Are There Classical Business Cycles? *

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Abstract

The aim of this paper is to test formally the classical business cycle hypothesis, using data from industrialized countries for the time period since 1960. The hypothesis is characterized by the view that the cyclical structure in GDP is concentrated in the investment series: fixed investment has typically a long cycle, while the cycle in inventory investment is shorter. To check the robustness of our results, we subject the data for 15 OECD countries to a variety of detrending techniques. While the hypothesis is not confirmed uniformly for all countries, there is a considerably high number for which the data display the predicted pattern. None of the countries shows a pattern which can be interpreted as a clear rejection of the classical hypothesis.

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1 Introduction

Over the last years, it has become increasingly common to test business cycle theories against some stylized facts. Identifying a set of key stylized facts, or empirical regularities of economic fluctuations, is therefore not only interesting in its own right, it also has important implications for theoretical research.

With respect to these empirical regularities, there is a significant difference between the view of modern business cycles researchers and that of their classical predecessors. In the classical tradition, fluctuations were seen as genuine cycles, that is as recurrent phenomena with characteristic periodicities. This tradition originated in the 19th century with the seminal work of Juglar (1862) and was continued until the 1950s, comprising, among many others, the works of Aftalion (1909), Kitchin (1923), Kuznetz (1926) and Schumpeter (1939). An important aspect of the classical view is that cycles of different frequencies can be found in different series, mainly the investment series. In contrast, the perception of most modern macroeconomists is that economic time series do typically not have a pronounced cyclical pattern around the business cycle frequencies. According to the modern view, the defining property of business cycles is the strong *coherence* of many important economic time series at business cycle frequencies, i.e., their tendency to move together (cf. the discussion in Sargent, 1987, Ch. XI.11). The modern view was greatly influenced by the work of Burns and Mitchell (1946) and the NBER methodology.

The aim of this paper is to define an exact hypothesis that represents the classical view, and to test it formally. Within the classical tradition, there seems to have a consensus emerged around 1950 that at least three types of cycles can be found in the data: i) a three- to four-year cycle (Kitchin cycle), typically found in inventory investment; ii) a seven- to ten-year cycle (Juglar cycle), typically found in equipment investment; iii) a cycle of about 20 years (Kuznetz cycle) in building investment. The existence of long waves (Kondratieff cycle) of about 50 years was debated. This view is documented for example in Davis (1941, Ch. 7) and Matthews (1959, Ch. XII). The first two cycles roughly coincide with NBER minor and major cycles. In Section 2 we will propose a definition of what it means that an economic time series exhibits a cycle in a specified frequency range. Since our data are too short to identify cycles of 20 years or more, we will focus on cycles of type i) and ii).

Tests on the cyclical structure of economic time series are not very powerful, mainly because the available time series are short compared to the cycle periods under consideration. In the present paper, this problem is somewhat alleviated, because we do not look for cyclicality in general, but test a very specific hypothesis, namely that of the classical business cycle. In addition, we use data from 15 industrialized countries, which should partially compensate for the small sample size. The data used are from 1960 onwards, which means that the sample starts many years after the classical view has been established. In fact, one can almost say that by 1960 this view had already been abandoned. We think this is an additional argument why the obtained results are not just a product of chance.

Previous empirical studies (cf. Hillinger and Sebold-Bender, 1992 and Woitek, 1996) have led us to believe that the classical view contains an important element of truth. This is confirmed by the formal tests in this paper. While the methodology used here is from time series analysis, the results should also have implications for future attempts to build structural models of economic fluctuations. Most of the recent contributions to business cycle theory (see, e.g., Cooley, 1995) do not address the question whether economic fluctuations have a cyclical structure or not. Some recent models do produce genuine cycles, for example Gale (1996). However, the aspect of genuine cyclicality versus more general fluctuations is usually not stressed by the authors, and it is not discussed whether this is a realistic or unrealistic feature of their model. (One notable exception is Wen, 1998, who develops a time-to-build model in order to explain a seven-year cycle in the fixed investment/GDP ratio.) In fact, the widely used practice to confront models and data by means of low order auto- and cross-correlations is not suitable for this purpose. To investigate the cyclical structure, one has to use frequency domain techniques, as for example in Watson (1993). The present results indicate that it is worthwhile doing so.

The plan of the paper is as follows: Section 2 provides an exact statement of the classical business cycle hypothesis. As a preliminary step for the empirical analysis, Section 3 discusses the problem of detrending. Section 4 presents suitable tests statistics, states their asymptotic properties and investigates their small sample properties. The empirical results are in Section 5, and Section 6 concludes.

2 The Definition of Classical Business Cycles

To engage in formal statistical testing, it is necessary to define precisely what is meant by a cycle in the classical sense. Since classical writers saw business cycles as recurrent phenomena with typical frequencies, one could define the existence of a cycle as a peak in the spectrum of a time series in the specified range. This, however, is not yet precise enough, because the spectral density of a process may have many local maxima, and there is no widely accepted definition of what is a “significant maximum”. In addition, since economic time series do obviously not show a very regular periodicity, one should not require the existence of a very sharp peak. It seems more natural to concentrate on the spectral mass contained in specified frequency ranges, and we therefore propose the following

Definition 1. *An economic time series exhibits a Classical Long (Short) Cycle if there is significantly more spectral mass in the business cycle range 7-10 (3-5) years than in the other relevant frequency ranges.*

To make this definition operative, we have to clarify what we mean by "the other relevant frequency ranges". Why don't we just say "significantly more mass than the average of all frequencies"? The problems of such a definition can be illustrated by the following picture, which displays the (stylized) graph of the spectral density of a hypothetical quarterly economic time series:

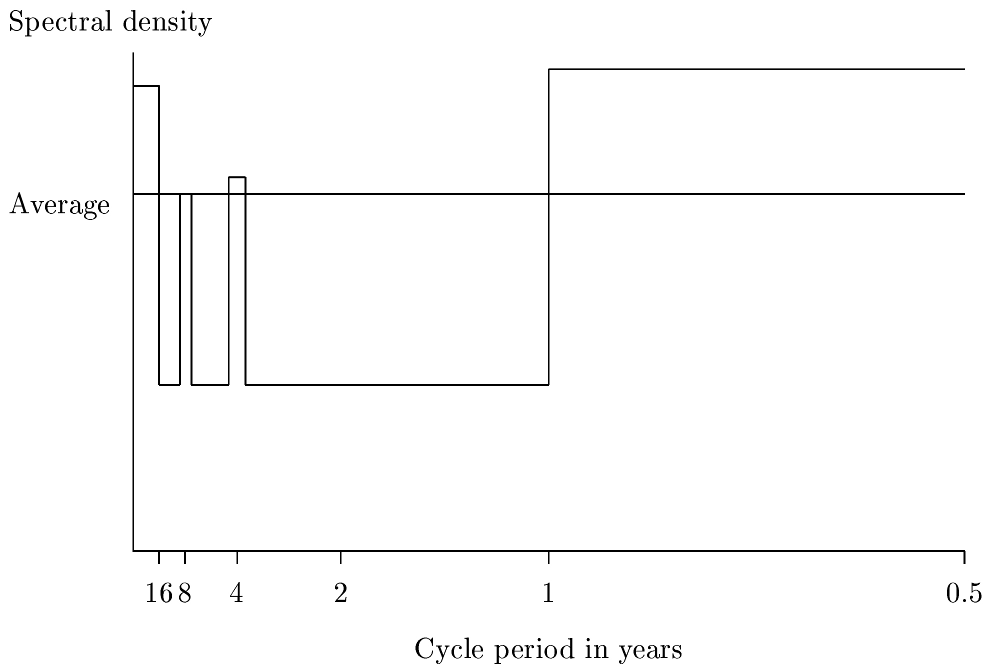

Figure 1: Spectral density of hypothetical economic time series

The x-axis in the picture measures frequency, but the numbers on the axis indicate the cycle period, for convenience. Recall that the frequency range of the periodogram of a quarterly series is from zero to 2 (half-year cycle). Our hypothesized series has a very strong seasonal component, depicted by a high spectral density for periods smaller than 1. It also has a high density in the low frequency range, for cycles of period greater than 16 years, perhaps due to inadequate detrending. In the intermediate range, there is low density, except around the business cycle periods 4 and 8. This picture would certainly indicate classical business cycles, despite the fact that the spectral density at these ranges is only about average. The average is so high because of the high seasonal and long-run components. The strength of seasonal cycles, however, is irrelevant for the question whether business cycles are genuine cycles or not, and our definition of business cycles should therefore not depend on it. The spectral mass in the very low frequency range should also not affect our measurement of cycles, since it mainly reflects how the time series was detrended. We will therefore compare the business cycle frequency ranges to the frequencies conforming to periods between 1 and 15 years. The upper boundary was chosen as 15 years because this is well above the business cycle periods, but the spectral estimates are not yet too sensitive to detrending, with the available data series.

