News Shocks and Business Cycles: Bridging the Gap from Different Methodologies*

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Abstract

A significant challenge faced by the news driven view of the business cycle formalized by Beaudry and Portier (2004), is the lack of agreement between different—VAR and DSGE—methodologies over the empirical plausibility of this view. We show that VAR and DSGE methodologies provide a broadly consistent assessment of the empirical relevance of news shocks once we augment a standard DSGE model with a financial channel that provides amplification to news shocks. Both methodologies suggest news shocks to the future growth prospects of the economy to be significant drivers of U.S. business cycles in the post-Greenspan era (1990-2011), explaining as much as 50% of the forecast error variance in hours worked in cyclical frequencies.

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1 Introduction

Motivated by the U.S. investment boom–bust episode of the 1990s, news shocks about future total factor productivity (TFP) have been proposed as a potentially important source of fluctuations (Beaudry and Portier (2004), Jaimovich and Rebelo (2009)). However, conflicting estimates in the literature, question the empirical plausibility of the “news” view of fluctuations. In the context of Vector autoregressive (VAR) methodologies, Beaudry and Portier (2006) and Beaudry and Lucke (2010) find that TFP news shocks are important drivers of business cycles, while Barsky and Sims (2011) and Forni et al. (2012) find they are not. The estimated DSGE methodology (Fujiwara et al. (2011), Khan and Tsoukalas (2012), Schmitt-Grohe and Uribe (2012)), find them to be negligible sources of fluctuations. In this paper we show that in the post–Greenspan era (1990-2011), different empirical methodologies, namely DSGE and VAR, yield a unified answer that provides strong support for the “news” view. The essential element is a strong link between financial markets and real activity that results in amplification of news shocks.

The financial channel we favor is one with leveraged lenders a-la Gertler and Karadi (2011) and Gertler and Kiyotaki (2010) introduced in a two sector New Keynesian (NK) model that has been shown in a companion paper to account well for the dynamics of U.S. business cycles (see Görtz and Tsoukalas (2015)). The model features a final goods (consumption) and a capital goods (investment) sector with (different) sector specific technologies. The final goods sector buys goods from the capital goods sector, acting as a demand source for the latter. Following the anticipation of a future permanent increase in its own TFP, the final goods (consumption) sector demands capital goods from the investment sector, and the latter responds by hiring more hours worked to satisfy demand, bidding up the price of investment goods and the price of capital. Financing the demand for capital is facilitated by intermediaries which face a leverage constraint tied to their equity. Intermediaries earn an excess return from holding capital, an object
we measure with the corporate bond spread. A higher price of capital boosts equity capital, relaxes the constraint and stimulates more lending, in turn providing a source of financial amplification. Procyclical capital prices are thus key to the amplification since they imply equity gains and a strong lending boom. In the model, the corporate bond spread declines, and activity rises following an anticipated change in the future productivity of capital. This transmission, in which investment demand drives the cycle, is strongly favored when the model is taken to the data.\(^1\).

Our main objective is to investigate whether the proposed financial channel can bring in line DSGE and VAR methodologies over the empirical significance of TFP news shocks. To accomplish this, we undertake three comparisons. First, a comparison of the DSGE based and VAR based impulse response functions to a TFP news shock. Both methodologies predict a statistically significant expansion of hours, consumption, output and investment and a decline in the corporate bond spread to an expected future TFP improvement. Second, we investigate whether the empirical VAR responses following the news shock, can be replicated by responses of VARs estimated on artificial model samples (assuming the DSGE model to be the data generating process), and we find this to be the case for the majority of the empirical VAR responses examined. Third, a comparison of the shares in forecast error variance of key macro aggregates accounted for by the TFP news shock. We find those shares to be quite similar across the two methodologies, strikingly so in the case of output and hours. These findings suggest both methodologies produce a consistent assessment of TFP news shocks that supports them as a significant driver of fluctuations in the post–1990 era.

Our paper contributes to the ongoing debate on the importance of news shocks for aggregate fluctuations and highlights a new—financial—channel that can provide recon-

\(^1\)This transmission is consistent with the traditional “news” view of fluctuations formalized by Beaudry and Portier (2004). This amplification is missing from standard NK (see Christiano et al. (2008), Khan and Tsoukalas (2012)) or RBC models (e.g. Jaimovich and Rebelo (2009)), even though the latter can in theory produce the traditional news driven business cycle, characterized by the comovement of macro aggregates in response to a news shock.
ciliation between DSGE and VAR methodologies over the empirical assessment of news shocks.

The rest of the paper is organized as follows. Section 2 describes the model economy. Section 3 describes the estimation methodology, data, and briefly discusses estimation results. Section 4 discusses the relation with VAR–based findings. Section 5 concludes.

2 The Two Sector Model

In this section we provide a basic overview of the two sector model and abstract from the detailed description of parts that are standard in the literature. All details are presented in Appendix C.

The two sectors in the model produce consumption and investment goods. The latter are used as capital inputs in each sectors’ production process, while the former enter only into households utility functions. Households consume, save in interest bearing deposits and supply labor on a monopolistically competitive labor market. A continuum of sector specific intermediate goods firms produce distinct investment and consumption goods using labor and capital services. They are subject to sector specific Calvo contracts when setting prices. Capital producers use investment goods and existing capital to produce new sector specific capital goods. Leverage constrained financial intermediaries (as in Gertler and Karadi (2011)) collect deposits from households and finance capital acquisitions. A monetary policy authority controls the nominal interest rate.

2.1 Intermediate and final goods production

Intermediate goods in the consumption sector are produced by a monopolist according to the production function,

\[ C_t(i) = \max \left\{ A_t(L_{C,t}(i))^{1-a_c}(K_{C,t}(i))^{a_c} - A_t V_t^{\frac{a_{ec}}{1-\mu}} F_C; 0 \right\}. \]
Intermediate goods in the investment sector are produced by a monopolist according to the production function,

\[ I_t(i) = \max \{ V_t(L_{I,t}(i))^{1-a_{I}}(K_{I,t}(i))^{a_{I}} - V_t^{\frac{1}{1-a_{I}}} F_t; 0 \}, \]

where \( K_{x,t}(i) \) and \( L_{x,t}(i) \) denote the amount of capital services and labor services rented by firm \( i \) in sector \( x = C, I \) and \( a_c, a_i \in (0, 1) \) denote capital shares in production.\(^2\) The variables \( A_t \) and \( V_t \) denote the (non-stationary) level of TFP in the consumption and investment sector respectively, and \( z_t = \ln \left( \frac{A_t}{A_{t-1}} \right) \) and \( v_t = \ln \left( \frac{V_t}{V_{t-1}} \right) \) denote corresponding (stationary) stochastic growth rates of TFP. For ease of exposition, these latter processes, along with all other exogenous processes introduced in various parts of the model will be described in Section 2.6. The model includes sectoral nominal price rigidities as intermediate goods producers set prices according to Calvo (1983) contracts.

Final goods, \( C_t \) and \( I_t \), in the consumption and investment sector respectively, are produced by perfectly competitive firms combining a continuum—\( C_t(i) \) and \( I_t(i) \)—of intermediate goods, according to the technology,

\[ C_t = \left[ \int_0^1 (C_t(i))^{\frac{1}{1+\lambda_{C,t}}} \, di \right]^{1+\lambda_{C,t}}, \quad I_t = \left[ \int_0^1 (I_t(i))^{\frac{1}{1+\lambda_{I,t}}} \, di \right]^{1+\lambda_{I,t}}, \]

The elasticities \( \lambda_{C,t} \) and \( \lambda_{I,t} \) are the exogenous stochastic process of (sectoral) price markup over marginal cost. As is standard in NK models, prices of final goods, \( P_{C,t} \) and \( P_{I,t} \), are CES aggregates of intermediate good prices. Details about price setting are provided in Appendix C as this is standard in the literature.

### 2.2 Households

Households consist of two member types, workers (relative size \( 1-f \)) and bankers (relative size \( f \)). Workers supply (specialized) labor, indexed by \( j \), and earn wages while bankers

\(^2\)Fixed costs of production, \( F_C, F_I > 0 \), ensure that profits are zero along a non-stochastic balanced growth path and allow us to dispense with the entry and exit of intermediate good producers (Christiano et al. (2005)). The fixed costs are assumed to grow at the same rate as output in the consumption and investment sector to ensure that they do not become asymptotically negligible.
manage a financial intermediary. Both member types return their respective earnings back to the household. This set-up is identical to Gertler and Karadi (2011) except for the fact that workers have monopoly power in setting wages. The household maximizes,

\[
E_0 \sum_{t=0}^{\infty} \beta^t b_t \left[ \ln(C_t - hC_{t-1}) - \varphi \frac{(L_{C,t}(j) + L_{I,t}(j))^{1+\nu}}{1+\nu} \right], \quad \beta \in (0,1), \quad \varphi > 0, \quad \nu > 0,
\]

where \( E_0 \) is the conditional expectation operator, \( \beta \) is the discount factor and \( h \) is the degree of (external) habit formation. The inverse Frisch labor supply elasticity is denoted by \( \nu \), while \( \varphi \) is a free parameter which allows to calibrate total labor supply in the steady state.\(^3\) The variable \( b_t \) is a intertemporal preference shock. The household’s flow budget constraint (in consumption units) is,

\[
C_t + \frac{B_t}{P_{C,t}} \leq \frac{W_t(j)}{P_{C,t}}(L_{C,t}(j) + L_{I,t}(j)) + R_{t-1} \frac{B_{t-1}}{P_{C,t}} - \frac{T_t}{P_{C,t}} + \frac{\Psi_t(j)}{P_{C,t}} + \frac{\Pi_t}{P_{C,t}},
\]

where \( B_t \) is holdings of risk free bank deposits, \( \Psi_t \) is the net cash flow from household’s portfolio of state contingent securities, \( T_t \) is lump-sum taxes, \( R_t \) the (gross) nominal interest rate paid on deposits and \( \Pi_t \) is the net profit accruing to households from ownership of all firms. Notice above, the wage rate, \( W_t \), is identical across sectors due to perfect labor mobility. Household’s wage setting is subject to nominal rigidities as in Erceg et al. (2000). The desired markup of wages over the household’s marginal rate of substitution (or wage mark-up), \( \lambda_{w,t} \), follows an exogenous stochastic process.

### 2.3 Capital goods production

**Physical capital production.** Capital is sector-specific. Our assumption is motivated by evidence in Ramey and Shapiro (2001) who report significant costs of reallocating capital across sectors. Capital producers in sector \( x = C, I \) use a fraction of investment goods from final goods producers and undepreciated capital from capital services producers to

\(^3\)Consumption is not indexed by \( (j) \) because the existence of state contingent securities ensures that in equilibrium, consumption and asset holdings are the same for all households.
produce new capital goods, subject to investment adjustment costs (IAC) as proposed by Christiano et al. (2005). Solving their optimization problem yields a standard capital accumulation equation,

\[
\bar{K}_{x,t} = (1 - \delta_x)\xi^K_{x,t} \bar{K}_{x,t-1} + \left(1 - S\left(\frac{I_{x,t}}{I_{x,t-1}}\right)\right) I_{x,t}, \quad x = C, I, \tag{1}
\]

where \(\delta_x\) denotes the sectoral depreciation rate, \(S\left(\frac{I_{x,t}}{I_{x,t-1}}\right)\) denotes IAC, where \(S(\cdot)\) satisfies the following: \(S(1) = S'(1) = 0\), \(S''(1) = \kappa > 0\), and \(\xi^K_{x,t}\) is explained below.

**Capital services producers.** These agents purchase—using funds from financial intermediaries—physical capital from physical capital producers and transform it to capital services by choosing the utilization rate. They rent capital services—in perfectly competitive markets—to intermediate goods producers earning a rental rate equal to \(R_{K_{x,t}}/P_{C,t}\) per unit of capital. They sell the un-depreciated portion of capital at the end of period \(t + 1\) at price \(Q_{x,t+1}\) to physical capital producers.\(^5\) The utilization rate, \(u_{x,t}\), transforms physical capital into capital services according to

\[
K_{x,t} = u_{x,t} \xi^K_{x,t} \bar{K}_{x,t-1}, \quad x = C, I,
\]

and incurs a cost denoted by \(a_x(u_{x,t})\) per unit of capital. This function has the properties that in the steady state \(u = 1\), \(a_x(1) = 0\) and \(\chi_x \equiv \frac{a''_x(1)}{a'_x(1)}\), denotes the cost elasticity.

In the transformation above, we allow for a capital quality shock (as in Gertler and Karadi (2011)), \(\xi^K_{x,t}\). This disturbance shifts the demand for capital and directly affects its value—equivalently the value of assets held by intermediaries since they provide finance for capital acquisitions. For this reason we interpret it as a financial shock.\(^6\)

\(^4\)Sector specific capital implies that installed capital is immobile between sectors. Two sector models with sector specific capital include, among others, Boldrin et al. (2001), Ireland and Schuh (2008), Huffman and Wynne (1999) and Papanikolaou (2011). Limited factor mobility is shown to be able to correct many counterfactual predictions of one sector models with respect to both aggregate quantities and asset returns.

\(^5\)The price of capital, equivalent to Tobin’s marginal \(Q\), is \(Q_{x,t} = \frac{\Phi_{x,t}}{\Lambda_t}\), where \(\Lambda_t\), \(\Phi_{x,t}\), are the lagrange multipliers on the households’ budget constraint, and capital accumulation constraint respectively.

\(^6\)Other studies that consider this type of shock include for example Gourio (2012), Sannikov and Brunnermeier (2014), Gertler and Kiyotaki (2010) and Gertler et al. (2012).
These producers solve,

\[
\max_{u_{x,t+1}} \left[ \frac{R_{x,t+1}^K}{P_{C,t+1}} u_{x,t+1} x_{x,t+1} K_{x,t} - a_x(u_{x,t+1}) x_{x,t+1} K_{x,t} A_{t+1} V_{t+1}^{\kappa - 1} \right] \quad x = C, I.
\]

Total receipts of capital services producers in period \( t + 1 \) are equal to,

\[
R_{x,t+1}^B Q_{x,t} K_{x,t},
\]

with

\[
R_{x,t+1}^B = \frac{R_{x,t+1}^K x_{x,t+1} + Q_{x,t+1} \xi_{x,t+1} (1 - \delta_x) - a_x(u_{x,t+1}) x_{x,t+1} A_{t+1} V_{t+1}^{\kappa - 1}}{Q_{x,t}^{\kappa - 1}}, \tag{2}
\]

As in Gertler and Karadi (2011), capital services producers finance their purchase of capital at the end of each period with funds from financial intermediaries (to be described below). The stochastic return earned by financial intermediaries is denoted by \( R_{x,t+1}^B \) (for details of the derivation see Appendix C).

### 2.4 Financial sector

Financial intermediaries use deposits from households and their own equity capital to finance the acquisitions of physical capital by capital services producers. The financial sector in the model is a special case of Gertler and Kiyotaki (2010) where banks lend in specific islands (sectors) and cannot switch between them. Alternatively we can interpret the financial sector as a single intermediary with two branches, each specializing in providing financing to one sector only, where the probability of lending specialization is equal across sectors and independent across time. Each branch maximizes equity from financing the specific sector.\(^7\) Since we follow closely Gertler and Karadi (2011), we only briefly describe the essential mechanics (Appendix C provides all the equations). These can be described with three key equations. The balance sheet identity, the demand for

\(^7\)For example, within an intermediary there are divisions specializing in consumer or corporate finance.
assets that links equity capital with the value of assets (physical capital), and finally, the
evolution of equity capital.

The **balance sheet** (in nominal terms) of a branch that lends in sector \( x = C, I \), is,

\[
Q_{x,t}P_{C,t}S_{x,t} = N_{x,t}P_{C,t} + B_{x,t},
\]

where \( S_{x,t} \) denotes the quantity of financial claims on capital services producers held by
the intermediary and \( Q_{x,t} \) denotes the price per unit of claim. The variable \( N_{x,t} \) denotes
equity capital (or wealth) at the end of period \( t \), \( B_{x,t} \) are households deposits and \( P_{C,t} \) is
the consumption sector price level.

Financial intermediaries are limited from infinitely borrowing household funds by
a moral hazard/costly enforcement problem, where bankers can steal funds and transfer
them to households. Intermediaries maximize expected terminal wealth, i.e. the
discounted sum of future equity capital. The moral hazard problem introduces an endogenous **leverage constraint**, limiting the bank’s ability to acquire assets. This is
formalized in the equation that determines the demand for assets,

\[
Q_{x,t}S_{x,t} = \varrho_{x,t}N_{x,t}.
\] (3)

In the equation above, the value of assets which the intermediary can acquire depends
on equity capital, \( N_{x,t} \), scaled by the leverage ratio, \( \varrho_{x,t} \). With \( \varrho_{x,t} > 1 \), the leverage
constraint magnifies changes in equity capital on the demand for assets. Higher demand
for capital goods for example, which raises the price of capital, increases equity capital
(through the balance sheet identity) which in turn brings about further changes in the
demand for assets by intermediaries pushing the price of capital further. This amplification
turns out to be the key reason for the important role of news shocks we recover from
the estimated model.

Finally, the evolution of equity capital is described by the following **law of motion**

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[8] The leverage ratio (bank’s intermediated assets to equity) is a function of the marginal gains of
expanding assets (holding equity constant), expanding equity (holding assets constant), and the gain
from diverting assets.
for equity capital,

\[ N_{x,t+1} = \left( \theta_B \left[ (R_{x,t+1}^{B} \pi_{C,t} - R_t) g_{x,t} + R_t \right] \frac{N_{x,t}}{\pi_{C,t+1}} + \omega Q_{x,t+1} S_{x,t+1} \right). \]

where, \( \theta_B \) is the exit rate of bankers, \( \omega \) denotes the fraction of assets given to new bankers. It is useful to define the expected (nominal) excess return (or risk premium) on assets earned by banks as

\[ R_{x,t}^S = R_{x,t+1}^{B} \pi_{C,t+1} - R_t, \quad x = C, I. \]  

(4)

The presence of the financial intermediation constraint in equation (3), implies a non-negative excess return (equivalently wedge between the expected return on capital and the risk free interest rate), which varies over time with the equity capital of intermediaries.

Financing capital acquisitions by capital services producers. Capital services producers issue \( S_{x,t} \) claims equal to units of physical capital acquired, \( K_{x,t} \), priced at \( Q_{x,t} \).

Then, by arbitrage the following constraint holds,

\[ Q_{x,t} K_{x,t} = Q_{x,t} S_{x,t}, \]

where the left-hand side stands for the value of physical capital acquired and the right-hand side denotes the value of claims against this capital.\(^9\) Using the assumptions in Gertler and Karadi (2011) we can interpret these claims as one period state-contingent bonds which allows interpreting the excess return defined in equation (4) as a corporate bond spread.

