix} \pi_t \\ \widehat{Y}_t \\ \widehat{B}_t \end{pmatrix}$$

Recalling

$$H = \begin{pmatrix} \beta & 0 & 0 \\ \left(\frac{1}{\gamma_c} - \varphi\frac{\lambda}{1-\lambda}\right)^{-1} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}^{-1} \times \begin{pmatrix} 1 & -\kappa\left(\frac{1}{\gamma_c} + \varphi + \frac{\tau}{1-\tau}(\delta_2 - 1)\right) & -\frac{\tau}{1-\tau}\delta_1 \\ \left(\frac{1}{\gamma_c} - \varphi\frac{\lambda}{1-\lambda}\right)^{-1}\phi & 1 & 0 \\ \phi - \frac{1}{\beta} & -\frac{1}{\beta}\left(1 + (\delta_2 - 1)\right) & \frac{1}{\beta}\left(1 - \frac{\tau Y}{B}\delta_1\right) \end{pmatrix}$$

This system has two non-predetermined variables $\left\{\pi_t, \widehat{Y}_t\right\}$ and one predetermined variable $\left\{\widehat{B}_t\right\}$. Following Blanchard and Khan (1981), determinacy requires for H to have two eigenvalues outside the unit circle and one inside the unit circle.

Appendix B

This section presents the results of the Gali' et Al.(2007) model with no capital perfectly competitive labour market. *Ceteris paribus,* the notation is equivalent to the model presented above.

OTPC

Ricardian consumers behave like a traditional optimizer agent. Therefore they maximizes the expected utility

$$E_t \sum_{t=0}^{+\infty} \beta^t u\left(C_t^o, N_t^o\right) \tag{61}$$

subject to the sequence of budget constraints

$$P_t C_t^o + R_t^{-1} B_{t+1} = W_t P_t N_t^o + B_t + D_t - P_t T_t^o \tag{62}$$

The FOC's with respect consumption and labour are

$$1 = R_t \beta E_t \left(\frac{C_t^o}{C_{t+1}^o} \frac{P_t}{P_{t+1}} \right) \tag{63}$$

and

$$W_t = C_t^o (N_t^o)^{\varphi} \tag{64}$$

As in section 2

$$C_t^{r,o} = \left(\int_0^1 C_t^{r,o}(j)^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{65}$$

defining the price index as

$$P_t = \left(\int_0^1 P_t(j)^{1-\varepsilon} \, dj \right)^{\frac{1}{1-\varepsilon}} \tag{66}$$

The demand for each good is

$$C_t^{r,o}(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon} C_t^{r,o} \tag{67}$$

ROTC

Their utility maximization reduces to a static problem

$$U_t = \log C_t^r - \frac{(N_t^r)^{1+\varphi}}{1+\varphi} \tag{68}$$

subject to

$$P_t N_t^r W_t = P_t C_t^r + P_t T_t^r \tag{69}$$

so that the level of consumption is equal to the net real wage

$$C_t^r = W_t N_t^r - T_t^r \tag{70}$$

with

$$T_t^r \neq T_t^o \tag{71}$$

Furthermore as for the Ricardian consumer the labour supply must satisfy:

$$W_t = C_t^r \left(N_t^r\right)^{\varphi} \tag{72}$$

Aggregation

Aggregate consumption follows

$$C_t = \lambda C_t^r + (1-\lambda) C_t^o \tag{73}$$

and aggregate labour supply

$$N_t = \lambda N_t^r + (1-\lambda) N_t^o \tag{74}$$

and the total government revenues from tax collection

$$T_t = \lambda T_t^r + (1-\lambda)\, T_t^o \tag{75}$$

and the total output

$$Y_t = C_t + G_t \tag{76}$$

Monetary Policy

We assume the monetary authority fix the nominal interest rate in each period following the rule

$$r_t = r + \phi_\pi \pi_t \tag{77}$$

where $r_t \equiv R_t - 1$, r is the steady state interest rate, and $\phi_\pi > 1$ for the Taylor principle.

Fiscal Policy

The government budget constraint is

$$P_t T_t + R_t^{-1} B_{t+1} = B_t + P_t G_t \tag{78}$$

now letting $\widehat{T}_t = (T_t - T)/Y$, $\widehat{G}_t = \frac{(G_t - G)}{Y}$ and $\widehat{B}_t = \left(\frac{B_t}{P_{t-1}} - \frac{B}{P}\right)/Y$ we assume a simple fiscal rule of the type

$$t_t = \phi_b \widehat{B}_t + \phi_g \widehat{G}_t \tag{79}$$

with $\phi_b$and ϕ_g positive.