One should note that the above definition is, in one sense, rather weak. A time series with the "typical spectral shape" of Granger (1966), i.e., with spectral density that is high for low frequencies and decreases monotonically as we go to higher frequencies, might well qualify as showing a classical long cycle according to the above definition. This is certainly inadequate and one might therefore argue that a genuine cycle only exists if the average spectral density in the specified frequency range is not only higher than the average of all other relevant ranges, but also significantly higher than in both neighboring frequency ranges. However, in the formal analysis we stick to the above weak formulation, since we can test for it in a straightforward way by considering the null hypothesis of equality of the average spectral density in two regions (cf. Section 4.1). In contrast, the null hypothesis for the stronger condition would have to comprise several regimes, which considerably complicates the investigation of small sample properties of tests. In the empirical section, we will report the average spectral density of all frequency ranges, and thereby detect this kind of undesired spectral shape, if it exists.

Using the above definition, we can specify precisely what we mean by classical business cycles. We follow closely the description given in Matthews (1959). We do not consider the Kuznetz cycle and long waves, since our data do not allow the identification of cycles of this length. We differ from Matthew's account only in one detail, we define the range for the short cycle as 3–5 years rather than 3–4 years, since our data analysis has shown that 3–4 years is a too narrow range to fit the data of many different countries. Accordingly, denote by Ω_S the range of frequencies corresponding to periods of 3–5 years, by Ω_L the one corresponding 7–10 years, and by Ω_R the union of frequency ranges corresponding to 2–3, 5–7 and 10–15 years. The *Classical Business Cycle Hypothesis* is then characterized by the following three statements:

1. In fixed investment, the average spectral density in Ω_L is greater than that in Ω_R ("fixed investment has a long cycle").

2. In inventory investment, the average spectral density in Ω_S is greater than that in Ω_R ("inventory investment has a short cycle").

3. In GDP, the average spectral density in Ω_S as well as in Ω_L is greater than that in Ω_R ("GDP has a long and a short cycle").

3 Detrending

The most serious obstacle to the application of spectral methods in economics is that economic time series contain fluctuations as well as trend components, and that the nature of the trend component as well as its interaction with the cyclical component is not sufficiently understood. The discussion in Canova (1998a, 1998b) and Burnside (1998) makes clear that different detrending methods emphasize different frequency ranges in the data, and that many stylized facts are sensitive to the choice of detrending method. This is most obvious for a hypothesis of the sort that we investigate here, that a time series contains more spectral mass in a given frequency range than in others.

The Classical Business Cycle Hypothesis therefore seems ill defined, as long as there is no convincing way to identify the cyclical component of an economic time series. Formulating the hypothesis relative to a certain detrending method appears arbitrary. The only possible way is therefore to state the hypothesis in a stronger form: an economic time series contains a business cycle component in a certain frequency range, only if the spectral density has more mass in this range than in other ranges, *under a variety of detrending techniques* which are routinely employed by econometricians. If there is a strong cyclical component, it will dominate the effects of the detrending filter, and the results will be roughly equivalent for different methods.

In our application we employ first differencing, first differencing plus subtracting a linear trend, the Hodrick-Prescott filter HP100 (with smoothness parameter set to 100, since we use annual data) and the Baxter-King and modified Baxter-King filter[1] to identify the stationary series. We choose 1/20 as the cut-off frequency of the Baxter-King filter, that means, the filter is designed to eliminate fluctuations of duration greater than 20 years. The filter length is 3.

All filters may lead to spurious cycles. Figure 2 shows the gain functions for the different filters, valid for I(1) series (except for the difference filter, where the gain

[1]The Baxter-King filter from Baxter and King (1995) is a symmetric moving average bandpass filter to isolate business cycle components in non-stationary time series. The filter weights are determined in frequency domain by minimizing the sum of squared deviations of the gain of the filter from an ideal gain, i.e., one that perfectly isolates the business cycle frequency band. The modified Baxter-King filter was developed by one of us (Woitek 1998) and uses Lanczos' σ factors to deal with the problem of spurious side lobes, which invariably arises with finite length filters.

function is valid for I(0) series). First differencing plus subtracting a linear trend, which is not shown in the picture, is equivalent in large samples to first differencing, both for I(0) and I(1) processes.

In the empirical section, we will consider the results of the modified Baxter-King filter in more detail, because this filter appears to introduce the least distortions in the relevant frequency ranges, at least when applied to I(1) series. Furthermore, if this filter has a bias, it acts *against* the Classical Business Cycle Hypothesis, since it emphasizes the frequency range 5–7 years.

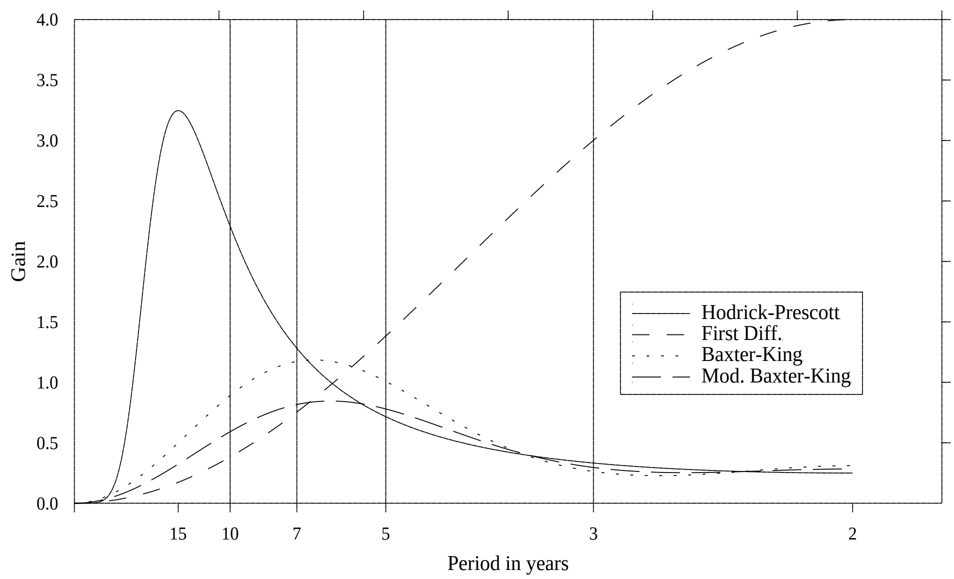

Figure 2: Gain function for different filters

4 Testing for Business Cycles

4.1 Theory

This section presents three simple tests which are designed to test the Classical Business Cycle hypothesis.

First we state formally a suitable null hypothesis. Let both Ω_1 and Ω_2 be the finite union of intervals of the frequency range $[0, \pi]$ such that $\Omega_1 \cup \Omega_2 = \emptyset$. Let $\|.\|$ denote Lebesgues measure and $h(\omega)$ the spectral density of a stochastic process. Then we consider the

Null hypothesis $\mathcal{H}_0$: the frequency ranges Ω_1 and Ω_2 have the same average spectral density,

$$\frac{\int_{\Omega_1} h(\omega)\, d\omega}{\|\Omega_1\|} = \frac{\int_{\Omega_2} h(\omega)\, d\omega}{\|\Omega_2\|} \tag{1}$$

Probably the standard test for the existence of cycles is Fisher's g-test for the significance of the highest peak in the periodogram. This test is not suitable to address $\mathcal{H}_0$, for two reasons. First, it is designed to detect purely periodic components, while we interpret economic time series as stochastic processes with a continuous spectrum. Second, we do not *search* for peaks in the spectrum, but have an a priori idea of where we expect increased business cycle activity: the classical ranges of 3-5 and 7-10 years.

