Debt Stabilization in a Non-Ricardian Economy*

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Abstract

In models with a representative infinitely lived household, modern versions of tax smoothing imply that the steady-state of government debt should follow a random walk. This is unlikely to be the case in OLG economies, where, the equilibrium interest rate may differ from the policy-maker's rate of time preference such that it may be optimal to reduce debt today to reduce distortionary taxation in the future. Moreover, the level of the capital stock (and therefore output and, possibly, consumption) in these economies is likely to be sub-optimally low, and reducing government debt will 'crowd in' additional capital. Using an elaborate version of the model of perpetual youth developed by Blanchard (1985) and Yaari (1965), we derive the optimal steady state level of government assets. We show how and why this level of government assets falls short of the level of debt that achieves the optimal capital stock and the level that eliminates income taxes.

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1 Introduction

The problems caused by excessive levels of public debt do not need enumerating. As governments around the world try to bring deficits under control, and subsequently to reduce levels of debt in relation to GDP, a natural question to ask is how far debt levels should be reduced, and how quickly, once any immediate crisis resulting from large default risk premia has diminished. In other words, what should be the ultimate target for the debt to GDP ratio, and how quickly should we get there? Until now,

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most analysis of this question has been undertaken using models in which consumers in effect live forever, by appropriately internalising the utility of their children. This tends to have the implication that the optimal level of debt depends upon the initial level of debt as policy makers seek to minimise the costs of distortionary taxation going forwards (see Barro (1979) and Chamley (1985,1986) for example). The implications of the benchmark result in such models is striking: once fears of default have receded, the optimum level of debt is closely tied to the historic debt level. This martingale process for debt has also re-emerged in New Keynesian style DSGE models, (see for example, Benigno and Woodford (2003) and Schmitt-Grohe and Uribe (2004a)), where policy makers also care about the costs of inflation in an sticky price environment as well as minimising the costs of tax distortions. These applications of tax smoothing all suggest that attempts to reduce the extent of distortionary taxation in the long run will require short run increases in these taxes whose cost outweighs the eventual gain.

However, within this literature there have been attempts to analysis the optimal quantity of debt by introducing additional costs or benefits associated with the level of government debt. For example, in Aiyagari et al (2002) implict risk premia in an economy with incomplete financial markets may encourage the government to accumulate sufficient assets to pay for (exogenously determined, but stochastic) government spending after eliminating distortionary taxation, although introducing ad hoc limits on the levels of assets held by the government will ensure policy is more akin to that described in the orginal tax smoothing result of Barro (1979). Aiyagari and McGratten (1998) allow for a role for government debt in that it can help alleviate households' borrowing constraints, while Shin (2006) allows for household heterogeneity and idiosyncratic income shocks to provide a role for government debt in facilitating precautionary saving. However, with the exception of Aiyagari and McGratten (1998), where risk premia due to incomplete financial markets drive interest rates above the rate of time preference in a production economy which utilises physical capital, these papers do not allow for one of the common worries associated with rising debt levels, namely that public debt crowds out private capital.

In overlapping generations economies where agents do not care about their children (or do not care about them enough), this effect is central to the desirability of stabilising debt. There are, in fact, two reasons why the random walk steady state debt result no longer holds in these Non-Ricardian economies. First, if the economy is not dynamically inefficient, then the real interest rate is likely to exceed the rate of time preference, which means that from a Ramsey planner's point of view it may be worth sacrificing some current utility in order to achieve a steady state where distortionary taxes are lower than they currently are (even if the current generation may lose out as a result). Second, as noted above, the level of the capital stock (and therefore output and consumption) in these economies is likely to be sub-optimally low, and reducing government debt will 'crowd in' additional capital.

This raises an immediate question: will the debt target in such models be the debt level that eliminates the need for distortionary taxes, or will it be the level that achieves the optimal capital stock? This is one of the issues we examine in this paper. Using an elaborate version of the model of perpetual youth developed by Blanchard (1985) and Yaari (1965), which allows us to vary the extent of Non-Ricardian behaviour parametrically, we derive the optimal steady state level of government assets. We show how and why this level of government assets falls short of both the level of debt that achieves the optimal capital stock and the level that eliminates income taxes. We also explore the non-linear path the policy maker follows in moving the economy from its current position to the desired long-run solution to the Ramsey problem.

Section 2 contrasts, in a highly simplified way, the steady state random walk debt result with the outcome when the rate of interest is above the rate of time preference, and where capital is below its optimal level. Section 3 outlines a quite rich version of the model of perpetual youth, which features sticky prices, exogenous growth, distortionary taxation, government consumption and public and private physical capital accumulation. In section 4, we discuss social welfare, the model's calibration, and our numerical results for both the steady-state of the Ramsey problem and the non-linear Ramsey dynamics. A final section concludes.

2 Optimal Debt and Optimal Capital

The benchmark model for optimal debt implies that there is no optimal level of debt. This benchmark assumes that individuals are effectively infinitely lived, and ignores the possibility of default. Taxation is distortionary, so if we could choose the level of government debt we inherit, it would be negative, and the interest payments on these government assets would pay for any government consumption. In the discussion below, we call this the zero-tax level of government assets, or A^T. Of course, without recourse to default or some equivalent expropriation mechanism, a government cannot choose the level of debt it inherits. (A Ramsey planner could in theory expropriate sufficient capital using a capital tax, and then commit to setting capital taxes to zero, but this commitment is unlikely to be credible.)

Suppose we inherit a level of debt different from A^T, the zero tax level of assets. In the absence of any other means of reducing debt except higher taxes or lower spending, then we have a choice between high taxes (or lower spending) now to reduce debt towards the optimal level, or accepting permanently positive taxes (or lower than optimal government spending) that will finance the interest payments on the inherited debt level, and so leave debt unchanged. If the costs of distortionary taxes or lower than optimal public spending are increasing at the margin, then we get a classic tax smoothing result, which is that it is optimal to keep the inherited level of debt.

However implicit in this argument is that the real rate of interest is equal to the rate at which we discount the future. We can show this formally as follows. Suppose social welfare can be represented as

$$W_t = -\sum_{i=0}^{\infty} \beta^i T_{t+i}^2 \tag{1}$$

where T is the level of distortionary taxes and β is the discount factor. The budget constraint is

$$A_t = (1+r)A_{t-1} + T_t - G \tag{2}$$

where A are government assets. The government inherits a debt level $B_{-1} > 0$, such that government assets are negative $A_{t-1} = -B_{t-1}$.

The Lagrangian is

$$L = \sum_{i=0}^{\infty} \beta^i [T_{t+i}^2 + 2\lambda_{t+i}(A_{t+i} - (1+r)A_{t+i-1} - T_{t+i})] \tag{3}$$

The first order condition for taxes is

$$T_{t+i} - \lambda_{t+i} = 0 \tag{4}$$

and for debt

$$\lambda_{t+i} - \beta(1+r)\lambda_{t+i+1} = 0 \tag{5}$$

Combining gives

$$\beta(1+r)T_{t+i+1} - T_{t+i} = 0 \tag{6}$$

If $\beta(1+r) = 1$, then the FOC for debt implies the Lagrange multiplier is constant, which in turn implies constant taxes. Taxes can only be constant if they are sufficient to satisfy the budget constraint if A is constant at $-B_{-1}$, which is the tax smoothing or random walk steady state debt result. (See, for example, Schmitt-Grohe and Uribe (2004a) and Benigno and Woodford (2003)). However, if $\beta(1+r) \neq 1$, then a steady state is possible only if taxes are zero. If $\beta(1+r) < 1$ debt will decline towards this value. In this situation it is always better to reduce debt each year, because the discounted benefits of lower future taxes exceed the cost of higher taxes today. The cost of permanently positive taxes will always outweigh the cost of reducing debt, because we discount at less than the rate of interest, and so we head towards the zero tax level of government assets. If $\beta(1+r) > 1$ then debt will follow an explosive path. For an example of this, see Kirsanova, Leith, and Wren-Lewis (2007).

In this simple model, when $\beta(1+r) = 1$, the tax smoothing result is time consistent. There is no reason to deviate from the inherited level of debt at any time. This result will not be robust to two natural extentions of the model: introducing nominal debt, or staying with real debt but allowing for sticky prices. If inflation is determined by a New Keynesian Phillips Curve, then Leith and Wren-Lewis (2007) show that there is a first period incentive to reduce inherited debt somewhat (but not completely). However, this incentive recurs as we move to the next period, and so the random walk result is not just modified, but is also time inconsistent. They also show that the time consistent policy involves a very rapid reduction in debt to its initial level following a positive shock. (This is true for a closed economy, or simple open economy with a flexible exchange rate, but rates of adjustment are slower under EMU - see Leith and Wren-Lewis (2010).) However, there are reasons for wanting to focus on the time inconsistent case. In reality governments do not rapidly correct any debt disequilibrium. This may be because the costs of doing so are not only high, but they are also short term, so an impatient government would have an incentive to stick to the time inconsistent plan.

When it comes to thinking about the optimal capital stock, we have another benchmark result, which is that if consumers are effectively infinitely lived, then in the absence of the taxation and other distortions, the level of the capital stock that would be chosen by a social planner would be the same as that produced by a market equilibrium. Because Ricardian Equivalence holds, any increase in government debt leads to a matching increase in private saving, with no impact on this capital stock. In short, debt does not crowd out capital. However, in an OLG economy, the economy will not in general generate an optimal capital stock, and government debt will crowd out capital.

The assumption that individuals leave bequests because they internalise the utility of the next generation (although with discounting), so that they are effectively infinitely lived, is a useful benchmark, but it may be at the extreme end of plausible degrees of inter-generational altruism. At the opposite extreme we have overlapping generation (OLG) models, which generally assume completely selfish generations that leave no intentional bequests. In the model of Perpetual Youth developed by Blanchard and Yaari, if income does not decline with age (and there is no retirement), the real rate of interest will be above the rate of time preference, and so the level of the capital stock is likely to be suboptimally low. In addition, higher government debt will crowd out private capital in OLG models, because Ricardian Equivalence no longer holds. Agents accumulate assets because it is optimal for them to do so as individuals, with no thought for the utility of future generations. It is a model of this kind we develop in the next section. Although the model we develop below is quite rich, the essence of the implications for government debt for the real interest rate can be understood by considering some key equations from a simplified version of the model. Ignoring the households' cash holdings and the tax on consumption, logarithmic utility implies that the aggregate consumption function is a linear function of human and financial wealth,

$$c_t = (1 - \gamma\beta)(lw_t + \frac{W_t}{P_t}) \tag{7}$$

where c is consumption, lw is human wealth, W/P financial assets, γ is the survival probability and β is the households' subjective discount factor. Human capital is given by

$$lw_t = (1-\tau_t^w)w_t l_t + OI_t + \gamma\frac{\pi_{t+1}}{R_t}lw_{t+1} \tag{8}$$

where $(1-\tau_t^w)w_t l_t$ is post-tax labour income, OI_t are other (exogenous) sources of income detailed in the model section and $\frac{R_t}{\pi_{t+1}}$ is the real interest rate. Agents hold portfolios of financial assets such that they effectively receive an additional return $1/\gamma$ on their assets, conditional on their surviving. The dynamics of aggregate financial wealth is given by

$$\frac{W_{t+1}}{P_{t+1}}\frac{\pi_{t+1}}{R_t} = \frac{W_t}{P_t} + (1-\tau_t^w)w_t l_t + OI_t - C_t \tag{9}$$

Combining these equations implies

$$C_t = \frac{C_{t+1}\pi_{t+1}}{\beta R_t} + \frac{(1-\gamma\beta)(1-\gamma)}{\gamma\beta}\frac{W_{t+1}}{P_{t+1}}\frac{\pi_{t+1}}{R_t} \tag{10}$$

Ignoring capital adjustment costs such that Tobin's q is always 1, when the only asset is capital, $\frac{W_{t+1}}{P_{t+1}}\frac{\pi_{t+1}}{R_t} = K_t$, then this equation clearly implies that in steady state $r > 1/\beta$. Individual agents are always saving, but the aggregate level of assets can be constant because those who die have positive assets and the newborn have none. This is the first important implication of allowing for finite lives with no bequests: the real rate of interest can differ from the rate of time preference even in steady state. (The implications of this point are discussed in Erosa and Gervais (2001).)

The second important difference an OLG model makes is that government debt can crowd out capital. Let $\frac{W_{t+1}}{P_{t+1}}\frac{\pi_{t+1}}{R_t} = K_t + B_t$, where B_t is government debt as before. In steady state, if consumption and real interest rates were unchanged, government debt would crowd out private capital one for one. In fact consumption is likely to fall if capital falls, increasing the extent of crowding out. However, a reduction in the capital stock will also raise real interest rates, which for given consumption levels will raise the overall level of aggregate assets, which moderates the degree of crowding out of capital. (In the infinite life case, which we approach as γ tends to one, any increase in government debt leads to an equal increase in savings, so there is no crowding out.)