Furthermore we assume that the variable $\widehat{G}_t$ follow an AR(1) process of the type

$$\widehat{G}_t = \rho_g \widehat{G}_{t-1} + \zeta_t \tag{80}$$

with 0< $\rho_g < 1$ and ζ_t following a i.i.d. behavior.

Linearized Version of the Model

The Euler equation for the OPTC can be rewritten as deviation from the steady state as

$$\widehat{C}_t^o = E_t \widehat{C}_{t+1}^o - \left(\widehat{R}_t - E_t \pi_{t+1}\right) \tag{81}$$

while the consumption of the rule of thumber is given up to a first order approximation by[12]

$$\widehat{C}_t^r = \left(\frac{WN^r}{C^r}\right)\left(\widehat{W}_t + \widehat{N}_t^r\right) - \left(\frac{Y}{C^r}\right)\widehat{T}_t^r \tag{82}$$

The aggregation

$$\widehat{C}_t = \lambda \widehat{C}_t^r + (1-\lambda)\widehat{C}_t^o \tag{83}$$

$$\widehat{N}_t = \lambda \widehat{N}_t^r + (1-\lambda)\widehat{N}_t^o \tag{84}$$

and after some algebra

$$\widehat{W}_t = \widehat{C}_t + \varphi \widehat{N}_t \tag{85}$$

$$\widehat{MC}_t = \widehat{W}_t \tag{86}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa(\widehat{C}_t + \varphi \widehat{N}_t) \tag{87}$$

With few iteractions the aggregate Euler equation becomes

$$\widehat{C}_t = E_t \widehat{C}_{t+1} - a_1\left(\widehat{R}_t - E_t \pi_{t+1}\right) - a_2 E_t \Delta \widehat{N}_{t+1} + a_3 E_t \Delta \widehat{T}_t^r \tag{88}$$

where $a_1 = (1-\lambda)\left(\frac{\Gamma}{\Gamma - \lambda(\omega + \omega\varphi)}\right)$, $a_2 = \frac{\lambda\omega\varphi(1+\varphi)}{\Gamma - \lambda(\omega+\omega\varphi)}$, $a_3 = \frac{\lambda(\mu\varphi)}{\Gamma-\lambda(\omega+\omega\varphi)}$, γ_c is the ratio of consumption over total output in steady state and $\Gamma = (\mu\gamma_c\varphi + \omega)$.The Production function

$$\widehat{Y}_t = \widehat{N}_t \tag{89}$$

[12]The derivation of the rule of thumb consumer as deviation from the steady state is in the Appendix

the market clearing condition

$$\widehat{Y}_t = \gamma_c \widehat{C}_t + \widehat{G}_t \tag{90}$$

Log linearizing eq (29) around the steady state with zero debt and a balanced primary budget we obtain

$$\widehat{B}_{t+1} = (1+\rho)\left(\widehat{B}_t + \widehat{G}_t + \widehat{T}_t\right) \tag{91}$$

where $\rho = \beta^{-1} - 1$ represents the steady state interest rate. Plugging the previous expression into the fiscal rule assumed above it yields

$$\widehat{B}_{t+1} = (1+\rho)(1-\phi_b)\,\widehat{B}_t + (1+\rho)\left(1-\phi_g\right)\widehat{G}_t \tag{92}$$

In order to have a stationary debt dynamic $(1+\rho)(1-\phi_b) < 1$ or equivalently

$$\phi_b > \frac{\rho}{\rho+1}$$

The dynamic system is then represented by (87),(88),(89),(90) and (92). In Table 1 are reported the parameter values of the calibration of the Gali' et Al(2007) baseline model.

Table 2: Calibration

Parameter	**Value**	**Description**
β	0.99	Discount factor
ρ_g	0.9	Shock persistence
ϕ_π	1.5	Monetary parameter
ϕ_b	0.2	Public debt fiscal parameter
ϕ_g	0.13	Public spending fiscal parameter
φ	0.2	Frisch inverse elasticity of labour supply
θ	0.75	Degree of price stickiness
μ	1.23	Steady state mark up
λ	0.5	Share of ROTC
γ_c	0.75	Steady state share of G
ρ	0.01	Steady state interest rate

