

# 3-Regime symmetric STAR modeling and exchange rate reversion

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## Abstract

The breakdown of the Bretton Woods system and the adoption of generalized floating exchange rates ushered in a new era of exchange rate volatility and uncertainty. This increased volatility lead economists to search for economic models able to describe observed exchange rate behavior. In the present paper we propose more general STAR transition functions which encompass both threshold nonlinearity and asymmetric effects. Our framework allows for a gradual adjustment from one regime to another, and considers threshold effects by encompassing other existing models, such as TAR models. We apply our methodology to three different exchange rate data-sets, one for developing countries, and official nominal exchange rates, the second emerging market economies using black market exchange rates and the third for OECD economies.

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# 1 Introduction

The breakdown of the Bretton Woods system and the adoption of generalised floating exchange rates ushered in a new era of exchange rate volatility and uncertainty. This increased volatility lead economists to search for economic models able to describe observed exchange rate behavior. Purchasing Power Parity (hereafter PPP) is often the relationship economists first turn to when trying to explain longer run exchange rate behavior and as a consequence it is probably one of the most investigated international parity conditions. Early empirical tests of PPP used linear models and were based on variants of the Dickey-Fuller (DF) regression. The empirical evidence from such "first generation" tests of PPP essentially failed to find much supportive evidence (see Meese and Rogoff (1988) and Mark (1990)). As an alternative, the empirical analysis of PPP shifted to testing for cointegration between nominal exchange rates and relative prices. For example, Lothian and Taylor (1996) argued that the lack of empirical evidence in favour of PPP was due to the low power of unit root tests in small samples. Following Lothian and Taylor (1996) researchers employed longer spans of data and found, in some cases, evidence supporting PPP. Engel (2000), however, criticised this approach since it involved using data spanning different exchange rates regimes and demonstrated that it can generate spurious rejection of the null hypothesis of a unit root.

"Second generation" tests of PPP advocated a different approach. Since the main problem with unit root tests is their lack of power in small samples, such second generation tests suggested pooling data together using both time series and cross sectional dimensions. The literature employing panel unit root and cointegration methods grew very rapidly producing consistent evidence in favour of PPP. However, O'Connell (1998) questioned this approach and showed that the empirical evidence of PPP from panel unit root and cointegration tests mainly arose from neglecting cross sectional dependence.

The econometric approaches noted above have considered PPP within a linear framework. However, there are now reasons to believe that the exchange rate is not in fact driven by a linear stochastic process. For example, Dumas (1992), Secru et al. (1995) and Berka (2004) show that transaction costs can create a band of inaction when the marginal cost of arbitrage exceeds the marginal benefit. In this circumstance, the existence of transaction costs and other impediments to trade - such as transportation costs, tariffs and quotas in international trade - drives a wedge between prices in different locations. That is, when the marginal benefit is greater than the cost in absolute value, trade takes place to exploit evident profit opportunities and PPP deviations are corrected. On the other hand, when the

marginal benefit is smaller than the marginal cost in absolute value, no trading takes place and PPP deviations are not corrected. In other words, in the presence of transactions costs, deviations from PPP will be non-equilibrium-reverting as long as they are smaller than the cost, and equilibrium reverting once they exceed costs. Based on this condition, the theoretical work cited above stresses the importance of these costs in modelling deviations from the equilibrium and provides a theoretical framework for nonlinear models used in empirical work.

Following more or less the same theoretical argument, many empirical models have implemented nonlinear adjustment for real exchange rates. For example, Obstfeld and Taylor (1997) and Sarno et al. (2004) employ a threshold autoregressive (hereafter TAR) model and Michael et al. (1997), Sollis et al. (2002), and Kapetanios et al. (2003) use smooth transition autoregressive (hereafter STAR) models. Within such frameworks, the nonlinear dynamics of the adjustment process can capture the effect of transaction costs. In a TAR model, an inaction bound is considered within which the exchange rate follows a random walk process. Outside the threshold, a symmetric type of adjustment takes place. One of the few papers which takes a different approach is Sollis et al. (2002), who allows for asymmetric mean reversion. However key main problem with the STAR models is that they only consider a narrow ‘inner’ regime, while assumptions underpinning PPP would suggest a ‘neutral’ band.