Just as government debt crowds out capital, if the government holds assets ($B < 0$), capital will be crowded in. If, when $A = 0$, capital is sub-optimally low, then accumulating government assets can be used to move to the optimal level of capital. We could define the level of government assets that achieve this optimum capital stock as the 'optimum capital' level of assets, or A^K. Unless the economy with $A = B = 0$ is dynamically inefficient[1], such a move would not represent a Pareto improvement, because the higher taxes that the government would require to accumulate assets would hit the current generation. However, as any debt policy is almost certain to disadvantage

[1] In this model of perpetual youth, $r > \theta$, so the economy is never dynamically inefficient. However introducing either government assets, or allowing income to decline with age, can allow the possibility that $r < \theta$, as we note below.

some generation, this should not prevent us considering using debt as a means of moving towards A^K.

Defining what is optimal in an OLG model of course involves deciding how to compare different generations. Since we are interested in formulating optimal policy for our economy populated with overlapping generations of finitely lived consumers we must face the tricky issue of constructing a welfare metric. We discuss the issues involved in defining a social welfare function below. However, we essentially follow Calvo and Obstfeld (1988) by splitting the problem into an intratemporal problem of how to allocate consumption across generations at a given point in time, and an intertemporal problem, of how to stabilise debt over time. Since we are primarily interested in the latter aspect of the problem, we abstract from the first by assuming that the policy maker ignores the intratemporal problem and only considers per capita variables when defining social welfare in an environment where government debt can crowd out private capital. In doing so we assume that the policy maker discounts welfare between generations at the same rate as household discount there own utility.

If the only implication of moving to an OLG framework was that there was some optimal capital stock, then we could simply calculate A^K, and this could become our long run debt 'target'. Indeed, if lump sum taxes were available, we could in theory immediately move to A^K: the additional tax payments would be exactly offset by interest payments on this debt. However, in the absence of lump sum taxes, any change in government assets/debt will, by changing capital, also change the real interest rate. This means that the level of government assets that would eliminate distortionary taxes (A^T) also becomes a potential 'target' for long run government debt.

In the case of the Ricardian model, the zero-tax level of assets A^T was irrelevant because of tax smoothing, as the real rate of interest was equal to the rate of time preference. However in general this condition will not hold in an OLG model. We can examine the implications of this for steady state debt in a highly oversimplified fashion as follows. Suppose social welfare can now be represented as

$$W_t = -\sum_{i=0}^{\infty} \beta^i \left[T_{t+i}^2 + \alpha \left(A_{t+i} - A^K \right)^2 \right] \tag{11}$$

where A are government assets. Welfare fails to reach the first-best allocation for two reasons (which for simplicity we assume are separable): taxes are distortionary, but also capital is away from its optimal level whenever government assets are different from A^K. We still have the budget constraint

$$A_t = (1+r)A_{t-1} + T_t - G \tag{12}$$

where we now allow the real interest rate to depend on government assets in the following simple way:

$$r_t = r_0 - \gamma A_{t-1}/2$$

which captures the idea that as government assets rise, the capital this crowds in reduces the real interest rate. As before, the government inherits a debt level $B_{-1} > 0$. We define

$$A^T = G/r$$

The Lagrangian is

$$L = \sum_{i=0}^{\infty} \beta^i \left[T_{t+i}^2 + \alpha \left(A_{t+i} - A^K\right)^2 + 2\lambda_{t+i} \left(A_{t+i} - (1 + r_0 - \gamma A_{t+i-1}/2)A_{t+i-1} - T_{t+i}\right)\right] \tag{13}$$

The FOC for taxes is

$$T_{t+i} - \lambda_{t+i} = 0 \tag{14}$$

and for debt

$$\alpha(A_{t+i} - A^K) + \lambda_{t+i} - \beta(1 + r_0 - \gamma A_{t+i})\lambda_{t+i+1} = 0 \tag{15}$$

Combining gives

$$\beta(1 + r_0 - \gamma \left(A_{t+i} - a^K\right) - \gamma A^K)T_{t+i+1} - T_{t+i} = \alpha(A_{t+i} - A^K) \tag{16}$$

which can be rewritten as

$$\beta(1 + \tilde{r})T_{t+i+1} - T_{t+i} = (\alpha + \beta\gamma T_{t+i+1})(A_{t+i} - A^K) \tag{17}$$

where $1 + \tilde{r} = 1 + r_0 + \gamma B^K$.

Consider the case where $\beta(1+\tilde{r}) = 1$ first. Whatever the level of steady state taxes, government assets will end up at the level that achieves the optimal capital stock i.e. $A^* = A^K$ where $A^*$is the steady state level of assets (and $B^* = -A^*$ the steady-state level of debt). Taxes will be given by

$$T^* = G + rB^*$$

We can think of this in the following way. The case where $\beta(1 + \tilde{r}) = 1$ is akin to tax smoothing, so A^* is not attracted to A^T. However, we do not get a random walk in steady state debt, because reducing debt has the benefit of increasing capital and therefore output.

If $\beta(1 + \tilde{r}) > 1$, we already know that tax smoothing does not apply, and there will be some history-independent debt target. There are two possibilities. First, taxes are positive in steady state, and so steady state government assets exceed the level required to obtain the optimal capital stock. We therefore have $A^T > A^* > A^K$. Second, taxes become negative in steady state, but despite this government assets are insufficient to achieve the optimal capital stock (providing $\alpha + \beta\gamma T_{t+i+1} > 0$). In this second case, it must be that $A^K > A^* > A^T$. In both cases, we can think about optimal government assets as being a compromise between the zero-tax level and the optimal capital level. The former matters, because tax smoothing does not apply.

Is $\beta(1 + \tilde{r}) < 1$ interesting? If the term multiplying the deviation of debt from the optimal capital level happened to be zero it would not be, because there would be no incentive to stabilise debt. However, as we have seen with the case of $\beta(1 + \tilde{r}) = 1$, the additional incentive to move debt towards the level that maximises the capital stock means that government assets can converge to this level. So providing $\beta(1 + \tilde{r})$ is not too far below one, a steady state is still possible. If it does exist, then if taxes are positive we will have $A^* < A^K$ and $A^* < A^T$. The reason is that if debt reached the optimal capital level, then there would be a tendency for debt to explode. The economy therefore stabilises when this incentive is exactly offset by the incentive to get capital a little higher. Another possibility is that $A^* > A^K$ and taxes are negative, implying $A^T < A^K < A^*$.

To sum up, in an OLG model the 'target' or 'steady state optimal' level of government assets A^* will depend on both the level of assets that delivers the optimum capital stock (A^K) and the level of assets that eliminate distortionary taxes (A^T) in ways that are likely to depend on the detailed structure and parameterisation of the model. If A^* is associated with a real interest rate below the rate of time preference, then it may be the case that A^* will not lie in between A^T and A^K. The next section sets out the model we will investigate, where we find this is indeed the case.[2]

3 The Complete Model

In this section we outline our model. Our economy is populated by overlapping generations of consumers who face a constant probability of death, such that, even if taxes were lump-sum, Ricardian Equivalence would not hold in our model.[3] These consumers supply labour to imperfectly competitive firms, who combine this labour with capital rented from a representative capital rental firm and public capital accumulated by the government, to produce a differentiated product. The firms producing these differentiated products are also subject to the constraints implied by Rotemberg (1982) quadratic adjustment costs. Consumers' labour income is taxed, and they hold financial wealth in the form of money, bonds and equities, as well as life-insurance contracts.

[2] However, equation (10) will hold, so whether $A^* <> K$ in steady state will be directly related to whether the real rate of interest at the optimal level of debt will be greater or less than the rate of time preference (after adjusting for growth effects).

[3] For recent analysis that investigates further the short term role that fiscal policy can play in this class of model, see Devereau (2010).

3.1 Consumers' Behaviour

Here we introduce the main departure from the canonical New-Keynesian model. While there is abundant evidence of a strong interaction among fiscal impulses and output (see, for example, Blanchard and Perotti (2002) or Fatas and Mihov (1998), standard dynamic general equilibrium models downplay the role of demand. The importance of the demand side of the economy is partially restored when there is slow adjustment in nominal and real variables, but still intertemporal substitution mechanisms and Ricardian equivalence leave consumption largely unresponsive to a temporary fiscal stimulus. Introducing a probability of death implies that consumers discount their future disposable income more heavily, such that the usual Ricardian experiment of a deficit-financed lump-sum tax cut now increases consumption.

Households face a constant probability of death $(1-\gamma)$. As this is a constant exogenous probability, and there is a continuum of households, they imply there is no aggregate uncertainty in our economy. This implies that a consumer born at time i, who is still alive at time t receives utility from consuming a basket of consumer goods at time t,

$$c_t^i = \left[\int_0^1 c_t^i(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}, \tag{18}$$

and holding real money balances, M_t^i/P_t, and suffers disutility from supplying labour to imperfectly competitive firms, l_t^i. We can write this household's expected utility function as,

$$\sum_{t=0}^{\infty} (\beta\gamma)^t \left[\ln c_t^i + \chi \ln \frac{M_t^i}{P_t} + \vartheta \ln g_t^c + \varkappa \ln(1-l_t^i) \right] \tag{19}$$

By reducing the household's discount factor by the survival probability γ we are implicitly conditioning on the survival of this particular household (otherwise there would be double-counting of the probability of death).

Due to the difficulties in conceptualising complete financial contracts amongst markets participants some of whom are as yet unborn, we assume that financial markets are incomplete, but in an economy without aggregate uncertainty. Accordingly, we assume that households can hold risk-free nominal one period government bonds which pay a gross interest rate of R_t regardless of the state of nature (including the survival of the bond holder), and non-interest bearing money. Households also buy shares, V_t^i in capital rental firms for a real price q_t^v which pay out their net cash flows as dividends, d_t. They can also enter into survival-contingent contracts with other households, which pay an agreed sum to other households in the event of the individual's death, but entitle the individual to similar payments from deceased households should the individual survive. The individual will construct a portfolio of money, bonds, equities and survival-contingent contracts such that the payoff from that portfolio should the individual die is zero. However, if household i is lucky enough to survive their combined return from risk-free bonds, equities and survival-contingent contracts written against those bonds and shares will be $B^i_{t-1}R_{t-1}/\gamma$ and $\frac{P_t(q^v_t+d_t)V^i_{t-1}}{\gamma}$, respectively, while the return to holding money is M^i_{t-1}/γ. This is simply an alternative means of capturing the insurance contracts usually undertaken within the Blanchard-Yaari set-up.

Consumers seek to maximise utility subject to the demand schedule for their labour services and their budget constraint, which in nominal terms can be written as

$$\begin{aligned} & M^i_t + B^i_t + P_t q^v_t V^i_t + P_t c^i_t \\ = \; & P_t(1-\tau^w_t)w_t l^i_t + \frac{R_{t-1}B^i_{t-1}}{\gamma} + \frac{M^i_{t-1}}{\gamma} + \frac{P_t(q^v_t+d_t)V^i_{t-1}}{\gamma} \\ & + (1-\gamma)\, P_t \int_0^1 \Omega_{jt} dj \end{aligned} \tag{20}$$

Here consumers earn after-tax income from their labour services $P_t(1-\tau^w_t)w_t l^i_t$, and receive their share of the profits of final goods producers, $(1-\gamma)\, P_t \int_0^1 \Omega_{jt} dj$, as well as household specific public transfers.

Let us define

$$H^i_t \equiv \left[(1-\tau^w_t)w_t l^i_t + (1-\gamma)\int_0^1 \Omega_{jt} dj\right] \tag{21}$$

and

$$W^i_t \equiv \frac{1}{\gamma}M^i_{t-1} + \frac{R_{t-1}}{\gamma}B^i_{t-1} + \frac{P_t(q^v_t+d_t)}{\gamma}V^i_{t-1} \tag{22}$$

as the non-financial and financial income of generation i households in period t. Then, the budget constraint can be written as

$$M^i_t\left(\frac{R_t-1}{R_t}\right) + Q_{t,t+1}W^i_{t+1} + P_t c^i_t = P_t H^i_t + W^i_t \tag{23}$$

W^i_t represents the payoff from the household's portfolio in all states of nature, but conditional on the household surviving, and $Q_{t,t+1} = \gamma R_t^{-1}$ is the price of receiving one unit of that payoff. Note that should the household not survive, the payoff from the portfolio is zero, such that the expected payoff from one unit of the portfolio across all states of nature, including the survival/non-survival of the household, is the risk free rate of interest R_t.