Appendix C

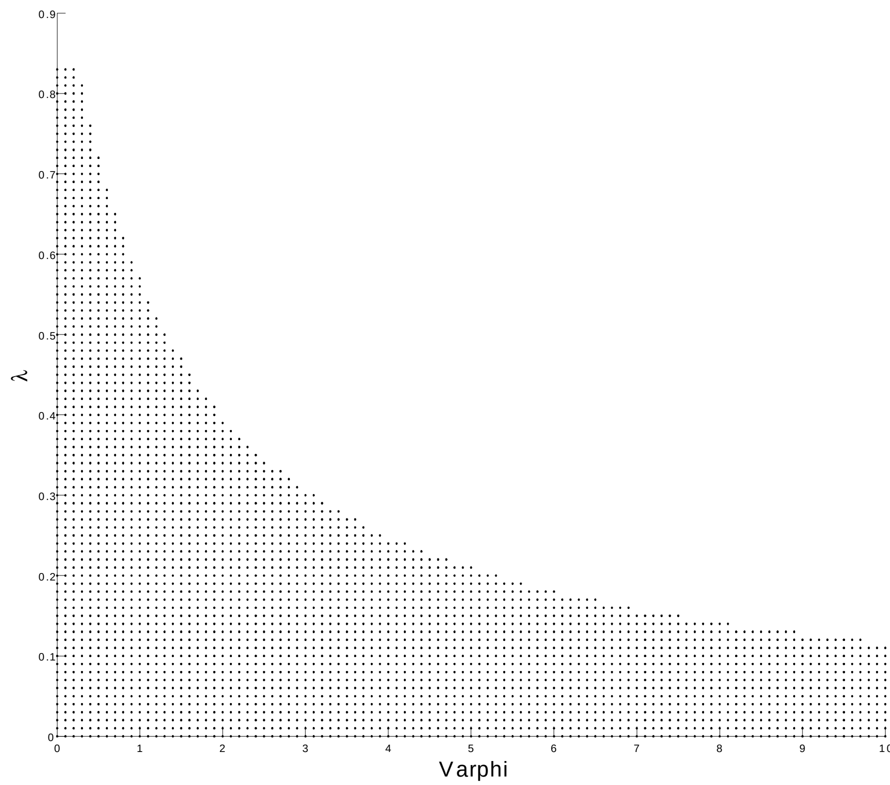

Figure 1: Combination of λ and φ that makes the economy non Keynesian (Black spot: Keynesian economy, white area: non-Keynesian economy).

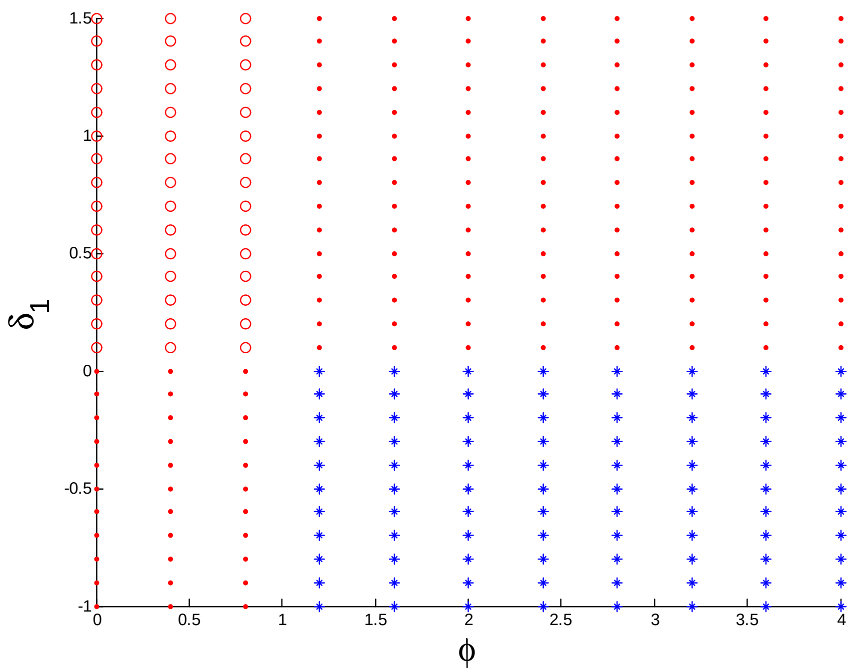

Figure 2: Determinacy area in the Keynesian side of the economy. (spot: determinacy, star: 1st order indeterminacy,circle:1st order instability, diamonds:2nd order instability)

