A DEFORMATION MODEL OF FLEXIBLE, HIGH AREA-TO-MASS RATIO DEBRIS FOR ACCURATE PROPAGATION UNDER PERTURBATION

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Outline

- Background
- Objective
- The model
- Simulation
- Results
- Conclusion and Future work
**Space Debris**

**Space debris**
- Artificial debris and natural debris.
- Orbit with hypervelocity that can threat to active spacecraft leading to catastrophic break-ups generating new space debris.
- Need to reduce the number of debris.

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**Experiment**

**Gravity movie**
Discover HAMR objects

Collision and explosion

- Fengyun1-C

Discovery in 2004 (GEO)

- High area-to-mass ratio (HAMR) objects
- Variations of light curves
- Variations of area-to-mass ratio (AMR)

- Iridium 33 and Cosmos 2251

(Collision and explosion (Anselmo, L. and C. Pardini, 2010)

Suspected objects

Experiment

- Shot micro satellite model

(Murakami, J., et al 2008)

Multi-layer insulation
Objective

1. Develop a model of a thin, highly flexible MLI-type membrane, in terms of multi-body dynamics, and solved by using fundamental Newtonian mechanics.

2. Investigate the orbital dynamics under J2 and the luni-solar third body gravitation and solar radiation pressure (SRP) by comparing with rigid body case.

3. Investigate a self-shadowing effect to the orbital dynamics of flexible debris.
flexible model
The flexible model

Flat plate

Consider deformation in 2 D of x-y plane
Flat plate

Dimension
1 x 1 square meter
l₁ = l₂ = 0.5 (m)

Torsional Damper

Torsional spring
Simplification

Triangular shape

Torsional Damper

Torsional spring
Multibody dynamics

Newtonian equation

\[ F_i + T_i + F_{s,i} + F_{d,i} = m_i a_i \]

Where \( i \) = mass of each (1, 2 and 3)

Spring and damper forces

\[ F_s = k_s \theta \quad F_d = c \dot{\theta} \]

\[ k_s = \frac{E I}{\text{Length}} \quad c = DF \sqrt{M k_s} \]

Where
- \( k_s \) = rotational spring constant,
- \( \theta \) = angle of deformation
- \( \dot{\theta} \) = angular velocity of the deformation,
- \( E \) = young modulus
- \( I \) = the moment of inertia of thin plate
- \( \text{Length} \) = the length of each rod and is
- \( c \) = Coefficient of torsion spring (N.m rad\(^{-1}\))
- \( DF \) = dissipation factor of material
- \( M \) = mass of rod (Kg)

Constrained equation

\[ (x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2 + (z_{i+1} - z_i)^2 = l_i^2 \]

- Length of a rod = 0.5 m
Simulation of the model
Initial shape to test the model

- First plane
- Second plane

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S. Channumsin
The simulation results without external force and damper by activating torsional spring
The simulation with torsional spring and damper
Orbital dynamics and Perturbation
The modified equinoctial elements

\[ \dot{p}_i = \frac{2p}{w} \sqrt{\frac{p}{\mu}} \Delta_{i,t} \]

\[ \dot{f}_i = \sqrt{\frac{p}{\mu}} [\Delta_{i,r} \sin L + [(w+1) \cos L + f] \frac{\Delta_{i,t}}{w} - (h \sin L - k \cos L) \frac{g \Delta_{i,n}}{w}] \]

\[ \dot{g}_i = \sqrt{\frac{p}{\mu}} [-\Delta_{i,r} \cos L + [(w+1) \cos L + g] \frac{\Delta_{i,t}}{w} - (h \sin L - k \cos L) \frac{f \Delta_{i,n}}{w}] \]

\[ \dot{h}_i = \sqrt{\frac{p}{\mu}} \frac{s^2}{2w} \cos L \Delta_{i,n} \]

\[ \dot{k}_i = \sqrt{\frac{p}{\mu}} \frac{s^2}{2w} \cos L \Delta_{i,n} \]

\[ \dot{L}_i = \sqrt{\mu p \left(\frac{w}{p}\right)^2 + \frac{1}{w}} \sqrt{\frac{p}{\mu}} (h \sin L - k \cos L) \Delta_{i,n} \]

Where

- \( \mu \) = gravitational constant
- \( \Omega \) = right ascension of ascending node degree
- \( e \) = eccentricity
- \( \omega \) = argument of perigee
- \( i \) = Inclination
- \( L \) = true longitude
- \( \nu \) = true anomaly
- \( a \) = semi-major axis(km)

\[ p = a(1 - e^2) \]

\[ h = \tan\left(\frac{i}{2}\right) \sin \Omega \]

\[ g = e \sin(\omega + \Omega) \]

\[ f = e \cos(\omega + \Omega) \]

\[ k = \tan\left(\frac{i}{2}\right) \sin \Omega \]

\[ L = \Omega + \omega + \nu \]
J2 Perturbations and Third body

J2 perturbations

\[
a_{j2,l} = \frac{\partial R_2}{\partial x_I} = -\frac{3\mu J_2 R^2 x_I}{2x^5} \left(1 - \frac{5x_k^2}{x^2}\right)
\]

\[
a_{j2,J} = \frac{\partial R_2}{\partial x_J} = -\frac{3\mu J_2 R^2 x_J}{2x^5} \left(1 - \frac{5x_K^2}{x^2}\right)
\]

\[
a_{j2,K} = \frac{\partial R_2}{\partial x_K} = -\frac{3\mu J_2 R^2 x_K}{2x^5} \left(3 - \frac{5x_K^2}{x^2}\right)
\]