Maximising household utility subject to the budget constraint yields the consumption Euler equation,

$$Q_{t,t+1} = \gamma\beta\left\{\frac{c^i_t P_t}{c^i_{t+1}P_{t+1}}\right\} \tag{24}$$

or equivalently,

$$1 = R_t\beta\frac{c^i_t P_t}{c^i_{t+1}P_{t+1}}, \tag{25}$$

a demand for money equation (where $m_t^i \equiv \frac{M_t^i}{P_t}$),

$$m_t^i = \chi \frac{R_t}{R_t - 1} c_t^i, \tag{26}$$

a labour supply condition,

$$(1 - \tau_t^w) w_t (1 - l_t^i) = \varkappa c_t^i, \tag{27}$$

and the no-arbitrage condition for equities,

$$q_t^v = \frac{\pi_{t+1}}{R_t} (q_{t+1}^v + d_{t+1}). \tag{28}$$

Using the household budget constraint, together with the money-demand equation, the Euler equation, and the no-arbitrage condition for perpetuities, we obtain the consumer's consumption function,

$$c_t^i = \frac{1 - \gamma\beta}{1 + \chi} \left[\frac{W_t^i}{P_t} + \sum_{s=0}^{\infty} (\gamma)^s \left(\prod_{i=0}^{s-1} \frac{\pi_{t+i+1}}{R_{t+i}} \right) H_{t+s}^i \right] \tag{29}$$

where the household discounts future labour and profit income more heavily than its straight rate of time preference, as it will not receive that income should it die, but expectations are taken over all states of nature, other than the survival/non-survival of the household. We can further write this as,

$$c_t^i = \frac{1 - \gamma\beta}{1 + \chi} \left[\frac{W_t^i}{P_t} + lw_t^i \right]$$

where lw_t^i represents the generation i's human wealth, given as the discounted value of labor income and profits, where the effective discount factor accounts for the probability of survival,

$$lw_t^i \equiv H_t^i + \sum_{s=1}^{\infty} (\gamma)^s \left(\prod_{i=0}^{s-1} \frac{\pi_{t+i+1}}{R_{t+i}} \right) H_{t+s}^i = H_t^i + \gamma \left(\frac{\pi_{t+1}}{R_t} \right) lw_{t+1}^i \tag{30}$$

3.2 Aggregating across Consumers and Consumption Dynamics.

If the size of each cohort when born is 1, then the size of a cohort i at time t is given by, γ^{t-i}. Therefore the total size of the population is given by[4],

$$\sum_{i=-\infty}^{t} \gamma^{t-i} = \frac{1}{1 - \gamma}. \tag{31}$$

[4] Note that this implies that an infinitesimally small number of consumers will live-forever. This is why this means of introducing non-Ricardian behaviour is sometimes called the 'perpetual youth model'.

Aggregate variables are defined as, $x_t = \sum_{i=-\infty}^{t} \gamma^{t-i} x_t^i$. Aggregating consumers' labour supply yields,

$$(1-\tau_t^w)w_t\left(\frac{1}{1-\gamma}-l_t\right)=\varkappa c_t \tag{32}$$

The aggregate demand for money is given by,

$$m_t = \chi \frac{R_t}{R_t-1} c_t \tag{33}$$

It is similarly possible to aggregate across consumers from different generations to generate an aggregate consumption function,

$$c_t = \frac{1-\gamma\beta}{1+\chi}\left[\frac{W_t}{P_t}+lw_t\right] \tag{34}$$

and aggregate human wealth is given by,

$$lw_t = H_t + \gamma \frac{\pi_{t+1}}{R_t} lw_{t+1}$$

where

$$H_t \equiv \left[(1-\tau_t^w)w_t l_t + s_t + \int_0^1 \Omega_{jt} dj\right] \tag{35}$$

It should be noted that the aggregate of financial wealth, $W_t = M_{t-1} + R_{t-1}B_{t-1} + P_t(q_t^v + d_t)V_{t-1}$, takes account of the fact that not all households will have survived from last period into the current one, implying that the households' aggregate budget constraint is given by,

$$\begin{aligned} M_t + B_t + P_t q_t D_t + P_t q_t^v V_t + P_t c_t &= P_t(1-\tau_t^w)w_t l_t + R_{t-1}B_{t-1} + M_{t-1} \\ &\quad + P_t(q_t^v + d_t)V_{t-1} + P_t \int_0^1 \Omega_{jt} dj \end{aligned} \tag{36}$$

3.3 The Capital Rental Firm's Behaviour

We assume that there is a single representative firm accumulating private capital for rental to the final goods firms. This firm seeks to maximise the discounted value of its cashflows. This objective function is consistent with maximising the value of the households' equity. Therefore the firm's objective function is to maximise the following expression,

$$P_t(q_t^v + d_t)V_{t-1} = p_t^k k_t - e_t + \left\{\sum_{z=1}^{\infty}\left(\prod_{i=0}^{z-1} R_{t+i}^{-1}\right)\frac{P_{t+z}}{P_t}\left[p_{t+z}^k k_{t+z} - e_{t+z}\right]\right\} \tag{37}$$

where p_t^k is the real rental cost of capital, k_t is the capital stock, e_t is real investment expenditure, and τ_t^k is the rate of taxation on the income from renting capital. The equation of motion of the capital stock is then given by,

$$k_{t+1} = e_t + (1-\delta)k_t. \tag{38}$$

The first order condition for investment is given by,

$$\lambda_t^k = 1 \tag{39}$$

where λ_t^k is the Lagrange multiplier associated with the equation of motion for the capital stock. Given the homogeneity of our profit function, this is equivalent to Tobin's q so that in the absence of capital adjustment costs, Tobin's q is one. Also, differentiating the Lagrangian with respect to k_{t+1} gives the equation of motion for Tobin's q,

$$1 = \frac{\pi_{t+1}}{R_t}\left(p_{t+1}^k + 1 - \delta\right) \tag{40}$$

The capital accumulated by this sector is then rented out to the imperfectly competitive firms producing final goods for consumers, as described below.

This marginal q can be related to average q (and therefore the value of household's equity) as follows. Firstly, use the equation of motion of the capital stock to rewrite as,

$$\lambda_t^k k_{t+1} = \frac{\pi_{t+1}}{R_t}\left(p_{t+1}^k k_{t+1} + \left(k_{t+2} - e_{t+1} + \Phi'\left(\frac{e_{t+1}}{k_{t+1}}\right)e_{t+1}\right)\lambda_{t+1}^k\right). \tag{41}$$

Then, using the first order condition for investment, we obtain

$$\lambda_t^k k_{t+1} = \frac{\pi_{t+1}}{R_t}\left(p_{t+1}^k k_{t+1} - e_{t+1} + k_{t+2}\lambda_{t+1}^k\right) \tag{42}$$

which implies that,

$$\lambda_t^k k_{t+1} + p_t^k k_t - e_t = (q_t^v + d_t)V_{t-1}$$

so we can define non-human wealth as,

$$W_t = M_{t-1} + R_{t-1}B_{t-1} + (P_t p_t^k + 1 - \delta)k_t$$

3.4 Capital and Labour Demand: Cost Minimization

The optimal combination of capital and labour employed in the production of final goods, is obtained from the cost minimization problem of the firm, given the production function it faces,

$$y_{jt} = A_t k_{jt}^{\alpha_1}(A_t^l l_{jt})^{\alpha_2}(k_t^p)^{\alpha_3}. \tag{43}$$

where k_{jt} is the private capital employed by the firm, l_{jt} is the labour employed by the firm, A_t^l is labour embodied technical progress and k_t^p is the public stock of capital accumulated by the government. We assume that this production function exhibits constrant returns to scale in its arguments, so that the firm faces diminishing returns in its private factors. Accordingly, we can experience exogenous growth through the exogenous growth of labour-embodied technical progress, ω such that $A^l_{t+1} = \omega A^l_t$.

This implies the following cost minimising combinations of labour and capital,

$$\frac{l_{jt}}{k_{jt}} = \frac{\alpha_2}{\alpha_1}\frac{p^k_t}{w_t}$$

and the real marginal cost, mc_t ,which is common across all firms, is given by,

$$mc_{jt} = (y_{jt})^{\frac{1-\alpha_1-\alpha_2}{\alpha_1+\alpha_2}} A_t^{-\frac{1}{\alpha_1+\alpha_2}} \alpha_1^{-\frac{\alpha_1}{\alpha_1+\alpha_2}} \alpha_2^{-\frac{\alpha_2}{\alpha_1+\alpha_2}} (p^k_t)^{\frac{\alpha_1}{\alpha_1+\alpha_2}} (w_t)^{\frac{\alpha_2}{\alpha_1+\alpha_2}} (A^l_t)^{-\frac{\alpha_2}{\alpha_1+\alpha_2}} \tag{44}$$

w_t is the real wage and p^k_t the rental cost of capital. Since all firms are identical, these can be related to aggregate variables and we have:

$$\frac{l_t}{k_t} = \frac{\alpha_2}{\alpha_1}\frac{p^k_t}{w_t}$$

$$mc_t = (y_t)^{\frac{1-\alpha_1-\alpha_2}{\alpha_1+\alpha_2}} A_t^{-\frac{1}{\alpha_1+\alpha_2}} \alpha_1^{-\frac{\alpha_1}{\alpha_1+\alpha_2}} \alpha_2^{-\frac{\alpha_2}{\alpha_1+\alpha_2}} (p^k_t)^{\frac{\alpha_1}{\alpha_1+\alpha_2}} (w_t)^{\frac{\alpha_2}{\alpha_1+\alpha_2}} (A^l_t)^{-\frac{\alpha_2}{\alpha_1+\alpha_2}} \tag{45}$$

and,

$$y_t = A_t k_t^{\alpha_1} (A^l_t l_t)^{\alpha_2} (k^p_t)^{\alpha_3}. \tag{46}$$

3.5 Price Setting of Final Goods Firms

We define Rotemberg price adjustment costs as,

$$\frac{\phi}{2}\left(\frac{p_t(j)}{\pi^* p_{t-1}(j)} - 1\right)^2 P_t y_t \tag{47}$$

where π^* is the steady-state inflation rate. The problem facing firm j is to maximise the discounted value of profits,

$$\max_{p_t(j)} \left[\Pi_t(j) + \sum_{z=1}^{\infty}\left(\prod_{i=0}^{z-1} R^{-1}_{t+i}\right)\Pi_{t+z}(j)\right]$$

where given the demand curve, $y_t(j) = (p_t(j)/P_t)^{-\varepsilon}\, y_t$, nominal profits are defined as,

$$\begin{aligned}\Pi_t(j) &\equiv p_t(j)y_t(j) - mc_t y_t(j) P_t - \frac{\phi}{2}\left(\frac{p_t(j)}{\pi^* p_{t-1}(j)} - 1\right)^2 P_t y_t \\ &= p_t(j)^{1-\varepsilon} P_t^{\varepsilon} y_t - mc_t p_t(j)^{-\varepsilon} P_t^{1+\varepsilon} y_t - \frac{\phi}{2}\left(\frac{p_t(j)}{\pi^* p_{t-1}(j)} - 1\right)^2 P_t y_t\end{aligned} \tag{48}$$

So that, in a symmetric equilibrium where $p_t(j) = P_t$, the first order conditions are given by,

$$(1-\varepsilon) + \varepsilon mc_t - \phi \frac{\pi_t}{\pi^*}\left(\frac{\pi_t}{\pi^*} - 1\right) + \phi \frac{\pi_{t+1}}{R_t}\frac{\pi_{t+1}}{\pi^*}\frac{y_{t+1}}{y_t}\left(\frac{\pi_{t+1}}{\pi^*} - 1\right) = 0 \tag{49}$$

which is the Rotemberg version of the Phillips curve relationship. Equilibrium real profits of all final goods producers are then given as,

$$\int_0^1 \Omega_{jt} dj \equiv P_t^{-1}\left(\int_0^1 \Pi_t(j) dj\right) = y_t \left[1 - mc_t - \frac{\phi}{2}\left(\frac{\pi_t}{\pi^*} - 1\right)^2\right]$$

That completes our derivation of the model, which is summarised in Appendix 1.

