A New Test of the Real Interest Rate Parity Hypothesis: 
Bounds Approach and Structural Breaks

George Bagdatoglou
Timberlake Consultants

Alexandros Kontonikas *
University of Glasgow Business School

Abstract

We test the real interest rate parity hypothesis using data for the G7 countries over the period 1970-2008. Our contribution is two-fold. First, we utilize the ARDL bounds approach of Pesaran et al. (2001) which allows us to overcome uncertainty about the order of integration of real interest rates. Second, we test for structural breaks in the underlying relationship using the multiple structural breaks test of Bai and Perron (1998, 2003). Our results indicate significant parameter instability and suggest that, despite the advances in economic and financial integration, real interest rate parity has not fully recovered from a breakdown in the 1980s.

Keywords: real interest rate parity, bounds test, structural breaks.

JEL classification: F21, F32, C15, C22

* Corresponding author: Alexandros Kontonikas. Postal address: University of Glasgow, Department of Economics, Glasgow, G12 8RT, U.K. Telephone No.++44(0)1413306866; Fax No. ++44(0)1413304940; E-mail: alexandros.kontonikas@glasgow.ac.uk.
1. Introduction

The real interest rate parity (RIP) hypothesis is one of the cornerstones of international macroeconomics and finance. It assumes Uncovered Interest Parity (UIP) and Purchasing power Parity (PPP) so that arbitrage in international financial and goods markets prevents domestic real rates of return from drifting apart from the ‘world’ real interest rate. RIP is a fundamental feature in early monetary models of exchange rate determination (see e.g. Frenkel, 1976). RIP has important implications for policy-makers. If RIP holds then the effectiveness of national monetary policy as stabilization tool would be restricted to the extent to which it can influence the world real interest rate (Mark, 1985).

A significant amount of empirical work has been devoted to investigating the validity of RIP, typically using the U.S. as the reference foreign economy. Early studies apply conventional regression analysis and reject the RIP-consistent zero and unity restrictions on the intercept and slope coefficients, respectively, in regressions of domestic on foreign real interest rates (see e.g. Cumby and Mishkin, 1986). These studies overlook or pre-date the developments in unit root tests and the modelling of non-stationary variables.

The more recent studies can be divided into two broad strands on the basis of opposite findings about the order of integration of real interest rates.¹ The first strand uses methodologies designed to deal with stationary series. For instance, Fujii and Chinn (2001) estimate the relationship between domestic and foreign rates using GMM and find weak evidence of RIP for the G7 countries. Jorion (1996) also provides evidence which is unfavourable to the RIP across the U.S., U.K. and Germany. The second strand treats real interest rates as non-stationary variables and applies either cointegration analysis or unit root tests on real interest rate

¹ For example, using the unit root test of Phillips and Perron (1988) Alexakis et al. (1997) fail to reject non-stationarity in the real interest rates of several developed markets over the period 1982-1993. In contrast, Fujii and Chinn (2001) find that all G7 short-run rates are stationary during 1976-2000 when applying the ADF-GLS test of Elliott et al. (1996). Goodwin and Grennes¹ (1994) report mixed results; they apply the Dickey and Fuller (1979) test and reject the unit root null in 5 out of 20 real rates that they examined over the period 1975-1987. These differences effectively reflect the well known power problems of unit root tests, especially against the alternative of a highly persistent stationary series, and their sensitivity to the sample size, time-period and type of test considered.
differentials. Results from studies that employ tests for cointegration between real rates provide strong support for a long-run relationship but little evidence for real interest rate equalization (see e.g. Goodwin and Grennes, 1994) whereas studies that apply panel unit root tests on real interest rate differentials tend to be quite supportive for RIP (see e.g. Wu and Chen, 1998). The evidence provided by the existing literature seems to be dependent on the choice of the RIP testing framework.

While the earlier studies did not pay attention to the possibility of structural breaks affecting the RIP tests, more recent studies find that accounting for structural breaks within the cointegration or unit root testing framework generates more favorable evidence for RIP. See for instance Fountas and Wu’s (1999) evidence from a cointegration-based RIP test which allows for endogenously determined structural breaks, and Arghyrou et al. (2009) who apply a time-series unit root test with structural breaks on real interest rate differentials. Camarero et al. (2010) extend the literature on panel data-based tests of real interest rate differentials and find evidence in favour of RIP, utilizing a panel stationarity test that allows for both cross-section dependence and structural breaks.

In this paper, we test for RIP using an empirical approach that not only overcomes the potentials problems associated with uncertainty regarding the stationarity of real interest rates but also accounts for structural breaks. In particular, we apply the autoregressive distributed lag (ARDL) bounds test approach of Pesaran et al. (2001) which allows to consistently test for and estimate the long-run relationship between real interest rates when it is not known with certainty whether the underlying series are purely I(0), purely I(1), or mutually cointegrated.

Footnotes:
2 As Goodwin and Grennes (1994) point out, the latter approach is conditional upon assuming unity slope in the cointegrating relationship between real rates.
3 It is well known that conventional regression analysis between non-stationary series can lead to spurious inference (Granger and Newbold, 1974). Likewise, methodologies designed for non-stationary series can be misleading when applied to stationary variables. For instance, Rahbek and Mosconi (1999) show that including I(0) variables in a Johansen-type VECM can lead to nuisance parameters in the asymptotic distribution of the trace tests for the cointegration rank.
4 Note that, to the best of our knowledge, no study has so far examined how the ARDL bounds test approach may be affected by long memory in the underlying series.
focuses on the G7 real interest rates during 1970-2008 proxying the ‘world’ interest rate by the U.S. rate. In line with the more recent literature on RIP we pay particular attention to the possibility of structural breaks. Specifically, we augment the ARDL approach with the multiple structural breaks testing approach of Bai and Perron (1998, 2003).

The layout of the paper is the following. Section 2 presents the theoretical background of our analysis. Section 3 describes the data. Sections 4 outlines the ARDL bounds testing approach and presents the empirical results. Section 5 describes the structural breaks testing approach and presents the related findings. Section 6 presents the robustness checks and Section 7 concludes.