Michael et al. (1997) argued that non-linear exchange rates models should consider a symmetric type of mean reversion because adjustments to deviations from PPP should be the same for both positive and negative deviations from equilibrium. However, Sollis et al. (2002) demonstrates empirically that estimates show stronger mean reversion when the real exchange rate is below the mean than when it is positive. An explanation for this could run along the following lines. Persistent and large deviations from PPP can have important implications for a country’s competitiveness and its net exports. In instances where a currency is overvalued governments are much more likely to intervene in foreign exchange markets and /or use interest rate changes to affect the potentially deleterious effect on competitiveness than they are when the currency is undervalued. These empirical results show the necessity of considering asymmetric effects together with an inaction band when modeling the nonlinear dynamics of PPP.

The contributions of this paper are threefold. First, we propose more general STAR transition functions which encompass both threshold nonlinearity and asymmetric effects. Our framework allows for a gradual adjustment from one regime to another, and considers threshold effects by encompassing other existing models, such

as TAR models. We allow the processes to follow a unit root in the band of inaction and test it against the alternative of a globally stationary STAR, by extending the infimum  $t$ -test recently suggested by Park and Shintani (2005). Second, we present some Monte Carlo simulations and show that the test has good size and power. Finally, we apply the proposed test to two different exchange rate data-sets, one for developing countries, and official nominal exchange rates, and the second for emerging market economies using black market exchange rates. Much of the extant testing of PPP has involved using data from developed industrial countries and little if any has been conducted using data from emerging market countries. The work that has been conducted uses official exchange rates and as Reinhart and Rogoff (2002) note, such rates can be profoundly misleading as they are unlikely to be market determined. However, one of the unique features of emerging markets economies is that they have very well developed black markets for foreign exchange and the rates determined in these markets are fully market determined. Such black market exchange rates have a long tradition and in many cases have also been supported by governments. In fact, generally, the volume of transactions in black markets is even larger than that in the official market. Although black market exchange rates play such a major role in emerging market economies, it is surprising to note that very few papers use this major source of information to investigate real exchange rates dynamics. The present study attempts to fill the existing gap in the literature. Our results provide evidence suggesting that for several currencies, the asymmetric STAR model characterizes well deviations from PPP. In turn, these results are consistent with previous studies on transaction costs in international market arbitrage and the importance of considering asymmetric adjustment in deviations from PPP.

The remainder of this paper is organized as follows. In the next section we provide an overview of the existing analysis of real exchange rate behaviour, from the basic theory to nonlinear empirics. We also present a theoretical justification for using the information conveyed by nonlinear and multi-regime approaches. Section 3 summarizes previous empirical work using nonlinear unit root tests and then proposes our models along with the estimation method and the properties of our proposed models. The empirical results of our real exchange rate modeling using black market exchange rates are contained in section 5. Section 6 concludes the paper.

## 2 Testing for PPP

The fundamental basis of PPP is the law of one price (hereafter LOP). In a two-country setup with homogenous traded goods, the LOP states that identical goods should sell at the same price when there are no impediment to international trade, such as transportation costs and tariffs. The LOP for good  $i$  may be expressed as:

$$P_t^i = S_t P_t^{i*},$$

where  $P^i$  denotes the price of the good  $i$ ,  $S$  is the nominal exchange rate (domestic price of foreign currency) and an asterisk represent a foreign magnitude. Therefore the country's nominal exchange rate is determined as the ratio of the price levels at home and abroad. Assuming a measure for the price level,  $P_t$  and  $P_t^*$ , we can therefore write,

$$S_t = \frac{P_t}{P_t^*}.$$

This relationship implies that movements in the nominal exchange rate should be proportional to the ratio of national price levels and therefore the real exchange rate should be constant. Defining the logarithm of the real exchange rate,  $q_t$ , in the conventional way as:

$$q_t = s_t - p_t + p_t^*, \quad (1)$$

where  $s_t$  is the logarithm of nominal exchange rate (domestic price of foreign currency) and  $p_t$  and  $p_t^*$  are the logarithms of domestic and foreign price levels, respectively. Therefore the real exchange rate can be seen as a measure of deviations from PPP. In practice, empirical applications of PPP use the real exchange rate according to the above definition and aggregate national price indices. The real exchange rate can be driven away from its PPP equilibrium value due to, for example, exchange rate market intervention or non-zero interest differentials. One way of capturing this idea is to use the real exchange rate model below and test for a unit root:

$$q_t = \rho q_t + \beta + \varepsilon_t, \quad (2)$$

where  $0 < \rho < 1$  is the parameter of mean reversion, the random error term,  $\varepsilon_t$ , is normally and independently distributed over time and  $\beta$  is constant. In terms of unit root tests, the idea is to search for the stationarity of the real exchange rate. That is, since the real exchange rate can be interpreted as a deviation from PPP, a necessary condition for PPP to hold is that the real exchange rate is stationary over time and not driven by permanent shocks.