2.5 Monetary policy and market clearing

The nominal interest rate \( R_t \), set by the monetary authority follows a feedback rule,

\[ \frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\rho_R} \left[ \left( \frac{\pi_{c,t}}{\pi_c} \right)^{\phi_\pi} \left( \frac{Y_t}{Y_{t-1}} \right)^{\phi_{\Delta Y}} \right]^{1-\rho_R} \eta_{mp,t}, \quad \rho_R \in (0, 1), \phi_\pi > 0, \phi_{\Delta Y} > 0, \]

where \( R \) is the steady state (gross) nominal interest rate and \( (Y_t/Y_{t-1}) \) is the gross growth rate in real GDP. The interest rate responds to deviations of consumption goods

\(^9\)We assume—in line with Gertler and Karadi (2011)—there are no frictions in the process of intermediation between non-financial firms and banks.
(gross) inflation from its target level, and real GDP growth and is subject to a monetary policy shock $\eta_{mp,t}$. GDP (in consumption units) is defined as,

$$Y_t = C_t + \frac{P_{I,t}}{P_{C,t}} I_t + G_t,$$

where $G_t$ denotes government spending (in consumption units) assumed to evolve ex-ogenously according to $G_t = \left(1 - \frac{1}{y_t}\right) Y_t$, and $g_t$ is a government spending shock. The sectoral resource constraints are as follows.

The resource constraint in the consumption sector is,

$$C_t + (a(u_{C,t})\xi^K_{C,t}K_{C,t-1} + a(u_{I,t})\xi^K_{I,t}K_{I,t-1}) \frac{A_tV_{I_t}^{\alpha_a/a_i}}{V_t^{1-a_i}} = A_tL_t^{1-a_i}K_{C,t}^{\alpha_i} - A_tV_t^{1-a_i}F_C.$$

The resource constraint in the investment sector is,

$$I_{I,t} + I_{C,t} = V_tL_t^{1-a_i}K_{I,t}^{\alpha_i} - V_t^{1-a_i}F_I.$$

Hours worked, $L_t$, and bank equity, $N_t$, are aggregated as,

$$L_t = L_{I,t} + L_{C,t}, \quad \text{and} \quad N_t = N_{I,t} + N_{C,t}.$$

### 2.6 Shocks and Information

We describe the shocks in the model and the timing assumptions that govern when agents learn about shocks. The baseline model includes the following shocks: sectoral shocks to the growth rate of TFP ($z_t$, $v_t$), sectoral price mark-up shocks ($\lambda_{p,t}^C$, $\lambda_{p,t}^I$), wage mark-up shock ($\lambda_{w,t}$), preference shock ($b_t$), sectoral capital quality shocks ($\xi^K_{C,t}$, $\xi^K_{I,t}$), monetary policy ($\eta_{mp,t}$) and government spending shocks ($g_t$). We model the log deviations of each shock from its steady state as a first order autoregressive (AR(1)) process and as standard in the literature innovations to the processes that are Gaussian, i.i.d (homoskedastic, zero mean). The only exception is the monetary policy shock, $\eta_{mp,t}$, where we set the first order autoregressive parameter to zero (details are provided in Appendix C).

**TFP news shocks.** The sectoral productivity growth processes follow,

$$z_t = (1 - \rho_z)g_a + \rho_zz_{t-1} + \varepsilon^z_t,$$  (5)
\[ v_t = (1 - \rho_v) g_v + \rho_v v_{t-1} + \varepsilon^v_t, \]  

(6)

The parameters \( g_a \) and \( g_v \) are the steady state growth rates of the two TFP processes above and \( \rho_z, \rho_v \in (0, 1) \) determine their persistence.

Our representation of news shocks is standard and follows for example Schmitt-Grohe and Uribe (2012) and Khan and Tsoukalas (2012). Specifically, we assume the respective innovation in the processes, (5) and (6), above are defined as

\[ \varepsilon^x_t = \varepsilon^x_{t,0} + \varepsilon^x_{t-4,4} + \varepsilon^x_{t-8,8}, \quad \text{and} \quad \varepsilon^v_t = \varepsilon^v_{t,0} + \varepsilon^v_{t-4,4} + \varepsilon^v_{t-8,8}, \]

where the first component, \( \varepsilon^x_{t,0} \), is unanticipated, where \( x = z, v \). The components \( \varepsilon^x_{t-4,4} \) and \( \varepsilon^x_{t-8,8} \) are anticipated and represent news about period \( t \) that arrives four and eight quarters ahead, respectively. As conventional in the literature, it is assumed that the anticipated and unanticipated components for sector \( x = C, I \) and horizon \( h = 0, 1, \ldots, H \) are i.i.d. with \( N(0, \sigma^2_{x,t-h}) \), \( N(0, \sigma^2_{v,t-h}) \) and uncorrelated across sector, horizon and time.

3 Data and Methodology

We estimate the DSGE model using quarterly U.S. data (1990 Q2 - 2011 Q1) on the following list of observables.

\[ \mathbf{Y}_t = [\Delta \log Y_t, \Delta \log C_t, \Delta \log I_t, \Delta \log W_t, \pi_{C,t}, \pi_{I,t}, \log L_t, R_t, R^S_{C,t}, R^S_{I,t}, \Delta \log N_t] \]

where \( Y_t, C_t, I_t, W_t, \pi_{C,t}, \pi_{I,t}, L_t, R_t, R^S_{C,t}, R^S_{I,t}, N_t \), denote, output (GDP), consumption, investment, real wage, consumption sector inflation, investment sector inflation, hours worked, nominal interest rate, consumption sector bond spread, investment sector bond spread and bank equity respectively, and \( \Delta \) denotes the first-difference operator. We use a subset of the variables above to identify the news shocks from the VAR, namely, GDP, consumption, investment, hours worked, consumption sector bond spread, consumption sector inflation. We also use a measure of utilization adjusted TFP provided by John Fernald of the San Francisco Fed. We provide details about the VAR method-
ology (adopted from Barsky and Sims (2011)) in Section 4. We briefly note, the VAR methodology uses very minimal restrictions and only identifies two shocks, namely unanticipated and news TFP, whereas the DSGE model identifies more shocks (through many cross equation restrictions implied by the equilibrium) considered previously in this literature. The agnostic nature of the VAR restrictions is very appealing, though potential non-fundamentalness of structural shocks (an issue likely to arise with news shocks) may invalidate inference.\textsuperscript{10} DSGE models on the other hand do not suffer from this limitation. We do not take the view that one methodology should be preferred over the other in this identification problem. A consistent answer regarding the dynamics generated by a news shock across the two however, suggests that we can build more confidence regarding the macroeconomic effects of news shocks. This is taken in Section 4.

The real and nominal variables are standard in business cycle analysis using the estimated DSGE methodology. The aggregate quantity variables are expressed in real, per capita terms. Our financial observables consist of sectoral (non-financial) corporate bond spreads and a publicly available measure of intermediaries’ equity capital reported by the Federal Financial Institutions Examination Council. The latter refers to total equity of all insured US commercial banks—it is also expressed in real per capita terms. To arrive at the sectoral bond spread information we allocate 2-digit industries from the North American Industry Classification System (NAICS) into sectors using the year 2005 Input-Output tables. We provide the details of the data construction in the data Appendix.

**Information from corporate bond spreads.** We inform the estimation with corporate bond spreads that in principle can help to identify news shocks as they are likely to contain advance information over and above what can be extracted from real macr...
nomic aggregates. Philippon (2009) argues that corporate bond spreads may contain news about future corporate fundamentals and provides evidence that information extracted from corporate bond markets, in contrast to the stock market, is very informative for U.S. business fixed investment. Gilchrist and Zakrajsek (2012) find that corporate bond spreads have predictive power for future GDP. A corporate bond spread is defined as the difference between a company’s corporate bond yield and the yield of a US Treasury bond with an identical maturity—information provided by Reuters’ Datastream. In constructing spreads we only consider non-financial corporations and only bonds traded in the secondary market. A detailed description of these data is provided in the data Appendix. We briefly mention that we only utilize investment grade bonds. This allows to be consistent with the model assumptions that abstract away from financial frictions in borrowers’ balance sheets. The series for the sectoral spreads are constructed by taking the average over all company level spreads available in a certain quarter. The dataset contains 5381 bonds of which 1213 are classified to be issued by companies in the consumption sector and 4168 issued by companies in the investment sector. The average duration is 30 quarters (consumption sector) and 28 quarters (investment sector) with an average rating for both sectoral bond issues between BBB+ and A-.

**Prior and posterior distributions.** We estimate a subset of parameters; standard parameters, such as depreciation rates, capital share in output, are calibrated. These are summarized in Table 5 in the Appendix. We demean the data prior to estimation.\(^{11}\) We use the Bayesian methodology to estimate parameters. Our prior distributions conform to the assumptions in Justiniano et al. (2010) and Khan and Tsoukalas (2012). We consider four and eight quarter ahead sector specific TFP news. This choice is guided by the

\(^{11}\)Removing sample means from the data guards against the possibility that counterfactual implications of the model for the low frequencies may distort inference on business cycle dynamics. For example, in the sample, consumption has grown by approximately 0.32% on average per quarter, while output has grown by 0.20% on average per quarter respectively. However, the model predicts that they grow at the same rate. Thus, if we hardwire a counterfactual common trend growth rate in the two series, we may distort inference on business cycle implications that is of interest to us.
desire to economize on the state space and consequently on parameters to be estimated while being flexible enough such that the news process is able to accommodate revisions in expectations. Similar news horizons are considered by Christiano et al. (2014), Schmitt-Grohe and Uribe (2012) and Khan and Tsoukalas (2012). The prior means assumed for the TFP news components are in line with the studies mentioned above and imply that the sum of the variance of news components is, evaluated at prior means, at most one half of the variance of the corresponding unanticipated component.\footnote{We report and discuss several robustness checks on the estimation and the implications for business cycle accounting; for example on the weight on news shocks placed by priors, Gamma distributed shock volatilities, excluding the Great Recession period from the estimation sample, and various others. We also conduct two tests to check for identification of the model parameters, proposed by, (i) Iskrev (2010) and (ii) Koop et al. (2013). Both tests indicate that the parameters are well identified. All of the details are reported in Göertz and Tsoukalas (2015).} Table 1 reports information on prior and posterior distribution of parameters. In the interest of space we do not discuss the estimated parameters in detail—parameters are broadly in line with parameter estimates from earlier work (Smets and Wouters (2007), Khan and Tsoukalas (2012) and Justiniano et al. (2010)). We briefly note the estimated volatilities for the news components imply that approximately 65% (14%) of the total variance in the innovation to the \( z(v) \) process is anticipated. It is interesting to compare the log marginal data density of the baseline model (\( \text{LogL} = -528 \)) against a standard model without a financial channel (\( \text{LogL} = -541 \)). This comparison indicates that the baseline model is preferred by the data.\footnote{For this comparison, we have estimated the models on the same dataset and so the vector of observables does not include financial variables since the standard model does not have implications for the latter.} A more extensive set of comparisons of the baseline model with various perturbations is reported in Göertz and Tsoukalas (2015). It is shown that the baseline model dominates alternative models in terms of its fit with the data.
Table 1: Prior and Posterior Distributions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Prior Distribution</th>
<th>Posterior Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distribution</strong></td>
<td>Mean</td>
<td>Std. dev.</td>
<td>Mean</td>
</tr>
<tr>
<td>$h$</td>
<td>Consumption habit</td>
<td>Beta</td>
<td>0.50</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Inverse labour supply elasticity</td>
<td>Gamma</td>
<td>2.00</td>
</tr>
<tr>
<td>$\xi_w$</td>
<td>Wage Calvo probability</td>
<td>Beta</td>
<td>0.66</td>
</tr>
<tr>
<td>$\xi_C$</td>
<td>C-sector price Calvo probability</td>
<td>Beta</td>
<td>0.66</td>
</tr>
<tr>
<td>$\xi_I$</td>
<td>I-sector price Calvo probability</td>
<td>Beta</td>
<td>0.66</td>
</tr>
<tr>
<td>$\xi_w$</td>
<td>Wage indexation</td>
<td>Beta</td>
<td>0.50</td>
</tr>
<tr>
<td>$\xi_C$</td>
<td>C-sector price indexation</td>
<td>Beta</td>
<td>0.50</td>
</tr>
<tr>
<td>$\xi_I$</td>
<td>I-sector price indexation</td>
<td>Beta</td>
<td>0.50</td>
</tr>
<tr>
<td>$\chi_I$</td>
<td>I-sector utilization</td>
<td>Gamma</td>
<td>5.00</td>
</tr>
<tr>
<td>$\chi_C$</td>
<td>C-sector utilization</td>
<td>Gamma</td>
<td>5.00</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Investment adj. cost</td>
<td>Gamma</td>
<td>4.00</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Taylor rule inflation</td>
<td>Normal</td>
<td>1.70</td>
</tr>
<tr>
<td>$\rho_R$</td>
<td>Taylor rule inertia</td>
<td>Beta</td>
<td>0.60</td>
</tr>
<tr>
<td>$\phi_{tX}$</td>
<td>Taylor rule output growth</td>
<td>Normal</td>
<td>0.25</td>
</tr>
</tbody>
</table>

**Shocks: Persistence**

| $\rho}_z$ | C-sector TFP | Beta | 0.40 | 0.20 | 0.7498 | 0.6973 | 0.801 |
| $\rho}_v$ | I-sector TFP | Beta | 0.40 | 0.20 | 0.1415 | 0.0455 | 0.2328 |
| $\rho}_b$ | Preference | Beta | 0.60 | 0.20 | 0.9136 | 0.8762 | 0.9542 |
| $\rho}_g$ | Government spending | Beta | 0.60 | 0.20 | 0.9826 | 0.9664 | 0.9993 |
| $\rho}_C$ | C-sector price markup | Beta | 0.60 | 0.20 | 0.0539 | 0.0145 | 0.0919 |
| $\rho}_I$ | I-sector price markup | Beta | 0.60 | 0.20 | 0.8871 | 0.8442 | 0.9337 |
| $\rho}_w$ | Wage markup | Beta | 0.60 | 0.20 | 0.0523 | 0.0087 | 0.0945 |
| $\rho}_K,C$ | C-sector capital quality | Beta | 0.60 | 0.20 | 0.8437 | 0.8133 | 0.8765 |
| $\rho}_K,I$ | I-sector capital quality | Beta | 0.60 | 0.20 | 0.0862 | 0.0215 | 0.1471 |

**Shocks: Volatilities**

| $\sigma}_z$ | C-sector TFP | Inv Gamma | 0.50 | 2 | 0.1721 | 0.1288 | 0.2147 |
| $\sigma}_4$ | C-sector TFP. 4Q ahead news | Inv Gamma | 0.5/√2 | 2 | 0.1174 | 0.0839 | 0.1521 |
| $\sigma}_8$ | C-sector TFP. 8Q ahead news | Inv Gamma | 0.5/√2 | 2 | 0.2014 | 0.1544 | 0.2470 |
| $\sigma}_v$ | I-sector TFP | Inv Gamma | 0.50 | 2 | 1.8718 | 1.5932 | 2.1517 |
| $\sigma}_4$ | I-sector TFP. 4Q ahead news | Inv Gamma | 0.5/√2 | 2 | 0.2959 | 0.2090 | 0.4712 |
| $\sigma}_8$ | I-sector TFP. 8Q ahead news | Inv Gamma | 0.5/√2 | 2 | 0.7001 | 0.5282 | 0.8661 |
| $\sigma}_b$ | Preference | Inv Gamma | 0.10 | 2 | 1.4524 | 1.1644 | 1.7339 |
| $\sigma}_g$ | Government spending | Inv Gamma | 0.50 | 2 | 0.5102 | 0.4357 | 0.5794 |
| $\sigma}_m$ | Monetary policy | Inv Gamma | 0.10 | 2 | 0.1204 | 0.1023 | 0.1386 |
| $\sigma}_C$ | C-sector price markup | Inv Gamma | 0.10 | 2 | 0.6045 | 0.5184 | 0.6839 |
| $\sigma}_I$ | I-sector price markup | Inv Gamma | 0.10 | 2 | 0.2282 | 0.1647 | 0.2863 |
| $\sigma}_w$ | Wage markup | Inv Gamma | 0.10 | 2 | 0.3689 | 0.3100 | 0.4274 |
| $\sigma}_K,C$ | C-sector capital quality | Inv Gamma | 0.50 | 2 | 0.3118 | 0.2237 | 0.3948 |
| $\sigma}_K,I$ | I-sector capital quality | Inv Gamma | 0.50 | 2 | 2.4029 | 2.0458 | 2.7600 |

Notes. The posterior distribution of parameters is evaluated numerically using the random walk Metropolis-Hastings algorithm. We simulate the posterior using a sample of 500,000 draws and discard the first 100,000 of the draws.
4 Reconciling DSGE and VAR findings

The estimated DSGE model described above selects TFP news shocks as a major source of fluctuations. They account for 37%, 30%, 31%, 50% of the variance in output, consumption, investment and hours worked respectively in business cycle frequencies—Table 2 reports a summary variance decomposition of the model. Consumption specific news shocks dominate these shares, accounting for 31%, 29%, 21%, 43% of the variance in the same macro variables. These same shocks account for a significant share of the variance in the corporate bond spreads, a key information variable in the analysis. As we explain in detail in Görtz and Tsoukalas (2015), the financial channel in the model provides, relative to a standard NK model, the missing amplification to the TFP news shock. The standard model (without the financial channel) predicts, consistent with earlier work (see e.g. Fujiwara et al. (2011), Khan and Tsoukalas (2012), Schmitt-Grohe and Uribe (2012)), a substantially reduced empirical role for news shocks. To gain intuition we discuss IRFs to selected variables following a 2 year ahead anticipated consumption specific TFP news shock. Figure 1 plots IRFs from the baseline model against IRFs from an estimated model without financial intermediation (shock normalized to be of equal size). In both models, the news shock generates co-movement of the main macro aggregates. However, amplification is significantly stronger in the model with the financial channel.
Table 2: Variance decomposition at posterior estimates—business cycle frequencies (6-32 quarters)