2. Theoretical background

RIP can be derived using UIP, Eq. (1), relative ex ante PPP, Eq. (2), and the ex ante Fisher closed conditions for domestic and foreign country, Eqs. (3) and (4), respectively:

\[ i_t - i_t^* = \Delta s_{t+1}^e \]  
\[ \Delta p_{t+1}^e - \Delta p_{t+1}^{e*} = \Delta s_{t+1}^e \]  
\[ r_t = i_t - \Delta p_{t+1}^e \]  
\[ r_t^* = i_t^* - \Delta p_{t+1}^{e*} \]

where *, e indicate foreign variables, expected values, respectively; i, r denote the nominal, real interest rate, respectively; s, p denote the natural logarithm of the nominal exchange rate (units of domestic currency per unit of foreign currency), price level, respectively; \( \Delta \) denotes the first-difference operator. Combining Eqs. (1)-(4) yields:

\[ r_t = r_t^* \]  

RIP is more likely to hold in the long-run as arbitrage in financial and goods markets is not expected to be instantaneous and hence the two building blocks of RIP, PPP and UIP, are not expected to hold continuously. Real interest rate equalisation is not possible in the presence of
long-run departures from the underlying parity conditions. Deviations of domestic from foreign
real interest rates can be decomposed into PPP failures and UIP failures:

\[ r_t = r_t^* + \Delta q^{\text{ett}}_{t,t+1} + \psi_t \]  (6)

where \( q_t = s_t + p_t^* - p_t \) denotes the natural logarithm of the real exchange rate. Under relative ex-
ante PPP, \( \Delta q^{\text{ett}}_{t,t+1} = 0 \). The last term in Eq. (6), \( \psi_t = i_t - i_t^* - \Delta q^{\text{ett}}_{t,t+1} \), denotes deviations from UIP.

The Balassa-Samuelson effect (Balassa, 1964; Samuelson, 1964) predicts that an increase
in the relative productivity in the tradable-goods sector will increase wages not only in this sector
but also in the non-tradables sector pushing up the relative price of non-tradables. This in turn
contributes to real appreciation, driving \( r_t \) below \( r_t^* \). On the other hand, positive real interest rate
differentials may indicate premia due to currency-depreciation risk or country specific premia
due to e.g. default risk, political risk, and differential tax treatment. Transaction costs reflecting
factors such as shipping costs and trade barriers limit the scope of arbitrage across international
goods markets upon which PPP is based (see Sarno, 2005), leading to deviations of \( r_t \) from \( r_t^* \).

3. Data

Quarterly data on price levels and nominal interest rates over the period 1970-2008 were
collected for the G7 countries: Canada, France, Germany, Italy, Japan, United Kingdom, and the
United States. The sample spans over almost four decades providing us with around 150
quarterly observations and hence the power to obtain reliable estimates. The potential drawback
is that it covers a period when significant changes occurred (flexible dollar exchange rates since
1973, increasing economic and financial market integration and changes in the underlying
macroeconomic environments and monetary policy frameworks) which may affect the stability
and precision of our estimates if they are not taken into account.

5 If, however, productivity growth is equally strong in both traded and non-traded sectors then it should have no
impact on the real exchange rate.
6 Our data sources are OECD: Main Economic Indicators and IMF: International Financial Statistics.
Onshore nominal interest rates are measured using the three-month Treasury Bill rate in Canada, France, Japan, U.K., and the U.S.\textsuperscript{7} Since Treasury Bill rate series for Germany and Italy start later than the other sample countries, we use the three-month money market rate instead in these two cases.\textsuperscript{8} Interest rates with short (three-month) maturity are utilized in order to minimize the influence of factors such as term-premia and foreign-exchange risk, whose role is more prominent in interest rates of longer-maturity. The quoted three-month annualised nominal interest rates are transformed into a three-month continuously compounded nominal rate of return. We calculate current ex-post real interest rates \((r_t)\) as the difference between the three-month continuously compounded nominal return observed in period \(t-1\) minus the rate of inflation between periods \(t-1\) and \(t\). Inflation is measured by the change in the log of the seasonally adjusted Consumer Price Index.

\textbf{[TABLE 1 HERE]}

Table 1 provides descriptive statistics on G7 real interest rates. On average, the highest real rate is observed in Italy (0.69%), closely followed by Canada (0.68%) and France (0.67%). Japan exhibits the lowest average real interest rate (0.01%), followed by the U.S. (0.36%). The British real interest rate is the most volatile in the sample, followed by the Italian rate, while the least volatile series is observed in Germany, followed by the U.S. The Jarque-Berra test statistic indicates violation of the null hypothesis of normality in the cases of Italy, Japan and the U.K.. Visual inspection of the time-series suggests that non-normality may very likely be the result of outliers.\textsuperscript{9} Impulse dummy variables will be introduced in the modelling stage in order to account for these extreme observations. Finally, the first order autocorrelation coefficient suggests that

\textsuperscript{7} While international arbitrageurs could in principle use various interest rate variables to evaluate their returns home and abroad, Treasury Bills offer the advantage of being fixed-maturity instruments available to foreign investors in domestic markets (Ferreira and Leon-Ledesma, 2007).

\textsuperscript{8} Money market rates are very closely correlated with three-month Treasury Bill rates in both countries. The correlation coefficient between the two interest rate variables is equal to 0.97 in Germany (1975Q3-2002Q7) and 0.99 in Italy (1977Q2-2008Q2).

\textsuperscript{9} In all cases the outliers are associated with large changes in the underlying inflation rates.
the most persistent real interest rate series is observed in France, followed by Italy, while Japan exhibits the least persistent real interest rate, followed by Germany.

4. Testing the long run relationship of domestic and foreign real interest rates

4.1 ARDL approach

In this subsection we summarize the ARDL bounds approach to testing for a long-run relationship between the domestic real interest rate, $r_t$, and the foreign (U.S.) real interest rate, $r_t^*$. For a more detailed exposition of the ARDL bounds test methodology see Pesaran et al. (2001).

Consider the vector autoregressive model of order $p$ (VAR($p$)):

$$\Phi(L)(y_t - \mu) = \varepsilon_t,$$  \hspace{1cm} (7)

where $L$ is the lag operator, $\Phi(L) = 1 - \sum_{i=1}^{p} \Phi_i L^i$ is a matrix lag polynomial with $1$ denoting the identity matrix, $y_t = [r_t, r_t^*]'$, $\mu = [\mu_r, \mu_{r*}]'$ is a vector of constant terms, and $\varepsilon_t = [\varepsilon_{r,t}, \varepsilon_{r*,t}]' \sim IN(0, \Omega)$ is the vector error process. The variance matrix of $\varepsilon_t$ is positive definite and given by:

$$\Omega = \begin{bmatrix}
\omega_{rr} & \omega_{rr*} \\
\omega_{rr*} & \omega_{r*r*}
\end{bmatrix}$$  \hspace{1cm} (8)

Given the focus on the conditional modelling of $r_t$, $\varepsilon_{r,t}$ can be expressed in terms of $\varepsilon_{r*,t}$:

$$\varepsilon_{r,t} = \omega_{rr*} \varepsilon_{r*,t}^{-1} \varepsilon_{r,t} + u_t,$$  \hspace{1cm} (9)

The VAR model in Eq. (7) can be can rewritten as a vector error correction model (ECM):

$$\Delta y_t = \delta + \Lambda y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \varepsilon_t,$$  \hspace{1cm} (10)

where $\delta = -\Lambda \mu$, $\Gamma_i = -\sum_{j=1}^{i-1} \Phi_j (i = 1, \ldots, p-1)$ denote short-run response coefficient matrices, and $\Lambda$ is the long-run multiplier matrix.
The ARDL bounds test approach constrains the maximum number of conditional long-run relationships between the domestic and foreign real interest rate to be equal to one. A crucial identifying assumption involves restricting one of the off-diagonal elements of $\Lambda$ to zero. Since the U.S. is employed as the reference large economy, its real interest rate is assumed to be the exogenous long-run forcing process for the rest of the G7 members’ rates.\(^{10}\) Hence, the restriction: $\lambda_{rr} = 0$ is imposed.