Recently, PPP researchers have attempted to incorporate non-linearities into real exchange rate behaviour. For example, in the presence of transaction costs and trade barriers, Dumas (1992) and Berka (2004) a non-linear adjustment process better describes exchange rates dynamics. In this context, traditional PPP is then defined as:

$$s_t = \pi + p_t - p_t^*$$

where  $\pi$  is the symmetric transportation costs or other impediments between the home and foreign country trade. Since the relative price fluctuates in a range  $-\pi < \frac{p_t}{p_t^*} < \pi$ , deviations from PPP are permissible as in:

$$-\pi < q_t < \pi.$$

To this argument Berka (2004) recently shows that if transportation costs depend on distance, the range of variation in the relative price will also depend on that distance. However, sunk costs may widen the band above and below that associated with simple trade restrictions. In this context, it is argued that deviations from PPP should follow a nonlinear mean-reverting process with the speed of mean reversion depending on the magnitude of the deviation from PPP.

Figure (1) graphically describes the properties of the band of inaction when  $p$  is the relative price of goods. In terms of the LOP,  $p$  can be then viewed as the real exchange rate. Figure (1) shows several important features of nonlinear exchange rates adjustment. As a function of current price, the expected change in prices is: (i) negative when the deviation from parity is positive and vice versa; (ii) a curvature near the edge suggests that larger deviations from parity imply faster adjustments; (iii) the shape of the function depends crucially on the relative risk aversion parameter. In fact, the lower the risk aversion, the less sensitive ex-ante benefits of diversification achieved by shipping. A low degree of risk aversion consequently makes rebalancing of physical capital less desirable, which implies a slower mean reversion.

Thus non-linear models better describe exchange rates dynamics and a substantial amount of empirical research has now employed them and found evidence supporting PPP. For example, Obstfeld and Taylor (1997), and Sarno et al. (2004) used TAR models. These models capture the effects of transaction costs on exchange rates dynamics. Michael et al. (1997), Sollis et al. (2002), and Kapetanios et al. (2003) use instead STAR models to capture non-mean-reverting regime. TAR and STAR models have been largely used in empirical applications and provided encouraging results supportive of PPP. However, and as already pointed out, most

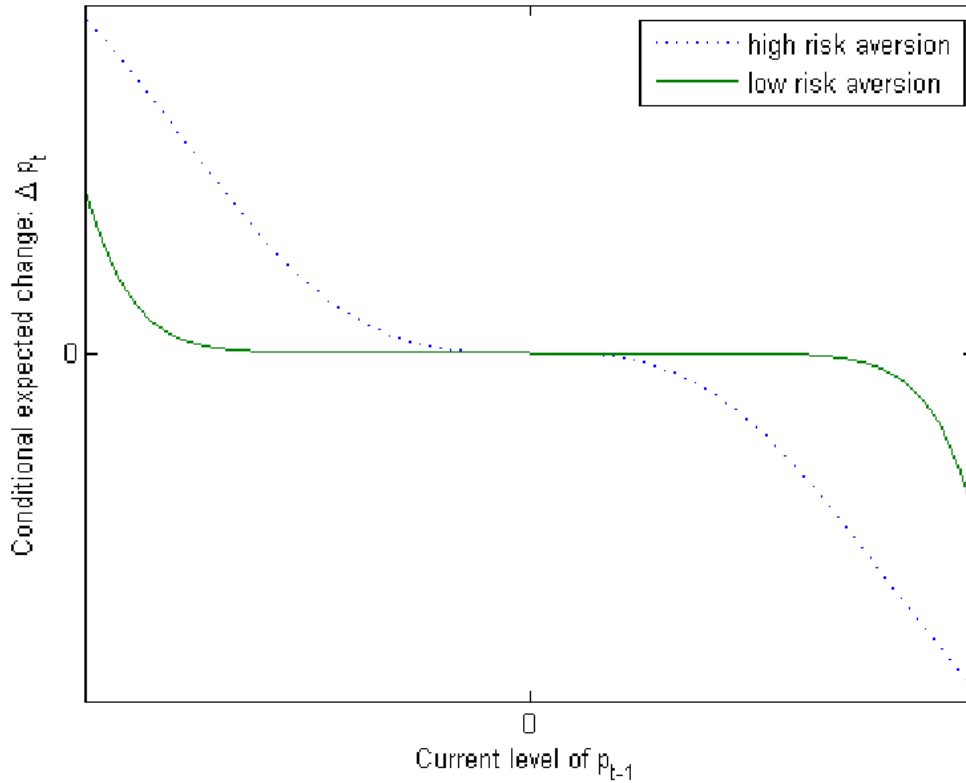


Figure 1: Conditional Expected Change of The Real Exchange Rate

of these models only consider symmetric adjustments except Sollis et al. (2002). Furthermore, STAR models only assume a narrow ‘inaction’ bound.