<table>
<thead>
<tr>
<th></th>
<th>TFP shocks:</th>
<th>financial shocks:</th>
<th>all other shocks</th>
<th>all TFP shocks</th>
<th>all TFP news shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$z$</td>
<td>$z^4$</td>
<td>$z^8$</td>
<td>$v$</td>
<td>$v^4$</td>
</tr>
<tr>
<td>Output</td>
<td>0.195</td>
<td>0.093</td>
<td>0.213</td>
<td>0.187</td>
<td>0.006</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.241</td>
<td>0.106</td>
<td>0.185</td>
<td>0.023</td>
<td>0.001</td>
</tr>
<tr>
<td>Investment</td>
<td>0.040</td>
<td>0.043</td>
<td>0.165</td>
<td>0.384</td>
<td>0.009</td>
</tr>
<tr>
<td>Total Hours</td>
<td>0.063</td>
<td>0.091</td>
<td>0.341</td>
<td>0.163</td>
<td>0.006</td>
</tr>
<tr>
<td>Real Wage</td>
<td>0.206</td>
<td>0.083</td>
<td>0.131</td>
<td>0.038</td>
<td>0.015</td>
</tr>
<tr>
<td>Nom. Interest Rate</td>
<td>0.036</td>
<td>0.070</td>
<td>0.466</td>
<td>0.122</td>
<td>0.003</td>
</tr>
<tr>
<td>C-Sector Inflation</td>
<td>0.006</td>
<td>0.017</td>
<td>0.231</td>
<td>0.103</td>
<td>0.007</td>
</tr>
<tr>
<td>I-Sector Inflation</td>
<td>0.028</td>
<td>0.044</td>
<td>0.315</td>
<td>0.226</td>
<td>0.003</td>
</tr>
<tr>
<td>C-Sector Spread</td>
<td>0.077</td>
<td>0.078</td>
<td>0.313</td>
<td>0.080</td>
<td>0.001</td>
</tr>
<tr>
<td>I-Sector Spread</td>
<td>0.093</td>
<td>0.083</td>
<td>0.329</td>
<td>0.140</td>
<td>0.001</td>
</tr>
<tr>
<td>Equity</td>
<td>0.190</td>
<td>0.088</td>
<td>0.324</td>
<td>0.056</td>
<td>0.002</td>
</tr>
</tbody>
</table>

$z =$ TFP in consumption sector, $z^x =$ $x$ quarters ahead consumption sector TFP news shock, $v =$ TFP in investment sector, $v^x =$ $x$ quarters ahead investment sector TFP news shock, $\xi_C$ and $\xi_I =$ capital quality shocks in the consumption and investment sector. Business cycle frequencies considered in the decomposition correspond to periodic components with cycles between 6 and 32 quarters. The decomposition is performed using the spectrum of the DSGE model and an inverse first difference filter to reconstruct the levels for output, consumption, total investment, the real wage, equity and the relative price of investment. The spectral density is computed from the state space representation of the model with 500 bins for frequencies covering the range of periodicities. We report median shares.
Amplification of news shocks is achieved through the impact of capital prices on intermediaries equity, which in turn generates a strong investment boom. Higher capital prices boost bank equity. Better capitalized banks demand more capital and this process further bids up capital prices. The strong investment demand is reflected in the relative price of investment which rises more sharply in the baseline model. Figure 1 illustrates that one significant (qualitative) difference in the dynamics between the two models are in the response of capital prices and in the credit spreads. In both models, capital prices rise in anticipation of the future rise in productivity. In the baseline model, due to the impact of intermediaries on the demand for capital, capital prices increase very strongly; for example, the price of consumption sector capital rises on impact by approximately nine times more compared to the standard model. Thereafter, as more capital gets installed capital prices and the return to capital are expected to decline. In the baseline model, thus in contrast to the model without the financial channel, credit spreads decline significantly and provide advance information about the future increase in the productivity of capital. Having established basic dynamic properties of the news shock in the DSGE model we now undertake a comparison with a VAR based identification of a consumption specific
TFP news shock.

To identify the latter we use the Barsky and Sims (2011) methodology and estimate a six variable VAR featuring a (utilization-adjusted) consumption specific TFP measure, consumption-sector corporate bond spread, consumption, output, hours and inflation in that order.\textsuperscript{14} The consumption specific TFP measure is derived from the growth accounting methodology of Basu et al. (2006), and corrects for unobserved capacity utilization (described in Fernald (2012)). However, it does not contain all the corrections as in Basu et al. (2006), namely imperfect competition or reallocation effects.\textsuperscript{15} The remaining series included in the VAR are identical to those used in estimating the DSGE model, except that they enter the VAR in levels consistent with the treatment in Barsky and Sims (2011). In a VAR with the TFP measure first in the ordering, the reduced form innovation serves as the surprise TFP shock, while the TFP news shock is identified as the shock orthogonal to the surprise component that best explains future movements in TFP over a finite horizon. We recover the TFP news shock by maximizing the share of the variance in TFP over horizons from 1 to 10 years. Our choice is guided by the DSGE model assumptions on the timing of arrival of news. The model implies only surprise TFP innovations can account for the variance in TFP over the first year while TFP news shocks can affect the variance of TFP only after the 1st year. The applicability of the VAR identification methodology rests on the assumption that only the two TFP shocks can explain all movements in TFP. To assess the validity of this assumption, Table 3 reports the variance shares of TFP accounted for unanticipated and news TFP shocks. First, there is a fraction of the variance in TFP that is left unexplained by the two TFP shocks.

\textsuperscript{14}We use Barsky and Sims (2011) method rather than a VAR method which incorporates more restrictions since the results reported therein challenged the traditional news view of business cycle emphasized by Beaudry and Portier (2006). This methodology is appealing because identification rests on very minimal assumptions. The results are qualitatively similar to smaller or larger VAR specifications (e.g. 4-7 variables, including for example the Michigan confidence indicator measure or using a weighted (taking into account both sectoral) spread series.

\textsuperscript{15}It is available from, http://www.frbsf.org/economic-research/economists/jfernald/quarterly_tfp.xls.
shocks combined. Nevertheless, from horizons 6 to 40, the two shocks combined explain the majority of the variance, that is, between 85% to 91% of the FEV of TFP.

<table>
<thead>
<tr>
<th>Horizon</th>
<th>6</th>
<th>12</th>
<th>20</th>
<th>24</th>
<th>32</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEV of TFP (unanticipated and news shock)</td>
<td>0.85</td>
<td>0.86</td>
<td>0.89</td>
<td>0.90</td>
<td>0.91</td>
<td>0.91</td>
</tr>
<tr>
<td>FEV of TFP (unanticipated only)</td>
<td>0.82</td>
<td>0.82</td>
<td>0.81</td>
<td>0.81</td>
<td>0.81</td>
<td>0.78</td>
</tr>
</tbody>
</table>

Sample is 1990Q2 to 2011Q1. Forecast error variance of TFP accounted for by the two identified shocks, unanticipated and news.

Figure 2 presents IRFs from the VAR specification described above conditional on a positive TFP news shock. The Figure shows the point estimate and +/- one standard deviation bootstrapped (shaded areas) bands as described in Kilian (1998). Note, first, consistent with the model, TFP begins to rise significantly above zero with a delay of about 10 quarters. Moreover, the VAR identified TFP news shock, in line with the model, creates a boom today: output, consumption, and hours increase significantly on impact. In addition, the corporate bond spread declines significantly suggesting that corporate bond markets anticipate future TFP. Investment also rises significantly in response to good news about future TFP (see Figure 3, based on an alternative VAR specification).

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16We note, the empirical VAR responses in the Figure stand in contrast to Barsky and Sims (2011) who use an aggregate TFP measure, or Nam and Wang (2012) who use sectoral TFP series like us and a much longer sample and find that hours and output decline in anticipation of a favorable aggregate (or consumption specific) TFP news shock. In on-going work (Gambetti et al. (2013)), with time–varying parameter VARs, we document a significant and qualitatively important difference in the IRFs of output and hours across time, detecting a break that occurs in the mid-1980s. Specifically, we find that while in a pre–1984 (1960-1984) sample, output and hours decline significantly (on impact) in response to a favorable consumption specific (or aggregate) TFP news shock (as in Barsky and Sims (2011)), in a post–1984 (1984-2011) sample, the same variables rise significantly on impact in response to the same two shocks. The longer sample, thus echoes the pre–1984 (1960-1984) sample results. We view the post 1984 sample results more relevant since the economy is thought to have entered the Great Moderation regime in the mid-1980s.

---
Inflation initially declines and rises with a delay in response to the news shock. How do the empirical VAR responses compare with the responses from the DSGE model? To facilitate illustration, in Figure 4 we plot the DSGE model responses to a 2 year ahead consumption specific TFP news shock along with the responses from the estimated VAR (as shown in Figure 2). The empirical VAR responses are qualitatively consistent with the model’s responses. There are some differences in terms of magnitudes, most notably in the output and inflation responses on impact, especially in the first periods.

Note however each methodology uses different moments (and implied restrictions on the moments) from the data to estimate the TFP news component. The DSGE model relies on a maximum likelihood (full information) estimation, taking all data moments into consideration, while the VAR uses a subset of moments, seeking a rotation of reduced form shocks that maximize the sum of variance of TFP over horizons 1 to 10 years.\footnote{Note that the VAR, in line with the convention in this literature, also utilizes an observable indicator of TFP to identify the news shock. In Görtz and Tsoukalas (2015), we have estimated the DSGE model using sectoral TFP as an observable and the results are quantitatively consistent with the baseline model. For space considerations and for consistency with the DSGE literature that typically does not use TFP as an observable we only report the results from the baseline.}

To make the comparison between VAR and DSGE model more precise, we investigate whether the empirical VAR responses could have been generated by the model, assuming the latter as the data generating process. To accomplish this, we generated 1,000 artificial model samples by drawing parameter values from the posterior distributions and simulated the model. We then compare the empirical VAR IRFs with those generated by identical VAR specifications (along with confidence bands) estimated on the artificial model samples.\footnote{We have simulated the model over 1084 periods. We construct the level of the resulting time series and discard all but the last 84 periods to minimize the impact of initial values. Note that the issue of non-invertibility does not seem to be particularly acute since the simulated VARs seem to pick the model responses, quite accurately, except for TFP.} Figure 5 shows this comparison for the consumption specific TFP news shock. Figure 6 shows the comparison for an aggregate TFP news shock, for comparison purposes with the majority of earlier VAR studies, which use the utilization adjusted ag-

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\textsuperscript{17} Note, that the VAR, in line with the convention in this literature, also utilizes an observable indicator of TFP to identify the news shock. In Görtz and Tsoukalas (2015), we have estimated the DSGE model using sectoral TFP as an observable and the results are quantitatively consistent with the baseline model. For space considerations and for consistency with the DSGE literature that typically does not use TFP as an observable we only report the results from the baseline.

\textsuperscript{18} We have simulated the model over 1084 periods. We construct the level of the resulting time series and discard all but the last 84 periods to minimize the impact of initial values. Note that the issue of non-invertibility does not seem to be particularly acute since the simulated VARs seem to pick the model responses, quite accurately, except for TFP.
Figure 2: Sample is 1990Q2-2011Q1. The solid line is the estimated impulse response to a consumption specific TFP news shock from a six variable VAR featuring consumption specific TFP, (investment grade) corporate bond spread, consumption, output, hours and inflation with 2 lags as suggested by the AIC criterion. The identification of the news shock is based on the method of Barsky and Sims (2011) with the truncation horizon set to $H=40$. The shaded gray areas are the +/- one standard deviation confidence band from 2000 bias-corrected bootstrap replications of the reduced form VAR. The horizontal axes refer to forecast horizons (quarters) and the units of the vertical axes are percentage deviations.

Figure 3: Sample is 1990Q2-2011Q1. The solid line is the estimated impulse response to a consumption specific TFP news shock from a six variable VAR featuring consumption specific TFP, (investment grade) corporate bond spread, consumption, investment, hours and inflation with 2 lags as suggested by the AIC criterion. The identification of the news shock is based on the method of Barsky and Sims (2011) with the truncation horizon set to $H=40$. The shaded gray areas are the +/- one standard deviation confidence band from 2000 bias-corrected bootstrap replications of the reduced form VAR. The horizontal axes refer to forecast horizons (quarters) and the units of the vertical axes are percentage deviations.
aggregate TFP measure (see for example, Beaudry and Portier (2006), Beaudry and Lucke (2010), Barsky and Sims (2011), Forni et al. (2012)). We observe that for both measures of TFP and in the majority of periods we plot, the empirical VAR responses are within the confidence bands generated by the simulated VAR responses taking the model as the data generating process. As mentioned above, an exception is inflation, where the empirical VAR predicts an initial decline in inflation, whereas the VAR on the simulated model data suggests an essentially zero response. These Figures suggest that the two methodologies produce consistency of VAR and DSGE responses. We believe, this consistency lends credibility to the financial channel we propose.

Figure 4: The line with diamonds is the impulse response to a 2 year ahead consumption specific TFP news shock from the DSGE model. The thick solid line is the impulse response to a consumption specific TFP news shock from a six variable VAR featuring TFP, (investment grade) corporate bond spread, consumption, output, hours and inflation. Sample is 1990Q2-2011Q1.

How does the quantitative role for TFP news compares across the two methodologies?

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19To generate an aggregate TFP measure from the model we take the weighted average of the consumption specific and investment specific measures using as weights their output shares, consistent with the methodology in Fernald (2012). We then use this aggregate measure in the VARs estimated on the artificial samples to identify an aggregate TFP news shock.
Figure 5: The thick solid line is the impulse response to a consumption specific TFP news shock from a six variable VAR featuring TFP, (investment grade) corporate bond spread, consumption, output, hours and inflation. The thin solid line (dotted lines) is the median (20%, 80% confidence bands) impulse response to a consumption specific TFP news shock estimated from a VAR (identical to the empirical VAR, 6 variables, 2 lags and 84 observations) on 1,000 samples, generated from the model. The horizontal axes refer to forecast horizons (quarters) and the units of the vertical axes are percentage deviations. Sample is 1990Q2-2011Q1.
Figure 6: Sample is 1990Q2-2011Q1. The thick solid line is the impulse response to an aggregate TFP news shock from a five variable VAR featuring TFP, (investment grade) corporate bond spread, consumption, output, hours. The thin solid line (dotted lines) is the median (20%, 80% confidence bands) impulse response to an aggregate TFP news shock estimated from a VAR (identical to the empirical VAR, 5 variables, 4 lags and 84 observations) on 1,000 samples, generated from the model. The horizontal axes refer to forecast horizons (quarters) and the units of the vertical axes are percentage deviations.

Figure 7 provides an answer, focusing on the empirical importance of TFP news shocks for the main macro aggregates, namely, output, hours worked, consumption and investment. It plots the variance shares accounted for by TFP news shocks, predicted by (i) the VAR, (ii) baseline DSGE model and (iii) DSGE model estimated without the financial channel discussed earlier in Figure 1. The variance shares predicted by the VAR and baseline DSGE model are very similar (especially in shorter horizons) for output and hours worked. For example, at the 12 quarter horizon, the baseline DGSE predicts 30% (41%) of the forecast error variance of output (hours worked) attributed to TFP news shocks, very similar to the variance shares in the same variables obtained from the VAR, equal to 31% (49%). By contrast, the differences between the VAR and the model without the financial channel are substantial, in fact the variance shares in the latter are several orders of magnitude smaller compared to the VAR. For investment, except for horizon 6, the baseline DSGE model predicts very similar variance shares accounted
for by the TFP news shock compared to the VAR. The comparison for consumption is somewhat less clear cut, where the DSGE model seems to under-predict, relative to the VAR, the variance share accounted for the TFP news shock in short horizons—but comes very close in longer horizons. Table 4 provides more information on the remaining variables. In business cycle frequencies (6-32 quarters), the VAR based TFP news shock (top panel) accounts for between 21%–38% in the variance of consumption, 28%–41% in the variance of the corporate bond spread, 15%–45% in the variance of investment and 21% in the variance of inflation. The middle panel reports the variance shares from the (identical) simulated VAR specifications estimated on the artificial model samples. This panel reports the median and (20%, 80%) confidence bands. Interestingly, the shares reported for the empirical VAR are, for the vast majority of horizons and variables, within the confidence bands generated by the VARs estimated on artificial data.

Figure 7: Sample is 1990Q2-2011Q1. Share of variance in output, total hours, consumption and total investment accounted for by consumption specific TFP news shocks in the VAR (light blue), the baseline DSGE model with the financial channel (red) and the baseline model without the financial channel (grey). The horizontal axis indicates the decomposition horizon in quarters. Median shares are reported for the DSGE models, point estimate for the VAR.
Table 4: Variance decompositions: TFP news—VARs and baseline DSGE

<table>
<thead>
<tr>
<th>Horizon</th>
<th>6</th>
<th>12</th>
<th>20</th>
<th>24</th>
<th>32</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Empirical VAR (point estimate)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>0.38</td>
<td>0.32</td>
<td>0.25</td>
<td>0.23</td>
<td>0.21</td>
</tr>
<tr>
<td>Output</td>
<td>0.36</td>
<td>0.31</td>
<td>0.25</td>
<td>0.23</td>
<td>0.21</td>
</tr>
<tr>
<td>Hours</td>
<td>0.61</td>
<td>0.49</td>
<td>0.33</td>
<td>0.29</td>
<td>0.25</td>
</tr>
<tr>
<td>Investment*</td>
<td>0.45</td>
<td>0.30</td>
<td>0.21</td>
<td>0.19</td>
<td>0.15</td>
</tr>
<tr>
<td>Bond Spread</td>
<td>0.41</td>
<td>0.31</td>
<td>0.30</td>
<td>0.29</td>
<td>0.28</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
</tr>
<tr>
<td><strong>VAR on simulated model data</strong> (medians with [20%,80%] bands)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>0.36</td>
<td>0.51</td>
<td>0.57</td>
<td>0.58</td>
<td>0.57</td>
</tr>
<tr>
<td>Output</td>
<td>[0.11, 0.59]</td>
<td>[0.22, 0.74]</td>
<td>[0.29, 0.78]</td>
<td>[0.30, 0.78]</td>
<td>[0.31, 0.79]</td>
</tr>
<tr>
<td>Hours</td>
<td>[0.20, 0.72]</td>
<td>[0.23, 0.76]</td>
<td>[0.25, 0.74]</td>
<td>[0.26, 0.75]</td>
<td>[0.27, 0.74]</td>
</tr>
<tr>
<td>Investment*</td>
<td>[0.17, 0.69]</td>
<td>[0.18, 0.64]</td>
<td>[0.21, 0.62]</td>
<td>[0.22, 0.62]</td>
<td>[0.23, 0.61]</td>
</tr>
<tr>
<td>Bond Spread</td>
<td>0.25</td>
<td>0.37</td>
<td>0.38</td>
<td>0.38</td>
<td>0.39</td>
</tr>
<tr>
<td>Inflation</td>
<td>[0.10, 0.69]</td>
<td>[0.14, 0.67]</td>
<td>[0.16, 0.65]</td>
<td>[0.18, 0.63]</td>
<td>[0.19, 0.63]</td>
</tr>
<tr>
<td><strong>DSGE (medians)</strong> (sum of 4 and 8 quarter ahead c-specific TFP news)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>0.13</td>
<td>0.07</td>
<td>0.20</td>
<td>0.29</td>
<td>0.37</td>
</tr>
<tr>
<td>Output</td>
<td>0.27</td>
<td>0.30</td>
<td>0.30</td>
<td>0.31</td>
<td>0.31</td>
</tr>
<tr>
<td>Hours</td>
<td>0.38</td>
<td>0.41</td>
<td>0.45</td>
<td>0.43</td>
<td>0.41</td>
</tr>
<tr>
<td>Investment</td>
<td>0.23</td>
<td>0.21</td>
<td>0.23</td>
<td>0.21</td>
<td>0.18</td>
</tr>
<tr>
<td>Bond Spread</td>
<td>0.07</td>
<td>0.25</td>
<td>0.40</td>
<td>0.41</td>
<td>0.43</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.04</td>
<td>0.18</td>
<td>0.25</td>
<td>0.26</td>
<td>0.26</td>
</tr>
</tbody>
</table>

Sample is 1990Q2 to 2011Q1. The decomposition in the top panel is obtained from a six variable VAR featuring consumption specific TFP, corporate bond spread, consumption, output, hours and inflation estimated with two lags, suggested by AIC and SC lag length criteria. * News share for investment is obtained from a six variable VAR as above but where investment replaces output. The decomposition in the middle panel is from identical VAR specifications (as in the top panel) run on 1,000 artificial samples from the model.
We undertake a final robustness check. Because a significant amount of information for the identification of TFP news shocks is contained in the corporate bond spread series we find it informative to investigate an alternative interpretation. Gilchrist and Zakrajsek (2012) argue that innovations in corporate bond spreads are, to a certain extent, driven by the excess bond premium (EBP), an indicator of disruptions in the supply of credit that may be orthogonal to corporate fundamentals. We investigate whether the EBP Granger causes the TFP news shocks. The results from this exercise are clear cut. We cannot reject the hypothesis that the EBP does not Granger cause either of the TFP news shocks at conventional significance levels. The Granger causality tests are carried out with 12 lags. These findings overall suggest both DGSE and VAR methodologies provide a consistent empirical assessment of TFP news shocks.