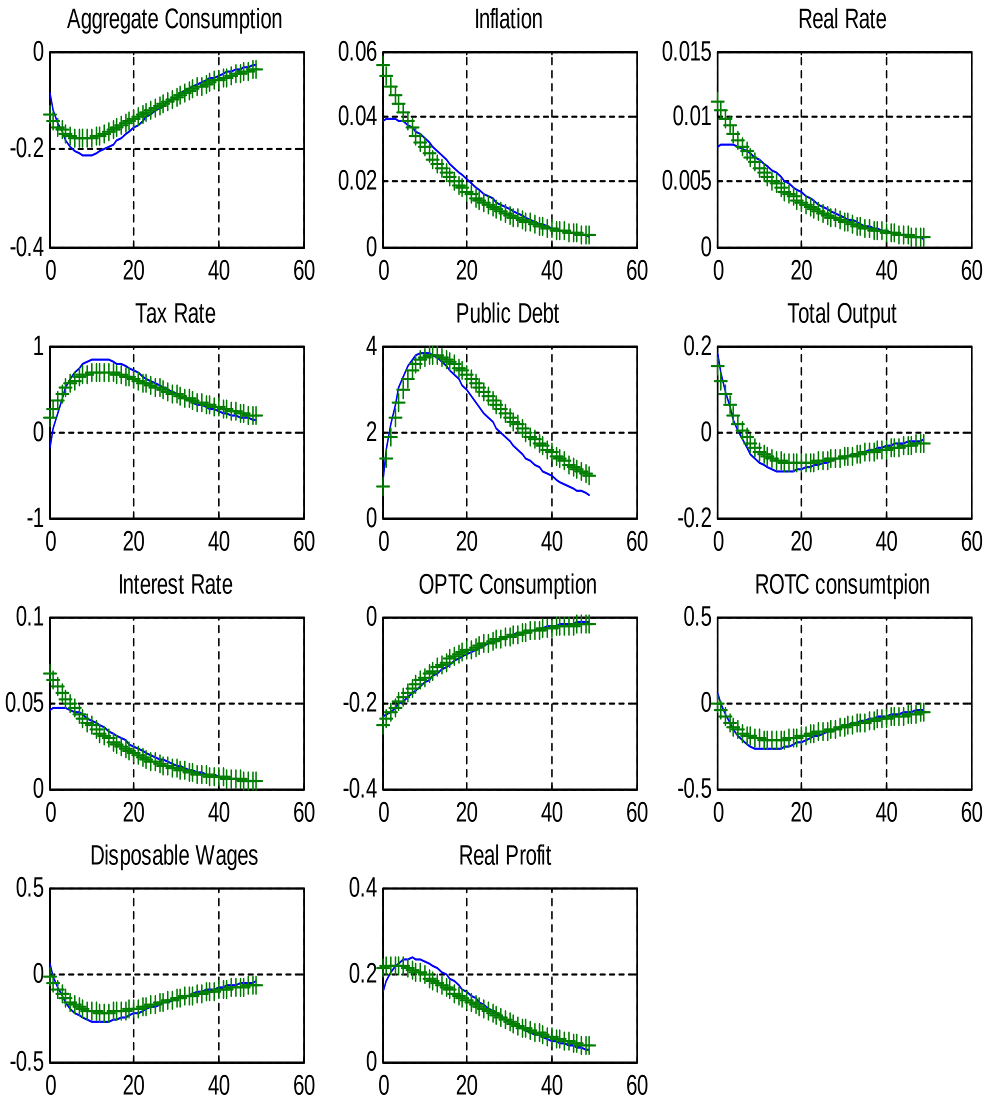

Figure 3: IRF to a SD Public Spending Shock with active monetary policy ($\phi = 1.2$) and passive fiscal policy($\delta_1 = 0.2$). Solid line $\delta_2^* = -1$, crosses $\delta_2^* = 1$.Horizontal axis: quarters.