Recently, Canova (1996) has proposed three tests for the existence of cycles. Relevant for our purpose is his third test statistic (Canova 1996, p.147),

$$D = \frac{\sum_{\omega \in \mathcal{F}(\Omega_1)} I_N(\omega)/\|\Omega_1\|_F}{\sum_{\omega \in \mathcal{F}(\Omega_2)} I_N(\omega)/\|\Omega_2\|_F} \tag{2}$$

where $I_N(\omega)$ is the periodogram estimate at frequency ω, $\mathcal{F}(\Omega)$ is the set of all Fourier frequencies in Ω, and $\|\Omega\|_F$ is the number of Fourier frequencies in Ω. Canova shows that, under $\mathcal{H}_0$, $\|\Omega_2\|_F \cdot D$ is asymptotically distributed as $\chi^2(2\|\Omega_1\|_F)$. The statistic D is perfectly suitable for testing $\mathcal{H}_0$ in large samples. Since the time series used in this paper are *very* short, we face a number of problems. The small sample distribution differs from the asymptotic one, of course. More serious is the fact that we have a complex null hypothesis, and different stochastic processes which all satisfy $\mathcal{H}_0$ may have different small sample distributions. We will investigate this problem in Section 4.2. Furthermore, the relevant frequency ranges contain only very few Fourier frequencies: the interval of 7–10 years contains indeed only 1 Fourier frequency for a sample of 34 annual data, as used in this paper. One can then hardly say that D tests a hypothesis about the *average* spectral density in this range. The outcome of the test depends sensitively on the exact location of the Fourier frequencies, which in turn depends on the sample size. More appropriate in the present context is therefore the following integral version of the test

$$D^{Int} = \frac{\int_{\Omega_1} I_N(\omega)\, d\omega/\|\Omega_1\|}{\int_{\Omega_2} I_N(\omega)\, d\omega/\|\Omega_2\|} \tag{3}$$

which integrates the periodogram over all frequencies in the ranges Ω_1 etc.

Since it is not clear how well the test performs in small samples, we also consider a test that is based on the difference of the periodogram averages, rather then the ratio. This new test statistic is defined as

$$Z = \sqrt{N} \frac{\int_{\Omega_1} I_N(\omega)\, d\omega/\|\Omega_1\| - \int_{\Omega_2} I_N(\omega)\, d\omega/\|\Omega_2\|}{\sqrt{\pi \left[\int_{\Omega_1} {I_N}^2(\omega)\, d\omega/\|\Omega_1\|^2 + \int_{\Omega_2} {I_N}^2(\omega)\, d\omega/\|\Omega_2\|^2\right]}} \tag{4}$$

The following result describes the asymptotic distribution of Z.

Proposition 1. *Under $\mathcal{H}_0$, the statistic Z in (4) is asymptotically distributed as standard normal.*

Proof. The asymptotic expectation of $\int_A I_N \, d\omega$ is equal to $\int_A h(\omega) \, d\omega$ (Priestley 1981, p.473), so that the numerator of Z has asymptotically zero expectation under the null hypothesis. In addition, it is normally distributed: the integrated cumulative periodogram is asymptotically distributed as a time-stretched Brownian motion (Priestley 1981, p.474). The asymptotic variance of $\int_A I_N \, d\omega$ is equal to $2\pi \int_A h^2(\omega) \, d\omega$ (Priestley 1981, p.474). Since $\frac{1}{2} \int_A {I_N}^2 \, d\omega$ is a consistent estimator for $\int_A h^2(\omega) \, d\omega$ (Priestley 1981, p.477), the claim follows. □

The following section will compare the small sample properties of the different test statistics.

4.2 Small sample properties

The international data series available for our purposes are very short and comprise only 34 annual observations. For such short time series, the distribution of the test statistics D, D^{Int} and Z is far from their asymptotic distribution. More importantly, the small sample distribution is not the same for all processes satisfying our composite hypothesis $\mathcal{H}_0$. In a first step, we therefore examine how strongly the distribution varies under a wide variety of processes which all satisfy the null hypothesis. Afterwards we investigate the power of the tests to detect deviations from $\mathcal{H}_0$.

The null hypotheses appropriate to test the Classical Business Cycle Hypothesis as defined at the end of the Introduction, are special cases of (1). In both cases, $\Omega_2 = \Omega_R = \left(\frac{2\pi}{15}, \frac{2\pi}{10}\right) \cup \left(\frac{2\pi}{7}, \frac{2\pi}{5}\right) \cup \left(\frac{2\pi}{3}, \frac{2\pi}{2}\right)$. For the test on the long cycle, $\Omega_1 = \Omega_L = \left(\frac{2\pi}{10}, \frac{2\pi}{7}\right)$, while for the short cycle, $\Omega_1 = \Omega_S = \left(\frac{2\pi}{5}, \frac{2\pi}{3}\right)$. The corresponding test statistics are denoted by D_{lc}, D_{sc} etc.

Distribution under $\mathcal{H}_0$

We use Monte-Carlo experiments to study the small sample distribution of the test statistics for a wide variety of processes that satisfy $\mathcal{H}_0$. We consider a white noise process as well as 24 different ARMA processes, where the order of the AR parts is between 1 and 5, and the order of the MA parts between 3 and 16. The processes 1–12 satisfy $\mathcal{H}_0$ for the long cycle, processes 13–24 for the short cycle. The processes have zero, one or two internal maxima. The spectral densities of all 24 processes are shown in Figures 4 and Figures 5 of Appendix A.

We take the white noise process as the reference case, from which we obtain the critical values. Table 1 presents critical values for different percentiles, as well as expected value and standard deviation. We see that the distributions are rather asymmetric. The

Table 1: Critical Values for White Noise, Sample Size $n = 34$

	0.01	0.05	0.10	0.25	0.50	0.75	0.90	0.95	0.99	μ	σ
D_{lc}	0.01	0.05	0.10	0.28	0.71	1.49	2.58	3.51	5.91	1.11	1.26
D_{sc}	0.23	0.36	0.45	0.65	0.97	1.42	2.01	2.45	3.58	1.13	0.70
${D^{Int}}_{lc}$	0.11	0.21	0.29	0.49	0.85	1.40	2.11	2.63	3.98	1.07	0.82
${D^{Int}}_{sc}$	0.26	0.38	0.48	0.67	0.96	1.36	1.87	2.24	3.16	1.09	0.61
Z_{lc}	-2.72	-2.22	-1.84	-1.06	-0.22	0.40	0.77	0.93	1.16	-0.38	0.98
Z_{sc}	-2.20	-1.71	-1.36	-0.77	-0.08	0.54	1.01	1.24	1.61	-0.13	0.90

lc: long cycle; sc: short cycle

median of neither D nor D^{Int} is close to one, and the median of Z is not close to zero. It is also obvious that the distribution of Z is far from standard normal.

Using the critical values obtained from the white noise process, Table 7 in the Appendix gives the rejection probabilities for the 24 different processes for the tests of nominal size 10, 5 and 1 percent. A summary of that information is presented in Figure 3. For each test statistic, the box plots in the figure describe the distribution of the rejection probability for the different models, namely processes 1 to 12 for the long cycle and processes 13 to 24 for the short cycle. The outer lines give the minimum and maximum, the intermediate lines the 25% and 75% percentiles, and the inner line gives the median of the rejection probability. Two conclusions emerge. First, the rejection probabilities differ strongly between processes. For example, using the critical value that leads to a 5 per cent rejection rate in the white noise case, the rejection rates of statistic D for the long cycle range between 0.051 and 0.177. Second, the right-sided test, i.e., the tests that spectral density is higher in the short or long cycle range, tend to reject more often than they should. The maximum of the rejection rate is always much greater than the intended size of the test. (For the long cycle, this is mainly due to one “outlier”, namely process 9, cf. Table 7). The absolute value of the rejection rate is less important than its variation across processes, and the results for statistic Z turns out to be more reliable on average than the other two test.

A look at the left-sided tests, however, reveals a serious asymmetry of Z, whose rejection probability is on average much smaller than the nominal size of the test. Since we will compare later the results of left- and right-sided tests, Z is biased in favor of the Classical Business Cycle Hypothesis hypothesis, and therefore inadequate. For the other test statistics, this bias is at least much smaller.