Using the restriction $\lambda_{rr} = 0$ and Eq. (9) the ECM in Eq. (10) becomes:

$$\Delta r_t = c_0 + \pi_1 r_{t-1} + \pi_2 r^*_{t-1} + \pi_3 \Delta r^*_{t-1} + \sum_{i=1}^{P-1} \theta_{r, r} \Delta r_{t-i} + \sum_{i=1}^{Q-1} \theta_{r, r} \Delta r^*_{t-i} + u_t$$

(12)

where $\pi_1 = \lambda_{rr}$ measures the speed of adjustment to a discrepancy between domestic and foreign real interest rates in the previous period, $\pi_2 = \lambda_{rr} - \omega \lambda_{rr}$ measures the combined effect from the current and past values of the foreign real interest rate on the current domestic rate, while $\pi_3 = \omega r^*_{t-1} \lambda_{rr}$ measures the instantaneous effect from changes in the foreign real interest rate.\(^{11}\)

Equation (12) can be consistently estimated by OLS irrespective of whether real interest rates are I(0), I(1) or mutually cointegrated. The null hypothesis of no long-run relationship is equivalent to $H_0$: $\pi_1 = \pi_2 = 0$ and can be tested by an $F$-test.\(^{12}\) Pesaran et al. (2001) point out that the asymptotic distribution of the test-statistic in non-standard under the null hypothesis and provide two sets of asymptotic critical values. The first set (lower bound) covers the case when

\(^{10}\)The use of the U.S. as the reference foreign economy is in line with the majority of the previous studies which test for RIP vis-à-vis the U.S (see e.g. Fujii and Chin, 2001; Camarero et al, 2010). The assumption of long-run exogeneity of U.S. real interest rates is not unreasonable since the U.S. has dominated the world economy and international financial markets during the period under investigation, with investors typically considering U.S. three-month Treasury Bills as the global risk-free asset. U.S. dominance is supported by Al Awad and Goodwin’s (1998) evidence from RIP tests.

\(^{11}\)Equation (12) corresponds to case III in Pesaran et al. (2001) where the intercept is unrestricted and there is no trend.

\(^{12}\)Pesaran et al. (2001) consider an additional test, that is, the Banerjee et al., (1998) $t$-test for the null hypothesis $\pi_1 = 0$. This test was also performed but not reported in order to save space. The results were identical in nature to those from the $F$-test and are available upon request.
all the regressors are $I(0)$ and the second set (upper bound) the case when all the regressors are $I(1)$. If the $F$-statistic exceeds the upper bound, the null hypothesis of no long-run relationship can be rejected. If the $F$-statistic is smaller than the lower bound, the null hypothesis cannot be rejected. In the case when the test statistic falls within the bound, no conclusive decision can be made without knowing the order of integration of the underlying series.

Under the alternative hypothesis of interest, $\pi_1 \neq 0$ and $\pi_2 \neq 0$, there is a conditional long-run level relationship between $r_t$ and $r^*_t$ defined by:

$$r_t = \alpha + \beta r^*_t + \nu_t$$  \hspace{1cm} (13)

where $\alpha = -c_0/\pi_1$ and $\beta = -\pi_2/\pi_1$ denote the long-run intercept and slope, respectively; $\nu_t$ is a zero mean stationary process. Note that the alternative hypothesis in the $F$-test allows for the possibility of degenerate cases such as $\pi_1 \neq 0$ and $\pi_2 = 0$ (Pesaran et al., 2001). If $\pi_2 = 0$, there is no impact from current and past values of the foreign real interest rate on the contemporaneous domestic rate. From the definition of the long-run slope, it follows that $\beta = 0$.

We consider two forms of RIP, weak-form and strong-form (see also Fountas and Wu, 1999). Both forms require that the null of no level relationship between $r_t$ and $r^*_t$ is rejected and that $\beta \neq 0$, i.e. the degenerate case of zero long-run effect of $r^*_t$ on $r_t$ is ruled out. With weak-form RIP domestic policymakers have some (imperfect) control over macro-stabilisation policies. Strong-form RIP, however, further restricts the long-run coefficients to $\alpha = 0$ & $\beta = 1$, so that real interest rates are equalised. Strong form RIP could fail when goods markets and/or capital markets are not perfectly integrated. Hence, violations of strong form RIP should be ultimately traced down to deviations from PPP and/or UIP, reflecting factors such as non-traded goods, risk premia and transaction costs (see Section 2).
4.2 ARDL approach: empirical evidence

Pesaran et al. (2001) suggest that the lag lengths in Eq. (12) should be selected in conjunction with some information criteria, such as the Akaike (AIC) and/or Schwarz Bayesian Information Criterion (SBC), and diagnostic tests for residual serial correlation. We allow for a maximum of four lags, corresponding to one year, and utilise the SBC and tests for residual serial correlation as means of model selection.\(^{13}\)

[TABLE 2 HERE]

The estimates of the preferred specifications are reported in Table 2. The adjusted \(R^2\) ranges from 0.31 (France) to 0.78 (Japan) and there is no evidence of residual serial correlation and heteroscedasticity. The \(F\)-test results show that the null hypothesis of no level relationship between domestic and foreign real interest rates is strongly rejected in five out of six cases with the test statistic exceeding the 1% level of significance upper critical bound. Thus, with the exception of Italy, the necessary condition for RIP is satisfied in all countries.

The coefficient of adjustment to past deviations of \(r_t\) from \(r_t^*, \pi_t\), is significantly different from zero in all cases. Its magnitude varies across countries, with Japanese real interest rates exhibiting the fastest degree of error correction, closely followed by Germany. These findings therefore reinforce the simple ACF results in Table 1 in that the Japanese and German real interest rates are the least persistent in our sample. Estimates of the immediate impact on domestic real interest rates from changes in U.S. rates indicate that this effect is strongest in Canada (\(\beta_3 = 0.74\)). In the remaining countries, the instantaneous effect is generally not as prominent, ranging from 0.25 (Japan) to 0.4 (France).

Table 3 reports the implied long-run coefficients. The long-run intercept, \(\alpha\), ranges from -0.09 in Japan to 0.47 in Germany. The average \(\alpha\) across countries is equal to 0.25 indicating the existence of a positive wedge in the long-run between U.S. and domestic real interest rates. The

\(^{13}\) Based on Monte Carlo simulations, Panopoulou and Pittis (2004) find that the SBC selects the appropriate lag length much more frequently than the AIC.
long-run response of domestic to U.S. real interest rates, $\beta$, ranges from 0.37 in Germany to 1.21 in the U.K., with an average value of 0.81.