5 Conclusions

The empirical evaluation of the “news” driven view of business cycles has been challenging on both modelling and econometric front (see Beaudry and Portier (2014)). Considerable disagreement exists between VAR based and DSGE based methodologies on the empirical relevance of this view. DSGE models, despite incorporating model frictions that in theory allow TFP news shocks to matter, estimate them to be unimportant as sources of business cycles. VARs often reach diametrically opposite conclusions on their empirical importance. In this paper we propose and empirically evaluate a financial channel that links in a parsimonious way leveraged lenders, capital prices and real activity in a DSGE model. When we discipline this channel with information from corporate bond markets, we find that TFP news shocks are important drivers of the U.S. business cycles in the post-Greenspan era. Importantly, we show that the financial channel can bring close in line the empirical estimates of TFP news shocks from DSGE and VAR methodologies and thus resolves an important empirical disconnect in the literature.
References


6 Appendix with supplementary material (Not for publication)

A Supporting details and results

A.1 Calibration and estimation

Calibration. Table 5 describes the calibrated parameters referred to in section 3. We set the quarterly depreciation rate to be equal across sectors, $\delta_C = \delta_I = 0.025$. From the steady state restriction $\beta = \pi_C / R$, we set $\beta = 0.9974$. The shares of capital in the production functions, $a_C$ and $a_I$, are assumed equal across sectors and fixed at 0.3. The steady state values for the ratios of nominal investment to consumption and government spending to output are calibrated to be consistent with the average values in the data. The steady state sectoral inflation rates are set to the sample averages and the sectoral steady state mark-ups are assumed to be equal to 15%. We also calibrate the steady state (deterministic) growth of TFP in the consumption/investment sectors in line with the sample average growth rates of output in the two sectors. This yields $g_a = 0.141\%$ and $g_v = 0.434\%$ per quarter. There are three parameters specific to financial intermediation. The parameter $\theta_B$, which determines the banker’s average life span does not have a direct empirical counterpart and is fixed at 0.96, very similar to the value used by Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). This value implies an average survival time of bankers of slightly over six years. The parameters $\varpi$ and $\lambda_B$ are fixed at values which guarantee that the steady state risk premium (the average of spreads across the two sectors) and the steady state leverage ratio matches their empirical counterparts. The average of the consumption sector and investment sector credit spreads are each equal to 50 basis points in the sample. The average leverage ratio in the data is computed from the ratio of assets (excluding loans to consumers, real estate and holdings of government bonds) to equity for all U.S. insured commercial banks and
Table 5: Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_C$</td>
<td>0.025</td>
<td>Consumption sector capital depreciation</td>
</tr>
<tr>
<td>$\delta_I$</td>
<td>0.025</td>
<td>Investment sector capital depreciation</td>
</tr>
<tr>
<td>$a_c$</td>
<td>0.3</td>
<td>Consumption sector share of capital</td>
</tr>
<tr>
<td>$a_I$</td>
<td>0.3</td>
<td>Investment sector share of capital</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.9974</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\pi_C - 1$</td>
<td>0.6722</td>
<td>Steady state consumption sector net inflation rate (percent quarterly)</td>
</tr>
<tr>
<td>$\pi_I - 1$</td>
<td>0.0245</td>
<td>Steady state investment sector net inflation rate (percent quarterly)</td>
</tr>
<tr>
<td>$\lambda_p$</td>
<td>0.15</td>
<td>Steady state price markup</td>
</tr>
<tr>
<td>$\lambda_w$</td>
<td>0.15</td>
<td>Steady state wage markup</td>
</tr>
<tr>
<td>$g_a$</td>
<td>0.141</td>
<td>Steady state C-sector TFP growth (percent quarterly)</td>
</tr>
<tr>
<td>$g_v$</td>
<td>0.434</td>
<td>Steady state I-sector TFP growth (percent quarterly)</td>
</tr>
<tr>
<td>$p_{it}$</td>
<td>0.399</td>
<td>Steady state investment / consumption</td>
</tr>
<tr>
<td>$\frac{G}{Y}$</td>
<td>0.19</td>
<td>Steady state government spending / output</td>
</tr>
<tr>
<td>$\theta_B$</td>
<td>0.96</td>
<td>Fraction of bankers that survive</td>
</tr>
<tr>
<td>$\varpi$</td>
<td>0.0021</td>
<td>Share of assets transferred to new bankers</td>
</tr>
<tr>
<td>$\lambda_B$</td>
<td>0.69</td>
<td>Fraction of funds bankers can divert</td>
</tr>
<tr>
<td>$\varrho$</td>
<td>5.47</td>
<td>Steady state leverage ratio</td>
</tr>
<tr>
<td>$R^B - R$</td>
<td>0.5</td>
<td>Steady state spread (percent quarterly)</td>
</tr>
</tbody>
</table>

Notes. $\beta$, $\pi_C$, $\pi_I$, $g_a$, $g_v$, $p_{it}$, $\varrho$, $R^B - R$ are based on sample averages. $\varpi$ and $\lambda_B$ are set to be consistent with the average values of the leverage ratio, $\varrho$, and $R^B - R$.

B Data Sources and Time Series Construction

Table 6 provides an overview of the data used to construct the observables. All the data transformations we have made in order to construct the dataset used for the estimation of the model are described in detail below. As described in the main body, a subset of variables are used for estimating the various VAR specifications and they enter in levels. The data series for aggregate and consumption specific TFP used to estimate the VARs are taken from John Fernald’s website (http://www.frbsf.org/economic – research/economists/jfernal and are described in Fernald (2012).

**Sectoral definition.** To allocate a sector to the consumption or investment category, we used the 2005 Input-Output tables. The Input-Output tables track the flows of goods
and services across industries and record the final use of each industry’s output into three broad categories: consumption, investment and intermediate uses (as well as net exports and government). First, we determine how much of a 2-digit industry’s final output goes to consumption as opposed to investment or intermediate uses. Then we adopt the following criterion: if the majority of an industry’s final output is allocated to final consumption demand it is classified as a consumption sector; otherwise, if the majority of an industry’s output is allocated to investment or intermediate demand, it is classified as an investment sector. Using this criterion, mining, utilities, transportation and warehousing, information, manufacturing, construction and wholesale trade industries are classified as the investment sector and retail trade, real estate, rental and leasing, professional and business services, educational services, health care and social assistance, arts, entertainment, recreation, accommodation and food services and other services except government are classified as the consumption sector.

**Real and nominal variables.** Consumption (in current prices) is defined as the sum of personal consumption expenditures on services and personal consumption expenditures on non-durable goods. The times series for real consumption is constructed as follows. First, we compute the shares of services and non-durable goods in total (current price) consumption. Then, total real consumption growth is obtained as the chained weighted (using the nominal shares above) growth rate of real services and growth rate of real non-durable goods. Using the growth rate of real consumption we construct a series for real consumption using 2005 as the base year. The consumption deflator is calculated as the ratio of nominal over real consumption. Inflation of consumer prices is the

---

20 The investment sectors’ NAICS codes are: 21 22 23 31 32 33 42 48 49 51 (except 491). The consumption sector NAICS codes are: 6 7 11 44 45 53 54 56 81. This information is provided by the Bureau of Economic analysis (Use Tables/Before Redefinitions/Producer Value ([http://www.bea.gov/industry/io_annual.htm](http://www.bea.gov/industry/io_annual.htm))). We have checked whether there is any migration of 2-digit industries across sectors for our sample. The only industry which changes classification (from consumption to investment) during the sample is “information” which for the majority of the sample can be classified as investment and we classify it as such.
growth rate of the consumption deflator. Analogously, we construct a time series for the investment deflator using series for (current price) personal consumption expenditures on durable goods and gross private domestic investment and chain weight to arrive at the real aggregate. The relative price of investment is the ratio of the investment deflator and the consumption deflator. Real output is GDP expressed in consumption units by dividing current price GDP with the consumption deflator.

The hourly wage is defined as total compensation per hour. Dividing this series by the consumption deflator yields the real wage rate. Hours worked is given by hours of all persons in the non-farm business sector. All series described above as well as the equity capital series (described below) are expressed in per capita terms using the series of non-institutional population, ages 16 and over. The nominal interest rate is the effective federal funds rate. We use the monthly average per quarter of this series and divide it by four to account for the quarterly frequency of the model. The time series for hours is in logs. Moreover, all series used in estimation (including the financial time series described below) are expressed in deviations from their sample average.

**Financial variables.** Data for sectoral credit spreads are not directly available. However, Reuters’ Datastream provides U.S. credit spreads for companies which we map into the two sectors using The North American Industry Classification System (NAICS) as explained above. A credit spread is defined as the difference between a company’s corporate bond yield and the yield of a US Treasury bond with an identical maturity. In constructing credit spreads we only consider non-financial corporations and only bonds traded in the secondary market. In line with Gilchrist and Zakrajsek (2012) we make the following adjustments to the credit spread data we construct: using ratings from Standard & Poor’s and Moody’s, we exclude all bonds which are below investment grade as well as the bonds for which ratings are unavailable. We further exclude all spreads with a duration below one and above 30 years and exclude all credit spreads below 10 and above 5000 basis points to ensure that the time series are not driven by a small number of extreme observations. The series for the sectoral credit spreads are constructed by taking
Table 6: Time Series used to construct the observables and steady state relationships

<table>
<thead>
<tr>
<th>Time Series Description</th>
<th>Units</th>
<th>Code</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross domestic product</td>
<td>CP, SA, billion $</td>
<td>GDP</td>
<td>BEA</td>
</tr>
<tr>
<td>Gross Private Domestic Investment</td>
<td>CP, SA, billion $</td>
<td>GPDI</td>
<td>BEA</td>
</tr>
<tr>
<td>Real Gross Private Domestic Investment</td>
<td>CVM, SA, billion $</td>
<td>GPDIC1</td>
<td>BEA</td>
</tr>
<tr>
<td>Personal Consumption Exp.: Durable Goods</td>
<td>CP, SA, billion $</td>
<td>PCDG</td>
<td>BEA</td>
</tr>
<tr>
<td>Personal Consumption Expenditures: Services</td>
<td>CVM, SA, billion $</td>
<td>PCESV</td>
<td>BEA</td>
</tr>
<tr>
<td>Personal Consumption Expenditures: Services</td>
<td>CVM, SA, billion $</td>
<td>PCESVC96</td>
<td>BEA</td>
</tr>
<tr>
<td>Personal Consumption Exp.: Nondurable Goods</td>
<td>CP, SA, billion $</td>
<td>PCND</td>
<td>BEA</td>
</tr>
<tr>
<td>Real Personal Consumption Exp.: Nondurable Goods</td>
<td>CVM, SA, billion $</td>
<td>PCNDGC96</td>
<td>BEA</td>
</tr>
<tr>
<td>Civilian Noninstitutional Population</td>
<td>NSA, 1000s</td>
<td>CNP160V</td>
<td>BLS</td>
</tr>
<tr>
<td>Nonfarm Business Sector: Compensation Per Hour</td>
<td>SA, Index 2005=100</td>
<td>COMPNFB</td>
<td>BLS</td>
</tr>
<tr>
<td>Nonfarm Business Sector: Hours of All Persons</td>
<td>SA, Index 2005=100</td>
<td>HOANBS</td>
<td>BLS</td>
</tr>
<tr>
<td>Effective Federal Funds Rate</td>
<td>NSA, percent</td>
<td>FEDFUNDS</td>
<td>BG</td>
</tr>
<tr>
<td>Total Equity</td>
<td>NSA</td>
<td>EQTA</td>
<td>IEC</td>
</tr>
<tr>
<td>Total Assets</td>
<td>NSA</td>
<td>H.8</td>
<td>FRB</td>
</tr>
<tr>
<td>All Employees</td>
<td>SA</td>
<td>B-1</td>
<td>BLS</td>
</tr>
<tr>
<td>Average Weekly Hours</td>
<td>SA</td>
<td>B-7</td>
<td>BLS</td>
</tr>
</tbody>
</table>


the average over all company level spreads available in a certain quarter. These two series are transformed from basis points into percent and divided by four to guarantee that they are consistent with the quarterly frequency of our model. After these adjustments the dataset (1990Q2-2011Q1) contains 5381 bonds of which 1213 are classified to be issued by companies in the consumption sector and 4168 issued by companies in the investment sector. This is equivalent to 35413 observations in the consumption and 115286 observations in the investment sector over the entire sample. The average duration is 30 quarters (consumption sector) and 28 quarters (investment sector) with an average rating for both sectoral bond issues between BBB+ and A-. The total number of firms in our sample is equal to 1696, where 516 firms belong to the consumption sector and 1180 firms belong to the investment sector.

Steady state financial parameters. The steady state leverage ratio of financial intermediaries in the model – which helps to pin down the parameters $\varpi$ and $\lambda_B$ – is
calculated by taking the sample average of the inverse of total equity over adjusted assets of all insured US commercial banks available from the Federal Financial Institutions Examination Council. The same body reports a series of equity over total assets. We multiply this ratio with total assets in order to get total equity for the U.S. banking sector that we use in estimation. Total assets includes consumer loans and holdings of government bonds which we want to exclude from total assets to be consistent with the model concept. Thus, to arrive at an estimate for adjusted assets we subtract consumer, real estate loans and holdings of government and government guaranteed bonds (such as government sponsored institutions) from total assets of all insured U.S. commercial banks.

C Model Details and Derivations

We provide the model details and derivations required for solution and estimation of the model. We begin with the pricing and wage decisions of firms and households, the financial sector followed by the normalization of the model to render it stationary, the description of the steady state and the log-linearized model equations.

C.1 Intermediate and Final Goods Producers

Intermediate producers pricing decision. A constant fraction $\xi_{p,x}$ of intermediate firms in sector $x = C, I$ cannot choose their price optimally in period $t$ but reset their price — as in Calvo (1983) — according to the indexation rule,

$$P_{C,t}(i) = P_{C,t-1}(i)\pi_{C,t-1}^{\text{pc}}\pi_{C}^{1-\text{pc}},$$

$$P_{I,t}(i) = P_{I,t-1}(i)\pi_{I,t-1}^{\text{pi}}\pi_{I}^{1-\text{pi}}\left[\left(\frac{A_t}{A_{t-1}}\right)^{-1}\left(\frac{V_t}{V_{t-1}}\right)^{1-\alpha}\right],$$

where $\pi_{C,t} \equiv \frac{P_{C,t}}{P_{C,t-1}}$ and $\pi_{I,t} \equiv \frac{P_{I,t}}{P_{I,t-1}}\left(\frac{A_t}{A_{t-1}}\right)^{-1}\left(\frac{V_t}{V_{t-1}}\right)^{1-\alpha}$ is gross inflation in the two sectors and $\pi_{C}, \pi_{I}$ denote steady state values. The factor that appears in the investment sector expression adjusts for investment specific progress.
The remaining fraction of firms, \((1 - \xi_{p,x})\), in sector \(x = C, I\) can adjust the price in period \(t\). These firms choose their price optimally by maximizing the present discounted value of future profits.

The resulting aggregate price index in the consumption sector is,

\[
P_{C,t} = \left(1 - \xi_{p,C}\right)\hat{P}_{C,t}^{\lambda_{p,t}^C} + \xi_{p,C}\left((\pi_{C,t-1}^{1-\rho_{p,C}}\pi_C^{-1-\rho_{p,C}}P_{C,t-1})^{\frac{1}{\lambda_{p,t}^C}}\right)^{\lambda_{p,t}^C}.
\]

The aggregate price index in the investment sector is,

\[
P_{I,t} = \left(1 - \xi_{p,I}\right)\hat{P}_{I,t}^{\lambda_{p,t}^I} + \xi_{p,I}\left((\pi_{I,t-1}^{1-\rho_{p,I}}\pi_I^{-1-\rho_{p,I}}(\frac{V_t}{V_{t-1}})^{\frac{1-\rho_{p,I}}{1-\rho_{p,I}}}P_{I,t-1})^{\frac{1}{\lambda_{p,t}^I}}\right)^{\lambda_{p,t}^I}.
\]

**Final goods producers.** Profit maximization and the zero profit condition for final good firms imply that sectoral prices of the final goods, \(P_{C,t}\) and \(P_{I,t}\), are CES aggregates of the prices of intermediate goods in the respective sector, \(P_{C,t}(i)\) and \(P_{I,t}(i)\),

\[
P_{C,t} = \left[\int_0^1 P_{C,t}(i)^{\frac{1}{\lambda_{p,t}^C}} \, di\right]^{\lambda_{p,t}^C}, \quad P_{I,t} = \left[\int_0^1 P_{I,t}(i)^{\frac{1}{\lambda_{p,t}^I}} \, di\right]^{\lambda_{p,t}^I}.
\]

The elasticity \(\lambda_{p,t}^x\) is the time varying price markup over marginal cost for intermediate firms. It is assumed to follow the exogenous stochastic process,

\[
\log(1 + \lambda_{p,t}^x) = (1 - \rho_{\lambda_{p,t}^x})\log(1 + \lambda_{p,t}^x) + \rho_{\lambda_{p,t}^x}\log(1 + \lambda_{p,t-1}^x) + \varepsilon_{p,t}^x,
\]

where \(\rho_{\lambda_{p,t}^x} \in (0, 1)\) and \(\varepsilon_{p,t}^x\) is i.i.d. \(N(0, \sigma_{\lambda_{p,t}^x}^2)\), with \(x = C, I\).