In the next sections we shall present a more general econometric framework which encompasses both the theoretical and empirical arguments mentioned above. We suggest a transition function which allows for threshold effects and asymmetrical adjustments when the real exchange rate is away from equilibrium.

### 3 Non-Linear Unit Root Tests

#### 3.1 The Model

Consider the following Dickey-Fuller (DF) regression

$$\Delta y_t = \beta y_{t-1} + u_t,$$

where  $y_t$  is mean corrected series and  $u_t \sim i.i.d.$ .

To accommodate non-linearity the following transition function  $S(y_{t-d}, \theta)$  is introduced. Here,  $y_{t-d}$  is the transition variable with lag delay  $d \geq 1$ ,  $\theta$  is a parameter



Model	Transition Function: $S(y_{t-d}, \theta)$	Parameter: $\theta$
ESTAR	$1 - \exp(-\gamma y_{t-d}^2)$	$\gamma$
Asymmetric STAR	$[1 + \exp\{(-\gamma_1^2 y_{t-d}^2)I_t + (-\gamma_2^2 y_{t-d}^2)(1 - I_t)\}]^{-1} - \frac{1}{2}$	$\gamma_1, \gamma_2$
3-Regime SETAR	$1\{y_{t-d} \leq c_1\} + 1\{y_{t-d} \geq c_2\}$	$c_1, c_2$

Table 1: Transition Functions

set that has to be estimated and  $S(y_{t-d}, \theta)$  is then a real value function that takes values between zero and one. The DF regression can be written as

$$\Delta y_t = \beta S(y_{t-d}, \theta) y_{t-1} + u_t, \quad (3)$$

where  $u_t \sim i.i.d.$ <sup>1</sup>

Using the DF regression above one can then test the unit root null hypothesis

$$H_0 : \beta = 0,$$

against the alternative

$$H_1 : \beta < 0.$$

The transition functions  $S(y_{t-d}, \theta)$  considered in the literature are given in Table (1). The unit root test with exponential smooth transition autoregressive (hereafter ESTAR) was suggested by Michael et al. (1997) and Kapetanios et al. (2003). In their framework, the function is bounded between 0 and 1, and its value depends on the value of the parameter  $\gamma$ . Transition between the central and outer regimes occurs with deviations of  $y_{t-d}$  from the mean,  $\mu$ , and the speed of transition increases with the value of  $\gamma$ . Specifically, when  $y_{t-d} = \mu$ , the transition function  $S(y_{t-d}, \theta)$  takes the value zero and the specification (3) follows an  $I(1)$  process. With the ESTAR the unit root regime is therefore an inner regime and mean-reversion an outer regime. This model collapses to a linear model with scale parameter,  $\gamma$ .

The asymmetric STAR was introduced in Sollis et al. (2002). The model has similar properties to the ESTAR but it allows asymmetric scale parameters,  $\gamma_1$  and  $\gamma_2$ . In addition, the transition function  $S(y_{t-d}, \theta)$  is bounded from 0 to 0.5

<sup>1</sup>In Dickey-Fuller framework,  $y_t = \lambda y_{t-1} + \varepsilon_t$ . When we consider a transition function,  $S(\cdot)$ , the model is reparameterized as

$$\Delta y_t = \phi y_{t-1} + \beta S(\cdot) y_{t-1} + \varepsilon_t$$

where  $\phi = \lambda - 1$ . Imposing  $\phi = 0$  our specification is given

$$\Delta y_t = \beta S(\cdot) y_{t-1} + \varepsilon_t$$

when the  $\gamma_1$  and  $\gamma_2$  have sufficiently large values. The fundamental properties of the asymmetric STAR movement between regimes are the same as the ESTAR function and, obviously, for  $\gamma_1 = \gamma_2$  it encompasses the symmetric model.