[TABLE 3 HERE]

To carry out hypothesis testing, we calculate the standard errors of the implied long-run coefficients using the Delta method.\textsuperscript{14} Results from the $t$-test for the zero long-run wedge hypothesis, $\alpha = 0$, as predicted by strong form RIP, indicates that it can be rejected at the 1% level of significance in Canada and Germany (Table 3, column 4). U.S. real interest rates significantly affect domestic rates in the long-run since $H_0: \beta = 0$ can be rejected at the 1% level of significance in all cases (Table 3, column 5). Thus, the relationship between real interest rates is not degenerate but consistent with, at least, weak-form RIP. Furthermore, in Canada, France and the U.K. the null hypothesis of unity long-run slope coefficient, as suggested by strong form RIP, cannot be rejected (Table 3, column 6). Finally, the Wald test for the more restrictive case $H_0: \alpha = 0 \& \beta = 1$ (Table 3, last column) indicates that the real interest rate equalisation hypothesis is not rejected only in France and the U.K. Thus, overall, our results suggest that while there is strong support for weak form RIP, evidence in favour of real interest rate equalisation is less prevalent. These results are consistent with the findings of existing studies that employ cointegration analysis as empirical framework to test RIP.

5. Structural breaks analysis

5.1 Multiple breaks testing approach

The empirical results in the previous section do not take the possibility of structural breaks into account. Given the long span of our sample and the important changes that took place over the last four decades it is important to investigate the case of regime shifts. Structural breaks may have occurred that could affect the long-run relationship between domestic and foreign real

\textsuperscript{14} See Appendix A for more details.
interest rates. Preliminary evidence from CUSUM and CUSUM of squares tests (Brown, Durbin and Evans, 1975) indicates some evidence of parameter instability in the ECM represented by Eq. (12). Consequently, the implied long-run parameters in Eq. (13) may also be affected since they based upon on the ECM.

Since the CUSUM approach is known to exhibit drawbacks such as poor power (see e.g. Hansen, 1992), we proceed by applying the procedure developed by Bai and Perron (1998, 2003) for multiple structural change models (henceforth, BP methodology), which determines the number of breaks and break dates endogenously. Bai and Perron’s (2001) simulation evidence indicates that the BP approach displays considerable power in locating structural breaks.

In place of the no-break ECM model, we consider a partial structural change model with \( m \) breaks \((m+1\) regimes) given by:

\[
\Delta r_t = X_t' D + Z_t' G_j + \zeta_t, \quad t = T_{j-1} + 1, \ldots, T_j
\]

(14)

where \( j = 1, \ldots, m+1; \ X_t = [\Delta r_t^* \ \Delta r_{t-1}^* \ \ldots \ \Delta r_{t-p+1}^* \ \Delta r_{t-1}^* \ \ldots \ \Delta r_{t-q+1}^*]' \) is the vector of regressors whose coefficients, contained in vector \( D \), are fixed across regimes; \( Z_t = [1 \ r_{t-1} \ r_{t-1}^*]' \) is the vector of regressors whose coefficients, contained in vector \( G_j \), are allowed to change; \( \zeta_t \) is the error term. Given our focus on the long-run relationship between domestic and foreign real interest rates, Eq. (14) considers the possibility of long-run shifts, with changes occurring in the intercept (via the constant term and/or the coefficient of \( r_{t-1} \)) and/or the slope (via the coefficient of \( r_{t-1}^* \) and/or \( r_{t-1}^* \)).

BP treat the breakpoints \((T_1, \ldots, T_m)\) as unknown and estimate them along with the regression coefficients using the least squares principle. Three types of tests have been developed by BP in order to identify the number of structural breaks. The sup\( F_T(k) \) test of no structural break \( (m=0) \) versus the alternative of \( k \) breaks. The double maximum tests, \( UD_{max} \) and \( WD_{max} \), for the null hypothesis of no structural break against the alternative of an unknown number of breaks

\[\text{Results are available upon request.}\]
(given some upper bound). Finally, the $\text{Sup}_{F_l}(l+1\mid l)$ test for $l$ versus $l+1$ breaks. BP recommend a sequential application of the $\text{Sup}_{F_l}(l+1\mid l)$ test, but also argue that there are some cases where the sequential procedure can be improved. Particularly, in the presence of multiple breaks, the $\text{Sup}_{F_l}(1\mid 0)$ statistic can exhibit low power causing the sequential procedure to break down. Therefore, BP suggest that, first, the $UD_{\text{max}}$ and $WD_{\text{max}}$ tests should be utilised to check if at least one break is present. If the double maximum statistics are significant, then the number of breaks should be determined by a sequential application of the $\text{Sup}_{F_l}(l+1\mid l)$ test, ignoring the $\text{Sup}_{F_l}(1\mid 0)$ statistic.

5.2 Multiple breaks testing approach: empirical evidence

Table 4 reports the results from the BP tests for structural breaks in the model linking domestic to foreign real interest rates. In 4/6 cases (Canada, France, Germany and U.K.) both double maximum statistics are significant at conventional significance levels, indicating strong evidence in favour of structural change in the link between real interest rates across countries. For these countries, the $\text{Sup}_{F_l}(2\mid 1)$ statistic is significant at the 1% level of significance, while the $\text{Sup}_{F_l}(3\mid 2)$ statistic is insignificant, suggesting two structural breaks (three regimes). Note that in Canada the $\text{Sup}_{F_l}(1\mid 0)$ statistic is not significant indicating zero breaks. However, following BP’s recommendation, since the double maximum statistics are significant we apply the sequential procedure, bypassing the $\text{Sup}_{F_l}(1\mid 0)$ test, and find two breaks. In Italy, the $WD_{\text{max}}$ double maximum statistic is significant at the 10% level, and the $\text{Sup}_{F_l}(4\mid 3)$ test suggests four

---

\[16\] We allow the variance of the residuals and the regressors’ moments matrices to differ across the different regimes. Consequently, following Bai and Perron’s (2001) suggestion, we use a trimming parameter of 0.15 and allow for up to 5 structural breaks. Estimations are carried out using the GAUSS program available from P. Perron’s homepage (http://people.bu.edu/perron/code.html).
structural breaks (five regimes). In Japan, no breaks are found since none of the BP statistics is significant at conventional significance levels. Regarding the timing of the breaks, some common patterns arise. Canada, Germany and Italy exhibit the first break towards the end of the 1970s, while in France and U.K. it takes place around the early-to-mid 1980s. When two breaks are present the second one occurs during the 1990s. The third and final regime commences at the mid-1990s in Canada, France and Germany and earlier in that decade in the U.K.