4 Social Welfare

Defining what is optimal in an OLG model involves deciding how to compare different generations. Since we are interested in formulating optimal policy for our economy populated with overlapping generations of finitely lived consumers we must face the tricky issue of constructing a welfare metric. Calvo and Obstfeld (1988) define the social welfare function at time 0 as,

$$W_0 = \sum_{s=0}^{\infty}\left[\sum_{t=s}^{\infty} u(s,t)(\gamma\beta)^{t-s}\right]\rho^s + \sum_{s=-\infty}^{0}\left[\sum_{t=0}^{\infty} u(s,t)(\gamma\beta)^{t-s}\right]\rho^s \tag{50}$$

where $u(s,t) = \ln c_t^s + \chi \ln \frac{M_t^s}{P_t} + \vartheta \ln g_t^c + \varkappa \ln(1-l_t^s)$ is the utility at time t of a household born at time s. The first summation is the utility of representative agents of generations yet to be born, discounted at the policy-maker's discount factor, ρ. The second is the expected utility of households currently alive. These utilities are discounted back to the birthdate of the currently living generations, rather than the current period. Calvo and Obstfeld (1988) note that this is necessary to avoid the time inconsistency in preferences that would otherwise emerge by treating generations asymmetrically. In other words, if the policy maker did not discount utilities back to birthdates, then he would wish to change the consumption plans he put in place for currently unborn generations the moment they are born.

By changing the order of summation the welfare function can be rewritten as,

$$W_0 = \sum_{t=0}^{\infty}\left[\sum_{s=-\infty}^{t} u(s,t)\left(\frac{\gamma\beta}{\rho}\right)^{t-s}\right]\rho^t \tag{51}$$

so that the instantaneous flow utility to the policy maker is given by the summation over generations of their instantaneous utility discounted by the private discount factor and adjusted by the public discount factor. These are then discounted over time using the policy maker's discount factor, ρ. This can be further rewritten as,

$$W_0 = \sum_{t=0}^{\infty} \left[\sum_{z=0}^{\infty} u(t-z,t) \left(\frac{\gamma\beta}{\rho} \right)^z \right] \rho^t \tag{52}$$

which allows us to decompose the policy-maker's problem into two parts. The first part involves the policy maker's optimal allocation of consumption and labour supply across households. The second relates to the intertemporal aspects of the problem. Since we are only interested in the macroeconomic effects of fiscal adjustment in an environment where government debt can potentially crowd-out private capital, we abstract from the intratemporal intergenerational problem and focus on the intertemporal problem, such that the social welfare function is given by,

$$W_0 = \sum_{t=0}^{\infty} \rho^t [\ln c_t + \chi \ln \frac{M_t}{P_t} + \vartheta \ln g_t^c + \varkappa \ln(1-l_t)] \tag{53}$$

where we assume that $\rho = \beta$ such that the policy maker discounts the future at the same rate as households, but without accounting for the probability of death. In solving its intertemporal problem the policy maker ignores the distribution of variables across generations at a given point in time by focusing on per capita variables.[5]

An additional complication we need to consider is that our model is non-stationary due to the exogenous increase in labour-embodied technical progress. Due to the fact that utility is logarithmic we can rewrite the objective function in terms of detrended variables as,

$$W_0 = E_t \sum_{t=0}^{\infty} \beta^t \ln(u_t)$$

where $\ln(u_t) = \ln A_0^l + \chi \ln(\widetilde{c}_t) + (1-\chi)\ln\left(\widetilde{g_t^c}\right) + \sum_{s=0}^{t-1} \ln(\omega) + \varphi \ln(1-l_t)$. This implies we can obtain an exact expression for discounted lifetime welfare,

$$W_t = \beta E_t W_{t+1} + \ln A_0^l + \chi \ln(\widetilde{c}_t) + (1-\chi)\ln\left(\widetilde{g_t^c}\right) + \frac{\beta}{1-\beta}\ln(\omega) + \varphi \ln(1-l_t)$$

[5] It should be noted that allowing the government to implement a (lump-sum) intratemporal redistribution scheme to maximise social welfare would effectively offset the differential tax treatment of different generations that the perpetual youth model relies on to break from Ricardian Equivalence. While allowing aggregate policy to consider distributional issues when implementing macro policy would require us to track the distribution of financial wealth across generations which is generally intractable due to the impact of birth of new generations on that distribution.

4.1 Optimal Monetary and Fiscal Policy

Given the social welfare function, the optimal policy problem can be set up in terms of a Lagrangian as,

$$L_0 = \max_{\mathbf{y}_t} E_0 \sum_{t=0}^{\infty} \beta^t [U(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{u}_t) - \boldsymbol{\lambda}_t f(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{u}_t)]$$

where $\mathbf{y}_t$ and $\mathbf{u}_t$ are vectors of the model's endogenous and exogenous variables, respectively, $U(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{u}_t) = \ln c_t + \chi \ln \frac{M_t}{P_t} + \vartheta \ln g_t^c + \varkappa \ln(1 - l_t) + tip$, where tip refers to terms in the productivity growth which are independent of policy, $f(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{u}_t) = 0$ are the model's equilibrium conditions, and $\boldsymbol{\lambda}_t$ is a vector of Lagrange multipliers associated with these constraints.

The optimisation implies the following first order conditions,

$$\left[\frac{\partial U(.)}{\partial \mathbf{y}_t} + \beta F \frac{\partial U(.)}{\partial \mathbf{y}_{t-1}} + \beta^{-1} \boldsymbol{\lambda}_{t-1} F^{-1} \frac{\partial f(.)}{\partial \mathbf{y}_{t+1}} + \boldsymbol{\lambda}_t \frac{\partial f(.)}{\partial \mathbf{y}_t} + \beta \boldsymbol{\lambda}_{t+1} F \frac{\partial f(.)}{\partial \mathbf{y}_{t-1}}\right] = 0 \tag{54}$$

where F is the lead operator, such that F^{-1} is a one-period lag. We can then solve these first order conditions in combination with the non-linear equilibrium conditions of the model, $f(\mathbf{y}_{s+1}, \mathbf{y}_s, \mathbf{y}_{s-1}, \mathbf{u}_s) = 0$. We do this fully non-linearly to obtain the steady-state of the policy makers problem. Since this is a perfect foresight economy, we can also solve for the non-linear transition dynamics using standard techniques, and we discuss those dynamic paths below.

In exploring optimal policy, we also consider the allocation that would be achieved by a social planner who simply implemented the first-best solution. The social planner's problem, in stationary form, is given by,

$$L_0 = \sum_{t=0}^{\infty} \beta^t [\ln \widetilde{c}_t + \vartheta \ln \widetilde{g}_t + \varkappa \ln(1 - l_t)] + tip \tag{55}$$

subject to,

$$\widetilde{y}_t = A_t \widetilde{k}_t^{\alpha_1} l_t^{\alpha_2} (\widetilde{k}_t^p)^{\alpha_3} \tag{56}$$

$$\omega \widetilde{k}_{t+1} = \widetilde{e}_t + (1 - \delta) \widetilde{k}_t \tag{57}$$

and

$$\omega \widetilde{k}_{t+1}^p = \widetilde{e}_t^p + (1 - \delta) \widetilde{k}_t^p \tag{58}$$

$$\widetilde{y}_t = \widetilde{c}_t + \widetilde{g}_t + \widetilde{e}_t + \widetilde{e}_t^p \tag{59}$$

Note that government debt does not exist in the social planner's problem, so the constraints involved in inheriting a positive debt level disappear. Deriving the focs and eliminating the associated lagrange multipliers gives us the optimal relationship between government spending and consumption,

$$\widetilde{g}_t = \vartheta \widetilde{c}_t \tag{60}$$

the labour allocation is given by,

$$\frac{\chi}{1-l_t} = \widetilde{c}_t^{-1} \alpha_2 \frac{\widetilde{y}_t}{l_t} \tag{61}$$

the intertemporal consumption/saving decision is given by,

$$\omega \widetilde{c}_t^{-1} = \beta \widetilde{c}_{t+1}^{-1} [1 - \delta + \alpha_1 \frac{\widetilde{y}_{t+1}}{\widetilde{k}_{t+1}}] \tag{62}$$

while the balance between public and private forms of capital is given by,

$$\alpha_1 \widetilde{k}_t^p = \alpha_3 \widetilde{k}_t \tag{63}$$

Simultaneously solving equations (56)-(63) then yields the social planner's allocation.

4.2 Calibration

In order to analyse the main implications of our model, we first calibrate our model based on empirically observed levels of real GDP growth, public and private capital, government consumption, labour income shares and government debt in the US. Between 1980 and 2008, the average annualised growth rate was 2.88%, private and public capital to GDP ratios were 2.3 and 0.6 respectively, government consumption was 16% of GDP, the labour income share was around 54% and government debt averaged 55.6% percent of GDP. Table 1 summarises the values of the calibrated baseline parameters, and Table 2 summarises the resultant steady-state..

The elasticity of demand with respect to price (ε) is set to 11, consistent with a steady-state mark-up, $\varepsilon/(\varepsilon - 1)$, equal to 1.1. The price adjustment cost parameter of $\phi = 100$ is standard and is set to ensure the log-linearised NKPC matches that obtained under Calvo (1983) pricing with empirically estimated contract duration probabilities such as those in Leith and Malley (2006). We assume a steady-state annualised inflation rate of 2%. Parameter $\varkappa$, measuring the weight on leisure in utility, was set to 1.14, which is generally consistent with households allocating about a third of their time to market activities. While the weight given to government consumption in utility, $\vartheta = 0.24$, implies that the policy maker would ensure that government consumption as a share of private consumption is similar to the patterns found in the US data.With a quarterly discount factor (β) of 0.9938, and a survival probability of $\gamma = 0.995$, implying an expected adult working life of 50 years[6] our model can match these steady-

[6] We focus on economically active individuals (from 15 to 64 years old). 50 years is then a compromise between the years that Europeans are active, which is the reference variable for labour, and

β	γ	χ	ϑ	$\varkappa$	α_1	α_2	δ	ε	ϕ	A
0.9938	0.995	0	0.24	1.14	0.59	0.34	0.021	11.0	100	1

Table 1: Calibration of baseline model - Parameters

b/y	ω	g/y	k/y	k^p/y	wN/y	π	r	τ
55.6%	2.88%	0.16	2.27	0.64	0.54	2%	5.44%	0.45

Table 2: Calibration of baseline model - Initial Steady State

state ratios with an elasticity of output with respect to labour and private capital of $\alpha_1 = 0.59$ and $\alpha_2 = 0.34$, respectively. This, in turns implies a coefficient on public capital in production of 0.07 which is slightly above the 0.05 adopted in Baxter and King (200X), but well within the range of estimates considered in the meta-analysis of Bom and Ligthart (2009). Finally, since seignoriage revenues play no significant role in debt stabilization, we assume that the economy approaches its cashless limit, $\chi \to 0$. The depreciation rate (δ) is equal to 0.021, as estimated by Christiano and Eichenbaum (1992).

It should be noted that this calibration is not based on the steady-state of the Ramsey problem, but the steady-state of the structural model equations given the levels of government consumption, investment and taxes needed to support observed levels of government spending, public capital and government debt as a proportion of gdp, as well as labour income shares, growth rates and private capital/output ratios. We shall see that when these variables, in conjunction with monetary policy, are chosen optimally the economy will move a long way from this starting point. For these reason we do not employ any approximation techniques in solving the model, such that steady-state solutions and dynamics of the model are all obtained as fully non-linear solutions to the Ramsey policy problem described above.

4.3 The Optimal Debt Target

In this section we examine the optimal level of steady state government assets implied by the simplified version of our model, using the calibration set out above. This solution is obtained by solving the non-linear equations of the model together with the first order conditions (54). In the steady-state solution the policy maker achieves their inflation target of 2%, so that in terms of the steady-state solution to the Ramsey problem it is as if this is a 'real' model, where the only distortions are monopolistic competition and income taxes, which are the only taxes available to the government. However, when

life expectancy which is probably a more relevant variable for consumption. We also set "economic" life expectancy equal to 50 years as a way of having a lower discount rate and, therefore, higher non-Ricardian effects. Nevertheless, in sensitivity analysis, we also consider the consequences of having a lower probability of death.

we consider the dynamics of the solution to the Ramsey problem, we shall see that monetary policy has a significant role to play in the short-run.

Before exploring folly optimal policy, we consider first the steady state associated with zero government assets/debt. Essentially, policy is optimal in the case, except that we replace the foc for government debt with a target of zero debt. Government spending continues to be set at its optimal level, conditional on zero debt. This is the third column in Table 3 labelled 'Zero Debt'. Here we can see that there is a slight increase in the ratio of private capital to output as a result of the reduction in government debt, but a quite dramatic fall in the ratio of public capital to output from the 0.64 found in the data to 0.37. Public consumption is a bit over a quarter of the level of private consumption, and tax rates have fallen to 31% encouraging time spent in work to rise from 0.34 to 0.36. The welfare implications of such a policy are that steady-state welfare is 16% lower than that achieved by the social planner, which is a significant improvement on the 63% reduction in welfare relative to the Social Planer's allocation implied by the initial, unoptimised, steady-state. .