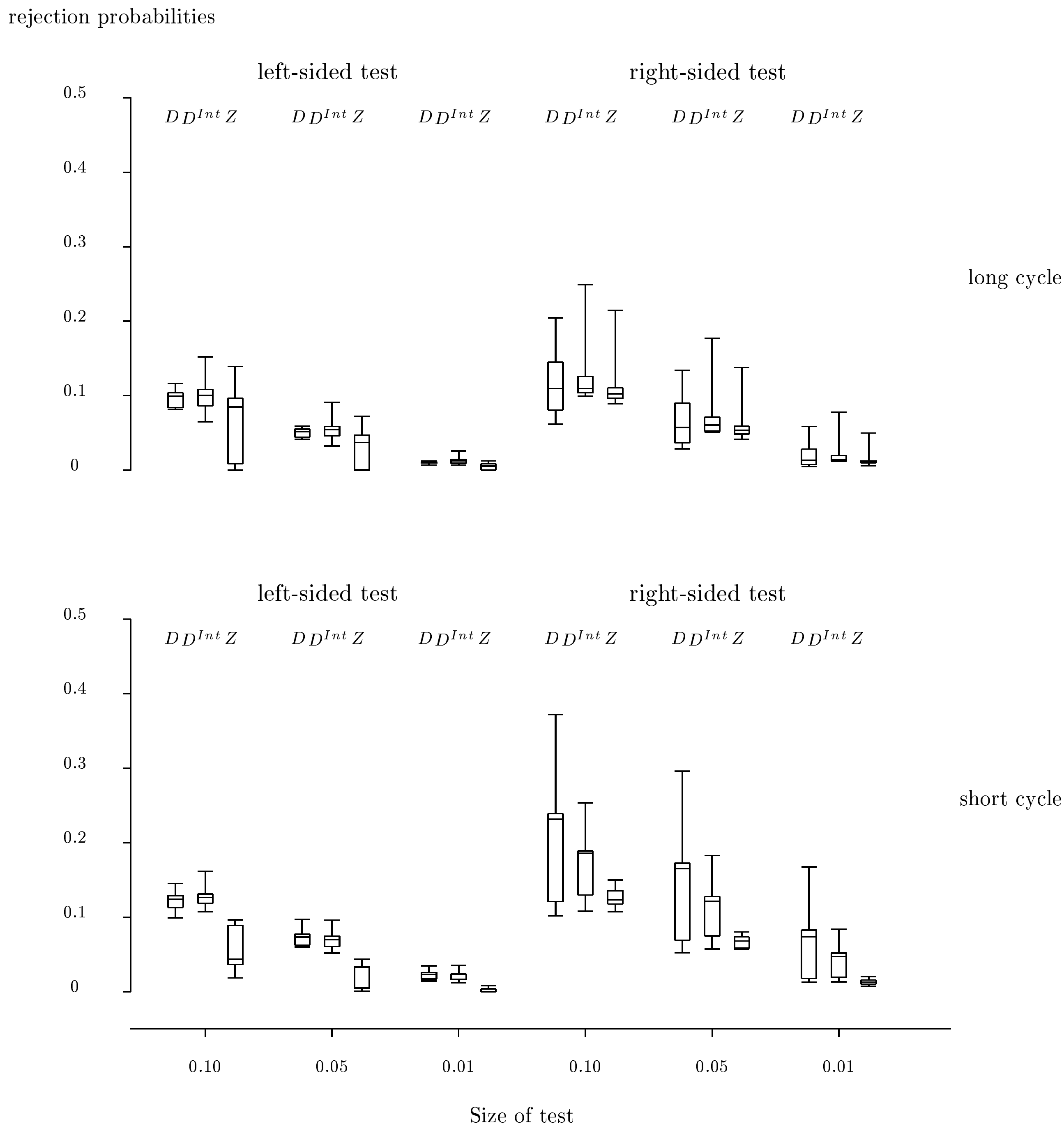

Figure 3: Rejection probabilities for 24 ARMA processes

Power

Next we investigate the power of our test statistic. We consider AR(2) processes that have a peak in one of the business cycle frequency ranges. Table 2 presents the simulation results. The columns "Period" and "ρ" denote the period (the inverse frequency) and the modulus of the roots of the characteristic polynomial of the process. The limit case of modulus $\rho = 0$ is equivalent to white noise, the higher ρ, the greater is the deviation from the null hypothesis. The deviation from $\mathcal{H}_0$ is also measured by Q_{long} and Q_{short}, which give the ratio of the spectral density in the long cycle region or short cycle region to the spectral density in the other relevant regions. These statistics are the theoretical counterparts to D^{Int}. As to be expected, increasing the modulus ρ increases the rejection probability of all three tests of the frequency range in which the process has its cycle. The rejection probability for the other frequency range is declining strongly in ρ, since a spectral peak in one range leads to increased spectral density in the neighboring ranges which belong to Ω_R. The power of D^{Int} to detect this kind of violation of the null

Table 2: Rejection Probabilities, AR(2)-Models,$n = 34$, Power: 0.05

Period	ρ	Q_{long}	Q_{short}	D_{lc}	${D^{Int}}_{lc}$	Z_{lc}	D_{sc}	${D^{Int}}_{sc}$	Z_{sc}
8.000	0.135	1.434	1.049	0.036	0.039	0.035	0.014	0.013	0.011
8.000	0.368	2.267	0.929	0.124	0.148	0.124	0.020	0.011	0.007
8.000	0.741	4.589	0.377	0.302	0.449	0.355	0.005	0.001	0.000
8.000	0.905	13.275	0.192	0.550	0.800	0.685	0.001	0.000	0.000
8.000	0.990	168.813	0.157	0.910	0.987	0.965	0.000	0.000	0.000
4.000	0.135	1.006	1.040	0.012	0.012	0.011	0.013	0.013	0.013
4.000	0.368	1.024	1.347	0.015	0.013	0.011	0.037	0.039	0.035
4.000	0.741	0.916	4.041	0.012	0.009	0.007	0.525	0.561	0.469
4.000	0.905	0.839	13.851	0.008	0.005	0.003	0.934	0.950	0.875
4.000	0.990	0.825	155.533	0.003	0.001	0.001	0.997	0.999	0.968
7.140	0.819	4.881	0.342	0.288	0.497	0.393	0.005	0.001	0.000
7.140	0.905	6.707	0.189	0.298	0.623	0.455	0.002	0.000	0.000
6.830	0.905	4.070	0.180	0.188	0.465	0.313	0.002	0.000	0.000
6.830	0.819	3.976	0.370	0.234	0.416	0.319	0.009	0.002	0.001

hypothesis is somewhat higher than that of Z, and considerably higher than that of D, in particular if the cycle is close to the boundary of the long cycle range. Statistic D^{Int} therefore appears to be the best compromise between power and reliability in the sense of the last subsection, and we have decided to use D^{Int} in the rest of the paper.

The differences between the three tests are not big, however, and the empirical results presented in the next section are not essentially altered if we use any of the other two test statistics.

5 Empirical business cycles in 15 OECD countries

5.1 Data

The data series contain annual observations from 1960 to 1993 of real GDP, gross fixed capital formation (GFCF) and inventory investment (II) of 15 OECD countries for which the data are available since 1960. We work with annual rather than quarterly data since the latter are not available since 1960 for most of the countries. The switch to higher-frequency observations would tend to increase the efficiency of the estimation, but this effect is probably more than outweighed by the reduction in the sample period, if one is concerned with identifying cycles of a fixed frequency range (here 3–5 and 7–10 years). This is because the number of Fourier frequencies in a given frequency range is not increased by a switch to higher frequency data.

5.2 Results

Table 3 presents a summary of the empirical results for the 15 OECD countries and for the 5 different detrending methods. For example, an entry n/m in the column II of the long cycle means that the statistic ${D^{Int}}_{lc}$ is significantly greater than 1 in the inventory investment series of n countries, and significantly smaller than 1 in m countries. The significance level is 5% (one-sided), but one should remember that the critical value was obtained from the white noise series (cf. Table 1), which probably underestimates the size of the test. Despite differences between detrending techniques, a very clear pattern emerges. First, the average spectral density of fixed investment in the long cycle range is often significantly higher than in the non-business cycle ranges, it is never significantly lower. A similar conclusion holds for GDP, but the result here seems more sensitive to detrending. Second, the average spectral density of inventory investment in the short cycle range is often significantly higher than in the non-business cycle ranges, and it is never significantly lower. For fixed investment and GDP, there are practically no significant results for the short cycle. This is exactly the pattern predicted by the classical view, with the exception that the inventory cycle is not found in GDP data. The results show that detrending *does* matter. In particular, the difference filter leads to fewer significant results. But it is also clear that detrending is not responsible for the main conclusion. The qualitative results are the same for all the detrending techniques used,

Table 3: Number of Significant Results

	long cycle			short cycle		
	II	*GFCF*	*GDP*	*II*	*GFCF*	*GDP*
HP	0/0	11/0	12/0	3/0	1/3	0/1
Diff	0/6	5/0	2/0	3/0	1/0	0/0
Diff+LinTr	0/6	5/0	1/0	3/0	1/0	0/0
Baxter-King	0/2	8/0	9/0	5/0	1/0	0/0
Mod. Baxter-King	0/3	8/0	6/0	4/0	2/0	0/0

n/m: statistic D^{Int} is significantly $>$ 1 in n countries, significantly $<$ 1 in m countries (significance level: 5 per cent, one-sided). Results for (modified) Baxter-King filter use critical values obtained for the truncated sample size (n=28).

and the systematic differences in the results for fixed investment, inventory investment and GDP cannot be caused by the detrending filter, since the same filter has been applied to all series.