[TABLE 4 HERE]

Having identified the break dates, we re-estimate the ECM model including relevant additive and multiplicative dummies to account for the regime shifts in the long-run relationship. In turn, we calculate the $F$-statistic for the null of no long-run relationship and the implied long-run coefficients together with their standard errors across all alternative regimes. Figure 1 shows the $F$-test along with the 5% level critical bounds. The null hypothesis is rejected in all cases, including Italy. Note that Italy was the only case for which we previously failed to reject the null hypothesis. Comparing the no-break $F$-statistics in Table 3 with those in Figure 1, it is evident that taking structural shifts into account increases substantially the magnitude of the test statistics implying stronger rejections of the no-relationship null. 

17 It must be acknowledged that evidence of structural breaks in Italy is not so straightforward since only one of the double maximum statistics is significant, while both the $\text{Sup} F_t (1|0)$ and the $\text{Sup} F_t (2|1)$ statistics are insignificant. Nevertheless, as Bai and Perron (2003) point out, one should keep in mind potential power problems in the presence of multiple breaks. This, together with the results from $\text{Sup} F_t (4|3)$ and $\text{Sup} F_t (4|0)$ tests, which are available upon request, led us to conclude that four breaks are present in Italy.

18 Recall that in the BP test we allowed for breaks in the constant term, the coefficient of $r_{t-1}$ and the coefficient of $r_{t-1}$. Thus, rejection of the null of, say, zero breaks implies that at least one of the coefficients of interest exhibits regime shift. Examination of the statistical significance of the break dummies reveals which parameters exhibit structural change. The results in Table 4 (final column) suggest that changes in the long-run coefficients are mainly driven by intercept shifts and/or changes in the coefficient of $r_{t-1}$, rather than changes in the autoregressive dynamics. The insignificant break dummies were excluded from the model to preserve degrees of freedom. The adjusted $R^2$ increased by 7% on average when the relevant dummies were included in the models indicating significant improvement of fit.

19 As Pesaran et al. (2001) point out, the asymptotic theory that they developed is not affected by the inclusion of one-off dummy variables, such as the ones employed in Italy and the U.K. They also argue that the critical values for the bounds test should be modified if the fraction of periods in which the dummy variables are non-zero does not tend to zero with the sample size. Our break dates imply that we have three cases where the related dummies take the value of one for more than 40% of the time (Canada 1977Q4-1995Q4; Germany 1979Q1-1995Q3; U.K. 1993Q1-2008Q2). We are confident that our inference is still valid in these cases since the $F$-statistic is quite large.
Figure 2 plots the long-run slope and the associated 95% confidence interval across the different regimes. In general, with the exception of Canada, the magnitude and significance of the long-run slope coefficient vary considerably. In particular, the $\beta$ coefficient in the cases of France, Italy and the U.K. during the mid-1980s to early/mid-1990s is not statistically different from zero implying no relationship between the real interest rate of these countries and that of the U.S. The same degenerate case arises in Italy during most of the 1970s. Germany during the 1970s is a special case as the $\beta$ coefficient is negative and statistically significant, indicating a tendency for German and U.S. real rates to diverge, inconsistent with RIP. From 1970 until the early/mid-1980s, the hypothesis that $\beta = 1$ cannot be rejected in two instances, that is, France and the U.K. Interestingly, the long-run slope is positive and statistically significant in all countries from early/mid-1990s onwards, although in none of the countries the unity restriction implied by the strong form RIP is satisfied.

Figure 3 shows the long-run constant and the associated 95% confidence interval across the different regimes. Like in the case of the long-run slope, the intercept exhibits significant variation. During the 1970s and early/mid 1980s, the $\alpha$ coefficient is statistically indistinguishable from zero. From the 1980s to early/mid-1990s, however, the constant term increases in magnitude and becomes statistically different from zero in all countries implying a positive gap between domestic and U.S. real interest rates. Finally, from the early/mid-1990s onwards, the constant drops but remains statistically different from zero unanimously, contrary to the strong-form RIP prediction.

relatively to the critical bound. For example, in Canada the $F$-statistic exceeds the upper bound by a factor of around six. The only exception is the U.K. for which during the first regime the null hypothesis of zero intercept is marginally rejected at the 5% level of significance, but not at the 1%.
Finally, Figure 4 shows the Wald test results for the joint hypothesis $\alpha = 0$ & $\beta = 1$. Not surprisingly, evidence in favour of the strong form RIP is scarce. It is only in France during the 1970s that the restrictions implied by the strong form RIP cannot be rejected. In most other cases, the restrictions can be comfortably rejected. This contrasts our ‘no-break’ results where we find support for strong-form RIP in both France and the U.K. It appears that failing to account for breaks may lead one to erroneously conclude in favour of real interest rate equalisation.

[FIGURE 4 HERE]

All in all, the relationship between international real interest rates is characterized by significant structural breaks. A general picture that emerges is that there was a stronger tendency for RIP to hold during the 1970s. The 1980s were a particularly bad period for the parity condition with even the weak form being rejected in several instances. Finally, from the early/mid-1990s onwards, we observe a tendency for both the constant and the slope to move closer to the RIP implied values. However, in all cases the restrictions required for real interest rate equalisation are still rejected, both individually and jointly.

The dismal performance of RIP during the 1980s could be attributed to PPP and/or UIP failures. Lothian (1998, p. 29) points out that the poor performance of PPP over the current float period is “…being largely confined to one time period – the early and mid-1980s – and one currency – the U.S. dollar”. He argues that dollar real exchange rates exhibited significant variation during the 1980s driven by a protracted dollar appreciation, which peaked around 1985, followed by an almost offsetting depreciation after the Plazza agreement came into effect. Additionally, UIP has also been documented to perform poorly during the 1980s. Particularly, Lothian and Wu (2005) examine the stability of UIP over a long time span and document that it

---

21 Of course, some caution should be applied since the present paper does not estimate a structural real interest rates differentials determination model. Hence, the explanations offered should be regarded as plausible interpretations of the identified stylized facts.
breaks down when the 1980s are included in the sample. They emphasize the destabilising role of slow adjustment of expectations to U.S. monetary policy changes that took place in the early 1980s with the goal of reducing inflation. Consistently with the previous authors, Flood and Rose (2001) find that UIP works better in the 1990s as compared to previous periods. Thus, a combination of PPP and UIP failure in the 1980s may help to explain the breakdown in the relationship between domestic and U.S. real interest rates.

A shift towards UIP in the 1990s, due to smaller risk premia and/or expectational errors, should then underlie some of the shifts in favour of RIP, such as the decrease in the long-run wedge separating domestic and U.S. real interest rates from the mid-1990s onwards. Such developments could originate from important policy changes that took place in the 1990s, which led to lower overall macroeconomic uncertainty. Nevertheless, the lack of empirical support for strong-form RIP during the final regime implies that deviations from UIP and/or PPP do persist. The existence of a positive wedge driving domestic real interest rates above U.S. rates during the last regime may be explained by the considerable real appreciation of the dollar from the mid-1990s to early 2000, itself potentially related to a productivity boom in the U.S. (see e.g. Tille et al., 2001).