Variable	Calibration	Zero Debt	$mc = 1$	+Lump Sum	Optimal	Lump Sum	$+mc = 1$	Soc. Planner
b/y	0.56	0	0	0	-2.84	-11.80	−2.90	n.a.
k/y	2.27	2.29	2.51	2.51	2.36	2.65	2.60	2.60
k^p/y	0.64	0.37	0.38	0.40	0.39	0.40	0.39	0.39
r	5.44%	5.85%	5.89%	5.89%	5.36%	3.83%	5.40%	n.a.
c/y	0.57	0.58	0.55	0.54	0.56	0.53	0.54	0.54
g/y	0.12	0.12	0.13	0.13	0.13	0.13	0.13	0.13
τ	0.45	0.31	0.29	n.a.	0.19	n.a.	n.a.	n.a.
h	0.34	0.36	0.40	0.49	0.41	0.47	0.49	0.49
Welfare	63%	16%	7%	1%	9%	3%	0	n.a.

Table 3: Steady State of Ramsey Problem

The next two columns look at the impact of the two steady-state distortions on the model, if government debt remains zero. Removing the monopoly distortion raises the level of all variables, although the impact on private capital is greatest. If we in addition allow lump sum taxes, so that the income tax rate is zero, then we substitute labour for capital, and as a result there is a substantial increase in the real interest rate and a massive increase in labour supply to 0.49. However private capital is still sub-optimally low in this economy and welfare falls only 1% short of the level achieved by the social planner. The reasons for this shortfall can be seen by comparing these numbers with the final column of the table, which gives the allocation that would be chosen by a social planner who fixed physical capital investment, output, consumption and labour supply only constrained by technology and the resource constraint.

The sixth column, labelled 'Optimal', restores both distortions, but sets government assets to their optimal level, assuming a discount rate equal to the rate of time preference. As expected, the optimal steady state debt target is negative. In fact, it is optimal for government assets to exceed the level of the capital stock, so that agents are net creditors. A direct implication (see equation (10)), is that the steady state real interest rate is slightly below the rate of time preference (after adjusting for growth). (The level of annualised real interest rates that would equate the two in the presence of steady-state growth of 2.88% is approximately 5.47%). Compared to the case where debt was zero, output is over 14% higher. Relative to this higher level of output, government consumption has risen to 13% of GDP, as the interest from government assets pays for a good proportion of this expenditure. The income tax rate has fallen from its calibrated rate of 45% to only 19%. However the fact that the income tax rate remains positive, implies that we are clearly well below the zero tax level of government assets.

This last finding enables us to interpret our results in terms of the analysis in section 2. We noted that if the real interest rate in steady state was below the rate of time preference, then it was possible that $A^* < A^K < A^T$. From the above we can see that $A^* < A^T$, although A^K is unobservable. However, if $A^* > A^K$ (government assets were greater than that required to achieve the optimum level of capital), then this would not be a steady state, because there would be an incentive to cut taxes and raise debt because the real interest rate was below the discount rate. We could observe A^K directly by eliminating distortionary taxes. This is done partially in the next column. The optimal level of government assets rises substantially, with a more moderate increase in capital, output and consumption. This clearly illustrates that A^K is greater than the A^* in the previous column. The steady-state of this 'optimal' policy falls 9% short of the levels of welfare enjoyed under the social planner's allocation. Allowing taxes to be lump-sum (column 7) leads to a massive increase in the steady-state level of government financial assets as tax smoothing ceases to be an issue, a dramatic reduction in the real interest rate and an overaccumulation of private sector capital. In this case, welfare is only 3% less than that attained by the social planner. While if we were to eliminate the monopolistic competition distortion as well, a policy maker with access to lump-sum taxes could achieve the social planner's allocation.

We noted in section 2 that the arguments for discounting the utility of future generations by the rate of time preference were not compelling. If we discounted utility at a lower rate, then clearly the optimal level of government assets would rise. In the most extreme case, where no discounting took place, the optimal level of capital would be the golden rule level that maximised steady state consumption. In this model, without discounting by the policy maker, the real interest rate would simply reflect technical progress. Whether this could be achieved with a sufficiently large level of government assets is unclear, but experiments with the model suggest that values of government assets in excess of 12 times the level of GDP continue to produce a positive real interest rate in excess of the rate of technical progress. In that sense, the levels of A^* presented in Table 3, although historically unprecedented, are not the upper bound of what might be socially optimal in an OLG economy.

We now undertake a sensitivity analysis of the optimal steady-state debt-gdp ratio as we vary the producitivity growth rate and the productivity of public capital, respectively. Figure 1 plots the steady-state debt-gdp ratio as a function of the annualised productivity growth rate. There is a strong dependence of the desired debt-gdp ratio on the growth rate, where a lower rate of productivity growth would substantially raise the desired stock of government assets. This is essentially driven by the fact that higher anticipated productivity growth reduces the socially optimal degree of capital accumulation such that there is less need to accumulate government assets in an attempt to crowd-in private investment. The desired stock of steady-state government assets also depends on the productivity of public capital - see Figure 2. The intuition is straightforward - as the productivity of public capital rises, relative to private capital there is less need for the government to attempt to reduce public debt in order to crowd in private-sector investment as public-sector investment is increasingly effective in raising output levels. We shall consider the impact of changing the probability of death on both the transition and steady-state below.

4.4 Transition paths

In this section we present a brief analysis of the optimal transition path to this steady state, using a simulation of the full non-linear Ramsey policy. Our simulation begins at the calibrated steady-state which features public and private capital to GDP ratios of 0.64 and 2.27, respectively, alongside a debt to gdp ratio of 0.56. Starting from that initial position, the Ramsey policy will move us towards the steady-state labelled 'optimal' in Table 3, where the long-run capital to GDP ratios for public and private capital are, 0.39 and 2.37 and the government debt to GDP ratio has fallen to -2.84. We break the transition between this initial state to the Ramsey steady-state into two stages. The first year impact of adopting optimal monetary and fiscal policies is shown in Figures 3 and 4, where the solid line details the paths followed by key variables in the initial year of the optimal policy. The most striking aspect of the early response to the switch to optimal policy is that it is desirable to underetake a very large privatisation programme, effectively transferring public capital to the private sector to the extent that the public capital to GDP ratio actually falls below its long-run value despite starting well above such a position.[7] In otherwords, although the initial ratio of public capital to gdp is well above the optimal long-run ratio, in the transition there is an overshooting

[7]Strictly speaking since we have single aggregate good in this economy which can be costlessly converted to and from use in consumption or either type of capital, this 'privatisation' does not involved a direct transfer of capital goods from the public to private sector. Nevertheless, the simultaneous reduction in public capital and increase in private capital mimics just such a transfer.

in the sell-off of public capital, and in the medium term public capital needs to be re-accumulated to achieve its long-run optimum. The proceeds of selling off public capital in this way give rise to a significant initial fall in government debt. There is also a mild initial rise in inflation, as policy makers take advantage of the fact that expectations are given in the initial period of the new optimal policy by raising inflation without fueling future inflation expectations and thereby slightly erode the real value of government debt. However, it is the effects of the selling public assets that have the biggest impact on initial debt dynamics.

Beyond the effects of the privatisation of public assets, the remainder of the fiscal adjustment is far smoother and is reported in Figures 5 and 6. which show that dynamic adjustment is very drawn out over time. Although a significant part of the debt reduction is achieved very quickly by selling public capital, it takes over 100 years to achieve the first 50% of the adjustment and complete adjustment takes around 500 years. This very long adjustment period is not surprising for two reasons. First, while complete tax smoothing no longer applies, the Blanchard-Yaari framework with realistic values for the probability of death gives only quantitatively minor deviations from Ricardian Equivalence, and so a large smoothing element is retained. Second, earlier analysis using models of this type suggest very long drawn out dynamics (e.g. Leith and Wren-Lewis (2000)). The result that debt adjustment should be very slow appears fairly robust (see Marcet and Scott (2008) for example).

The reduction in debt is achieved by above steady-state tax revenues, a short-run de-accumulation of public capital and cuts in government consumption. However, eventually once the debt has fallen sufficiently, tax rates fall and public investment and consumption can rise above current levels. Consumption declines for several years, which clearly shows why moving to the optimal level of debt is not a Pareto improvement. The current generation will be worse off as a result of raising the level of government assets. Most of the adjustment in debt is achieved through fiscal variables, and monetary policy generally succeeds in ensuring minimal deviations from its inflation target throughout the transition. .

Although the speed of adjustment is very slow, the size of adjustment required from current levels of debt is also very large. As a result, the implications for debt reduction for debt reduction today will still be significant. We should also note, however, that are starting point for adjustment does not involve interest rates at the zero lower bound and a large recession, so our analysis has no immediate implications for the current 'stimulus versus austerity' debate. However, we can contrast the transition paths for identical economies starting from different initial levels of public debt. Here we can see that all the economy will tend to the same steady-state level of government assets in the long-run, any initial shock to government debt will only be eliminated very slowly, with clear differences across the transition paths for at least 150 years. This implies that even although it may be optimal to substantially reduce government debt in the long-run, the fact that the recent financial crisis has raised government debt levels does not imply that that fiscal correction need be noticeably more rapid.

The dashed green line in the same Figures consider the transition path starting from a position where government debt is significantly higher. Here the privatisation programme is even more extensive, even although public capital will be built up again in the long-run. Additionally, the high debt stock tempts the policy maker to engineer a one-off suprise burst of inflation, such that inflation rises to an annualised rate of almost 40% upon adoption of the optimal policy, although the policy maker quickly returns inflation to target in subsequent periods. This application of surprise inflation in the initial period arises since we allowed the policy maker to exploit the fact that inflationary expectations are given prior to unexpectedly implementing optimal policy. It is interesting to note that this burst of inflation is achieved through a combination of loose monetary policy and an increase in distortionary tax rates, which is also inflationary. The fact that the policy maker is prepared to implement such a large rate of inflation demonstrates just how significant the time-inconsistency problems inherent in the optimal Ramsey policy must be (see Leith and Wren-Lewis (2007) for an exploration of the implications of not being able to commit to the optimal policy in the context of an infinite horizon New Keynesian economy). Beyond those initial attempts to reduce the debt burden the subsequent reduction in government debt remains gradual, alhough it is clearly more aggressive when debt levels are particularly large However it remains the case that it takes over 100 years to achieve half the desired reduction in the debt to gdp ratio and 500 years to achieve the full adjustment.

The absence of the random walk result stems from the fact that in our OLG economy interest rates typically deviate from consumers' rate of time preference since government debt constitutes an element of net worth. We assess the importance of the absence of Ricardian Equivalence by decreasing the expected working life from 50 years to 25 years and then 12.5 years in Figures 7-10.[8] Looking at the dynamic paths for debt in Figure 7 we can see that this has little impact on the ultimate long-run level of government assets, but significantly affects the speed of fiscal stabilisation. With the shortened household planning horizon of 12.5 years the debt stock turns negative almost instantly, thanks to the aggressive sell-off of public sector assets and deflation of the real value of debt due to an initial surprise inflation resulting from a relaxation of monetary policy and a sharp rise in distortionary taxation. Essentially, the policy maker acts quickly to offset the costly crowding out induced by the high levels of interest rates associated with high levels of government debt when the deviation from Ricardian equivalence is large.

Finally, we consider the robustness of these results to variations in other key parameters. Figure 11 plots the transition paths for government debt under the optimal policy for various changes in model parameters. In all subplots the benchmark calibration implies an optimal transition path given by the solid red line. In the first subplot,

[8]This is done by reducing the quarterly survival probability from 0.995 to 0.99 and then 0.98.

we then increase the markup from 10% to 12.% (green dashed line) and from there to 16.7% (blue dot-dashed line). This affects the level of output produced, cet. par. and so affects the initial debt-gdp ratio although the level of debt is held constant across simulations. Nevertheless the differences are small and ultimately a greater degree of imperfect competition results in a slightly higher level of debt-gdp largely reflecting the reduced level of output rather than any desire to increase the level of government borrowing. The next subplot varies the weight attached to government spending in utility from the benchmark to 0.3 (green dashed line) to 0.4 (blue dot-dashed line). As the weight attached to government consumption is increased there is some desire to increase the debt to gdp ratio relative to the becnhmark optimum, again reflecting the lower level of output implied by this reparameterisation. The third subplot, increases the weight on leisure in utility, from the benchmark to 1.2 (green dashed line) to 1.4 (blue dot-dashed line). This has a very small impact on the transition and ultimate steady-state, although again there is a tendency to have a slightly higher debt-to-gdp ratio as worker effort is reduced, reflecting the reduced output levels this implies. Finally, we vary the degree of price stickiness from the Rotemberg adjustment cost of 100 to 75 and then 50. This has a negligible impact on both transition dynamics and the optimal steady-state. Therefore, across all these variants of model parameterisation the optimal speed of fiscal correction and its ultimate accumulation of a large amount of financial wealth on the part of the government remain unchanged.