**C.1.1 Household’s wage setting**

Each household \(j \in [0, 1]\) supplies specialized labor, \(L_t(j)\), monopolistically as in Erceg et al. (2000). A large number of competitive “employment agencies” aggregate this specialized labor into a homogenous labor input which is sold to intermediate goods producers in a competitive market. Aggregation is done according to the following function,

\[
L_t = \left[\int_0^1 L_t(j)^{\frac{1}{1+\lambda_{w,t}}} \, dj\right]^{1+\lambda_{w,t}}.
\]
The desired markup of wages over the household’s marginal rate of substitution (or wage mark-up), \( \lambda_{w,t} \), follows the exogenous stochastic process,

\[
\log(1 + \lambda_{w,t}) = (1 - \rho_w) \log(1 + \lambda_w) + \rho_w \log(1 + \lambda_{w,t-1}) + \varepsilon_{w,t},
\]

where \( \rho_w \in (0, 1) \) and \( \varepsilon_{w,t} \) is i.i.d. \( N(0, \sigma^2_{\lambda_w}) \).

Profit maximization by the perfectly competitive employment agencies implies the labor demand function,

\[
L_t(j) = (W_t(j) - W_t)^{-1} W_t^{1-\nu} L_t,
\]

where \( W_t(j) \) is the wage received from employment agencies by the supplier of labor of type \( j \), while the wage paid by intermediate firms for the homogenous labor input is,

\[
W_t = \left[ \int_0^1 W_t(j)^{\frac{1}{\lambda_{w,t}}} dj \right]^{\lambda_{w,t}}.
\]

Following Erceg et al. (2000), in each period, a fraction \( \xi_w \) of the households cannot freely adjust its wage but follows the indexation rule,

\[
W_{t+1}(j) = W_t(j) \left( \pi_{c,t} e^{x_t + \frac{g_{a,t}}{1-\nu}} \right)^{\xi_w} \left( \pi_{c,t} e^{g_{a,t}} \right)^{1-\xi_w}.
\]

The remaining fraction of households, \( (1 - \xi_w) \), chooses an optimal wage, \( W_t(j) \), by maximizing,

\[
E_t \left\{ \sum_{s=0}^{\infty} \xi_{w,s}^s \beta^s \left[ -b_{t+s} W_{t+s}(j)^{1+\nu} \Lambda_{t+s} W_t(j) L_{t+s}(j) \right] \right\},
\]

subject to the labor demand function (C.1). The aggregate wage evolves according to,

\[
W_t = \left\{ (1 - \xi_w) (\tilde{W}_t)^{\lambda_w} + \xi_w \left( (\pi_{c,t-1} e^{x_{t-1} + \frac{g_{a,t-1}}{1-\nu}})^{1-\nu} W_{t-1} \right) \right\}^{\lambda_w},
\]

where \( \tilde{W}_t \) is the optimally chosen wage.

### C.2 Physical capital producers

Capital producers in sector \( x = C, I \) use a fraction of investment goods from final goods producers and undepreciated capital stock from capital services producers (as described
above) to produce new capital goods, subject to investment adjustment costs as proposed by Christiano et al. (2005). These new capital goods are then sold in perfectly competitive capital goods markets to capital services producers. The technology available for physical capital production is given as,

\[
O'_{x,t} = O_{x,t} + \left(1 - S\left(\frac{I_{x,t}}{I_{x,t-1}}\right)\right)I_{x,t},
\]

where \(O_{x,t}\) denotes the amount of used capital at the end of period \(t\), \(O'_{x,t}\) the new capital available for use at the beginning of period \(t+1\). The investment adjustment cost function \(S(\cdot)\) satisfies the following: \(S(1) = S'(1) = 0\) and \(S''(1) = \kappa > 0\), where \(''s\) denote differentiation. The optimization problem of capital producers in sector \(x = C, I\) is given as,

\[
\max_{I_{x,t}, O_{x,t}} \sum_{t=0}^{\infty} \beta^t \Lambda_t \left\{ Q_{x,t} \left[ O_{x,t} + \left(1 - S\left(\frac{I_{x,t}}{I_{x,t-1}}\right)\right)I_{x,t}\right] - Q_{x,t}O_{x,t} - \frac{P_{I,t}}{P_{C,t}}I_{x,t} \right\},
\]

where \(Q_{x,t}\) denotes the price of capital (i.e. the value of installed capital in consumption units). The first order condition for investment goods is,

\[
P_{I,t} = Q_{x,t} \left[1 - S\left(\frac{I_{x,t}}{I_{x,t-1}}\right) - S'\left(\frac{I_{x,t}}{I_{x,t-1}}\right)\left(\frac{I_{x,t}}{I_{x,t-1}}\right)\right] + \beta E_t Q_{x,t+1} \frac{\Lambda_{t+1}}{\Lambda_t} \left[S'\left(\frac{I_{x,t+1}}{I_{x,t}}\right)\left(\frac{I_{x,t+1}}{I_{x,t}}\right)^2\right].
\]

From the capital producer’s problem it is evident that any value of \(O_{x,t}\) is profit maximizing. Let \(\delta_x \in (0, 1)\) denote the depreciation rate of capital and \(K_{x,t-1}\) the capital stock available at the beginning of period \(t\) in sector \(x = C, I\). Then setting \(O_{x,t} = (1 - \delta)\xi^K_{x,t}K_{x,t-1}\) implies the available (sector specific) capital stock in sector \(x\), evolves according to,

\[
\bar{K}_{x,t} = (1 - \delta_x)\xi^K_{x,t}\bar{K}_{x,t-1} + \left(1 - S\left(\frac{I_{x,t}}{I_{x,t-1}}\right)\right)I_{x,t}, \quad x = C, I,
\]

as described in the main text.

### C.3 Financial Intermediaries

This section describes in detail how the setup of Gertler and Karadi (2011) is adapted for the two sector model and describes in detail how the equations for financial intermediaries in the main text are derived.
The balance sheet for the consumption or investment sector branch can be expressed as,

\[ P_{C,t}Q_{x,t}S_{x,t} = P_{C,t}N_{x,t} + B_{x,t}, \quad x = C, I, \]

where \( S_{x,t} \) denotes the quantity of financial claims held by the intermediary branch and \( Q_{x,t} \) denotes the sector specific price of a claim. The variable \( N_{x,t} \) represents the bank’s wealth (or equity) at the end of period \( t \) and \( B_{x,t} \) are the deposits the intermediary branch obtains from households. The sector specific assets held by the financial intermediary pay the stochastic return \( R_{B_{x,t}}^B \) in the next period. Intermediaries pay at \( t + 1 \) the non-contingent real gross return \( R_{t} \) to households for their deposits made at time \( t \). Then, the intermediary branch equity evolves over time as,

\[
N_{x,t+1} = P_{C,t+1}^B N_{x,t+1} + R_{t}^B (Q_{x,t}^B S_{x,t} - N_{x,t}) \]

\[
N_{x,t+1} = [(R_{x,t+1}^B \pi_{C,t}^B - R_{t}^B)Q_{x,t}^BS_{x,t} + R_{t}^B N_{x,t}] \frac{1}{\pi_{C,t+1}}. \]

The premium, \( R_{x,t+1}^B \pi_{C,t}^B - R_{t}, \) as well as the quantity of assets, \( Q_{x,t}^B S_{x,t}, \) determines the growth in bank’s equity above the riskless return. The bank will not fund any assets with a negative discounted premium. It follows that for the bank to operate in period \( i \) the following inequality must hold,

\[
E_t/\beta^i \Lambda_{t+1+i}^B (R_{x,t+1+i}^B \pi_{C,t+1+i} - R_{t+i}) \geq 0, \quad i \geq 0,
\]

where \( \beta^i \Lambda_{t+1+i}^B \) is the bank’s stochastic discount factor, with,

\[
\Lambda_{t+1}^B = \frac{\Lambda_{t+1}}{\Lambda_t},
\]

where \( \Lambda_t \) is the Lagrange multiplier on the household’s budget equation. Under perfect capital markets, arbitrage guarantees that the risk premium collapses to zero and the relation always holds with equality. However, under imperfect capital markets, credit constraints rooted in the bank’s inability to obtain enough funds may lead to positive risk premia. As long as the above inequality holds, banks will keep building assets by borrowing additional funds from households. Accordingly, the intermediary branch
objective is to maximize expected terminal wealth,

\[
V_{x,t} = \max E_t \sum_{i=0}^{\theta_B} (1 - \theta_B)^i \beta^i \Lambda_{t+1+i} \infty x_{x,t+1+i} \\
= \max E_t \sum_{i=0}^{\theta_B} (1 - \theta_B)^i \beta^i \Lambda_{t+1+i} \left[ (R_{x,t+1+i} \pi_{C,t+1+i} - R_{t+i}) \frac{Q_{x,t+i} S_{x,t+i}}{\pi_{C,t+1+i}} + \frac{R_{t+i} N_{x,t+i}}{\pi_{C,t+1+i}} \right],
\]

(C.3)

where \( \theta_B \in (0, 1) \) is the fraction of bankers at \( t \) that survive until period \( t + 1 \).

Following the setup in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) the banks are limited from infinitely borrowing additional funds from households by a moral hazard/costly enforcement problem. On the one hand, the agent who works in the bank can choose, at the beginning of each period, to divert the fraction \( \lambda_B \) of available funds and transfer it back to the household. On the other hand, depositors can force the bank into bankruptcy and recover a fraction \( 1 - \lambda_B \) of assets. Note that the fraction, \( \lambda_B \), which intermediaries can divert is the same across sectors to guarantee that the household is indifferent between lending funds between different branches.

Given this tradeoff, depositors will only lend funds to the intermediary when the latter’s maximized expected terminal wealth is larger or equal to the gain from diverting the fraction \( \lambda_B \) of available funds. This incentive constraint can be formalized as,

\[
V_{x,t} \geq \lambda_B Q_{x,t} S_{x,t}, \quad 0 < \lambda_B < 1.
\]

(C.4)

Using equation (C.3), the expression for \( V_{x,t} \) can be written as the following first-order difference equation,

\[
V_{x,t} = \nu_{x,t} Q_{x,t} S_{x,t} + \eta_{x,t} N_{x,t},
\]

with,

\[
\nu_{x,t} = E_t \{(1 - \theta_B) \Lambda_{t+1} (R_{x,t+1} \pi_{C,t+1} - R_{t+i}) + \theta_B \beta \lambda_{t+1} Z_{x,t+1}^{x} + \theta_{x,t}^{x} \},
\]

\[
\eta_{x,t} = E_t \{(1 - \theta_B) \Lambda_{t+1} R_{t+i} + \theta_B \beta \lambda_{t+1} Z_{x,t+1}^{x} \},
\]

and,

\[
Z_{x,t+1+i}^x = \frac{Q_{x,t+i} S_{x,t+i}}{Q_{x,t+i} S_{x,t+i}}, \quad Z_{x,t+1+i}^{x} = \frac{N_{x,t+i} N_{x,t+i}}{N_{x,t+i}}.
\]
The variable $\nu_{x,t}$ can be interpreted as the expected discounted marginal gain of expanding assets $Q_{x,t}S_{x,t}$ by one unit while holding wealth $N_{x,t}$ constant. The interpretation of $\eta_{x,t}$ is analogous: it is the expected discounted value of having an additional unit of wealth, $N_{x,t}$, holding the quantity of financial claims, $S_{x,t}$, constant. The gross growth rate in assets is denoted by $Z_{1,t+1}^x$ and the gross growth rate of net worth is denoted by $Z_{2,t+1}^x$.

Then, using the expression for $V_{x,t}$, we can express the intermediary’s incentive constraint (C.4) as,

$$\nu_{x,t} Q_{x,t} S_{x,t} + \eta_{x,t} N_{x,t}$$

As indicated above, under perfect capital markets banks will expand borrowing until the risk premium collapses to zero which implies that in this case $\nu_{x,t}$ equals zero as well. Imperfect capital markets however, limit the possibilities for this kind of arbitrage because the intermediaries are constrained by their equity capital. If the incentive constraint binds it follows that,

$$Q_{x,t}S_{x,t} = \frac{\eta_{x,t}}{\lambda_B - \nu_{x,t}} N_{x,t}$$

In this case, the quantity of assets which the intermediary can acquire depends on the equity capital, $N_{x,t}$, as well as the intermediary’s leverage ratio, $\varrho_{x,t}$, limiting the bank’s ability to acquire assets. This leverage ratio is the ratio of the bank’s intermediated assets to equity. The bank’s leverage ratio is limited to the point where its maximized expected terminal wealth equals the gains from diverting the fraction $\lambda_B$ from available funds. However, the constraint (C.5) binds only if $0 < \nu_{x,t} < \lambda_B$ (given $N_{x,t} > 0$). This inequality is always satisfied with our estimates.

Using the leverage ratio (C.5) we can express the evolution of the intermediary’s
wealth as,

\[ N_{x,t+1} = \left[ (R_{x,t+1}^{B}\pi_{C,t+1} - R_{t})q_{x,t} + R_{t} \right] \frac{N_{x,t}}{\pi_{C,t+1}}. \]

From this equation it also follows that,

\[ Z_{x,t+1}^2 = \frac{N_{x,t+1}}{N_{x,t}} = \left[ (R_{x,t+1}^{B}\pi_{C,t+1} - R_{t})q_{x,t} + R_{t} \right] \frac{1}{\pi_{C,t+1}}, \]

and,

\[ Z_{1,t+1}^x = \frac{Q_{x,t+1}^{x}S_{x,t+1}}{Q_{x,t}S_{x,t}} = \frac{q_{x,t+1}N_{x,t+1}}{q_{x,t}N_{x,t}} = \frac{q_{x,t+1}}{q_{x,t}} Z_{2,t+1}^x. \]

Financial intermediaries which are forced into bankruptcy are replaced by new entrants. Therefore, total wealth of financial intermediaries is the sum of the net worth of existing, \( N_{x,t}^{e} \), and new ones, \( N_{x,t}^{n} \),

\[ N_{x,t} = N_{x,t}^{e} + N_{x,t}^{n}. \]

The fraction \( \theta_{B} \) of bankers at \( t - 1 \) which survive until \( t \) is equal across branches. Then, the law of motion for existing bankers is given by,

\[ N_{x,t}^{e} = \theta_{B} [(R_{x,t}^{B}\pi_{C,t} - R_{t-1})q_{x,t-1} + R_{t-1}] \frac{N_{x,t-1}}{\pi_{C,t}}, \quad 0 < \theta_{B} < 1. \]  \( \text{(C.6)} \)

where a main source of variation is the ex-post excess return on assets, \( R_{x,t}^{B}\pi_{C,t} - R_{t-1} \).

New banks receive startup funds from their respective household, equal to a small fraction of the value of assets held by the existing bankers in their final operating period. Given that the exit probability is \( i.i.d. \), the value of assets held by the existing bankers in their final operating period is given by \( (1 - \theta_{B})Q_{x,t}S_{x,t} \). The transfer to new intermediaries is a fraction, \( \varpi \), of this value, leading to the following formulation for new banker’s wealth,

\[ N_{x,t}^{n} = \varpi Q_{x,t}S_{x,t}, \quad 0 < \varpi < 1. \]  \( \text{(C.7)} \)

Existing banker’s net worth (C.6) and entering banker’s net worth (C.7) lead to the law
of motion for total net worth,
\[ N_{x,t} = \left( \theta_B \left[ (R^B_{x,t} \pi_C,t - R_{t-1}) \varpi_{x,t-1} + R_{t-1} \right] \frac{N_{x,t-1}}{\pi_C,t} + \varpi Q_{x,t} \right) S_{x,t}. \]

The excess return, \( x = C, I \) can be defined as,
\[ R^S_{x,t} = R^B_{x,t+1} \pi_{C,t+1} - R_t. \]

Since \( R_t, \lambda_B, \varpi \) and \( \theta_B \) are equal across sectors, the institutional setup of the two representative banks in the two sectors is symmetric. Both branches hold deposits from households and buy assets from firms in the sector they provide specialized lending. Their performance differs because the demand for capital differs across sectors resulting in sector specific prices of capital, \( Q_{x,t} \), and nominal rental rates for capital, \( R^K_{x,t} \). Note that the institutional setup of banks does not depend on firm-specific factors. Gertler and Karadi (2011) show that this implies a setup with a continuum of banks is equivalent to a formulation with a representative bank. Owing to the symmetry of the banks this also holds for our formulation of financial intermediaries in the two-sector setup.

### C.4 Resource Constraints

The resource constraint in the consumption sector is,
\[ C_t + (a(u_C,t) \xi^K_{C,t} K_{C,t-1} + a(u_I,t) \xi^K_{I,t} K_{I,t-1}) \frac{A_t V_t^{1-a_C}}{V_t^{1-a_I}} = A_t L^{1-a_C}_{t} K_{c,t}^{a_C} - A_t V_t^{\frac{a_C}{1-a_C}} F_C. \]

The resource constraint in the investment sector is,
\[ I_{I,t} + I_{C,t} = V_t L^{1-a_I}_{I,t} K_{I,t}^{a_I} - V_t^{\frac{1}{1-a_I}} F_I. \]

Hours worked are aggregated as,
\[ L_t = L_{I,t} + L_{C,t}. \]

Bank equity is aggregated as,
\[ N_t = N_{I,t} + N_{C,t}. \]
C.5 Stationary Economy

The model includes two non-stationary TFP shocks, $A_t$ and $V_t$. This section shows how we normalize the model to render it stationary. Lower case variables denote normalized stationary variables.