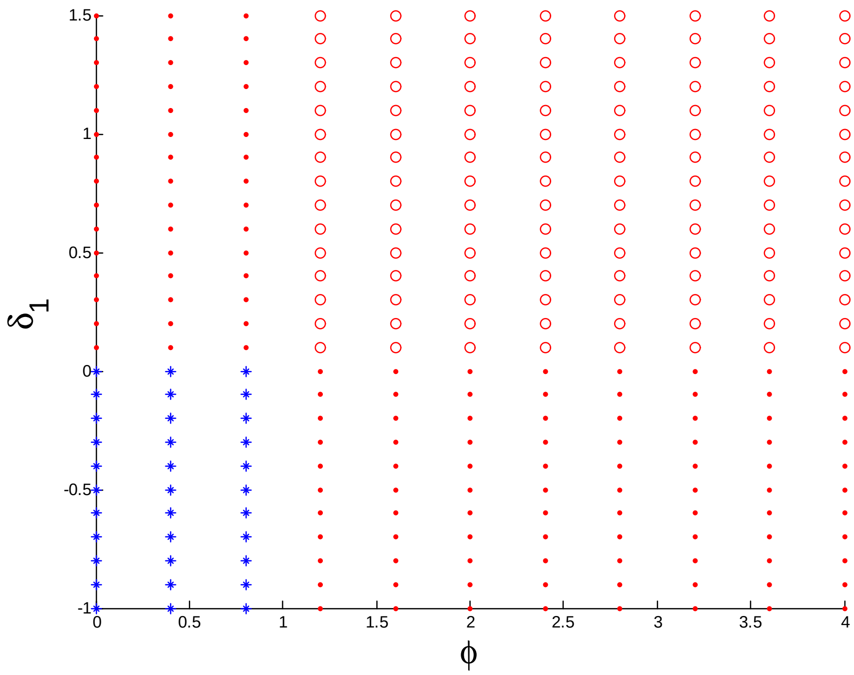

Figure 4: Determinacy area in the non-Keynesian side of the economy. $\delta_2 = 0$. (spot: determinacy, star: 1st order indeterminacy,circle:1st order instability, diamonds:2nd order instability)

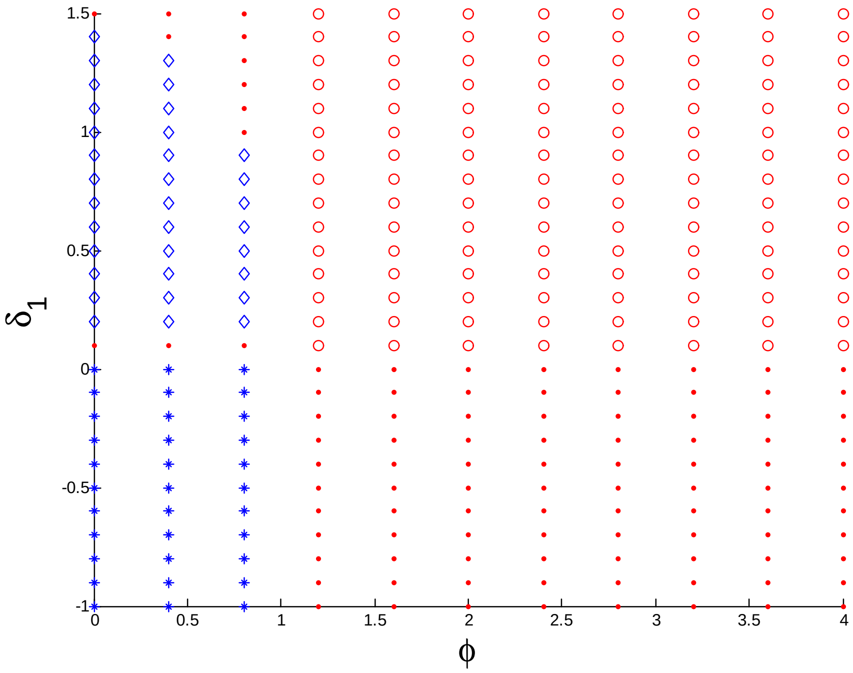

Figure 5: Determinacy area in the non-Keynesian side of the economy. $\delta_2 = 2$. (spot: determinacy, star: 1st order indeterminacy,circle:1st order instability, diamonds:2nd order instability)