5 Conclusions

In models without default where agents are effectively infinitely lived, there is no optimal debt target because the costs of reducing debt are always higher than the cost of accommodating the existing level of debt. In OLG models this is no longer true for two reasons. First, the real rate of interest is likely to be above the rate of time preference, so the benefits of future reductions in debt now outweigh the current costs of achieving lower debt. Second, the level of the capital stock is likely to be below the socially optimal level, and reductions in debt will crowd in capital.

In this paper we examine the optimal level of debt in one particular OLG model, the model of perpetual youth. We show that the optimal debt target in a calibrated version of this model involves positive government assets (i.e. a negative debt target), but these assets are below both the level required to eliminate distortionary taxes, and the level required to achieve the optimum capital stock. This is because, when the economy is distorted by monopolistic competition and income taxes, as debt declines the real rate of interest falls below the rate of time preference before the economy reaches the optimal capital stock. The optimal transition path towards this steady state is very drawn out, involving hundreds of years, but as the steady state involves historically unprecedented levels of government assets, the implications for debt adjustment in the short term may still be quantitatively significant.

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A Appendix – Summary of Aggregate Model

The aggregate demand for money is given by,

$$m_t = \chi \frac{R_t}{R_t - 1} c_t \tag{64}$$

where all variables are now in per capita terms.

The aggregate consumption function is

$$c_t = \frac{1 - \gamma\beta}{1 + \chi} \left[\frac{W_t}{P_t} + lw_t \right] \tag{65}$$

where aggregate financial wealth in real terms is

$$\frac{W_t}{P_t} = m_{t-1}\pi_t^{-1} + \frac{R_{t-1}}{\pi_t} b_{t-1} + \lambda_t^k k_{t+1} (p_t^k + 1 - \delta) k_t \tag{66}$$

where $m_t \equiv M_t/P_t$ and $b_t \equiv B_t/P_t$, and the aggregate human wealth is

$$lw_t = H_t + \gamma \frac{\pi_{t+1}}{R_t} lw_{t+1} \tag{67}$$

with

$$H_t \equiv (1 - \tau_t^w) w_t l_t + \int_0^1 \Omega_{jt} dj \tag{68}$$

The government budget constraint is given by

$$g_t + i_t^p + s_t = \tau_t^w w_t l_t + b_t - \frac{R_{t-1}}{\pi_t} b_{t-1} + m_t - \frac{m_{t-1}}{\pi_t} \tag{69}$$

The definition of profits (in real terms)

$$\int_0^1 \Omega_{jt} dj = y_t \left[1 - mc_t - \frac{\phi}{2} \left(\frac{\pi_t}{\pi^*} - 1 \right)^2 \right]$$

Combine the households' aggregate resource constraint with the government budget constraint and the definition of profits to obtain the aggregate resource constraint

$$g_t + i_t^p + c_t + e_t + \frac{\phi}{2} \left(\frac{\pi_t}{\pi^*} - 1 \right)^2 y_t = y_t \tag{70}$$

Labour supply is

$$(1 - \tau_t^w) w_t \left(\frac{1}{1 - \gamma} - l_t \right) = \varkappa c_t \tag{71}$$

The equation of motion of the private and public capital stocks are given by,

$$k_{t+1} = e_t + (1 - \delta) k_t \tag{72}$$

$$k_{t+1}^p = i_t^p + (1-\delta)k_t^p \quad (73)$$

respectively, and the first order condition for investment is given by,

$$1 = \frac{\pi_{t+1}}{R_t}\left(p_{t+1}^k + 1 - \delta\right) \quad (74)$$

Inflation is described by,

$$(1-\varepsilon) + \varepsilon mc_t - \phi\frac{\pi_t}{\pi^*}\left(\frac{\pi_t}{\pi^*} - 1\right) + \phi\frac{\pi_t}{R_t}\frac{y_{t+1}}{y_t}\frac{\pi_{t+1}}{\pi^*}\left(\frac{\pi_{t+1}}{\pi^*} - 1\right) = 0 \quad (75)$$

Technology,

$$\frac{l_t}{k_t} = \frac{\alpha_2}{\alpha_1}\frac{p_t^k}{w_t}$$

$$mc_t = (y_t)^{\frac{1-\alpha_1-\alpha_2}{\alpha_1+\alpha_2}} A_t^{-\frac{1}{\alpha_1+\alpha_2}} \alpha_1^{-\frac{\alpha_1}{\alpha_1+\alpha_2}} \alpha_2^{-\frac{\alpha_2}{\alpha_1+\alpha_2}} (p_t^k)^{\frac{\alpha_1}{\alpha_1+\alpha_2}} (w_t)^{\frac{\alpha_2}{\alpha_1+\alpha_2}} (A_t^l)^{-\frac{\alpha_2}{\alpha_1+\alpha_2}} \quad (76)$$

and,

$$y_t = A_t k_t^{\alpha_1}(A_t^l l_t)^{\alpha_2}(k_t^p)^{\alpha_3}. \quad (77)$$

A.1 Stationary Model

With an exogenous growth rate in labour-embodied technical progress of ω such that $A_{t+1}^l = \omega A_t^l$, we can render the equilibrium stationary by deflating the following variables $\{y_t, m_t, c_t\frac{W_t}{P_t}, k_t, e_t, b_t, D_t\}$ by the level of labour-embodied technical progress..

The aggregate demand for money is given by,

$$\widetilde{m}_t = \chi\frac{R_t}{R_t - 1}\widetilde{c}_t \quad (78)$$

The aggregate consumption function is

$$\widetilde{c}_t = \frac{1-\gamma\beta}{1+\chi}\left[\frac{\widetilde{W}_t}{P_t} + \widetilde{lw}_t\right] \quad (79)$$

where aggregate financial wealth in real terms is

$$\frac{\widetilde{W}_t}{P_t} = \frac{\widetilde{m}_{t-1}}{\omega_{t-1}}\pi_t^{-1} + \frac{R_{t-1}}{\pi_t\omega_{t-1}}\widetilde{b}_{t-1} + (p_t^k + 1 - \delta)\widetilde{k}_t \quad (80)$$

where $m_t \equiv M_t/P_t$ and $b_t \equiv B_t/P_t$, and the aggregate human wealth is

$$\widetilde{lw}_t = \widetilde{H}_t + \gamma\frac{\pi_{t+1}}{R_t}\omega\widetilde{lw}_{t+1} \quad (81)$$

with

$$\widetilde{H}_t \equiv (1-\tau_t^w)\widetilde{w}_t l_t + \int_0^1 \widetilde{\Omega}_{jt}dj \quad (82)$$

The government budget constraint is given by,

$$\widetilde{g}_t + \widetilde{i}_t^p = \tau_t^w \widetilde{w}_t l_t + \widetilde{b}_t - \frac{R_{t-1}}{\omega_{t-1}\pi_t}\widetilde{b}_{t-1} + \widetilde{m}_t - \frac{\widetilde{m}_{t-1}}{\omega_{t-1}\pi_t} \tag{83}$$

The definition of profits (in real terms)

$$\int_0^1 \widetilde{\Omega}_{jt} dj = \widetilde{y}_t \left[1 - mc_t - \frac{\phi}{2}\left(\frac{\pi_t}{\pi^*} - 1\right)^2\right]$$

Combine the households' aggregate resource constraint with the government budget constraint and the definition of profits to obtain the aggregate resource constraint

$$\widetilde{g}_t + \widetilde{i}_t^p + \widetilde{c}_t + \widetilde{e}_t + \frac{\phi}{2}\left(\frac{\pi_t}{\pi^*} - 1\right)^2 \widetilde{y}_t = \widetilde{y}_t \tag{84}$$

Labour supply is

$$(1 - \tau_t^w)\widetilde{w}_t \left(\frac{1}{1-\gamma} - l_t\right) = \varkappa \widetilde{c}_t \tag{85}$$

The equation of motion of the private and public capital stocks is given by,

$$\omega \widetilde{k}_{t+1} = \widetilde{e}_t + (1-\delta)\widetilde{k}_t \tag{86}$$

$$\omega \widetilde{k}_{t+1}^p = \widetilde{i}_t^p + (1-\delta)\widetilde{k}_t^p \tag{87}$$

respectively, and the first order condition for investment is given by,

$$1 = \frac{\pi_{t+1}}{R_t}\left((1 - \tau_{t+1}^k)p_{t+1}^k + (1-\delta)\right) \tag{88}$$

Inflation is described by,

$$(1-\varepsilon) + \varepsilon mc_t - \phi\frac{\pi_t}{\pi^*}\left(\frac{\pi_t}{\pi^*} - 1\right) + \phi\frac{\pi_t}{R_t}\frac{\omega\widetilde{y}_{t+1}}{\widetilde{y}_t}\frac{\pi_{t+1}}{\pi^*}\left(\frac{\pi_{t+1}}{\pi^*} - 1\right) = 0 \tag{89}$$

Cost minimisation,

$$\frac{l_t}{\widetilde{k}_t} = \frac{\alpha_2}{\alpha_1}\frac{p_t^k}{\widetilde{w}_t}$$

$$mc_t = (\widetilde{y}_t)^{\frac{1-\alpha_1-\alpha_2}{\alpha_1+\alpha_2}} A_t^{-\frac{1}{\alpha_1+\alpha_2}} \alpha_1^{-\frac{\alpha_1}{\alpha_1+\alpha_2}} \alpha_2^{-\frac{\alpha_2}{\alpha_1+\alpha_2}} (p_t^k)^{\frac{\alpha_1}{\alpha_1+\alpha_2}} (\widetilde{w}_t)^{\frac{\alpha_2}{\alpha_1+\alpha_2}} (\widetilde{k}_t^p)^{-\frac{\alpha_3}{\alpha_1+\alpha_2}} \tag{90}$$

and the production function,

$$\widetilde{y}_t = A_t \widetilde{k}_t^{\alpha_1} (l_t)^{\alpha_2} (\widetilde{k}_t^p)^{\alpha_3}. \tag{91}$$