The model variables can be stationarized as follows:

$$\begin{align*}
    k_{x,t} &= \frac{K_{x,t}}{V_t^{1-a_i}}, \quad \bar{k}_{x,t} = \frac{\bar{K}_{x,t}}{V_t^{1-a_i}}, \quad k_t = \frac{K_t}{V_t^{1-a_i}}, \quad (C.8) \\
    i_{x,t} &= \frac{I_{x,t}}{V_t^{1-a_i}}, \quad i_t = \frac{I_t}{V_t^{1-a_i}}, \quad c_t = \frac{C_t}{A_t V_t^{1-a_i}}, \quad (C.9) \\
    r^K_{C,t} &= \frac{R^K_{C,t}}{P_{C,t}} A_t^{-1} V_t^{1-a_i}, \quad r^K_{I,t} = \frac{R^K_{I,t}}{P_{C,t}} A_t^{-1} V_t^{1-a_i}, \quad w_t = \frac{W_t}{P_{C,t} A_t V_t^{1-a_i}}. \quad (C.10)
\end{align*}$$

From

$$\begin{align*}
P_{I,t} &= \frac{mc_{C,t} 1 - a_c A_t \left( \frac{K_{I,t}}{L_{I,t}} \right)^{-a_i} \left( \frac{K_{C,t}}{L_{C,t}} \right)^{a_c}}{mc_{I,t} 1 - a_i \bar{V}_t \left( \frac{K_{I,t}}{L_{I,t}} \right)^{-a_i} \left( \frac{K_{C,t}}{L_{C,t}} \right)^{a_c}} \\
&= \frac{mc_{C,t} 1 - a_c A_t V_t^{1-a_i} \left( \frac{k_{I,t}}{L_{I,t}} \right)^{-a_i} \left( \frac{k_{C,t}}{L_{C,t}} \right)^{a_c}}{mc_{I,t} 1 - a_i A_t V_t^{1-a_i} \left( \frac{k_{I,t}}{L_{I,t}} \right)^{-a_i} \left( \frac{k_{C,t}}{L_{C,t}} \right)^{a_c}},
\end{align*}$$

follows that,

$$p_{I,t} = \frac{P_{I,t}}{P_{C,t}} A_t^{-1} V_t^{1-a_i}. \quad (C.11)$$

and the multipliers are normalized as,

$$\lambda_t = \Lambda_t A_t V_t^{1-a_c}, \quad \phi_{x,t} = \Phi_{x,t} V_t^{1-a_c}. \quad (C.12)$$

where $\Phi_{x,t}$ denotes the multiplier on the respective capital accumulation equation. Using the growth of investment, it follows that the prices of capital can be normalized as,

$$q_{x,t} = Q_{x,t} A_t^{-1} V_t^{1-a_i}.$$ 

with the price of capital in sector $x$, defined as,

$$q_{x,t} = \phi_{x,t} / \lambda_t, \quad x = C, I.$$
Using the growth of capital, it follows,

\[ s_{x,t} = \frac{S_{x,t}}{V_t^{\frac{1}{1-a_i}}}. \]

Then, it follows from entering bankers wealth equation (C.7) that,

\[ n_{x,t}^n = N_{x,t} A_t^{-1} V_t^{\frac{1}{1-a_i}}. \]

Total wealth, wealth of existing and entering bankers has to grow at the same rate,

\[ n_{x,t}^e = N_{x,t}^e A_t^{-1} V_t^{\frac{1}{1-a_i}}, \quad n_{x,t} = N_{x,t} A_t^{-1} V_t^{\frac{1}{1-a_i}}. \]

### C.5.1 Intermediate goods producers

Firm’s production function in the consumption sector:

\[ c_t = L^{1-a_c} k^{a_c}_{C,t} - F_C. \]  

(C.13)

Firm’s production function in the investment sector:

\[ i_t = L^{1-a_i} k^{a_i}_{I,t} - F_I. \]  

(C.14)

Marginal costs in the consumption sector:

\[ m_{CC,t} = (1 - a_c)^{a_c-1} a_c^{1-a_c} (r_{C,t})^{a_c} w_t^{1-a_c}. \]  

(C.15)

Marginal costs in the investment sector:

\[ m_{CI,t} = (1 - a_i)^{a_i-1} a_i^{1-a_i} (r_{I,t})^{a_i} p_t^{-1}, \quad \text{with} \quad p_{i,t} = \frac{P_{I,t}}{P_{C,t}}. \]  

(C.16)

Capital labour ratios in the two sectors:

\[ \frac{k_{C,t}}{L_{C,t}} = \frac{w_t}{r_{C,t}} \frac{a_c}{1 - a_c}, \quad \frac{k_{I,t}}{L_{I,t}} = \frac{w_t}{r_{I,t}} \frac{a_i}{1 - a_i}. \]  

(C.17)

### C.5.2 Firms’ pricing decisions

Price setting equation for firms that change their price in sector \( x = C, I \):

\[ 0 = E_t \left\{ \sum_{s=0}^{\infty} \xi^{s}_{p,x} \beta^{s} \lambda_{t+s} \tilde{x}_{t+s} \left[ \tilde{p}_{x,t} \tilde{\Pi}_{t,t+s} - (1 + \lambda_{p,t+s}) m_{C,x,t+s} \right] \right\}. \]  

(C.18)
with
\[ \tilde{\Pi}_{t,t+s} = \prod_{k=1}^{s} \left( \frac{\pi_{x,t+k-1}}{\pi_{x}} \right)^{\xi_{x}} \left( \frac{\pi_{x,t+k}}{\pi_{x}} \right)^{-1} \] and \[ \tilde{x}_{t+s} = \left( \frac{\tilde{P}_{x,t} \tilde{\Pi}_{t,t+s}}{\tilde{P}_{x,t}} \right)^{-1+\lambda_{p,t+s}} \tilde{x}_{t+s} \]
and \[ \frac{\tilde{P}_{x,t}}{P_{x,t}} = \tilde{p}_{x,t}. \]

Aggregate price index in the consumption sector:
\[ 1 = \left( 1 - \xi_{x,p} \right) \pi_{p,t}^{\frac{1}{\lambda_{p,t}}} + \xi_{x,p} \left( \frac{\pi_{x,t-1}}{\pi_{x}} \right)^{\xi_{x,p}} \left( \frac{\pi_{x,t}}{\pi_{x}} \right)^{-1} \pi_{p,t}^{\frac{1}{\lambda_{p,t}}}. \]

It further holds that
\[ \frac{\pi_{I,t}}{\pi_{C,t}} = \frac{p_{i,t}}{p_{i,t-1}}. \]

(C.19)

C.5.3 Household’s optimality conditions and wage setting

Marginal utility of income:
\[ \lambda_{t} = \frac{b_{t}}{c_{t} - h_{C,t-1} \left( \frac{A_{t-1}}{A_{t}} \right) \left( \frac{V_{t-1}}{V_{t}} \right)^{\frac{n_{u}}{1-n_{u}}} - \beta h \left( \frac{A_{t+1}}{A_{t}} \right) \left( \frac{V_{t+1}}{V_{t}} \right)^{\frac{n_{u}}{1-n_{u}}} - h_{C,t}}. \]

(C.20)

Euler equation:
\[ \lambda_{t} = \beta E_{t+1} \left( \frac{A_{t}}{A_{t+1}} \right) \left( \frac{V_{t}}{V_{t+1}} \right)^{\frac{n_{u}}{1-n_{u}}} R_{t} \frac{1}{\pi_{C,t+1}}. \]

Labor supply
\[ \lambda_{t} w_{t} = b_{t} \varphi (L_{C,t} + L_{I,t})^{\nu}, \]

C.5.4 Capital services

Optimal capital utilization:
\[ r_{C,t}^{K} = a_{C}^{*} (u_{C,t}), \quad r_{I,t}^{K} = a_{I}^{*} (u_{I,t}). \]

Definition of capital services:
\[ k_{C,t} = u_{C,t} \xi_{C,t}^{K} \tilde{k}_{C,t-1} \left( \frac{V_{t-1}}{V_{t}} \right)^{\frac{1}{\lambda_{C,t}}}, \quad k_{I,t} = u_{I,t} \xi_{I,t}^{K} \tilde{k}_{I,t-1} \left( \frac{V_{t-1}}{V_{t}} \right)^{\frac{1}{\lambda_{I,t}}}. \]

(C.21)
Optimal choice of available capital in sector \( x = C, I \):

\[
\phi_{x,t} = \beta E_t \phi_{x,t+1} \left\{ \lambda_{t+1} \left( \frac{V_t}{V_{t+1}} \right)^{-\frac{1}{\alpha_w}} \left( r_{x,t+1} K_{x,t+1} u_{x,t+1} - a(u_{x,t+1}) \right) + (1 - \delta) E_t \phi_{x,t+1} \left( \frac{V_t}{V_{t+1}} \right)^{-\frac{1}{\alpha_w}} \right\},
\]  

(C.22)

C.5.5 Physical capital producers

Optimal choice of investment in sector \( x = C, I \):

\[
\lambda_{t} p_{x,t} = \phi_{x,t} \left[ 1 - S \left( \frac{i_{x,t}}{i_{x,t-1}} \left( \frac{V_t}{V_{t-1}} \right)^{-\frac{1}{\alpha_w}} \right) - S' \left( \frac{i_{x,t}}{i_{x,t-1}} \left( \frac{V_t}{V_{t-1}} \right)^{-\frac{1}{\alpha_w}} \right) \right] + \beta E_t \phi_{x,t+1} \left( \frac{V_t}{V_{t+1}} \right)^{-\frac{1}{\alpha_w}} \left[ S' \left( \frac{i_{x,t+1}}{i_{x,t}} \left( \frac{V_{t+1}}{V_t} \right)^{-\frac{1}{\alpha_w}} \right) \left( \frac{i_{x,t+1}}{i_{x,t}} \left( \frac{V_{t+1}}{V_t} \right)^{-\frac{1}{\alpha_w}} \right)^2 \right].
\]  

(C.23)

Accumulation of capital in sector \( x = C, I \):

\[
\tilde{k}_{x,t} = (1 - \delta_x) \bar{k}_{x,t} \tilde{k}_{x,t-1} \left( \frac{V_{t-1}}{V_t} \right)^{-\frac{1}{\alpha_w}} + \left( 1 - S \left( \frac{i_{x,t}}{i_{x,t-1}} \left( \frac{V_t}{V_{t-1}} \right)^{-\frac{1}{\alpha_w}} \right) \right) i_{x,t},
\]  

(C.24)

C.5.6 Household’s wage setting

Household’s wage setting:

\[
E_t \sum_{s=0}^{\infty} \beta^s \kappa_s \lambda_{t+s} \tilde{l}_{t+s} \left[ \bar{w}_t \tilde{\Pi}_{t+l+s}^w - (1 + \lambda_{w,t+s}) \bar{b}_{t+s} \phi_{t+s} \tilde{\Lambda}_{t+s} \right] = 0,
\]  

(C.25)

with

\[
\tilde{\Pi}_{t+l+s}^w = \prod_{k=1}^{s} \left( \frac{\pi_{C,t+k-1} \alpha_{t+k-1} + \alpha_w \pi_{w,t+k-1}}{\pi_{c,t+k} \alpha_t + \alpha_w \pi_{w,t+k}} \right)^{\frac{\alpha_{t+k} - \alpha_w}{1 - \alpha_w}}
\]

and

\[
\tilde{L}_{t+s} = \left( \frac{\bar{w}_t \tilde{\Pi}_{t+l+s}^w}{\bar{w}_{t+s}} \right)^{-\frac{1}{\lambda_{w,t+s}}} \tilde{L}_{t+s}.
\]

Wages evolve according to

\[
w_t = \left\{ (1 - \xi_{w}) \bar{w}_t \left[ \frac{\pi_{c,t-1}}{\pi_{c,t} \alpha_t + \alpha_w \pi_{w,t}} \right]^{\frac{\alpha_{t-1} + \alpha_w}{1 - \alpha_w}} \left[ \frac{\pi_{c,t} \alpha_{t+k} + \alpha_w \pi_{w,t+k}}{\pi_{c,t+k} \alpha_{t+k} + \alpha_w \pi_{w,t+k}} \right] \bar{w}_t \left[ \frac{\pi_{c,t+k} \alpha_{t+k} + \alpha_w \pi_{w,t+k}}{\pi_{c,t+k} \alpha_{t+k} + \alpha_w \pi_{w,t+k}} \right] \right\}^{\frac{1}{\lambda_{w,t}}}.
\]
C.5.7 Financial Intermediation

The stationary stochastic discount factor can be expressed as,

\[ \lambda_{t+1}^B = \frac{\lambda_{t+1}}{\lambda_t}. \]

Then, one can derive expressions for \( \nu_{x,t} \) and \( \eta_{x,t} \),

\[
\nu_{x,t} = E_t \{ (1 - \theta_B) \lambda_{t+1}^B \frac{A_t}{A_{t+1}} \left( \frac{V_t}{V_{t+1}} \right)^\frac{\sigma_c}{\sigma_n} \left( R_{x,t+1}^B \pi_{C,t+1} - R_t \right) + \theta_B \beta z_{1,t+1}^x \nu_{x,t+1} \},
\]

\[
\eta_{x,t} = E_t \{ (1 - \theta_B) \lambda_{t+1}^B \frac{A_t}{A_{t+1}} \left( \frac{V_t}{V_{t+1}} \right)^\frac{\sigma_c}{\sigma_n} R_t + \theta_B \beta z_{2,t+1}^x \eta_{x,t+1} \},
\]

with

\[
z_{1,t+1+i}^x = \frac{q_{x,t+1+i}s_{x,t+1+i}}{q_{x,t+i}s_{x,t+i}} \frac{A_{t+1}}{A_t} \left( \frac{V_{t+1}}{V_t} \right)^\frac{\sigma_c}{\sigma_n}, \quad z_{2,t+1+i}^x = \frac{n_{x,t+1+i}}{n_{x,t+i}} \frac{A_{t+1}}{A_t} \left( \frac{V_{t+1}}{V_t} \right)^\frac{\sigma_c}{\sigma_n}.
\]

It follows that if the bank’s incentive constraint binds it can be expressed as,

\[
\nu_{x,t}q_{x,t}s_{x,t} + \eta_{x,t}n_{x,t} = \lambda_B q_{x,t}s_{x,t}
\]

\[
\iff q_{x,t}s_{x,t} = q_{x,t}n_{x,t},
\]

with the leverage ratio given as,

\[
\varrho_{x,t} = \frac{\eta_{x,t}}{\lambda_B - \nu_{x,t}}.
\]

It further follows that:

\[
z_{2,t+1}^x = \frac{n_{x,t+1+i}}{n_{x,t+i}} \frac{A_{t+1}}{A_t} \left( \frac{V_{t+1}}{V_t} \right)^\frac{\sigma_c}{\sigma_n} = \left[ (R_{x,t+1}^B \pi_{C,t+1} - R_t) \varrho_{x,t} + R_t \right] \frac{1}{\pi_{C,t+1}},
\]

and

\[
z_{1,t+1}^x = \frac{q_{x,t+1+i}s_{x,t+1+i}}{q_{x,t+i}s_{x,t+i}} \frac{A_{t+1}}{A_t} \left( \frac{V_{t+1}}{V_t} \right)^\frac{\sigma_c}{\sigma_n} = \frac{q_{x,t+1+i}n_{x,t+1+i}}{q_{x,t+i}n_{x,t+i}} \frac{A_{t+1}}{A_t} \left( \frac{V_{t+1}}{V_t} \right)^\frac{\sigma_c}{\sigma_n} = \frac{q_{x,t+1+i}n_{x,t+1+i}}{q_{x,t+i}n_{x,t+i}} \varrho_{x,t} z_{2,t+1}^x.
\]

The normalized equation for bank’s wealth accumulation is,

\[
n_{x,t} = (\theta_B [(R_{x,t}^B \pi_{C,t} - R_{t-1}) \varrho_{x,t-1} + R_{t-1}] \frac{A_{t-1}}{A_t} \left( \frac{V_{t-1}}{V_t} \right)^\frac{\sigma_c}{\sigma_n} n_{x,t-1} + \omega q_{x,t}s_{x,t}).
\]

The borrow in advance constraint:

\[
\bar{k}_{x,t+1} = s_{x,t}.
\]
The leverage equation:

\[ q_{x,t} s_{x,t} = q_{x,t} n_{x,t}. \]

Bank’s stochastic return on assets can be described in normalized variables as:

\[
R^B_{x,t+1} = \frac{r^K_{x,t+1} u_{x,t+1} + q_{x,t+1} (1 - \delta_x) - a(u_{x,t+1}) \xi^K_{x,t+1}}{q_{x,t}} A_{t+1} \left( \frac{V_{t+1}}{V_t} \right)^{-\frac{1-a_c}{1-a_i}},
\]

knowing from the main model that

\[
r^K_{x,t} = \frac{R^K_{x,t}}{P_{x,t}} V_t^{-\frac{1-a_c}{1-a_i}}.
\]

C.5.8 Monetary policy and market clearing

Monetary policy rule:

\[
\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\rho_R} \left[ \frac{\pi_{C,t}}{\pi_C} \right]^{\phi_x} \left( \frac{y_t}{y_{t-1}} \right)^{\phi y} (1-\rho_R) \eta_{mp,t},
\]

Resource constraint in the consumption sector:

\[
c_t + (a(u_{C,t}) \xi^K_{C,t} \bar{k}_{C,t-1} + a(u_I,t) \xi^K_{I,t} \bar{k}_{I,t-1}) \left( \frac{V_{t-1}}{V_t} \right)^{\frac{1}{1-a_i}} = L^{1-a_c} k^{a_c}_{C,t} - F_C.
\]

Resource constraint in the investment sector:

\[
i_t = L^{1-a_i} k^{a_i}_{I,t} - F_I.
\]

Definition of GDP:

\[
y_t = c_t + p_t i_t + \left( 1 - \frac{1}{g_t} \right) y_t. \tag{C.26}
\]

Moreover

\[
L_t = L_{I,t} + L_{C,t}, \quad i_t = i_{C,t} + i_{I,t}, \quad n_t = n_{C,t} + n_{I,t}.
\]

C.6 Steady State

This section describes the model’s steady state.
From the optimal choice of available capital (C.22) and the optimal choice of investment (C.23) in both sectors:

\[ r^K_C = \left( e^{\frac{1-a_C}{\beta}} g - (1 - \delta_C) \right) p_i, \]
\[ r^K_I = \left( e^{\frac{1-a_I}{\beta}} g - (1 - \delta_I) \right) p_i. \]

(C.27)  
(C.28)

From firm’s price setting in both sectors (C.18),

\[ mc_C = \frac{1}{1 + \lambda_p}(1 - a_C) \frac{1}{\beta} - (1 - \delta_C) - a_c \]  
\[ mc_I = \frac{1}{1 + \lambda_p}(1 - a_I) \frac{1}{\beta} - (1 - \delta_I) - a_i. \]

(C.29)

Using equations (C.29) and imposing knowledge of the steady state expression for \( r^K_C \) and \( r^K_I \), one can derive expressions for the steady state wage from the equations that define marginal costs in the two sectors ((C.15) and (C.16)).