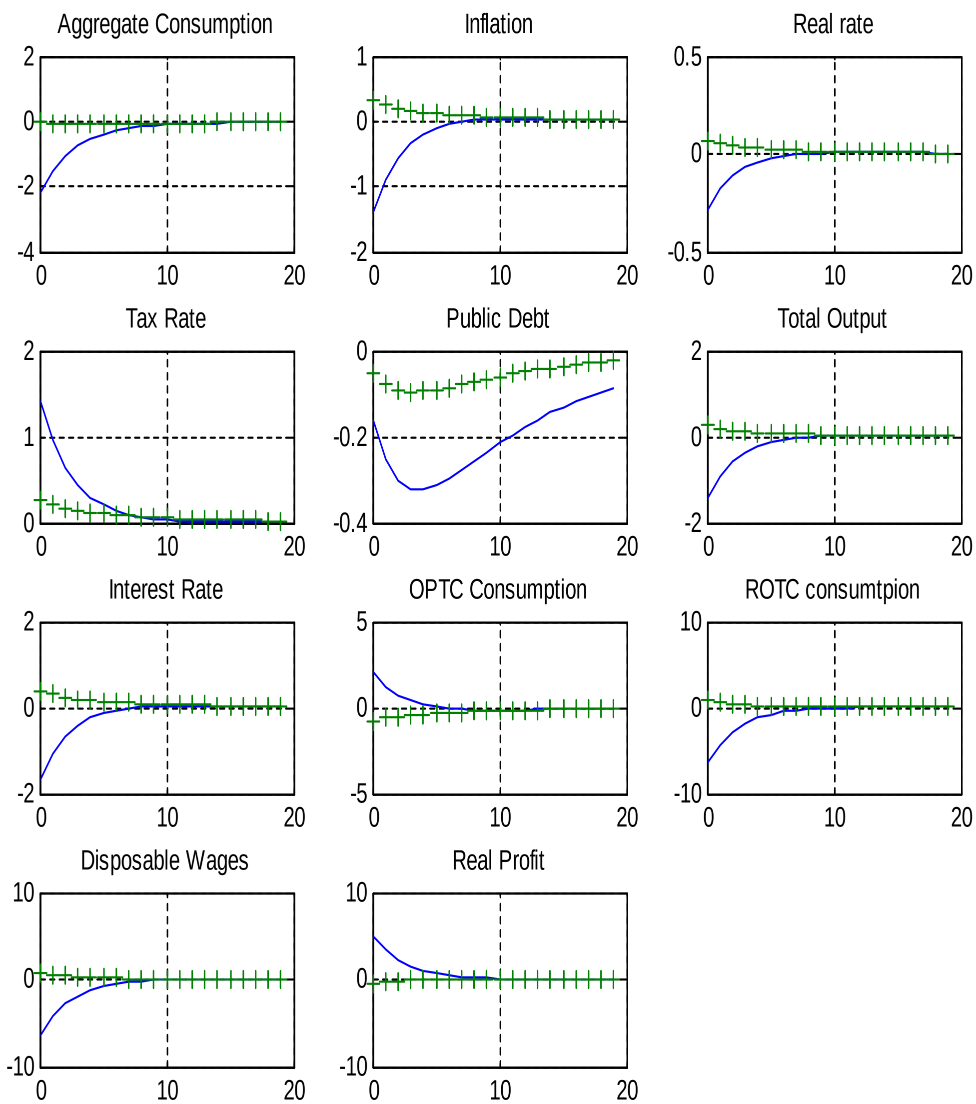

Figure 6: IRF to a SD Public Spending Shock with active monetary policy ($\phi = 1.2$) and active fiscal policy($\delta_1 = -0.2$). Solid line $\delta_2^* = -1$,crosses $\delta_2^* = 1$.Horizontal axis: quarters.