The detailed results for the modified Baxter-King filter are presented in Table 4 (detailed results for the other filters are presented in Tables 8–11 in the Appendix). Of the nine significant long cycles in GFCF, six are significant at the 1% level, while the short cycle in II is significant at the 1% level only in Germany. Italy, Sweden and the US are the only countries that do not significantly fulfill the classical hypothesis for either cycle.

Table 4: Modified Baxter-King Filter, D^{Int}

	long cycle			short cycle		
	II	$GFCF$	GDP	II	$GFCF$	GDP
AUS	$0.16^{\diamond\diamond}$	$5.55^{\star\star\star}$	$2.51^{\star}$	0.92	$4.79^{\star\star\star}$	1.37
AUT	0.82	$3.85^{\star\star}$	$2.58^{\star}$	$2.32^{\star}$	1.26	1.66
BEL	1.07	0.94	$3.51^{\star\star}$	$2.24^{\star}$	0.85	1.44
CAN	0.53	$5.01^{\star\star\star}$	1.79	1.32	0.99	1.07
DNK	0.66	$3.30^{\star\star}$	$2.71^{\star}$	$3.05^{\star\star}$	0.93	$2.08^{\star}$
FRA	0.64	$13.44^{\star\star\star}$	$6.54^{\star\star\star}$	$2.09^{\star}$	1.21	1.67
WGR	1.34	$6.29^{\star\star\star}$	$3.76^{\star\star}$	$4.29^{\star\star\star}$	0.89	$2.09^{\star}$
ITA	0.35	1.65	2.08	1.37	0.73	0.93
JPN	0.34	$5.58^{\star\star\star}$	$3.31^{\star\star}$	1.53	0.86	1.01
NDL	0.39	$2.26^{\star}$	1.79	$1.96^{\star}$	0.66	1.50
NZL	1.03	$6.05^{\star\star\star}$	$2.95^{\star\star}$	$2.10^{\star}$	0.88	1.35
NOR	0.50	0.84	$2.75^{\star}$	1.32	1.27	0.52
SWE	$0.09^{\diamond\diamond}$	1.56	0.96	1.61	$2.14^{\star}$	1.39
UK	0.31	0.73	2.07	$2.62^{\star\star}$	1.03	0.99
USA	$0.22^{\diamond}$	1.59	0.92	0.96	0.58	0.58

$^{\star}/^{\star\star}/^{\star\star\star}$: statistic is significantly > 1 (10/5/1 per cent significance level)

$^{\diamond}/^{\diamond\diamond}/^{\diamond\diamond\diamond}$: statistic is significantly < 1 (10/5/1 per cent significance level)

The absolute values of the test statistics in Table 4 show that the variations in average spectral density are not only statistically significant, but of substantial size. For example, in France the average spectral density of GFCF in the long cycle range is 12.78 times as high than in the non-cycle ranges. The importance of the fluctuations in the classical frequency ranges can perhaps be illustrated even better by the fraction of the spectral mass (variance) in these ranges, compared to the total spectral mass (Cf. Table 13 of the appendix). While the length of the frequency range of the long cycle is only 9.89% of the total range from 1/15 to 1/2, it contains a much higher fraction of the total spectral mass. For the 9 countries with significant long cycle in GFCF, this fraction ranges from 21.0% in the case of Australia to 57.9% in the case of France. The fraction is higher than one third also in Canada, West Germany, Japan and New Zealand. The length of the short cycle range is 30.77%, but the fraction of the spectral mass of II it contains ranges from 48.2% in New Zealand to 64.5% in Germany (considering only the countries with significant short cycle).

To provide more detailed information on the characteristics of the spectra of GFCF and II, Table 5 lists the average spectral densities for all 5 frequency ranges.

Table 5: Average Spectral Densities, MBK Filter

	II					GFCF				
	$[\frac{1}{15},\frac{1}{10}]$	$[\frac{1}{10},\frac{1}{7}]$	$[\frac{1}{7},\frac{1}{5}]$	$[\frac{1}{5},\frac{1}{3}]$	$[\frac{1}{3},\frac{1}{2}]$	$[\frac{1}{15},\frac{1}{10}]$	$[\frac{1}{10},\frac{1}{7}]$	$[\frac{1}{7},\frac{1}{5}]$	$[\frac{1}{5},\frac{1}{3}]$	$[\frac{1}{3},\frac{1}{2}]$
AUS	0.002	0.005	0.023	0.010	0.016	0.208	0.385	0.206	0.243	0.108
AUT	0.004	0.018	0.028	0.028	0.007	0.220	1.095	0.415	0.192	0.122
BEL	0.002	0.006	0.004	0.009	0.006	1.738	0.276	0.145	0.259	0.146
CAN	0.009	0.005	0.013	0.005	0.006	0.686	1.035	1.006	0.194	0.120
DNK	0.006	0.005	0.005	0.006	0.004	2.446	5.116	2.320	0.654	0.182
FRA	0.002	0.005	0.001	0.010	0.006	0.178	0.892	0.168	0.048	0.033
WGR	0.001	0.015	0.009	0.014	0.003	0.757	2.626	0.631	0.171	0.041
ITA	0.000	0.007	0.003	0.012	0.012	0.600	0.627	0.822	0.229	0.045
JPN	0.002	0.001	0.002	0.003	0.002	1.239	1.714	0.119	0.163	0.093
NDL	0.002	0.005	0.005	0.009	0.005	0.515	1.341	0.732	0.087	0.073
NZL	0.014	0.049	0.026	0.069	0.066	8.662	5.959	0.352	0.659	0.222
NOR	0.037	0.015	0.114	0.045	0.014	1.812	0.883	0.650	0.673	0.179
SWE	0.006	0.004	0.088	0.033	0.004	0.147	0.035	0.058	0.199	0.041
UK	0.002	0.003	0.015	0.012	0.004	0.306	0.296	0.490	0.125	0.072
USA	0.001	0.002	0.007	0.004	0.003	0.224	1.495	2.995	0.242	0.096

We have mentioned in Section 2 that our definition of cycle is weak in the sense that it does not require the spectral density of the long or short cycle range to be higher than that of both neighboring ranges. The table shows that this stronger condition is actually fulfilled for 8 of the nine cases with a significant long cycle in GFCF. The only exception is New Zealand, where the bulk of the spectral mass is located in the range of 10–15 years, and which is therefore not really a candidate for a classical long cycle. For the short cycle in II, the exception is UK, where average spectral density is slightly higher in the range 5–7 than 3–5 years, and in Austria it is about the same in both ranges.

For the cases without significant results, different patterns emerge. For GFCF, we see that in Belgium and Norway the mass is concentrated in the range 10–15 years, while for Sweden it is highest in the range 3–5 years. In Italy, UK and the US, spectral density is highest in the range 5–7 years. This, however, depends on the filter used. In Italy and the US, spectral density is highest in the range 7–10 years if the HP filter is applied (cf. Table 12 in the Appendix). If the data do not show a clear pattern, the results are dominated by the detrending filter.

Finally, it might be interesting to compare the empirical results with the results obtained from simulating the most basic neoclassical business cycle model. Table 6 reports the results for the RBC model of Cooley and Prescott (1995). The fixed investment series was converted to 34 annual observations. The simulation uses the original calibration of parameters, which is for the USA. We see that the model produces a time series whose first differences have an approximately flat spectrum. Applying different detrending filters we then obtain results which correspond to the respective filter gain functions. As we know from Cogley and Nason (1995), the dynamics of output and its aggregates in this model is not significantly different from the dynamics of the exogenous technological shock. The model does not produce cycles as they are found in the data of most countries, unless we postulate the existence of cycles in technological progress.