Consumption sector:

\[ w = \left( \frac{1}{1 + \lambda_p}(1 - a_C) \frac{1}{\beta} - (1 - \delta_C) - a_c \right)^{\frac{1}{1-a_C}}. \]

(C.30)

Investment sector:

\[ w = \left( \frac{1}{1 + \lambda_p}(1 - a_I) \frac{1}{\beta} - (1 - \delta_I) - a_i \right)^{\frac{1}{1-a_I}}. \]

(C.31)

Since labour can move across sectors the steady state wage has to be the same in the consumption and investment sector. The equality is verified by \( p_i \). An expression for \( p_i \) can be found by setting (C.30) equal to (C.31):

\[ \left( \frac{1}{1 + \lambda_p}(1 - a_C) \frac{1}{\beta} - (1 - \delta_C) - a_c \right)^{\frac{1}{1-a_C}} = \left( \frac{1}{1 + \lambda_p}(1 - a_I) \frac{1}{\beta} - (1 - \delta_I) - a_i \right)^{\frac{1}{1-a_I}}. \]

\[ \Leftrightarrow \left( \frac{1}{1 + \lambda_p}(1 - a_C) \frac{1}{\beta} - (1 - \delta_C) - a_c \right)^{\frac{1}{1-a_C}} = \left( \frac{1}{1 + \lambda_p}(1 - a_I) \frac{1}{\beta} - (1 - \delta_I) - a_i \right)^{\frac{1}{1-a_I}}. \]

\[ \Leftrightarrow p_i = \left[ \frac{1}{1 + \lambda_p}(1 - a_C) \frac{1}{\beta} - (1 - \delta_C) - a_c \right] \left[ \frac{1}{1 + \lambda_p}(1 - a_I) \frac{1}{\beta} - (1 - \delta_I) - a_i \right]^{-\frac{1}{1-a_i}}. \]

(C.32)

Knowing \( w, r^K_C \) and \( r^K_I \), the expressions given in (C.17) can be used to find the steady
state capital-to-labour ratios in the two sectors:

\[
\frac{k_C}{L_C} = \frac{w}{r_C^k} \frac{a_c}{1 - a_c}, \quad (C.33)
\]

\[
\frac{k_I}{L_I} = \frac{w}{r_I^k} \frac{a_i}{1 - a_c}. \quad (C.34)
\]

The zero profit condition for intermediate goods producers in the consumption sector, \( c - r_C^k k_C - w L_C = 0 \), and (C.13) imply:

\[
L_C^{1-a_c} k_C^{a_c} - F_C - r_C^k k_C - w L_C = 0
\]

\[
\Leftrightarrow \frac{F_C}{L_C} = \left( \frac{k_C}{L_C} \right)^{a_c} - r_C^k \frac{k_C}{L_C} - w.
\]

Analogously the zero profit condition for intermediate goods producers in the investment sector, \( i - r_I^k k_I - w L_I = 0 \), and (C.14) imply:

\[
\frac{F_I}{L_I} = \left( \frac{k_I}{L_I} \right)^{a_i} - r_I^k \frac{k_I}{L_I} - w.
\]

These expressions pin down the steady state consumption-to-labour and investment-to-labour ratios which follow from the intermediate firms’ production functions ((C.13) and (C.14)):

\[
\frac{c}{L_C} = \left( \frac{k_C}{L_C} \right)^{a_c} - \frac{F_C}{L_C}, \quad \frac{i}{L_I} = \left( \frac{k_I}{L_I} \right)^{a_i} - \frac{F_I}{L_I}.
\]

\[
1 + \lambda_p^C = \frac{c + F_C}{c} \Leftrightarrow \lambda_p^C c = F_C, \quad \text{and} \quad 1 + \lambda_p^I = \frac{i + F_I}{i} \Leftrightarrow \lambda_p^I i = F_I.
\]

This and the steady state consumption-to-labour ratio can be used to derive an expression for steady state consumption:

\[
c = \left( \frac{k_C}{L_C} \right)^{a_c} L_C - F_C
\]

\[
\Leftrightarrow c = \left( \frac{k_C}{L_C} \right)^{a_c} L_C - \lambda_p^C c
\]

\[
\Leftrightarrow c = \frac{1}{1 + \lambda_p^C} \left( \frac{k_C}{L_C} \right)^{a_c} L_C.
\]
Analogously one can derive an expression for steady state investment:

\[ i = \frac{1}{1 + \lambda_p^i} \left( \frac{k_I}{L_I} \right)^{a_i} L_I. \]

Combining these two expressions leads to,

\[ \frac{i^c}{p_i} = \frac{1}{1 + \lambda_p^c} \left( \frac{k_C}{L_C} \right)^{a_c} L_C \]

\[ \Rightarrow \frac{L_I}{L_C} = \left( 1 + \frac{i^c}{1 + \lambda_p^c} \left( \frac{k_C}{L_C} \right)^{a_c} p_i^{i^c-1} \right)^{-1}. \]

Total labour \( L \) is set to unity in the steady state. However, since \( a_i \) and \( a_c \) are not necessarily calibrated to be equal one needs to fix another quantity in addition to \( L = 1 \). We fix the steady state investment-to-consumption ratio, \( p_i^i \), which equals 0.399 in the data. This allows us to derive steady state expressions for labour in the two sectors.

Steady state labour in the investment sector is given by

\[ L_I = 1 - L_C, \quad (C.35) \]

and the two equations above imply that steady state labour in the consumption sector can be expressed as,

\[ L_C = \left( 1 + \frac{i^c}{1 + \lambda_p^c} \left( \frac{k_C}{L_C} \right)^{a_c} p_i^{i^c-1} \right)^{-1}. \quad (C.36) \]

The steady state values for labour in the two sectors imply:

\[ k_C = \frac{k_C}{L_C} L_I, \quad k_I = \frac{k_I}{L_I} L_I, \quad c = \frac{c}{L_C} L_C, \quad i = \frac{i}{L_I} L_I, \quad F_C = \frac{F_C}{L_C} L_C, \quad F_I = \frac{F_I}{L_I} L_I. \]

It follows from (C.21) that,

\[ k_C = \bar{k}_C e^{-\frac{1}{1-\nu}g_v}, \quad \text{and} \quad k_I = \bar{k}_I e^{-\frac{1}{1-\nu}g_v}. \]

The accumulation equation of available capital (C.24) can be used to solve for investment in the two sectors:

\[ i_C = k_C \left( e^{\frac{1}{1-\nu}g_v} - (1 - \delta_C) \right), \quad (C.37) \]

\[ i_I = k_I \left( e^{\frac{1}{1-\nu}g_v} - (1 - \delta_I) \right). \quad (C.38) \]
From the definition of GDP (C.26):

\[ y = c + p_i \left( 1 - \frac{1}{g} \right) y. \]

From the marginal utility of income (C.20):

\[ \lambda = \frac{1}{c - h c e^{-g_a - \frac{a}{1-g} g_v}} - \frac{\beta h}{c e^{g_a + \frac{a}{1-g} g_v} - h c}. \]

From the household’s wage setting (C.25)

\[ \sum_{s=0}^{\infty} \beta^s \xi^s \lambda L \left[ w - (1 + \lambda_w) \varphi \frac{L^\nu}{\lambda} \right] = 0, \]

follows the expression for \( L \):

\[ w - (1 - \lambda_w) \varphi \frac{L^\nu}{\lambda} = 0 \Rightarrow L = \left( \frac{w \lambda}{(1 + \lambda_w) \varphi} \right)^{\frac{1}{\nu}}. \]

This expression can be solved for \( \varphi \) to be consistent with \( L = 1 \):

\[ 1 = \left( \frac{w \lambda}{(1 + \lambda_w) \varphi} \right)^{\frac{1}{\nu}}, \]

\[ \Leftrightarrow \varphi = \frac{\lambda w}{1 + \lambda_w}. \]

It further holds from equation (C.19) that,

\[ \frac{\pi I}{\pi C} = e^{g_a - \frac{a}{1-g} g_v} \]

A system of 10 equations (C.27, C.28, C.30, C.32, C.33, C.34, C.35, C.36, C.37, C.38) can be solved for the 10 steady state variables \( k_C, k_I, w, i_C, i_I, r_C^k, r_I^k, L_C, L_I \) and \( p_i \).

The steady state values for the remaining variables follow from the expressions above.

Given these steady state variables, the remaining steady state values which are mainly related to financial intermediaries can be derived as follows.

The nominal interest rate is given from the Euler equation as,

\[ R = \frac{1}{\beta} e^{g_a + \frac{a}{1-g} g_v} \pi_C. \]

The bank’s stationary stochastic discount factor can be expressed in the steady state

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as

$$\lambda^B = 1.$$  

The steady state borrow in advance constraint implies that

$$\bar{k}_x = s_x.$$  

The steady state price of capital is given by

$$q_{x,t} = p_{i,t}.$$  

The steady state leverage equation is set equal to it’s average value in the data over the sample period.

$$\frac{q_x s_x}{n_x} = \varrho_x = 5.47.$$  

The parameters $\varpi$ and $\lambda_B$ help to align the value of the leverage ratio and the corporate bond spread with their empirical counterparts. Using the calibrated value for $\theta_B$, the average value for the leverage ratio (5.47) and the weighted quarterly average of the corporate spreads ($R^B_x - R = 0.5\%$) allows calibrating $\varpi$ using the bank’s wealth accumulation equation,

$$\varpi = \left[1 - \theta_B \pi_C (R^B_x - R)\varrho_x + R e^{-g_a - \frac{g_v}{1 - \gamma} - g_v} \frac{1}{\pi_C} \left(\frac{q_x s_x}{n_x}\right)^{-1}\right].$$  

Given the non-linearity in the leverage ratio, we solve numerically for the steady state expressions for $\eta$ and $\nu$ using,

$$\nu_x = (1 - \theta_B) \lambda^B e^{-g_a - \frac{g_v}{1 - \gamma} - g_v} (R^B_x \pi_C - R) + \theta_B \beta z_1^x \nu_x,$$

$$\eta_x = (1 - \theta_B) \lambda^B e^{-g_a - \frac{g_v}{1 - \gamma} - g_v} R + \theta_B \beta z_2^x \varrho_x,$$

with

$$z_2^x = \left[(R^B_x \pi_C - R)\varrho_x + R\right] \frac{1}{\pi_C},$$

and

$$z_1^x = z_2^x,$$

and the steady state leverage ratio,

$$\varrho_x = \frac{\eta_x}{\lambda_B - \nu_x}.$$  

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C.7 Log-linearized Economy

This section collects the log-linearized model equations. The log-linear deviations of all variables are defined as

\[ \hat{\varsigma}_t \equiv \log \varsigma_t - \log \varsigma, \]

except for

\[ \hat{z}_t \equiv z_t - g_z, \]
\[ \hat{v}_t \equiv v_t - g_v, \]
\[ \hat{\lambda}^C_{p,t} \equiv \log(1 + \lambda^C_{p,t}) - \log(1 + \lambda^C_p), \]
\[ \hat{\lambda}^I_{p,t} \equiv \log(1 + \lambda^I_{p,t}) - \log(1 + \lambda^I_p), \]
\[ \hat{\lambda}_w,t \equiv \log(1 + \lambda_{w,t}) - \log(1 + \lambda_w). \]

C.7.1 Firm’s production function and cost minimization

Production function for the intermediate good producing firm \((i)\) in the consumption sector:

\[ \hat{c}_t = \frac{c}{c} + F_I [a_c \hat{k}_{C,t} + (1 - a_c) \hat{L}_{C,t}]. \]

Production function for the intermediate good producing firm \((i)\) in the investment sector:

\[ \hat{i}_t = \frac{i}{i} + F_I [a_i \hat{k}_{I,t} + (1 - a_i) \hat{L}_{I,t}]. \]

Capital-to-labour ratios for the two sectors:

\[ \hat{r}^K_{C,t} - \hat{w}_t = \hat{L}_{C,t} - \hat{k}_{C,t}, \quad \hat{r}^K_{I,t} - \hat{w}_t = \hat{L}_{I,t} - \hat{k}_{I,t}. \]  \(\text{(C.39)}\)

Marginal cost in both sectors:

\[ \hat{m}c_{C,t} = a_c \hat{r}^K_{C,t} + (1 - a_c) \hat{w}_t, \quad \hat{m}c_{I,t} = a_i \hat{r}^K_{I,t} + (1 - a_i) \hat{w}_t - \hat{p}_{i,t}. \]  \(\text{(C.40)}\)
C.7.2 Firm’s prices

Price setting equation for firms that change their price in sector $x = C, I$:

$$0 = E_t \left\{ \sum_{s=0}^{\infty} \xi_{p,x}^s \beta^s \left[ \hat{p}_{x,t} \hat{\Pi}_{t,t+s} - \hat{\lambda}^x_{p,t+s} - \hat{m}_{c,x,t+s} \right] \right\},$$

with

$$\hat{\Pi}_{t,t+s} = \sum_{k=1}^{s} \left[ t_{p_x} \hat{\pi}_{t+k-1} - \hat{\pi}_{t+k} \right].$$

Solving for the summation

$$\frac{1}{1 - \xi_{p,x} \beta} \hat{p}_{x,t} = E_t \left\{ \sum_{s=0}^{\infty} \xi_{p,x}^s \beta^s \left[ - \hat{\Pi}_{t,t+s} + \hat{\lambda}^x_{p,t+s} + \hat{m}_{c,x,t+s} \right] \right\}
= - \hat{\Pi}_{t,t} + \hat{\lambda}^x_{p,t} + \hat{m}_{c,x,t} - \frac{\xi_{p,x} \beta}{1 - \xi_{p,x} \beta} \hat{\Pi}_{t,t+1}
+ \xi_{p,x} \beta E_t \left\{ \sum_{s=1}^{\infty} \xi_{p,x}^{s-1} \beta^{s-1} \left[ - \hat{\Pi}_{t+1,t+s} + \hat{\lambda}^x_{p,t+s} + \hat{m}_{c,x,t+s} \right] \right\}
= \hat{\lambda}^x_{p,t} + \hat{m}_{c,x,t} + \frac{\xi_{p,x} \beta}{1 - \xi_{p,x} \beta} E_t \left[ \hat{p}_{x,t+1} - \hat{\Pi}_{t,t+1} \right],$$

where we used $\hat{\Pi}_{t,t} = 0$.

Prices evolve as

$$0 = (1 - \xi_{p,x})\hat{p}_{x,t} + \xi_{p,x}(t_{p_x} \hat{\pi}_{t-1} - \hat{\pi}),$$

from which we obtain the Phillips curve in sector $x = C, I$:

$$\hat{\pi}_{x,t} = \frac{\beta}{1 + t_{p_x} \beta} E_t \hat{\pi}_{x,t+1} + \frac{t_{p_x}}{1 + t_{p_x} \beta} \hat{\pi}_{x,t-1} + \kappa_x \hat{m}_{c,x,t} + \kappa_x \hat{\lambda}^x_{p,t}, \quad (C.41)$$

with

$$\kappa_x = \frac{(1 - \xi_{p,x} \beta)(1 - \xi_{p,x})}{\xi_{p,x}(1 + t_{p_x} \beta)}.$$

From equation (C.19) it follows that

$$\hat{\pi}_{t,t} - \hat{\pi}_{C,t} = \hat{p}_{I,t} - \hat{p}_{I,t-1}.$$
C.7.3 Households

Marginal utility:

\[
\hat{\lambda}_t = \frac{e^G}{e^G - h\beta} \left[ \hat{b}_t + \left( \frac{a_c}{1 - a_i} \hat{v}_t \right) - \left( \frac{e^G}{e^G - h} \left( \hat{c}_t + \hat{z}_t \frac{a_c}{1 - a_i} \hat{v}_t \right) - \frac{h}{e^G - h} \hat{c}_t \right) \right] \\
- \frac{h\beta}{e^G - h\beta} E_t \left[ \hat{b}_{t+1} - \left( \frac{e^G}{e^G - h} \left( \hat{c}_{t+1} + \hat{z}_{t+1} \frac{a_c}{1 - a_i} \hat{v}_{t+1} \right) - \frac{h}{e^G - h} \hat{c}_{t+1} \right) \right] \\
\Rightarrow \hat{\lambda}_t = \alpha_1 E_t \hat{c}_{t+1} - \alpha_2 \hat{c}_t + \alpha_3 \hat{c}_{t-1} + \alpha_4 \hat{z}_t + \alpha_5 \hat{b}_t + \alpha_6 \hat{v}_t, 
\]

(C.42)

with

\[
\alpha_1 = \frac{h\beta e^G}{(e^G - h\beta)(e^G - h)}, \quad \alpha_2 = \frac{e^{2G} + h^2\beta}{(e^G - h\beta)(e^G - h)}, \quad \alpha_3 = \frac{he^G}{(e^G - h\beta)(e^G - h)}, \\
\alpha_4 = \frac{h\beta e^G \rho_z - he^G}{(e^G - h\beta)(e^G - h)}, \quad \alpha_5 = \frac{e^G - h\beta \rho_b}{e^G - h\beta}, \quad \alpha_6 = \frac{(h\beta e^G \rho_v - he^G) \frac{a_c}{1 - a_i}}{(e^G - h\beta)(e^G - h)}, \\
e^G = e^{ga + \frac{a}{1 - a_i}g\nu}.
\]

This assumes the shock processes for \( \hat{z}_t \) and \( \hat{b}_t \).

Euler equation:

\[
\hat{\lambda}_t = \hat{R}_t + E_t \left( \hat{\lambda}_{t+1} - \hat{z}_{t+1} - \hat{v}_{t+1} \frac{a_c}{1 - a_i} - \hat{\pi}_{C,t+1} \right). 
\]

(C.43)

C.7.4 Investment and Capital

Capital utilization in both sectors:

\[
\hat{r}^K_{C,t} = \chi_C \hat{u}_{C,t}, \quad \hat{r}^K_{I,t} = \chi_I \hat{u}_{I,t}, \quad \text{where} \quad \chi_x = \frac{a_x''(1)}{a_x'(1)}. 
\]

(C.44)

Choice of investment for the consumption sector:

\[
\hat{q}_{C,t} = e^{2\left(\frac{1}{1-a_i}g\nu\right)\kappa} \left( \hat{i}_{C,t} - \hat{i}_{C,t-1} + \frac{1}{1 - a_i} \hat{v}_t \right) - \beta e^{2\left(\frac{1}{1-a_i}g\nu\right)\kappa} E_t \left( \hat{i}_{C,t+1} - \hat{i}_{C,t} + \frac{1}{1 - a_i} \hat{v}_{t+1} \right) \\
+ \hat{p}_{t+1}. 
\]

(C.45)
with \( \hat{q}_{C,t} = \hat{\phi}_{C,t} - \hat{\lambda}_t \).

Choice of investment for the investment sector:
\[
\hat{q}_{I,t} = e^{2\left(\frac{1}{e} - \hat{a}_t\right)K_1} \left(\hat{i}_{I,t} - \hat{i}_{I,t-1} + \frac{1}{1 - \hat{a}_t} \hat{v}_t\right) - \beta e^{2\left(\frac{1}{e} - \hat{a}_t\right)K_2}Ke \left(\hat{i}_{I,t+1} - \hat{i}_{I,t} + \frac{1}{1 - \hat{a}_t} \hat{v}_{t+1}\right) + \hat{p}_{I,t},
\]
(C.46)

with \( \hat{q}_{I,t} = \hat{\phi}_{I,t} - \hat{\lambda}_t \).