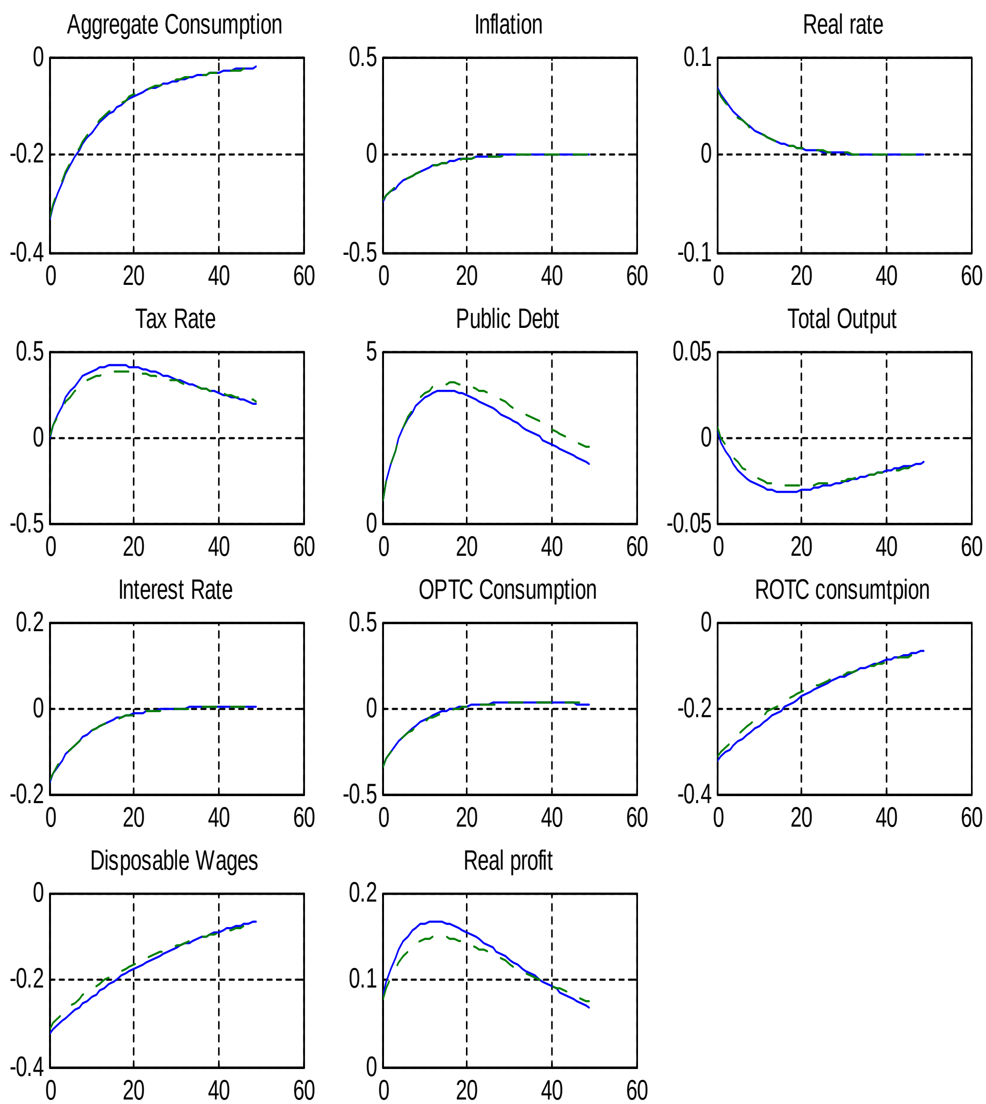

Figure 7: IRF to a SD Public Spending Shock with passive monetary policy ($\phi = 0.8$) and passive fiscal policy($\delta_1 = 0.2$). Solid line $\delta_2^* = -1$, dashed line $\delta_2^* = 1$.Horizontal axis: quarters.