In a TAR model, initially proposed by Tong (1990), a change in the autoregressive structure occurs when the level of the series reaches a particular threshold value. Since the introduction of TAR models there have been several variations of them, such as the 3-regime self-excited TAR (hereafter SETAR) introduced in Kapetanios and Shin (2003). The threshold variable considered in such a model is taken to be the lagged value of the time series itself,  $y_{t-d}$ . In the central state, when  $-c_1 < y_{t-d} < c_2$ ,  $S(y_{t-d}, \theta) = 0$ , and in the limiting outer states, when  $y_{t-d} \leq c_1$  and  $y_{t-d} \geq c_2$ ,  $S(y_{t-d}, \theta) = 1$ .

### 3.2 Symmetric Transition Function

We propose a transition function that should bridge the gap between the PPP theory and the existing empirical evidence. We specify a transition function  $S(y_{t-d}, \theta)$  with a middle-regime value of  $\theta$  that occurs when  $-c < y_{t-d} < c$ . Crucially, this middle-regime is the infimum of the function, so that the process is less persistent either side of its equilibrium threshold rather than just one side. We add an indicator function to the logistic function to allow it to take certain values either sides of the threshold. Consider, for example, the Heavyside indicator function  $\mathbf{I}_t$ ,<sup>2</sup>

$$\mathbf{I}_t = \begin{cases} 1 & \text{if } y_{t-1} < 0 \\ 0 & \text{if } y_{t-1} \geq 0 \end{cases}$$

with the logistic function

$$S(y_{t-d}, \theta) = [1 + \exp\{\gamma(y_{t-d} - c)\mathbf{I}_t - \gamma(y_{t-d} + c)(1 - \mathbf{I}_t)\}]^{-1} \quad (4)$$

where the parameter set  $\theta$  includes the scale parameter  $\gamma$  and the threshold  $c$ .

The function (4) should allow for both threshold effects and smooth transition movements of  $y_{t-d}$ . In the central regime, when  $-c < y_{t-d} < c$ ,  $S(y_{t-d}, \theta) = 0$ , the random variable considered follows an  $I(1)$  process. In the limiting outer regimes, when  $y_{t-d} < -c$  and  $c < y_{t-d}$ ,  $S(y_{t-d}, \theta) = 1$  it follows an  $I(0)$  mean reverting process. The specification given by (4) allows for a random walk in the central regime and the limiting outer regime of the model is a stationary autoregression.

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<sup>2</sup>The Heavyside indicator has been used by Enders and Granger (1997) who introduced TAR methodology into Dickey-Fuller test, in which the change in autoregressive structure under the alternative hypothesis takes place instantaneously as the lagged level of the series in a standard Dickey-Fuller specification reaches a particular threshold, or not at all.

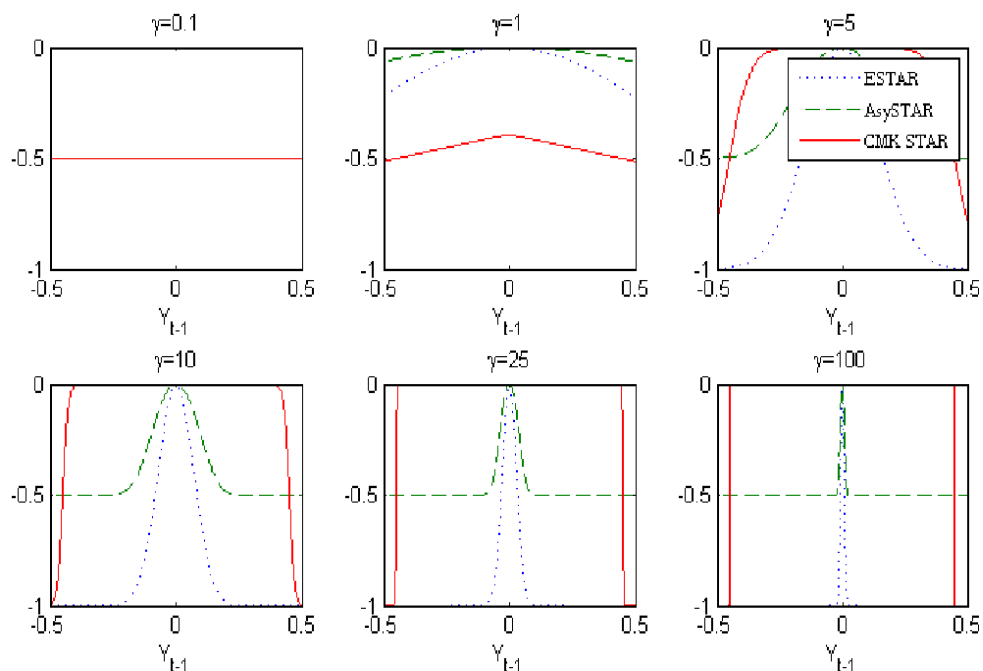


Figure 2: Properties of ESTAR, Asymmetric STAR, CMK-STAR Functions

Note that this type of approach is also consistent with a 3-regime SETAR.