Capital services input in both sectors:
\[
\hat{k}_{C,t} = \hat{u}_{C,t} + \xi_{K,t} + \hat{k}_{C,t-1} - \frac{1}{1 - \hat{a}_t} \hat{v}_t, \quad \hat{k}_{I,t} = \hat{u}_{I,t} + \xi_{K,t} + \hat{k}_{I,t-1} - \frac{1}{1 - \hat{a}_t} \hat{v}_t.
\]
(C.47)

Capital accumulation in the consumption and investment sector:
\[
\hat{\bar{k}}_{C,t} = (1 - \delta_C)e^{-1 - \hat{a}_t \hat{v}_t} \left(\hat{\bar{k}}_{C,t-1} + \xi_{K,t} - \frac{1}{1 - \hat{a}_t} \hat{v}_t\right) + \left(1 - (1 - \delta_C)e^{-1 - \hat{a}_t \hat{v}_t}\right) \hat{i}_{C,t},
\]
(C.48)
\[
\hat{\bar{k}}_{I,t} = (1 - \delta_I)e^{-1 - \hat{a}_t \hat{v}_t} \left(\hat{\bar{k}}_{I,t-1} + \xi_{K,t} - \frac{1}{1 - \hat{a}_t} \hat{v}_t\right) + \left(1 - (1 - \delta_I)e^{-1 - \hat{a}_t \hat{v}_t}\right) \hat{i}_{I,t}.
\]
(C.49)

### C.7.5 Wages

The wage setting equation for workers renegotiating their salary:
\[
0 = E_t \left\{ \sum_{s=0}^{\infty} \xi_s ^{\beta_s} \left[ \hat{w}_t + \hat{\Pi}_{w,t+s} - \hat{\lambda}_{w,t+s} - \hat{b}_{t+s} - \nu \hat{L}_{t+s} + \hat{\lambda}_{t+s} \right] \right\},
\]

with
\[
\hat{\Pi}_{w,t+s} = \sum_{k=1}^{s} \left[ t_w \left(\hat{\pi}_{c,t+k-1} + \hat{z}_{t+k-1} + \frac{a_c}{1 - \hat{a}_t} \hat{v}_{t+k-1}\right) - \left(\hat{\pi}_{c,t+k} + \hat{z}_{t+k} + \frac{a_c}{1 - \hat{a}_t} \hat{v}_{t+k}\right) \right],
\]
and
\[
\hat{L}_{t+s} = \hat{L}_{t+s} - \left(1 + \frac{1}{\lambda_{w}}\right) \left(\hat{w}_t + \hat{\Pi}_{w,t+s} - \hat{w}_{t+s}\right).
\]
Then using the labor demand function,

\[
0 = E_t \left\{ \sum_{s=0}^{\infty} \xi_w^s \beta^s \left[ \hat{\tilde{w}}_t + \hat{\Pi}_{t,t+s}^w - \hat{\lambda}_{t+s} - \hat{b}_{t+s} \right] \right. \\
- \nu \left( \hat{L}_{t+s} - \left( 1 + \frac{1}{\lambda_w} \right) (\hat{\hat{w}}_t + \hat{\Pi}_{t,t+s}^w - \hat{w}_{t+s}) \right) \right\}
\]

\[
\Leftrightarrow 0 = E_t \left\{ \sum_{s=0}^{\infty} \xi_w^s \beta^s \left[ \hat{\tilde{w}}_t \left( 1 + \nu \left( 1 + \frac{1}{\lambda_w} \right) \right) + \hat{\Pi}_{t,t+s}^w - \hat{\lambda}_{w,t+s} - \hat{b}_{t+s} \right] \right. \\
- \nu \left( \hat{L}_{t+s} - \left( 1 + \frac{1}{\lambda_w} \right) (\hat{\hat{w}}_t + \hat{\Pi}_{t,t+s}^w - \hat{w}_{t+s}) \right) \right\}.
\]

Solving for the summation,

\[
\frac{\nu_w}{1 - \xi_w \beta} \hat{\tilde{w}}_t = E_t \left\{ \sum_{s=0}^{\infty} \xi_w^s \beta^s \left[ \left( 1 + \nu \left( 1 + \frac{1}{\lambda_w} \right) \right) \hat{\Pi}_{t,t+s}^w + \hat{\psi}_{t+s} \right] \right. \\
= - \nu_w \hat{\Pi}_{t,t+1}^w + \hat{\psi}_t + E_t \left\{ \sum_{s=0}^{\infty} \xi_w^s \beta^s \left[ - \nu_w \hat{\Pi}_{t,t+s}^w + \hat{\psi}_{t+s} \right] \right. \\
= \hat{\psi}_t - \frac{\xi_w \beta}{1 - \xi_w \beta} \nu_w \hat{\Pi}_{t,t+1}^w + \xi_w \beta E_t \left\{ \sum_{s=0}^{\infty} \xi_w^s \beta^s \left[ - \nu_w \hat{\Pi}_{t,t+s}^w + \hat{\psi}_{t+s} \right] \right. \\
= \hat{\psi}_t + \frac{\xi_w \beta}{1 - \xi_w \beta} \nu_w E_t \left[ \hat{\tilde{w}}_{t+1} - \hat{\Pi}_{t,t+1}^w \right].
\] (C.50)

where

\[
\hat{\psi}_t \equiv \hat{\lambda}_{w,t} + \hat{b}_t + \nu \hat{L}_t + \nu \left( 1 + \frac{1}{\lambda_w} \right) \hat{\tilde{w}}_t - \hat{\lambda}_t,
\] (C.51)

\[
\nu_w \equiv 1 + \nu \left( 1 + \frac{1}{\lambda_w} \right),
\]

and recall that \( \hat{\Pi}_{t,t}^w = 0. \)

Wages evolve as,

\[
\hat{\tilde{w}}_t = (1 - \xi_w) \hat{\tilde{w}}_t + \xi_w \left[ \hat{\tilde{w}}_{t-1} + \xi_w \left( \hat{\tilde{w}}_{t-1} + \lambda w \hat{\hat{z}}_{c,t-1} + \lambda w \left( \hat{\hat{z}}_{t-1} + \frac{a_c}{1 - a_t} \hat{v}_{t-1} \right) - \hat{\hat{z}}_{c,t} - \hat{z}_t - \frac{a_c}{1 - a_t} \hat{v}_t \right) \right] \\
\Leftrightarrow \hat{\tilde{w}}_t = (1 - \xi_w) \hat{\tilde{w}}_t + \xi_w (\hat{\tilde{w}}_{t-1} + \hat{\Pi}_{t,t-1}^w).
\] (C.52)

Equation (C.52) can be solved for \( \hat{\tilde{w}}_t \). This expression, as well as the formulation for \( \hat{\psi}_t \) given in (C.51) can be plugged into equation (C.50). After rearranging this yields the wage Phillips curve,
\[ \hat{w}_t = \frac{1}{1 + \beta} \hat{w}_{t-1} + \frac{\beta}{1 + \beta} E_t \hat{w}_{t+1} - \kappa_w \hat{g}_{w,t} + \frac{\tau_w}{1 + \beta} \hat{\pi}_{c,t-1} - \frac{1 + \beta t_w}{1 + \beta} \hat{\pi}_{c,t} + \frac{\beta}{1 + \beta} E_t \hat{\pi}_{c,t+1} + \kappa_w \hat{\lambda}_{w,t} + \frac{\tau_w}{1 + \beta} (\hat{z}_{t-1} + \frac{a_c}{1 - a_t} \hat{v}_{t-1}) - \frac{1 + \beta t_w - \rho_z \beta}{1 + \beta} \hat{z}_t - \frac{1 + \beta t_w - \rho_v \beta}{1 + \beta} \frac{a_c}{1 - a_t} \hat{v}_t, \]  

(C.53)

where

\[ \kappa_w \equiv \frac{(1 - \xi_w \beta)(1 - \xi_c)}{\xi_w (1 + \beta) \left(1 + \nu (1 + \frac{1}{\lambda_w})\right)}, \]

\[ \hat{g}_{w,t} \equiv \hat{w}_t - (\nu \hat{L}_t + \hat{b}_t - \hat{\lambda}_t). \]

### C.7.6 Financial sector

The part of the economy concerned with the banking sector is described by the following equations:

The stochastic discount factor:

\[ \hat{\lambda}_t^B = \hat{\lambda}_t - \hat{\lambda}_{t-1}. \]  

(C.54)

Definition of \( \nu \) for \( x = C, I \):

\[ \hat{v}_{x,t} = (1 - \theta_B \beta z^x_1)[\hat{\lambda}_{t+1}^B - \hat{z}_{t+1} - \frac{a_c}{1 - a_t} \hat{v}_{t+1}] + \frac{1 - \theta_B \beta z^x_1}{R^B \pi_C - R} [R^B_x \pi_C \hat{R}_{x,t+1}^B + R^B_x \pi_C \hat{\pi}_{C,t+1} - R \hat{R}_t] + \theta_B \beta z^x_1 [\hat{z}^x_{t+1} + \hat{v}_{x,t+1}]. \]  

(C.55)

Definition of \( \eta \):

\[ \hat{\eta}_{x,t} = (1 - \theta_B \beta z^x_2)[\hat{\lambda}_{t+1}^B - \hat{z}_{t+1} - \frac{a_c}{1 - a_t} \hat{v}_{t+1} + \hat{R}_t] + \theta_B \beta z^x_2 \hat{z}^x_{t+1} + \hat{\eta}_{t+1}], \quad x = C, I. \]  

(C.56)

Definition of \( z_1 \):

\[ \hat{z}^x_{1,t} = \hat{\varphi}_{x,t} - \hat{\varphi}_{x,t-1} + \hat{z}^x_{2,t}, \quad x = C, I. \]  

(C.57)
Definition of $z_2$ for $x = C, I$:

$$
\hat{z}_{2,t} = \frac{\pi_C}{(R_x - R)g_x + R} [R_B^x \hat{\varphi}_x [\hat{R}_{x,t}^B + \hat{\pi}_{C,t}] + \frac{R}{\pi_C} (1- \varphi_x) \hat{R}_{t-1} + (R_B^x \pi_C - R) \hat{\varphi}_x \hat{R}_{x,t-1} - \hat{\pi}_{C,t}].
$$

(C.58)

The leverage ratio:

$$
\hat{\varphi}_{x,t} = \hat{\eta}_{x,t} + \frac{\nu}{\lambda_B - \nu} \hat{\varpi}_{x,t}, \quad x = C, I.
$$

(C.59)

The leverage equation:

$$
\hat{q}_{x,t} + \hat{s}_{x,t} = \hat{\varphi}_{x,t}, \quad x = C, I.
$$

(C.60)

The bank’s wealth accumulation equation

$$
\hat{n}_{x,t} = \theta_B \hat{\varphi}_x \pi_C e^{-g_a - \frac{a_c}{\pi_C} g_v} [R_B^x \pi_C [\hat{R}_{x,t}^B + \hat{\pi}_{C,t}] + \left( \frac{1}{\varphi_x} - 1 \right) R \hat{R}_{t-1} + (R_B^x \pi_C - R) \hat{\varphi}_x \hat{R}_{x,t-1} ]
$$

$$
+ \frac{\theta_B}{\pi_C} e^{-g_a - \frac{a_c}{\pi_C} g_v} [(R_B^x \pi_C - R) \varphi_x + R] \left[ - \hat{\varpi}_t - \frac{a_c}{1-a_i} \hat{\varpi}_t + \hat{n}_{x,t-1} - \hat{\pi}_{C,t} \right]
$$

$$
+ (1 - \frac{\theta_B}{\pi_C} e^{-g_a - \frac{a_c}{\pi_C} g_v} [(R_B^x \pi_C - R) \varphi_x + R]) \left[ \hat{q}_{x,t} + \hat{s}_{x,t} \right], \quad x = C, I.
$$

(C.61)

The borrow in advance constraint:

$$
\hat{k}_{x,t+1} = \hat{s}_{x,t}, \quad x = C, I.
$$

(C.62)

The bank’s stochastic return on assets in sector $x = C, I$:

$$
\hat{R}_{x,t}^B = \frac{1}{r_x^K + q_x (1 - \delta_x)} [r_x^K (\hat{r}_x^K + \hat{a}_{x,t}) + q_x (1 - \delta_x) \hat{q}_{x,t}] - \hat{q}_{x,t-1} + \hat{a}_x \hat{s}_{x,t} + \hat{\varpi}_t - \frac{1 - a_c}{1-a_i} \hat{\varpi}_t.
$$

(C.63)

Excess (nominal) return:

$$
\hat{R}_{x,t}^S = \frac{R_B^x \hat{\pi}_C}{R_B^x \pi_C - R} (\hat{R}_{x,t+1}^B + \hat{\pi}_{C,t+1}) - \frac{R}{R_B^x \pi_C - R} \hat{R}_t, \quad x = C, I.
$$

(C.64)

C.7.7 Monetary policy and market clearing

Monetary policy rule:

$$
\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R) \left[ \phi_{\pi} \hat{\pi}_{c,t} + \phi_{\Delta Y} (\hat{y}_t - \hat{y}_{t-1}) \right] + \hat{\eta}_{mp,t}
$$

(C.65)
Resource constraint in the consumption sector:

\[
\hat{c}_t + \left( r_c \frac{\dot{k}^C}{c} \hat{u}_{C,t} + r_I \frac{\dot{k}^I}{c} \hat{u}_{I,t} \right) e^{-\frac{1}{1-a} g^v} = \frac{c + F_c}{c} \left[ a_c \dot{k}^C_t + (1 - a_c) \dot{L}^C_t \right] \tag{C.66}
\]

Resource constraint in the investment sector:

\[
\hat{i}_t = \frac{i + F_I}{i} \left[ a_i \dot{k}^I_t + (1 - a_i) \dot{L}^I_t \right] \tag{C.67}
\]

Definition of GDP:

\[
\hat{y}_t = c^C_t + \frac{p_i}{i} \hat{p}^I_t + \frac{p_i}{i} (\hat{i}_t + \hat{p}_I + \hat{y}_t). \tag{C.68}
\]

Market clearing:

\[
\frac{L^C_t}{L} \hat{L}^C_t + \frac{L^I_t}{L} \hat{L}^I_t = \hat{L}_t, \quad \frac{\dot{i}^C_t}{\dot{i}^C_t} + \frac{\dot{i}^I_t}{\dot{i}^I_t} = \hat{i}_t, \quad \frac{n^C_t}{n} \hat{n}^C_t + \frac{n^I_t}{n} \hat{n}^I_t = \hat{n}_t. \tag{C.69}
\]

### C.7.8 Exogenous processes

The 10 exogenous processes of the model can be written in log-linearized form as follows:

**Price markup in sector** \(x = C, I\):

\[
\hat{\lambda}_{p, t}^x = \rho_p \hat{\lambda}_{p, t - 1}^x + \epsilon_{p, t}^x. \tag{C.70}
\]

The TFP growth (consumption sector):

\[
\hat{z}_t = \rho_z \hat{z}_{t - 1} + \epsilon_z^t. \tag{C.71}
\]

The TFP growth (investment sector):

\[
\hat{v}_t = \rho_v \hat{v}_{t - 1} + \epsilon_v^t. \tag{C.72}
\]

**Wage markup**:

\[
\hat{\lambda}_{w, t} = \rho_w \hat{\lambda}_{w, t - 1} + \epsilon_{w, t}. \tag{C.73}
\]

**Preference**:

\[
\hat{b}_t = \rho_b \hat{b}_{t - 1} + \epsilon_b^t. \tag{C.74}
\]

**Monetary policy**:

\[
\hat{\eta}_{mp, t} = \epsilon_{mp}^t. \tag{C.75}
\]
Government spending:
\[
\hat{g}_t = \rho g \hat{g}_{t-1} + \varepsilon_t^g. \tag{C.76}
\]

Capital quality in sector \(x = C, I\):
\[
\hat{\varsigma}_{Kx,t} = \rho \varsigma_{Kx,t} \hat{\varsigma}_{Kx,t-1} + \varepsilon_{\varsigma,t}^K \tag{C.77}
\]

The entire log-linear model is summarized by equations (C.39) - (C.49) and (C.53) - (C.69) as well as the shock processes (C.70) - (C.77).

### C.8 Measurement equations

For estimation, model variables are linked with observables using measurement equations. Letting a superscript "d" denote observable series, then the model’s measurement equations are as follows:

Real consumption growth,
\[
\Delta C^d_t \equiv \log \left( \frac{C_t}{C_{t-1}} \right) = \log \left( \frac{c_t}{c_{t-1}} \right) + \hat{z}_t + \frac{a_c}{1 - a_i} \hat{v}_t,
\]

Real investment growth,
\[
\Delta I^d_t \equiv \log \left( \frac{I_t}{I_{t-1}} \right) = \log \left( \frac{i_t}{i_{t-1}} \right) + \frac{1}{1 - a_i} \hat{v}_t,
\]

Real wage growth,
\[
\Delta W^d_t \equiv \log \left( \frac{W_t}{W_{t-1}} \right) = \log \left( \frac{w_t}{w_{t-1}} \right) + \hat{z}_t + \frac{a_c}{1 - a_i} \hat{v}_t,
\]

Real output growth,
\[
\Delta Y^d_t \equiv \log \left( \frac{Y_t}{Y_{t-1}} \right) = \log \left( \frac{y_t}{y_{t-1}} \right) + \hat{z}_t + \frac{a_c}{1 - a_i} \hat{v}_t,
\]

Consumption sector inflation,
\[
\pi^d_{C,t} \equiv \pi_{C,t} = \hat{\pi}_{C,t} \quad \text{and} \quad \hat{\pi}_{C,t} = \log(\pi_{C,t}) - \log(\pi_C),
\]

Investment sector inflation,
\[
\pi^d_{I,t} \equiv \pi_{I,t} = \hat{\pi}_{I,t} \quad \text{and} \quad \hat{\pi}_{I,t} = \log(\pi_{I,t}) - \log(\pi_I),
\]
Total hours worked,

\[ L_t^d \equiv \log L_t = \hat{L}_t, \]

Nominal interest rate (federal funds rate),

\[ R_t^d \equiv \log R_t = \log \hat{R}_t, \]

Consumption sector corporate spread,

\[ R_{C,t}^{S,d} \equiv \log R_{C,t}^S = \frac{R_{B}^R \pi_C}{R_{B}^R \pi_C - R} (\log \hat{R}_{C,t+1}^B + \log \hat{\pi}_{C,t+1}) - \frac{R}{R_{B}^R \pi_C - R} \log \hat{R}_t, \]

Investment sector corporate spread,

\[ R_{I,t}^{S,d} \equiv \log R_{I,t}^S = \frac{R_{B}^R \pi_C}{R_{B}^R \pi_C - R} (\log \hat{R}_{I,t+1}^B + \log \hat{\pi}_{C,t+1}) - \frac{R}{R_{B}^R \pi_C - R} \log \hat{R}_t, \]

Real total equity capital growth,

\[ \Delta N_t^d \equiv \log \left( \frac{N_t}{N_{t-1}} \right) = e^{g_n + \alpha_{n_i} g_v} \left( \frac{n_C}{n_C + n_I} (\hat{n}_{C,t} - \hat{n}_{C,t-1}) + \frac{n_I}{n_C + n_I} (\hat{n}_{I,t} - \hat{n}_{I,t-1}) + \hat{z}_t + \frac{a_c}{1 - a_i} \tilde{v}_t \right). \]