Figures

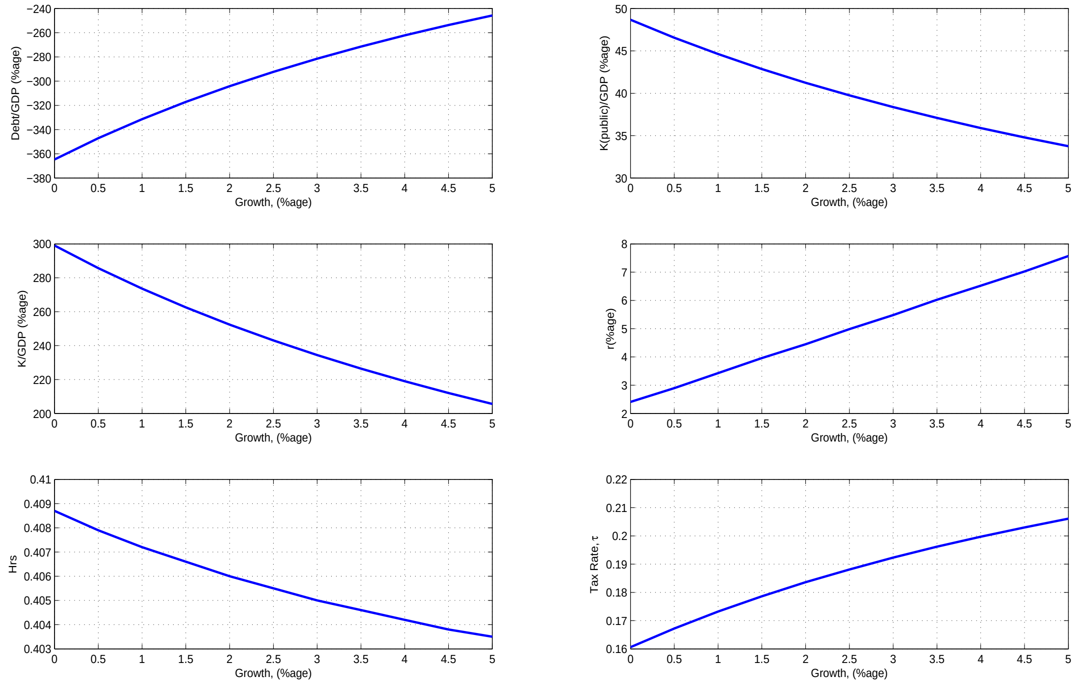

Figure 1: Ramsey Steady-State and the Rate of Growth

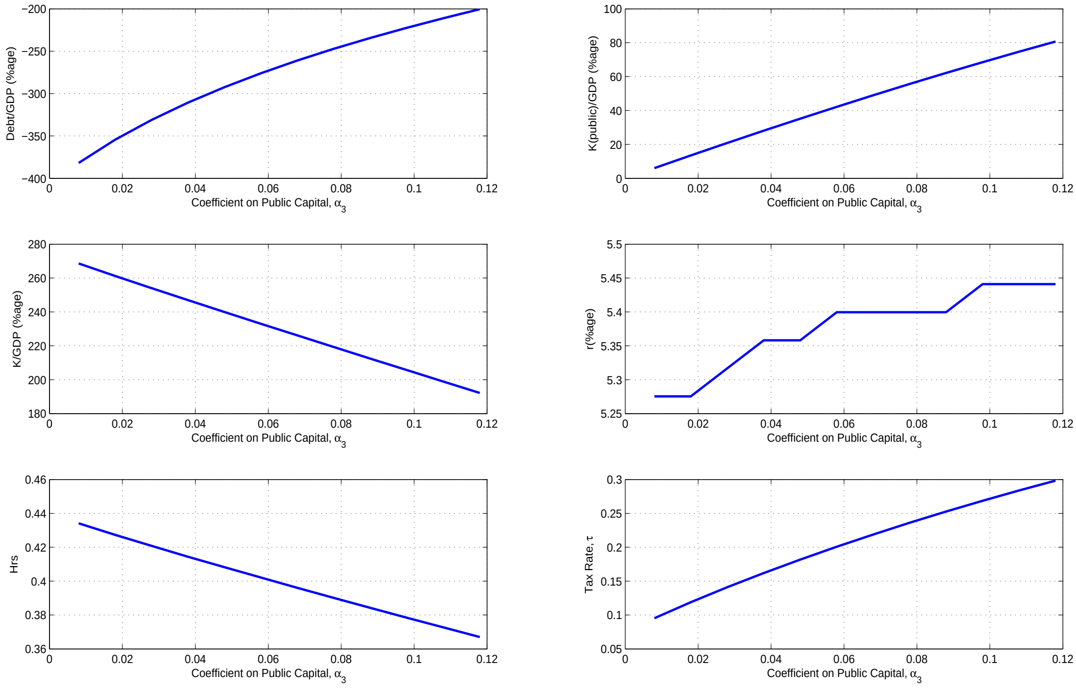

Figure 2: Ramsey Steady-State and Public Capital

Notes to Dynamics Figures 3–6: Solid red line - initial debt of 54% of gdp, dashed blue line - initial debt of 110%, dotted green line - initial debt of 160%.

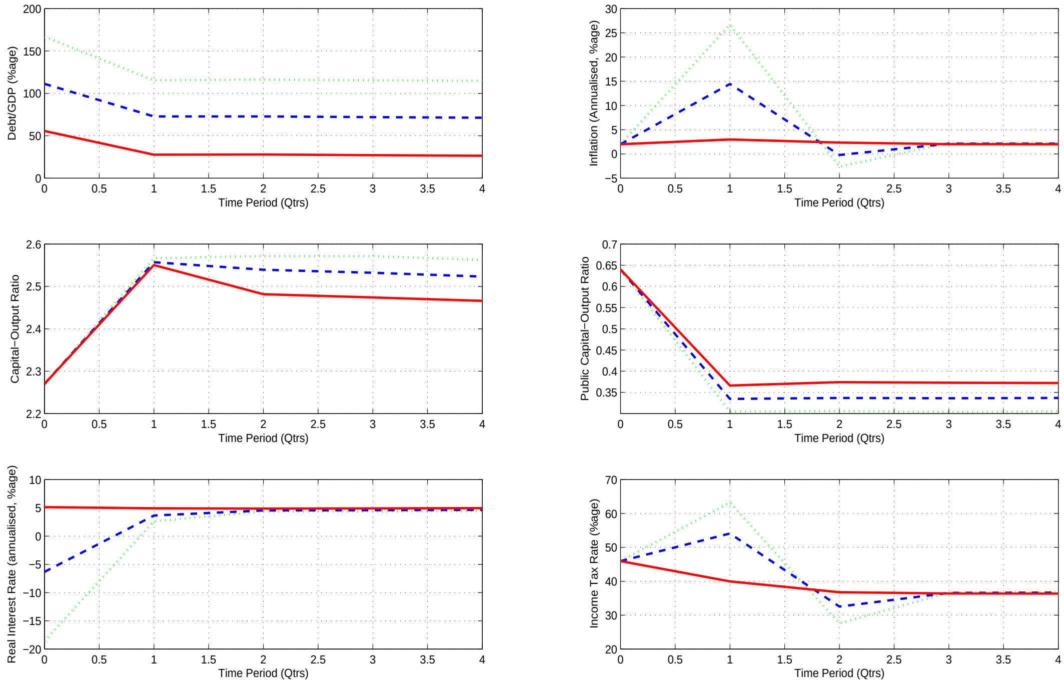

Figure 3: Ramsey Dynamics in the First Year I

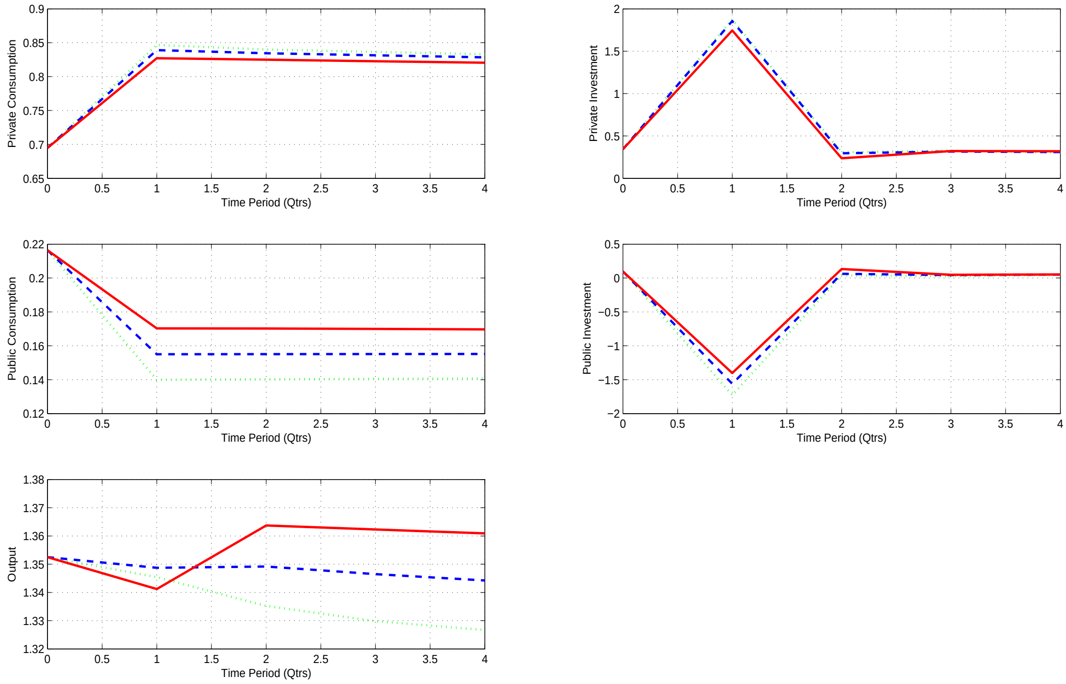

Figure 4: Ramsey Dynamics in the First Year II

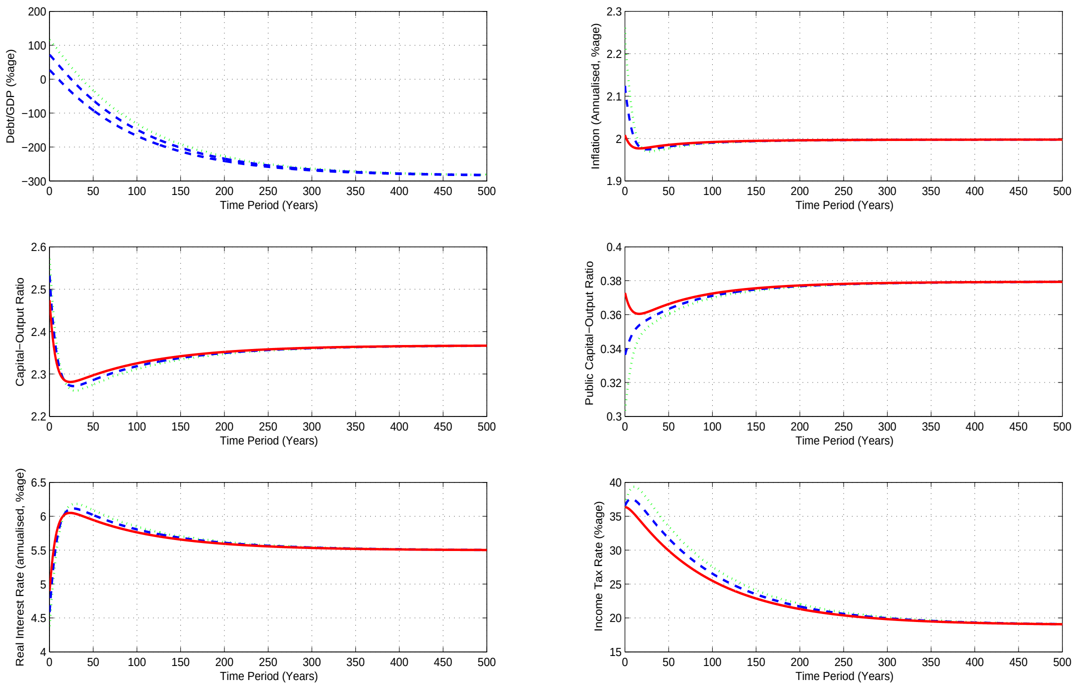

Figure 5: Ramsey Dynamics Beyond the First Year I

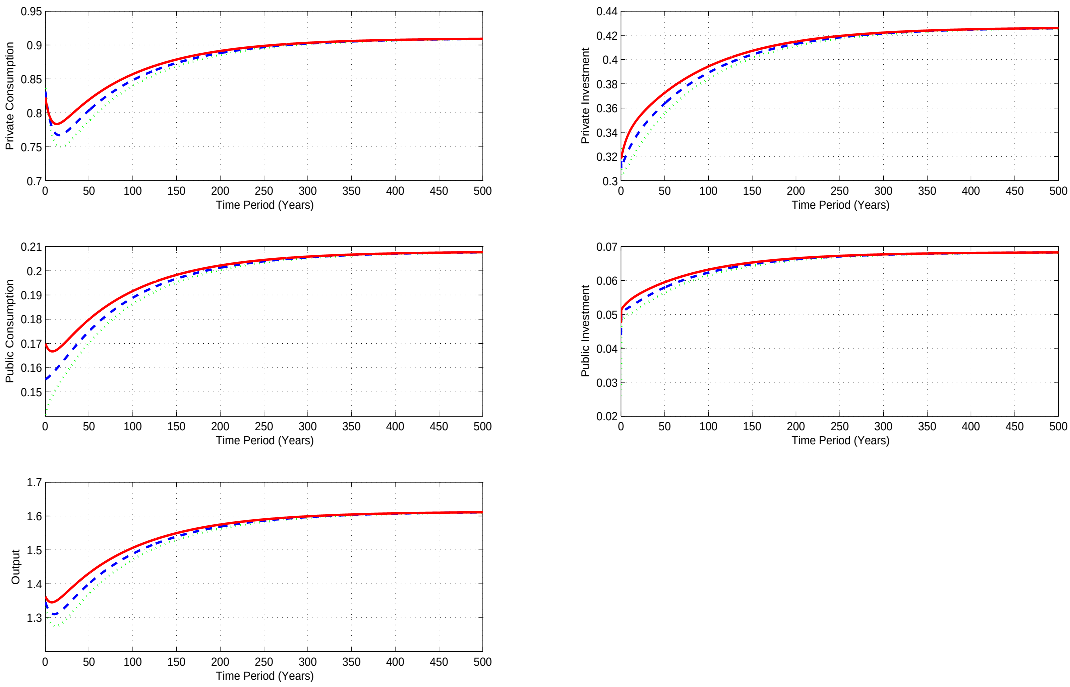

Figure 6: Ramsey Dynamics Beyond the First Year II

Notes to Figures 7-10: Expected Lifetime of 12.5 years - green dotted line, 25 years - blue dashed line and 50 years - red solid line.