### 3.3 Asymmetric Transition Function

We now consider asymmetric effects and change the transition function as follows

$$S(y_{t-d}, \theta) = [1 + \exp\{\gamma_1(y_{t-d} - c_1)\mathbf{I}_t - \gamma_2(y_{t-d} - c_2)(1 - \mathbf{I}_t)\}]^{-1} \quad (5)$$

where the parameter set,  $\theta$  includes the scale parameter  $\gamma_i$  and threshold  $c_i$  when  $i = 1, 2$ .

The desired neutral band, implied by the PPP theory, occurs when  $c_1 < y_{t-d} < c_2$ . This function is also consistent with a symmetric transition. However, if  $\gamma_1 \neq \gamma_2$  and  $c_1 \neq c_2$ , then with changes in  $y_{t-d}$ , the transition function  $S(y_{t-d}, \theta)$  is asymmetric.

To illustrate and compare the nature of our proposed models (4) and (5) with other STAR models, we perform a simulation with our CMK-STAR, ESTAR and asymmetric ESTAR. Since the parameters of an asymmetric function include that of symmetric, the functions in Figure (2) are simply plotted for the same symmetric threshold values of  $y_{t-d}$ , where  $d = 1$  with six different scale parameters  $\gamma$ . We consider a sequence of  $y_{t-1} \in [-0.5, 0.5]$ , threshold parameter  $c = 0.4$  and various values of the speed parameter  $\gamma$  ranging from 0.1 to 100. Figure (2) shows the results.

When the function moves between 0 and  $-1$  as  $y_{t-1}$  changes, the shape is determined by the size of  $\gamma$ . As expected small values of  $\gamma$ , for example,  $\gamma = 0.1$  generate slow transitions (near unit root), whereas large values, say  $\gamma = 100$ , generate rapid transitions. While all the functions tend to become flat as the scale parameter goes to zero, the exponential and CMK-STAR are close in the medium scale parameter such as 5 or 25. On the other hand, as the value of the scale parameter,  $\gamma$ , increases, the shape of the transition function become different and the CMK-STAR, as expected, tend to become discontinuous. Thus we are able to trace many observations in the immediate neighborhood of the threshold value  $c$ .

### 3.4 Estimation Method

With nonlinear models, consistent estimation of parameters can be obtained by ordinary least squares or, equivalently, maximum likelihood under the Gaussian assumption. The estimation technique begins by setting a proper grid over the parameters and at each point in the grid minimizing the residual sum of squares with respect to the remaining parameters in the model. In the presence of autocorrelation we suggest using the following modified Dickey and Fuller (1979) regression:

$$\Delta y_t = \beta S(y_{t-d}, \theta) y_{t-1} + \sum_{i=1}^p \rho_i \Delta y_{t-i} + \varepsilon_t, \quad (6)$$

where  $\varepsilon_t \sim i.i.d.$  and  $S(y_{t-d}, \theta)$  the symmetric or asymmetric function described above.

Consider for simplicity the case when  $p = 0$  in the equation above. In the central regime the model follows an  $I(1)$  process, since  $S(y_{t-d}, \theta) = 0$ . On the other hand, outside the inner regime, the model becomes  $\Delta y_t = \beta y_{t-1} + \varepsilon_t$  since  $S(y_{t-d}, \theta) = 1$ . This specification therefore allows for an  $I(1)$  central regime and the limiting outer case of the model is a stationary autoregression. The appropriate parameters to be estimated are  $\beta$ ,  $\delta$  and the parameter set of transition function,  $\theta$ .<sup>3</sup> We estimate these parameters considering various values for  $d$  in descending order and choose the value of  $d$  obtained in the model with the smallest residual sum of squares. This approach was also used in Peel et al. (2001). The coefficient,  $p$  is determined using a general-to-specific approach at the 10% level of significance.