6. Robustness analysis

We conduct a robustness check with respect to three issues related to our dataset. First, we use a later starting point to obtain onshore ex post real rates estimates, 1973Q1,

22 New monetary policy regimes were introduced in the early 1990s with Canada and the U.K. adopting explicit inflation targeting. The emphasis on inflation control and the resulting reduction in inflation uncertainty was more generalised, observed both in targeting and non-targeting countries. Following the ERM crisis of the early 1990s, the Euro-area block of the G7 (Germany, Italy and France) underwent a convergence process which peaked up in the mid-1990s culminating with the adoption of single currency and common monetary policy on 1/1/1999.

23 The U.S. productivity speed-up in the 1990s is inherently linked to the technological advances in computer and communication. Cova et al.’s (2008) evidence indicates that over the period 1995-99 the increase in productivity of the U.S. tradables sector far exceeded that in the non-tradables sector. A productivity boom concentrated in the traded sector could then explain the real appreciation of the dollar via Balassa-Samuelson effects. Note however that the debate on the nature of the U.S. productivity boom is still not settled and views that it has focused on the non-tradables, rather than the tradables sector, have also been expressed.
corresponding to the start of the period of flexible exchange rates.\textsuperscript{24} Second, we calculate onshore ex ante real interest rates, using one step-ahead forecasted inflation rates from an autoregressive model of inflation. More specifically, the inflation forecasts are generated from a 40-quarter rolling AR(4) model where at every estimation window we apply the general-to-specific approach to select a parsimonious specification. Third, we use Eurocurrency (offshore) rates to calculate ex post real interest rates. Eurocurrency rates are not affected by reserve requirements and capital controls (see e.g. Wu and Chen (1998) but become available at a later date, 1975Q1, with the exception of Italy and Japan where they start at 1978Q3.

The results are shown in Tables B1-B3 in Appendix B and indicate that in the most cases the finding of three regimes, with the second one commencing at the end of the 1970s or the early-to-mid 1980s and the third one during the 1990s, is overall quite robust. Specifically, comparison of the baseline results in Table 4 with those in Table B1 indicates that using onshore ex post rates and commencing the estimation at 1973 the estimated structural breaks and implied timing of the regimes are very similar in almost all cases. The onshore ex ante rates and offshore ex post results in Tables B2 and B3, respectively, agree with the baseline results that in most instances three regimes are identified. Finally, the finding of no breaks in the case of Japan is shown to be robust across all three alternative sensitivity checks.

7. Summary and conclusions

This paper has tested for weak-form and strong-form RIP using quarterly data from the G7 over the period 1970-2008 and the U.S. as the reference country. We build upon previous literature on real interest rate convergence and extend it to two important new directions. First, we utilize the ARDL bounds approach of Pesaran et al. (2001), which ensures that our results are robust to uncertainty about the order of integration of real interest rates. Overall, we find that

\textsuperscript{24} The Bretton Woods system of fixed exchange rates dissolved in August 1971 when the U.S. announced the suspension of the dollar’s convertibility into gold. Floating of the major currencies against each other commenced by March 1973.
strong-form RIP receives considerably less empirical support as compared to its weak-form version. Second, we turn our attention to the possibility of structural breaks in the long-run relationship between domestic and U.S. real interest rates, using the multiple structural breaks test of Bai and Perron (1998, 2003).

Results from structural breaks analysis have indicated significant parameter instability. We have accounted for it accordingly, allowing us to examine the pattern of real interest rate convergence over time. Our findings turn out to be quite interesting and novel. As we move from the 1970s towards the end of the sample, support for RIP does not increase monotonically as one would perhaps expect on the basis of reductions in trade barriers and barriers to capital mobility that occurred throughout the sample period. Instead, we document a stronger tendency for RIP to hold during the 1970s. Early RIP success is followed by widespread failure over the 1980s, with the relationship between domestic and U.S. real interest rates breaking down completely in some cases. From the early/mid-1990s onwards, a new regime emerged which is more favorable for RIP since its weak-form is now satisfied in all countries. Nevertheless, our results suggest that the conditions that are necessary for real interest rate equalization are still not met.

These findings in association with existing empirical evidence on PPP and/or UIP suggest that shifts in these parity relationships underlie changes in RIP performance over time such as the break-down over the 1980s and the partial rehabilitation towards the end of the sample period. It appears that despite the advances in economic and financial market integration during the last decades, perfect integration is yet to be achieved implying room for domestic stabilization policies.
References


Appendix A: Delta method

We use the Delta method to calculate the variance and associated standard error of the implied long-run coefficients from the estimates of the ECM model in Eq. (12). For example, the long-run intercept is:

\[ \alpha = -c_0 / \pi_1 \]  

(A1)

The variance of \( \alpha \) is equal to:

\[
Var(\alpha) = \begin{bmatrix}
\frac{\partial \alpha}{\partial c_0} & \frac{\partial \alpha}{\partial \pi_1} \\
\frac{\partial \alpha}{\partial c_0} & \frac{\partial \alpha}{\partial \pi_1}
\end{bmatrix}
\begin{bmatrix}
Var(c_0) & Cov(c_0, \pi_1) \\
Cov(\pi_1, c_0) & Var(\pi_1)
\end{bmatrix}
\begin{bmatrix}
\frac{\partial \alpha}{\partial c_0} \\
\frac{\partial \alpha}{\partial \pi_1}
\end{bmatrix}
\]  

(A2)

where the partial derivatives of \( \alpha \) with respect to \( c_0 \) and \( \pi_1 \) are given by:

\[
\frac{\partial \alpha}{\partial c_0} = -1 / \pi_1 
\]  

(A3)

\[
\frac{\partial \alpha}{\partial \pi_1} = c_0 / \pi_1^2 
\]  

(A4)

Substituting (A3) and (A4) in (A2) and using the information from the estimated ECM we can obtain an estimate of \( Var(\alpha) \). We follow the same procedure to calculate the variance of the long-run slope parameter.
## Appendix B: Robustness analysis

### Table B1: Bai and Perron (1998, 2003) tests for multiple structural breaks, onshore ex post real rates, starting at 1973

| Country | $UD_{\text{max}}$ | $WD_{\text{max}}$ | $\text{Sup}(1|0)$ | $\text{Sup}(2|1)$ | $\text{Sup}(3|2)$ | $\text{Sup}(4|3)$ | $\text{Sup}(5|4)$ | Structural Breaks |
|---------|-------------------|-------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| Canada  | 14.74 **          | 19.17 ***         | 14.74 **         | 9.39             | 5.59             | 3.83             | 2.06             | 1996Q1           |
| Germany | 13.24 *           | 17.60 **          | 13.24 *          | 12.30 *          | 11.02            | 11.02            | 6.43             | 1979Q1, 1992Q4   |
| Italy   | 8.78              | 13.64 *           | 8.09             | 6.36             | 24.59 ***        | 15.45 *          | 3.05             | 1978Q4, 1984Q1, 1992Q2, 1997Q3 |
| Japan   | 7.62              | 9.74              | 7.62             | 6.39             | 4.16             | 1.92             | 1.92             | No breaks        |
| U.K.    | 15.48 **          | 16.21 **          | 15.48 **         | 18.38 ***        | 10.55            | 10.55            | -                | 1985Q2, 1993Q1   |

Note: The sample period is 1973Q1-2008Q2. Structural breaks were allowed for in the intercept and the coefficients associated with $r_{t-1}$ and $r_{t-1}$. $UD_{\text{max}}$ and $WD_{\text{max}}$ denote the BP double maximum test statistics for the null hypothesis of no structural breaks versus the alternative of an unknown number of breaks given some upper bound $M$. The reported $WD_{\text{max}}$ corresponds to the 5% level of significance. $\text{Sup}(l|l-1)$ denotes the BP test for $l$ versus $l+1$ breaks. - indicates that given the location of the breaks from the global optimization there was no more place to insert an additional break given the minimal length requirement. Critical values for the tests are provided in BP. ***, * indicate rejection of the null hypothesis at the 1, 5, 10% level of significance.