Table 6: Average Spectral Densities GFCF, RBC model

Trend	$[\frac{1}{15}, \frac{1}{10}]$	$[\frac{1}{10}, \frac{1}{7}]$	$[\frac{1}{7}, \frac{1}{5}]$	$[\frac{1}{5}, \frac{1}{3}]$	$[\frac{1}{3}, \frac{1}{2}]$
First Differences	1.999	2.083	2.003	1.626	1.106
HP	4.83	3.464	1.948	0.832	0.335
BK	1.549	2.321	2.323	0.992	0.309
MBK	1.110	1.706	1.801	0.897	0.310

Note: Average of 1000 repetitions.

6 Conclusions

The aim of this paper was to demonstrate that the classical view of business cycles, as we have interpreted it, has an important element of truth. Obviously, the classical view is not correct in a very strict sense: not every country has significant cyclical structure in the same frequency range. One could not plausibly expect such a strong proposition to hold. Too different are the economic structures in the different countries, and the economic shocks they are subjected to. Nevertheless, the data support the predictions of the classical writers: fixed investment tends to have more spectral mass in the frequency range of 7–10 years, inventory behavior in the range of 3–5 years. Looking at each country separately, these regularities are often not significant in the statistical sense, which is not too surprising given the shortness of the available time series. The significance comes mainly from the similarity of results across countries.

We find it remarkable that international data starting in 1960 support a hypothesis that has been established until the 1950s, using data from the 19th and the first half of the 20th century. We think that the existence of these cycles is a stylized fact that current business cycle theory should pay more attention to. The dynamics of fixed investment is not explained by the most standard RBC model, since investment series contain more dynamics than neoclassical theories predict (Sensenbrenner 1991). Wen (1998) is a recent attempt to explain the cycle in fixed investment by a theory based on investment complementarities.

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A Appendix: Detailed Tables and Figures

Table 7: Rejection probabilities for 24 different stochastic processes

	Prob. of greater value					Prob. of greater value			
Process	0.95	0.10	0.05	0.01	Process	0.95	0.10	0.05	0.01
Statistic D									
1	0.950	0.108	0.056	0.013	13	0.932	0.121	0.069	0.018
2	0.948	0.129	0.073	0.019	14	0.924	0.228	0.162	0.072
3	0.956	0.161	0.095	0.036	15	0.923	0.236	0.168	0.078
4	0.946	0.076	0.037	0.008	16	0.927	0.139	0.084	0.025
5	0.942	0.081	0.036	0.007	17	0.918	0.248	0.180	0.089
6	0.949	0.145	0.090	0.029	18	0.927	0.239	0.168	0.075
7	0.945	0.088	0.041	0.010	19	0.939	0.102	0.052	0.013
8	0.959	0.159	0.090	0.028	20	0.903	0.239	0.172	0.083
9	0.959	0.204	0.134	0.059	21	0.937	0.372	0.296	0.168
10	0.941	0.062	0.029	0.005	22	0.934	0.113	0.067	0.017
11	0.948	0.111	0.059	0.014	23	0.909	0.250	0.188	0.088
12	0.945	0.091	0.044	0.009	24	0.940	0.204	0.135	0.055
Statistic D^{Int}									
1	0.945	0.104	0.051	0.013	13	0.937	0.130	0.075	0.019
2	0.947	0.123	0.069	0.017	14	0.925	0.187	0.122	0.046
3	0.954	0.166	0.101	0.034	15	0.930	0.189	0.128	0.050
4	0.935	0.099	0.051	0.012	16	0.935	0.146	0.087	0.026
5	0.941	0.112	0.062	0.013	17	0.926	0.200	0.134	0.059
6	0.968	0.226	0.149	0.057	18	0.926	0.186	0.120	0.048
7	0.937	0.105	0.060	0.014	19	0.948	0.108	0.057	0.013
8	0.954	0.126	0.071	0.019	20	0.904	0.186	0.127	0.052
9	0.959	0.249	0.177	0.078	21	0.930	0.254	0.183	0.084
10	0.909	0.101	0.053	0.012	22	0.939	0.127	0.071	0.018
11	0.946	0.107	0.057	0.013	23	0.911	0.196	0.133	0.056
12	0.945	0.105	0.056	0.013	24	0.939	0.153	0.092	0.030
Statistic Z									
1	0.953	0.102	0.049	0.012	13	0.964	0.117	0.064	0.012
2	0.992	0.109	0.059	0.011	14	0.995	0.136	0.076	0.016
3	0.999	0.144	0.079	0.020	15	0.996	0.138	0.076	0.017
4	0.945	0.095	0.046	0.010	16	0.967	0.122	0.063	0.014
5	0.965	0.097	0.048	0.006	17	0.996	0.133	0.070	0.011
6	0.999	0.184	0.107	0.029	18	0.995	0.140	0.080	0.020
7	0.953	0.097	0.050	0.011	19	0.956	0.107	0.058	0.012
8	0.998	0.111	0.058	0.011	20	0.994	0.118	0.058	0.007
9	1.000	0.215	0.138	0.050	21	0.999	0.150	0.074	0.010
10	0.927	0.089	0.042	0.006	22	0.965	0.122	0.066	0.015
11	0.961	0.103	0.054	0.012	23	0.994	0.119	0.058	0.007
12	0.954	0.102	0.054	0.013	24	0.993	0.125	0.070	0.016