To overcome the problem of unidentified parameters raised in Davies (1987), Leybourne et al. (1998) suggested calculating the test statistics over a grid set of possible values with summary statistics. The estimation of  $\beta$  in equation (6) can be

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<sup>3</sup>Apart from  $d$  and  $p$

obtained by using OLS as

$$\hat{\beta}(\theta) = \left( \sum_{t=1}^T x_t(\theta)x_t(\theta)' \right)^{-1} \left( \sum_{t=1}^T x_t(\theta)\Delta y_t \right),$$

with residuals  $\varepsilon_t = y_t - \hat{\beta}(\theta)'x_t(\theta)$  where  $x_t(\theta) = [S(y_{t-d}, \theta)y_{t-1}, \Delta y_{t-1}, \dots, \Delta y_{t-p}]$ .

Note that under the assumption that  $\varepsilon_t$  is normally distributed, the resulting estimates are equivalent to the maximum likelihood estimates. Finally, the parameters of interest can be estimated by the following conditional least squares,

$$\tilde{\theta} = \arg \min_{\theta} \sum_{t=1}^T (y_t - \hat{\beta}(\theta)'x_t(\theta))^2 = \arg \min_{\theta} \hat{\sigma}^2(\theta), \quad (7)$$

Leybourne et al. (1998) argue that this method reduces the dimensionality of the nonlinear least square estimation problem considerably. However, from the simulation experiments undertaken using the GAUSS OPTIMUM library, convergence was found to be difficult to achieve because of the initial value problem and parameter dimensionality in asymmetric specifications.

To circumvent local minima and parameter dimensionality problems, the estimation method used is based on the infimum  $t$ -test recently suggested by Park and Shintani (2005). Therefore, we first calculate:

$$s(\hat{\beta}(\theta)) = \hat{\sigma}^2 \left( \sum_{t=1}^T x_t(\theta)x_t(\theta)' \right)^{-1},$$

where  $\hat{\sigma}^2 = \sum_{t=1}^T \varepsilon_t^2 / (T - p - 1)$ .

The relevant infimum  $t$ -statistic is then given by

$$t(\hat{\beta}) = \frac{\hat{\beta}(\theta)}{s(\hat{\beta}(\theta))},$$

where  $s(\hat{\beta}(\theta))$  is the standard error of the estimate  $\hat{\beta}(\theta)$ . Since  $\sum_{t=1}^T x_t(\theta)x_t(\theta)'$  depends on the parameters  $\theta$ , the  $t$ -statistic is conditional. The infimum of  $t(\theta)$  is therefore taken over all values of  $\theta$ . Following Park and Shintani (2005) we define  $\hat{\theta}$  by

$$\hat{\theta} = \arg \max \left\{ t^2(\theta) \mid \hat{\beta}(\theta) < 0, \theta \right\}.$$

In the presence of unidentified parameters, the parameter values for the optimiza-

tion are obtained by grid search over  $c$  and  $\gamma$ . A meaningful set of values for the threshold parameter  $c$  is then defined as sample percentiles of the transition variable as suggested by Caner and Hansen (2001). For the threshold parameter  $c$  of the model, we therefore set the parameter space as

$$[Q(15), Q(85)], \quad (8)$$

where  $Q(15)$ ,  $Q(85)$  are the 15th and 85th percentiles of  $y_{t-d}$  respectively.

At the same time, to determine a useful set of scale parameter  $\gamma$ , Dijk et al. (2002) suggested re-scaling the transition function with the sample standard deviation, which makes  $\gamma$  approximately scale-free. That is, the transition parameter was standardized through by its sample variance. We therefore estimate the scale parameter  $\gamma$  over the interval given by:

$$[10^{-1}P_n, 10^3P_n], \quad (9)$$

where  $P_n = \left(\sum_{t=1}^n \frac{y_t^2}{n}\right)^{-\frac{1}{2}}$ .

However, the estimate of  $\gamma$  may be rather imprecise and often appears to be insignificant because of the fact that even large changes in  $\gamma_i$  only have a small effect on the shape of the transition function. As shown in the figure (2), we need to trace many observations in the immediate neighborhood of  $c$ . Therefore, at each step, the parameters set were estimated so as to maximize the sup-Wald test statistics. The combination of parameters,  $c$  and  $\gamma$  values that provide the overall maximum of the sup-Wald test statistics were then chosen as the estimated parameters for the model.