The third body

\[
\ddot{a}_k = -G \sum_{k=1,2} M_k \left[ \frac{\ddot{x} - \dddot{x}_k}{|\dddot{x} - \dddot{x}_k|^3} + \frac{\dddot{x}_k}{\dddot{x}_k^3} \right]
\]

Where \( k = 1 \) and 2 (Sun and Moon)
Solar radiation pressure force

\[ F_{rad} = \sum_{i=1}^{N} \frac{p_{sr} A_i \cos \varnothing}{c} \left\{ 2 \left( \frac{C_{Rd_i}}{3} + C_{Rs_i} \cos(\varnothing_{inc_i}) \right) \hat{n} + (1 - C_{Rs_i}) \hat{s} \right\} \]
Average solar radiation pressure

Rigid body case

Average SRP force

\[ F_{\text{avg}} = \frac{1}{4\pi} \int_0^\pi \int_0^{2\pi} \vec{F}_{\text{rad}, j} \, d\lambda_s \, d\delta_s \]

Equivalent area

\[ A_{\text{eq}} = \frac{F_{\text{avg}}}{P_{SP}(R)} \]

Where

\[ P_{SP}(R) = \frac{E}{C} \frac{A_\oplus^2}{|\vec{x}_i - \vec{x}_\oplus|^2} \]

Therefore

\[ F_{\text{AVG}} = -A_{\text{eq}} P_{SP}(R) \frac{\vec{x}_i - \vec{x}_\oplus}{|\vec{x}_i - \vec{x}_\oplus|} \]
**Self-shadowing**

\[ p = l - \frac{d + n \cdot l}{n \cdot (v - l)} (v - l) \]

or \( P = Mv \)

\[
M = \begin{bmatrix}
  n.l + d - l_x n_x & -l_x n_y & -l_x n_z & -l_x d \\
  -l_y n_x & n.l + d - l_y n_y & -l_y n_z & -l_y d \\
  -l_z n_x & -l_z n_y & n.l + d - l_z n_z & -l_z d \\
  -n_x & -n_y & -n_z & n.l
\end{bmatrix}
\]

Where

\( p \) = the projection of vertex \( v \)

\( v \) = Vertex on the plane: \( n \cdot x + d = 0 \)

\( l \) = a location of light source

The planar shadow projection, the original technique invented by Blinn [15], allows shadows to be cast on plane surface
Self-shadowing

a) Change position of light source

b) Change the geometry of debris
Simulation
## Material properties

<table>
<thead>
<tr>
<th>Material type</th>
<th>AMR [m²/kg]</th>
<th>Young’s Modulus [N/m²]</th>
<th>Cs, Cd, Ca</th>
</tr>
</thead>
<tbody>
<tr>
<td>PET coated</td>
<td>111.11</td>
<td>8.81x10⁹</td>
<td>0.60 0.26 0.14</td>
</tr>
<tr>
<td>Kapton coated</td>
<td>26.30</td>
<td>2.50x10⁹</td>
<td>0.60 0.26 0.14</td>
</tr>
<tr>
<td>uncoated</td>
<td>26.30</td>
<td></td>
<td>0.00 0.10 0.90</td>
</tr>
</tbody>
</table>

(Sheldahl, *The red book* (2012))
Initial position

Geosynchronous Earth orbit (GEO)

<table>
<thead>
<tr>
<th>Six element</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semi-major axes (km)</td>
<td>42,164</td>
</tr>
<tr>
<td>Mean anomaly (degree)</td>
<td>270°</td>
</tr>
<tr>
<td>Argument of perigee (degree)</td>
<td>90°</td>
</tr>
<tr>
<td>Ascending node (degree)</td>
<td>60°</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>0.0001</td>
</tr>
<tr>
<td>Inclination (degree)</td>
<td>5°</td>
</tr>
</tbody>
</table>

Propagation in 12 days
J2 and SRP
PET without self-shadowing

10 minutes

Graphs showing data over time.
PET with self-shadowing

10 minutes
Comparison

PET without self-shadowing

PET with self-shadowing
Orbital dynamics under J2, third body and SRP
Orbital dynamics 12 days

Graphs showing the change in eccentricity and inclination over 12 days.
PET Euler angles

1st panel

2nd panel
Kapton Euler angles

1st panel

2nd panel
Conclusion and Future work
1. Orbital dynamics of flexible debris is different from that of rigid debris due to the effective area.

2. Direct solar radiation pressure is the most effect to the orbital dynamics of HAMR flexible model.

3. Self-shadowing effect lead to irregular attitude dynamics and deformation
Future work

To set the deformation experiment to validate the flexible model

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Space Glasgow

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Thank you

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