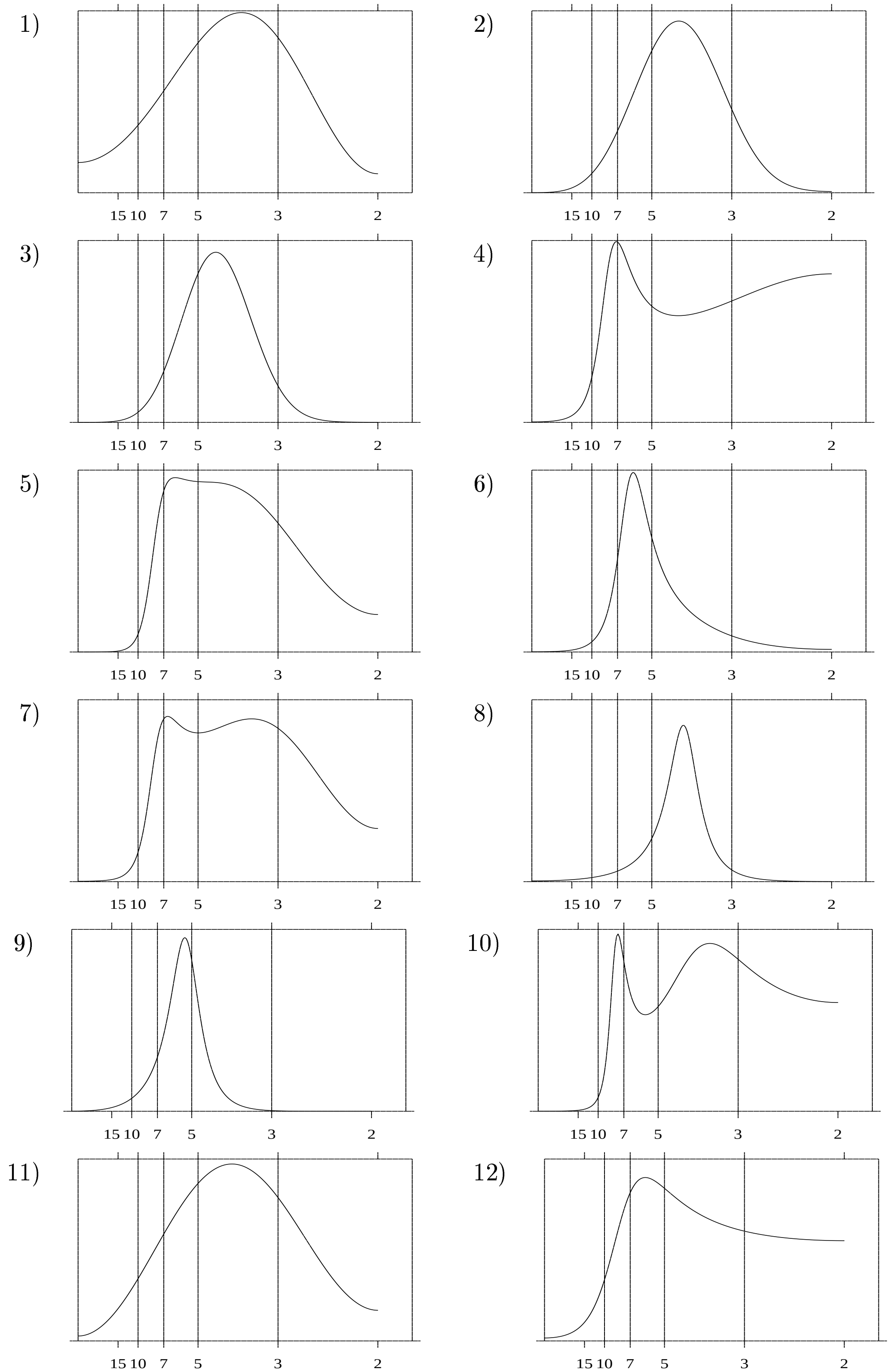

Figure 4: Spectral densities of test processes 1–12

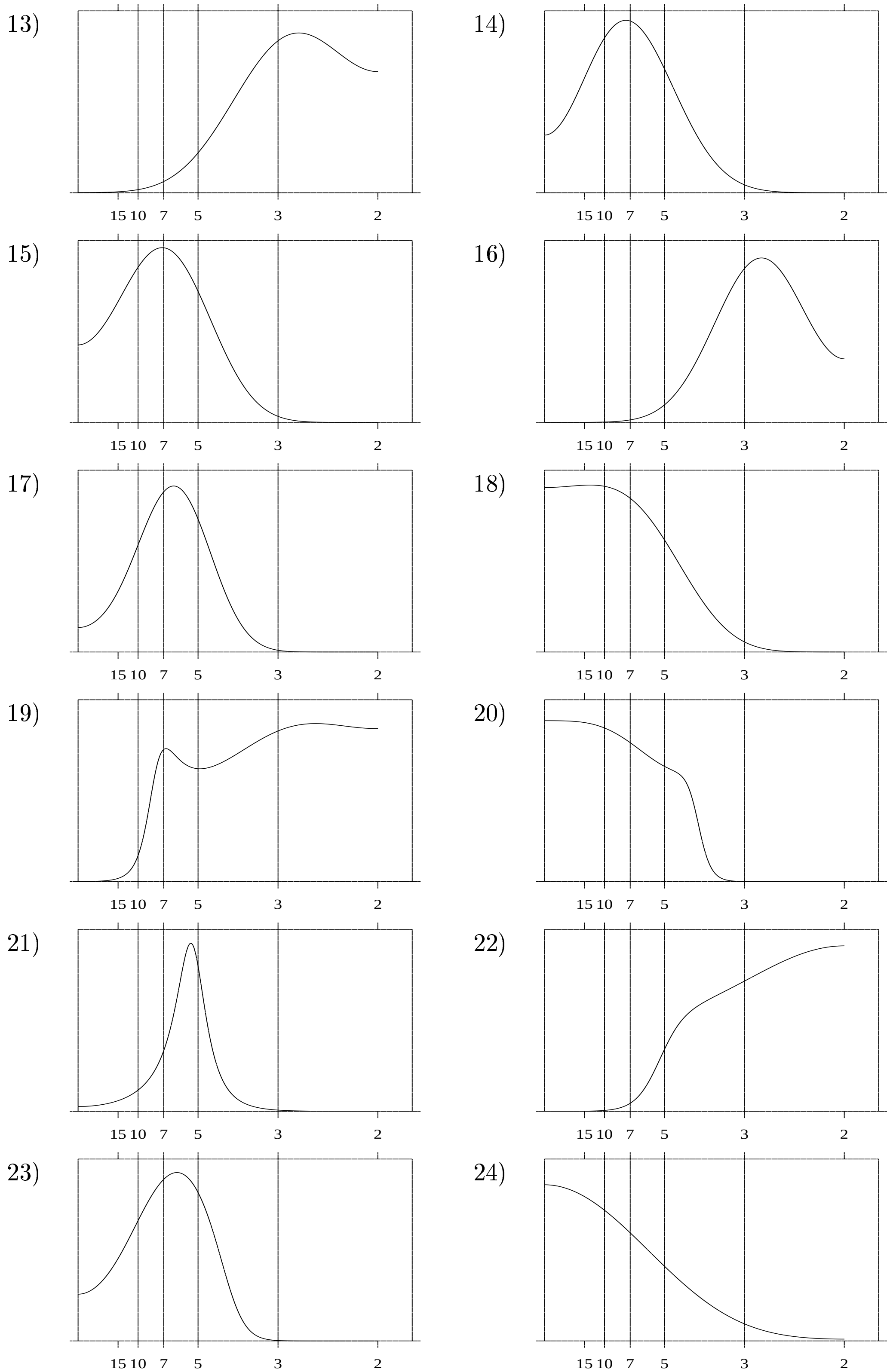

Figure 5: Spectral densities of test processes 13–24

Table 8: Hodrick-Prescott Filter, D^{Int}

	long cycle			short cycle		
	II	$GFCF$	GDP	II	$GFCF$	GDP
AUS	0.40	7.57***	3.91**	0.85	2.54**	1.17
AUT	1.83	4.40***	3.38**	2.49**	0.77	0.91
BEL	1.96	1.15	5.60***	2.05*	0.24$^{\diamond\diamond\diamond}$	0.73
CAN	1.29	6.06***	2.97**	0.92	0.43$^{\diamond}$	0.40$^{\diamond}$
DNK	0.51	3.64**	3.31**	1.17	0.48	1.18
FRA	2.01	11.66***	7.99***	1.35	0.48	0.64
WGR	2.23*	6.90***	3.62**	2.72**	0.55	0.98
ITA	0.97	3.66**	4.39***	1.33	0.44$^{\diamond}$	0.56
JPN	1.60	7.35***	4.18***	1.59	0.36$^{\diamond\diamond}$	0.44$^{\diamond}$
NDL	0.44	3.43**	1.14	1.60	0.42$^{\diamond}$	0.48
NZL	2.00	3.28**	3.20**	1.79	0.35$^{\diamond\diamond}$	0.52
NOR	1.16	1.22	4.41***	1.12	0.59	0.35$^{\diamond\diamond}$
SWE	0.24$^{\diamond}$	2.02	1.79	1.40	0.53	0.53
UK	1.18	2.15*	3.36**	2.82**	0.43$^{\diamond}$	0.41$^{\diamond}$
USA	0.69	3.47**	2.32*	0.92	0.58	0.39$^{\diamond}$

$^{\star}/^{\star\star}/^{\star\star\star}$: statistic is significantly > 1 (10/5/1 per cent significance level)
$^{\diamond}/^{\diamond\diamond}/^{\diamond\diamond\diamond}$: statistic is significantly < 1 (10/5/1 per cent significance level)

Table 9: Difference Filter, D^{Int}

	long cycle			short cycle		
	II	$GFCF$	GDP	II	$GFCF$	GDP
AUS	0.15$^{\diamond\diamond}$	3.14**	1.74	0.51	2.80**	1.04
AUT	0.63	1.99	2.18*	2.54**	1.01	1.30
BEL	0.38	0.53	2.50*	1.34	0.60	0.83
CAN	0.46	3.74**	1.57	0.91	0.75	0.95
DNK	0.19$^{\diamond\diamond}$	1.94	1.83	0.92	1.16	1.83
FRA	0.37	6.50***	4.26***	1.20	0.71	0.94
WGR	0.47	4.54***	2.91**	2.66**	1.03	1.58
ITA	0.15$^{\diamond\diamond}$	2.40*	1.39	1.00	0.70	0.81
JPN	0.37	3.58**	2.37*	1.61	0.53	0.62
NDL	0.18$^{\diamond\diamond}$	1.19	1.00	1.46	0.92	0.44$^{\diamond}$
NZL	0.37	2.53*	1.65	1.19	0.93	0.92
NOR	0.46	0.46	1.97	1.30	1.12	0.59
SWE	0.13$^{\diamond\diamond}$	1.69	1.48	1.41	0.71	0.69
UK	0.53	0.73	1.88	2.73**	1.14	0.94
USA	0.15$^{\diamond\diamond}$	2.30*	1.12	0.85	0.94	0.80

$^{\star}/^{\star\star}/^{\star\star\star}$: statistic is significantly > 1 (10/5/1 per cent significance level)
$^{\diamond}/^{\diamond\diamond}/^{\diamond\diamond\diamond}$: statistic is significantly < 1 (10/5/1 per cent significance level)