### Table B2: Bai and Perron (1998, 2003) tests for multiple structural breaks, onshore ex ante real rates

| Country | $UD_{\text{max}}$ | $WD_{\text{max}}$ | $\text{Sup}(1|0)$ | $\text{Sup}(2|1)$ | $\text{Sup}(3|2)$ | $\text{Sup}(4|3)$ | $\text{Sup}(5|4)$ | Structural Breaks |
|---------|-------------------|-------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| Canada  | 20.99 ***         | 24.47 ***         | 15.05 **         | 30.42 ***        | 6.83             | 4.44             | 2.21             | 1991Q1, 1996Q2   |
| Italy   | 15.74 **          | 24.82 ***         | 9.3              | 26.92 ***        | 16.03 **         | 15.04            | 2.45             | 1980Q2, 1986Q1, 1992Q4 |
| Japan   | 11.65             | 12.91             | 11.65            | 9.78             | 10.05            | 8.12             | -                | No breaks        |

Note: See Table 2 notes.
Table B3: Bai and Perron (1998, 2003) tests for multiple structural breaks, Eurocurrency ex post real rates

| Country | $UD_{max}$ | $WD_{max}$ | $SupF(1|0)$ | $SupF(2|1)$ | $SupF(3|2)$ | $SupF(4|3)$ | $SupF(5|4)$ | Structural Breaks |
|---------|------------|------------|-------------|-------------|-------------|-------------|-------------|------------------|
| Germany | 16.11 **   | 16.39 **   | 11.16       | 5.2         | 3.34        | 3.34        | 3.34        | 1992Q4           |
| Italy   | 17.09 **   | 19.21 ***  | 12.98       | 15.24 **    | 5.82        | 5.82        | 5.82        | 1983Q2, 1997Q3   |
| Japan   | 4.17       | 4.87       | 2.65        | 5.69        | 1.88        | -           | -           | No breaks        |
| U.K.    | 6.75       | 10.43      | 5.59        | 7.06        | 10.13       | 13.29       | -           | No breaks        |

Note: The sample period is 1975Q1-2008Q2 for all cases apart from Italy and Japan where it is 1978Q3-2008Q2. See also Table 2 notes.
### Table 1: Descriptive statistics, G7 real interest rates

<table>
<thead>
<tr>
<th></th>
<th>Canada</th>
<th>France</th>
<th>Germany</th>
<th>Italy</th>
<th>Japan</th>
<th>U.K.</th>
<th>U.S.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.68</td>
<td>0.67</td>
<td>0.61</td>
<td>0.69</td>
<td>0.01</td>
<td>0.58</td>
<td>0.36</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.91</td>
<td>0.81</td>
<td>0.63</td>
<td>1.24</td>
<td>1.09</td>
<td>1.32</td>
<td>0.75</td>
</tr>
<tr>
<td>JB</td>
<td>0.11</td>
<td>0.29</td>
<td>0.85</td>
<td>0.00 ***</td>
<td>0.00 ***</td>
<td>0.00 ***</td>
<td>0.67</td>
</tr>
<tr>
<td>ACF(1)</td>
<td>0.66</td>
<td>0.77</td>
<td>0.43</td>
<td>0.71</td>
<td>0.37</td>
<td>0.48</td>
<td>0.67</td>
</tr>
</tbody>
</table>

Note: The sample period is 1970Q1-2008Q2. ACF(1) denotes the first-order autocorrelation coefficient. JB denotes the \( p \)-value from the Jarque-Bera test for the null hypothesis that the series is normally distributed. *** indicates rejection of the null at the 1\% level of significance.
Table 2: ARDL model estimates and bounds test for level relationship

<table>
<thead>
<tr>
<th>Country</th>
<th>Estimated equation</th>
<th>$Ad. R^2$</th>
<th>SBC</th>
<th>$Q(4)$</th>
<th>Het</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>$\Delta r_t = 0.12 - 0.4r_{t-1} + 0.37 r_{t-1}^* + 0.74 \Delta r_{t-1}^* - 0.11 \Delta r_{t-2}$</td>
<td>0.51</td>
<td>1.61</td>
<td>2.11</td>
<td>0.96</td>
<td>17.6</td>
</tr>
<tr>
<td></td>
<td>(2.33) (5.92) (4.37) (9.59) (1.74)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>$\Delta r_t = 0.08 - 0.25r_{t-1} + 0.22r_{t-1}^* + 0.4\Delta r_{t-1}^* - 0.24\Delta r_{t-1}$</td>
<td>0.31</td>
<td>1.38</td>
<td>4.1</td>
<td>4.76</td>
<td>9.6</td>
</tr>
<tr>
<td></td>
<td>(1.58) (4.23) (3.38) (5.89) (3.34)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td>$\Delta r_t = 0.27 - 0.58r_{t-1} + 0.21r_{t-1}^* + 0.3\Delta r_{t-1}^* - 0.15\Delta r_{t-1}^* + 0.3\Delta r_{t-4}$</td>
<td>0.46</td>
<td>1.53</td>
<td>1.60</td>
<td>1.22</td>
<td>30.6</td>
</tr>
<tr>
<td></td>
<td>(4.79) (7.83) (3.14) (4.14) (2.07) (5.04)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Italy</td>
<td>$\Delta r_t = -0.03 - 0.16r_{t-1} + 0.25r_{t-1}^* + 0.26\Delta r_{t-1}^* - 0.27\Delta r_{t-1} + 4.64D_{1965Q3} + 3.04D_{1980Q2}$</td>
<td>0.43</td>
<td>2.36</td>
<td>6.84</td>
<td>9.13</td>
<td>3.2</td>
</tr>
<tr>
<td></td>
<td>(0.44) (2.26) (2.28) (2.31) (3.83) (6.10) (4.05)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>$\Delta r_t = -0.06 - 0.61r_{t-1} + 0.39r_{t-1}^* + 0.25\Delta r_{t-1}^* - 0.33\Delta r_{t-1} + 0.17\Delta r_{t-4} - 3.1D_{1975Q2} - 6.54D_{1974Q1} - 2.17D_{1975Q2}$</td>
<td>0.78</td>
<td>1.92</td>
<td>6.50</td>
<td>9.75</td>
<td>53.3</td>
</tr>
<tr>
<td></td>
<td>(1.02) (10.32) (5.02) (2.89) (6.74) (4.29) (5.34) (11.19) (3.83)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>$\Delta r_t = 0.08 - 0.36r_{t-1} + 0.43r_{t-1}^* + 0.36\Delta r_{t-1}^* - 0.43\Delta r_{t-1} - 0.33\Delta r_{t-2} - 0.39\Delta r_{t-3} + 0.16\Delta r_{t-4} - 4.85D_{1975Q2} + 3.84D_{1980Q3}$</td>
<td>0.70</td>
<td>2.48</td>
<td>1.74</td>
<td>14.2</td>
<td>13.1</td>
</tr>
<tr>
<td></td>
<td>(1.12) (4.88) (4.02) (3.19) (5.25) (4.18) (5.10) (2.40) (6.26) (4.88)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The sample period is 1970Q1-2008Q2. $D_{dum}$ denotes outlier dummy variables which take the value of one at the given date and zero otherwise. Figures in parentheses denote the absolute value of the $t$-statistic for the null hypothesis the coefficient is equal to zero. $Ad. R^2$ denotes the Adjusted $R^2$. SBC indicates the Schwarz Bayesian Information Criterion. $Q(4)$ denotes the Ljung-Box test statistics for serial correlation of order 4. Het denotes the $(nR^2)$ Breusch-Pagan-Godfrey test statistic for heteroscedasticity. $F$ denotes the $F$-statistic for the joint null hypothesis $\pi_1 = \pi_2 = 0$. The 10%, 5%, 1% critical values for the $F$-test are equal to 4.78, 5.73, 7.84, respectively, when all series are $I(1)$. The 10%, 5%, 1% critical values for the $F$-test are equal to 4.04, 4.94, 6.84, respectively, when all series are $I(0)$. Critical values are provided in Pesaran et al. (2001) Table CI(iii).
Table 3: Implied long-run coefficients and tests for real interest rate equalisation