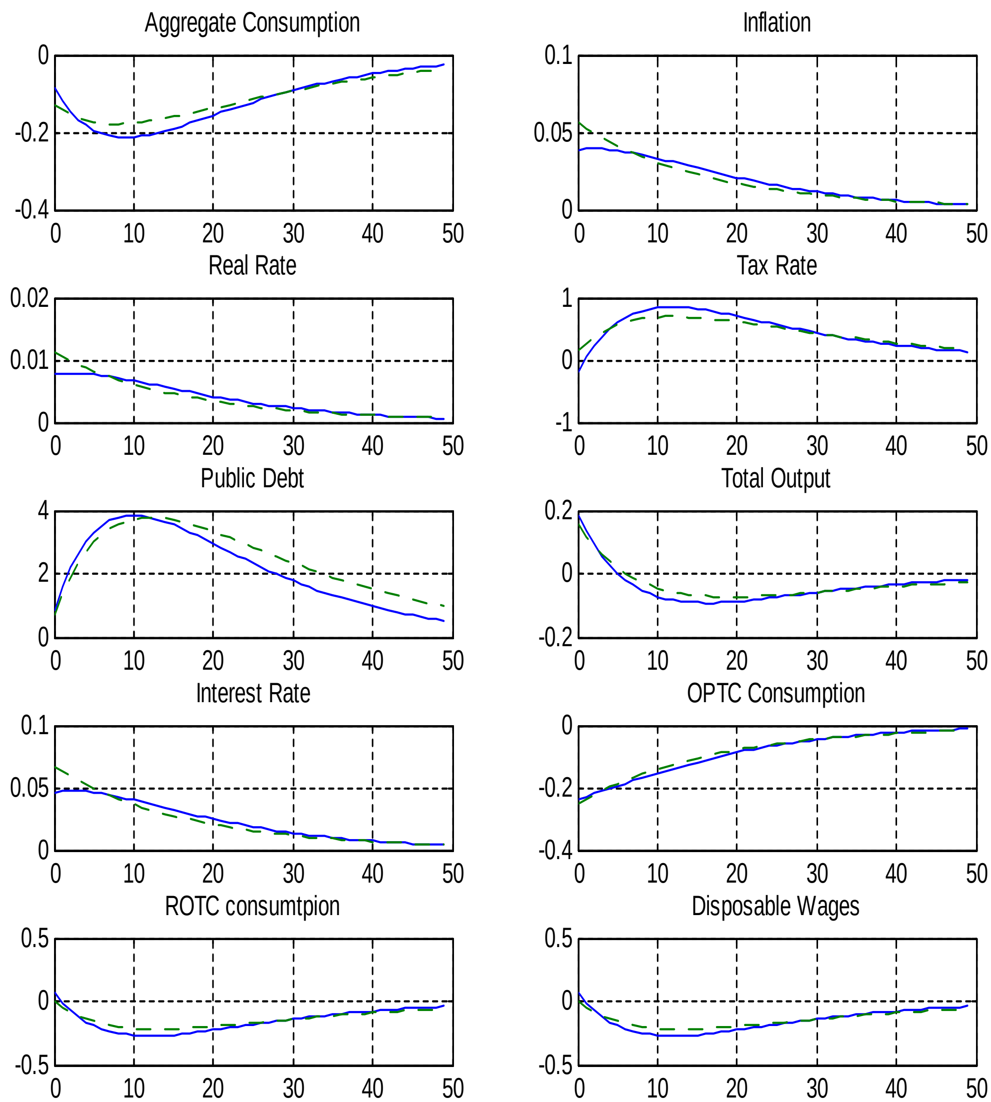

Figure 8: IRF to a SD government spending shock of the baseline model with distortive taxation and Galí *et al.*(2007) parameters values. Solid line $\delta_2^* = 1$, dashed line $\delta_2^* = -1$.Horizontal axis: quarters.

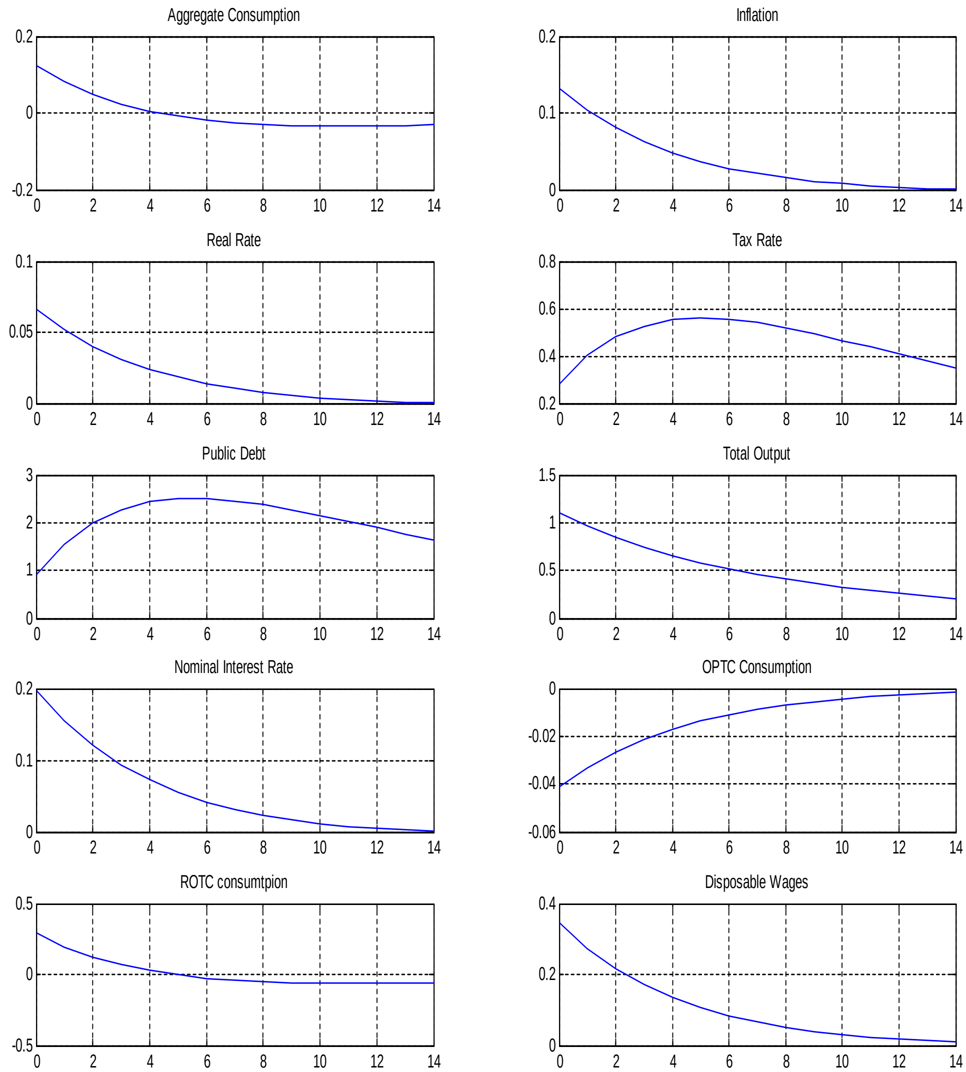

Figure 9: IRF to a SD government spending shock in the Galí *et al.* (2007) model. Horizontal axis: quarters.