Table 10: Difference Filter + Linear Trend, D^{Int}

	long cycle			short cycle		
	II	$GFCF$	GDP	II	$GFCF$	GDP
AUS	$0.15^{\diamond\diamond}$	$2.96^{\star\star}$	1.36	0.50	$3.05^{\star\star}$	1.16
AUT	0.61	1.53	1.14	$2.53^{\star\star}$	0.90	1.44
BEL	0.36	0.55	1.69	1.35	0.61	0.98
CAN	0.46	$4.11^{\star\star\star}$	1.75	0.91	0.75	1.01
DNK	$0.20^{\diamond\diamond}$	2.06	1.37	0.94	1.22	$1.90^{\star}$
FRA	0.31	$6.76^{\star\star\star}$	$3.66^{\star\star}$	1.22	0.80	1.24
WGR	0.44	$3.99^{\star\star\star}$	1.91	$2.66^{\star\star}$	0.97	1.66
ITA	$0.14^{\diamond\diamond}$	$2.43^{\star}$	1.91	1.00	0.73	1.00
JPN	0.34	$3.75^{\star\star}$	$2.17^{\star}$	1.59	0.63	0.93
NDL	$0.17^{\diamond\diamond}$	1.31	0.38	1.48	0.97	0.48
NZL	0.37	$2.33^{\star}$	1.34	1.19	0.94	0.93
NOR	0.44	0.58	$2.32^{\star}$	1.31	1.08	0.64
SWE	$0.12^{\diamond\diamond}$	1.71	0.88	1.42	0.80	0.99
UK	0.53	0.92	$2.12^{\star}$	$2.69^{\star\star}$	1.24	1.06
USA	$0.16^{\diamond\diamond}$	$2.14^{\star}$	1.18	0.85	0.96	0.82

$^{\star}/^{\star\star}/^{\star\star\star}$: statistic is significantly > 1 (10/5/1 per cent significance level)

$^{\diamond}/^{\diamond\diamond}/^{\diamond\diamond\diamond}$: statistic is significantly < 1 (10/5/1 per cent significance level)

Table 11: Baxter-King Filter, D^{Int}

	long cycle			short cycle		
	II	$GFCF$	GDP	II	$GFCF$	GDP
AUS	$0.20^{\diamond}$	$6.75^{\star\star\star}$	$3.07^{\star\star}$	0.99	$4.78^{\star\star\star}$	1.45
AUT	0.96	$4.50^{\star\star}$	$3.10^{\star\star}$	$2.10^{\star}$	1.23	1.45
BEL	1.42	1.04	$4.20^{\star\star}$	$2.42^{\star\star}$	0.79	1.35
CAN	0.63	$5.27^{\star\star\star}$	1.96	1.32	0.89	0.94
DNK	0.82	$3.63^{\star\star}$	$3.11^{\star\star}$	$3.09^{\star\star}$	0.76	1.81
FRA	0.86	$14.65^{\star\star\star}$	$7.24^{\star\star\star}$	$2.08^{\star}$	1.07	1.47
WGR	1.64	$6.62^{\star\star\star}$	$4.07^{\star\star}$	$4.11^{\star\star\star}$	0.76	1.82
ITA	0.46	1.75	$2.32^{\star}$	1.30	0.64	0.78
JPN	0.45	$6.32^{\star\star\star}$	$3.75^{\star\star}$	1.40	0.78	0.88
NDL	0.47	$2.51^{\star}$	2.02	1.91	0.53	1.40
NZL	1.40	$6.34^{\star\star\star}$	$3.42^{\star\star}$	$2.31^{\star}$	0.75	1.24
NOR	0.56	0.95	$2.94^{\star\star}$	1.26	1.15	0.46
SWE	$0.10^{\diamond\diamond}$	1.66	1.08	1.57	1.94	1.37
UK	0.36	0.81	$2.25^{\star}$	$2.62^{\star\star}$	0.96	0.91
USA	0.28	1.72	1.01	0.91	0.51	0.50

$^{\star}/^{\star\star}/^{\star\star\star}$: statistic is significantly > 1 (10/5/1 per cent significance level)

$^{\diamond}/^{\diamond\diamond}/^{\diamond\diamond\diamond}$: statistic is significantly < 1 (10/5/1 per cent significance level)

Table 12: Average Spectral Densities, HP Filter

	II					GFCF				
	$[\frac{1}{15},\frac{1}{10}]$	$[\frac{1}{10},\frac{1}{7}]$	$[\frac{1}{7},\frac{1}{5}]$	$[\frac{1}{5},\frac{1}{3}]$	$[\frac{1}{3},\frac{1}{2}]$	$[\frac{1}{15},\frac{1}{10}]$	$[\frac{1}{10},\frac{1}{7}]$	$[\frac{1}{7},\frac{1}{5}]$	$[\frac{1}{5},\frac{1}{3}]$	$[\frac{1}{3},\frac{1}{2}]$
AUS	0.005	0.006	0.017	0.013	0.018	0.282	1.110	0.304	0.372	0.066
AUT	0.010	0.019	0.024	0.026	0.005	0.619	0.969	0.344	0.169	0.098
BEL	0.001	0.007	0.002	0.007	0.004	4.663	0.941	0.431	0.200	0.185
CAN	0.017	0.008	0.006	0.006	0.004	1.724	2.436	0.588	0.172	0.074
DNK	0.005	0.003	0.007	0.007	0.006	4.595	3.893	1.409	0.514	0.249
FRA	0.009	0.011	0.003	0.007	0.005	0.590	1.677	0.144	0.070	0.054
WGR	0.005	0.009	0.007	0.011	0.003	0.714	2.382	0.953	0.192	0.063
ITA	0.003	0.006	0.004	0.008	0.008	0.662	1.871	1.637	0.223	0.094
JPN	0.005	0.004	0.002	0.004	0.002	1.365	2.424	0.388	0.120	0.103
NDL	0.003	0.003	0.013	0.010	0.004	1.150	1.097	0.541	0.134	0.079
NZL	0.012	0.063	0.017	0.056	0.040	11.472	5.985	0.709	0.637	0.280
NOR	0.039	0.036	0.088	0.035	0.010	2.828	0.795	0.819	0.383	0.158
SWE	0.009	0.004	0.064	0.025	0.004	2.692	1.410	1.039	0.374	0.183
UK	0.006	0.005	0.011	0.013	0.002	2.242	1.116	0.807	0.221	0.076
USA	0.001	0.002	0.005	0.002	0.002	0.610	1.701	1.665	0.283	0.062

Table 13: spectral mass in frequency ranges, as fraction of total spectral mass in range 1/2–1/15, MBK Filter

	II					GFCF				
	$[\frac{1}{15},\frac{1}{10}]$	$[\frac{1}{10},\frac{1}{7}]$	$[\frac{1}{7},\frac{1}{5}]$	$[\frac{1}{5},\frac{1}{3}]$	$[\frac{1}{3},\frac{1}{2}]$	$[\frac{1}{15},\frac{1}{10}]$	$[\frac{1}{10},\frac{1}{7}]$	$[\frac{1}{7},\frac{1}{5}]$	$[\frac{1}{5},\frac{1}{3}]$	$[\frac{1}{3},\frac{1}{2}]$
AUS	0.009	0.018	0.220	0.318	0.435	0.022	0.210	0.069	0.563	0.135
AUT	0.011	0.058	0.208	0.514	0.209	0.025	0.279	0.232	0.285	0.179
BEL	0.004	0.076	0.081	0.497	0.342	0.263	0.098	0.141	0.277	0.221
CAN	0.061	0.049	0.200	0.387	0.302	0.132	0.356	0.205	0.218	0.089
DNK	0.008	0.041	0.145	0.587	0.218	0.089	0.271	0.283	0.238	0.119
FRA	0.034	0.048	0.032	0.494	0.391	0.095	0.579	0.082	0.162	0.081
WGR	0.014	0.065	0.143	0.645	0.134	0.054	0.418	0.307	0.184	0.037
ITA	0.007	0.033	0.053	0.401	0.507	0.066	0.167	0.500	0.228	0.039
JPN	0.042	0.031	0.129	0.429	0.369	0.157	0.391	0.138	0.188	0.126
NDL	0.004	0.031	0.206	0.488	0.270	0.109	0.219	0.313	0.198	0.160
NZL	0.008	0.076	0.058	0.482	0.377	0.248	0.409	0.065	0.185	0.094
NOR	0.016	0.047	0.403	0.387	0.147	0.120	0.078	0.276	0.366	0.160
SWE	0.008	0.008	0.461	0.451	0.072	0.170	0.110	0.150	0.469	0.102
UK	0.025	0.021	0.215	0.563	0.175	0.181	0.073	0.310	0.322	0.114
USA	0.009	0.024	0.305	0.325	0.338	0.031	0.169	0.565	0.192	0.042