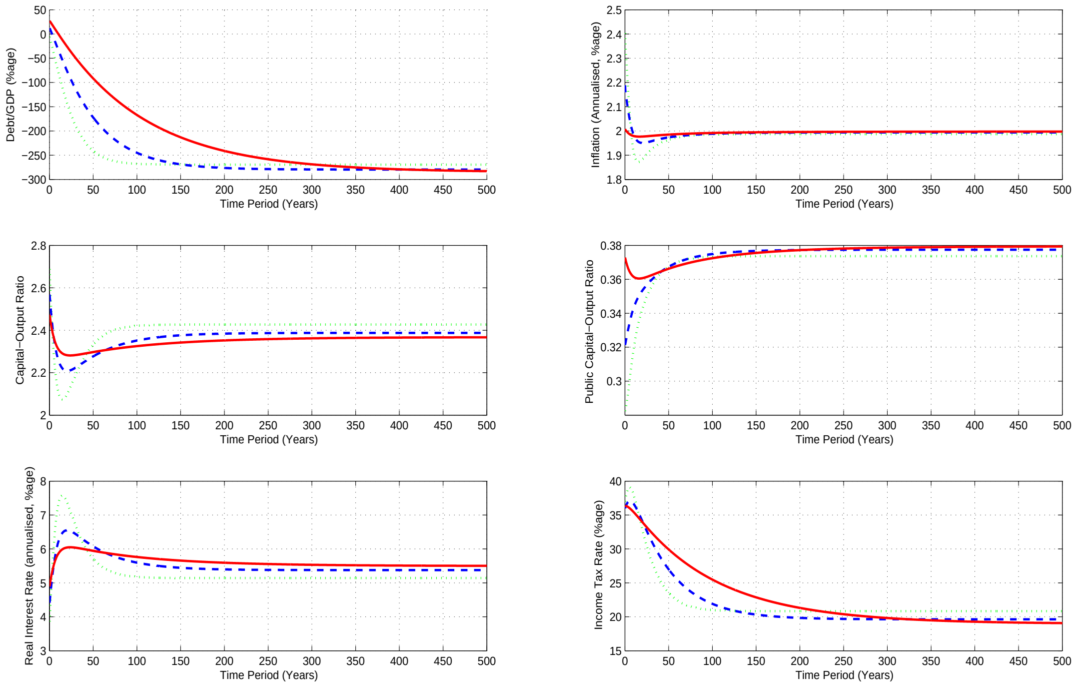

Figure 7: Ramsey Dynamics and Non-Ricardian Consumers I

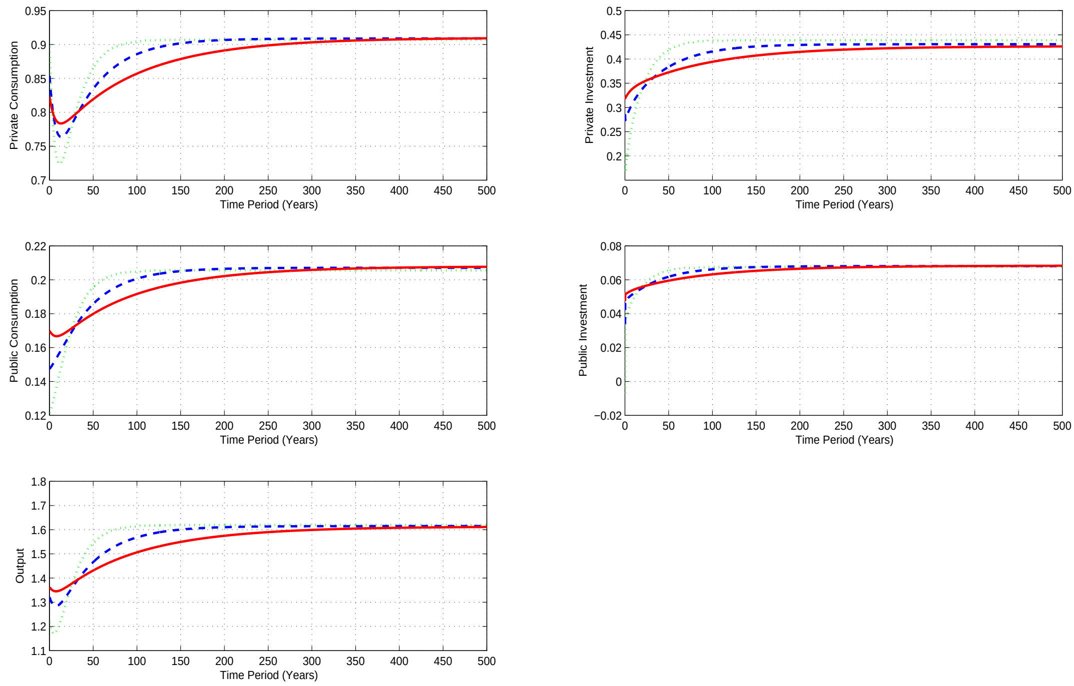

Figure 8: Ramsey Dynamics and Non-Ricardian Consumers II

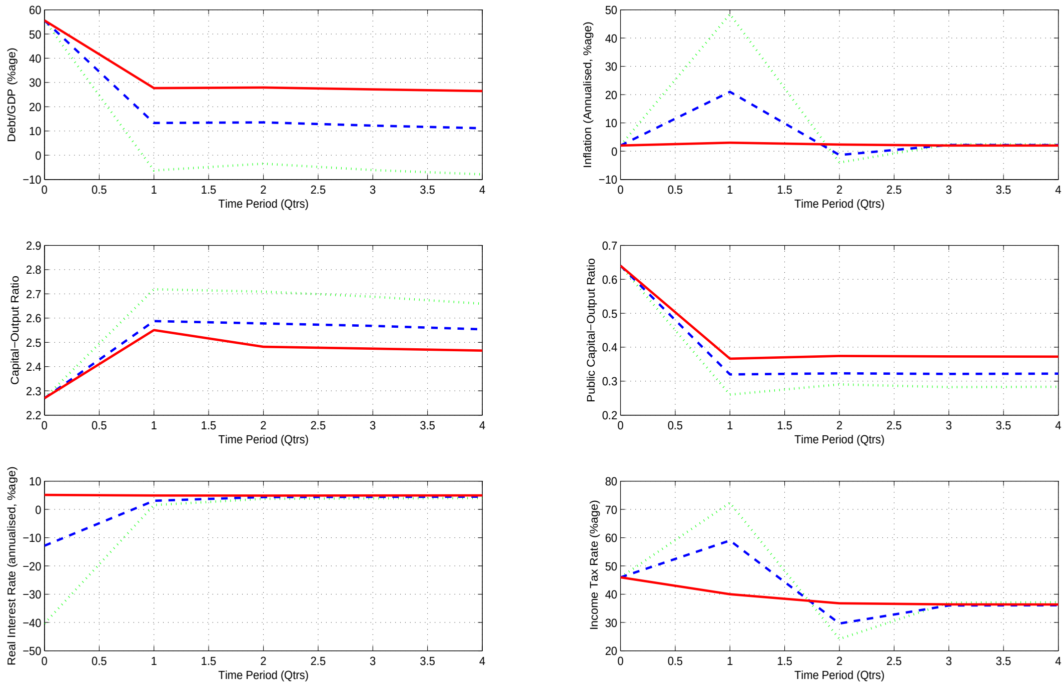

Figure 9: Ramsey Dynamics and Non-Ricardian Consumers - First Year I

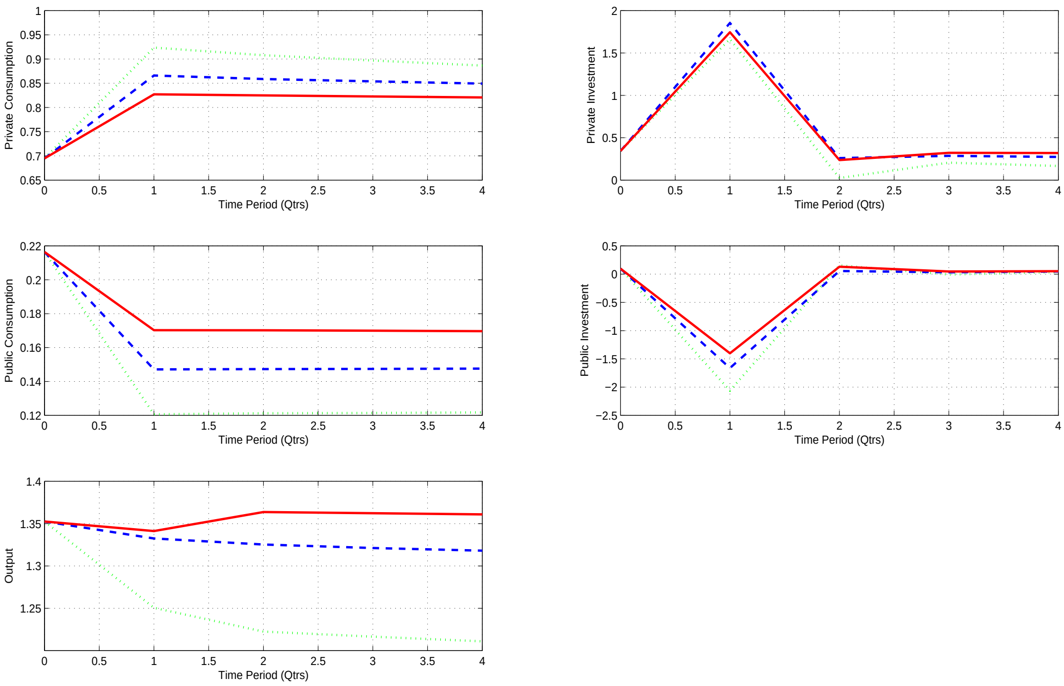

Figure 10: Ramsey Dynamics and Non-Ricardian Consumers - First Year II

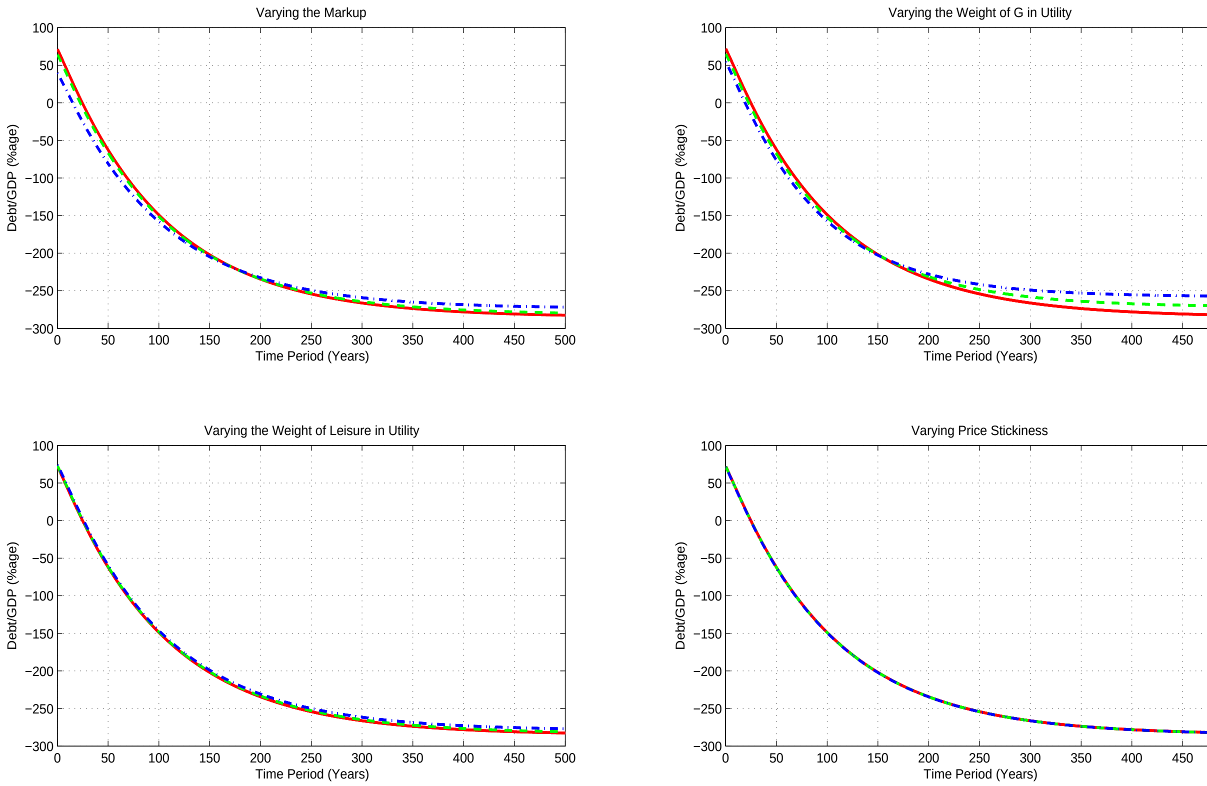

Figure 11: Robustness of Optimal Policy Across Alternative Parameterisations