| Country | $\alpha$ | $\beta$ | $|t_\alpha|$ | $|t^0_\beta|$ | $|t^1_\beta|$ | $W$ |
|---------|----------|---------|--------------|--------------|--------------|-----|
| Canada  | 0.31     | 0.93    | 2.62 ***     | 6.02 ***     | 0.48         | 2.98 * |
| France  | 0.31     | 0.90    | 1.83 *       | 3.96 ***     | 0.45         | 1.4  |
| Germany | 0.47     | 0.37    | 5.99 ***     | 3.44 ***     | 9.4 ***      | 13.58 *** |
| Japan   | -0.09    | 0.63    | 1.03         | 5.74 ***     | 3.34 ***     | 7.86 *** |
| U.K.    | 0.23     | 1.21    | 1.18         | 4.61 ***     | 0.79         | 1.86 |

Note: The sample period is 1970Q1-2008Q2. $t_\alpha$ denotes the $t$-statistic for the null hypothesis $\alpha = 0$. $t^0_\beta$ denotes the $t$-statistic for the null hypothesis $\beta = 0$. $t^1_\beta$ denotes the $t$-statistic for the null hypothesis $\beta = 1$. The standard errors that we used to calculate the $t$-tests were computed using the Delta method. $W$ denotes the Wald statistic for the joint null hypothesis $\alpha = 0 \& \beta = 1$. ***, **, * indicate rejection of the null hypothesis at the 1, 5, 10% level.
Table 4: Bai and Perron (1998, 2003) tests for multiple structural breaks

| Country  | $UD_{max}$ | $WD_{max}$ | SupF(1|0) | SupF(2|1) | SupF(3|2) | SupF(4|3) | SupF(5|4) | Structural Breaks          |
|---------|------------|------------|----------|----------|----------|----------|----------|-----------------------------|
| Canada  | 13.9 *     | 17.25 **   | 8.53     | 21.87 ***| 7.12     | 4.39     | 3.57     | 1977Q4, 1996Q1              |
| France  | 27.85 ***  | 33.43 ***  | 13.64 *  | 33.62 ***| 5.31     | 2.84     | 1.23     | 1983Q1, 1995Q3              |
| Germany | 17.00 **   | 20.64 ***  | 15.81 ** | 18.48 ***| 5.93     | 7.84     | -        | 1979Q1, 1995Q4              |
| Italy   | 10.87      | 15.43 *    | 7.73     | 6.34     | 18.53 ** | 18.53 ** | -        | 1979Q3, 1985Q2, 1991Q4, 1997Q3 |
| Japan   | 10.92      | 10.92      | 10.92    | 8.12     | 3.32     | 1.87     | 1.94     | No breaks                   |
| U.K.    | 18.83 ***  | 18.83 ***  | 18.83 ***| 18.38 ***| 10.55    | 10.55    | -        | 1985Q2, 1993Q1              |

Note: The sample period is 1970Q1-2008Q2. Structural breaks were allowed for in the intercept and the coefficients associated with $r_{t-1}$ and $r_{t-1}^+$. $UD_{max}$ and $WD_{max}$ denote the BP double maximum test statistics for the null hypothesis of no structural breaks versus the alternative of an unknown number of breaks given some upper bound $M$. The reported $WD_{max}$ corresponds to the 5% level of significance. SupF($l+1|l$) denotes the BP test for $l$ versus $l+1$ breaks. - indicates that given the location of the breaks from the global optimization there was no more place to insert an additional break given the minimal length requirement. Critical values for the tests are provided in BP. ***, * indicate rejection of the null hypothesis at the 1, 5, 10% level of significance.
Figure 1: Bounds test across regimes

Canada

France

Germany
Note: The plots show the $F$-test statistic of Pesaran et al. (2001) and the 5% level of significance critical bound. The regimes are based upon break dates located using the Bai and Perron (1998, 2003) tests.
Figure 2: Long-term slope across regimes

Canada

France

Germany
Note: The plots show the long-run slope, $\beta$, and the associated 95% confidence interval, calculated using the Delta method standard errors. The regimes are based upon break dates located using the Bai and Perron (1998, 2003) tests.
Figure 3: Long-term intercept across regimes

Canada

France

Germany
Note: The plots show the long-run intercept, $\alpha$, and the associated 95% confidence interval, calculated using the Delta method standard errors. The regimes are based upon break dates located using the Bai and Perron (1998, 2003) tests.
Figure 4: Wald statistic across regimes

Canada

- W-Statistic
- - - 5% CV

France

Germany
Note: The plots show the Wald test statistic for the joint null hypothesis $\alpha = 0 \& \beta = 1$, and the associated 5% level of significance critical value. The regimes are based upon break dates located using the Bai and Perron (1998, 2003) tests.