## 4 Monte Carlo Experiments

In order to clarify the advantage of our model with respect to alternatives we perform an additional simulation and compare the proposed model with representative regime switching models, such as, ESTAR and 3-regime SETAR, using a sequence of  $y_{t-1} \in [-0.5, 0.5]$ ,  $\beta = -0.3$  and, for simplicity, symmetric value of threshold parameter,  $c = 0.5$  and scale parameter,  $\gamma = 5$ .

In terms of theoretical implications, Figure (3) shows that our proposed model, CMK-STAR, most closely mimics the behavior of the real exchange rate movement predicted by Dumas (1992) and Berka (2004) when the level of relative risk aversion is low. On the other hand, the ESTAR is not able to capture these dynamics (i.e.

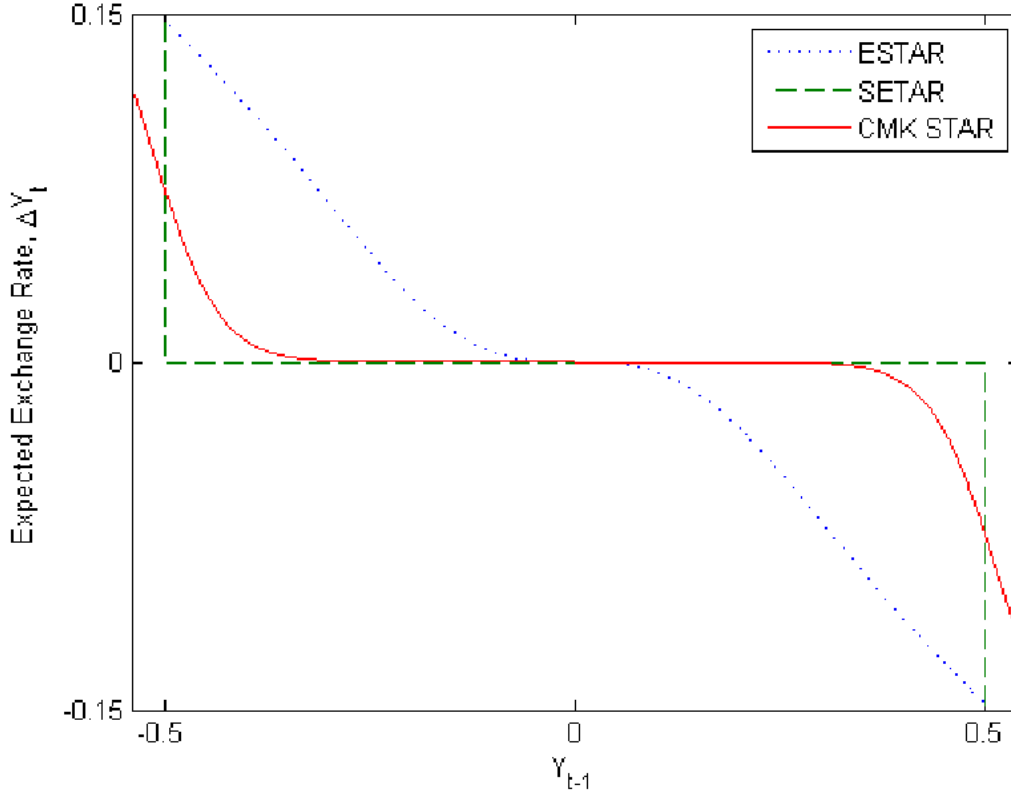


Figure 3: Simulated Conditional Expected Change Functions

the inaction bound) under any parameterization. The main limitation with 3-regime SETAR models is that the change is restricted to take place instantaneously, or not at all. That is, while the 3-regime SETAR offers an improvement over the ESTAR by considering a neutral band, it is still misspecified if the transition is gradual rather than instantaneous.

The critical values associated with our symmetric and asymmetric CMK-STAR models can be calculated using the same estimation procedure, as suggested above. The null distribution of the test was therefore simulated using Monte Carlo simulation methods under the random walk assumption. Therefore, a driftless random walk with standard normal error term,  $u_t \sim i.i.d$  was chosen as data generating process (hereafter DGP) with  $d = 1$ . A sample sizes of 1,000 observations and 10,000 replications were considered. Critical values at 1%, 5% and 10% significant levels are given in Table (2). The critical values for all of the symmetric and asymmetric tests are, in general, more negative than those for the corresponding standard Dickey-Fuller test.

We now report size and power analysis and compare our test with the DF test. For the size all results are empirical rejection frequencies from 10,000 replications

Transition function	Asymptotic Critical Values					
	1%	5%	10%	90%	95%	99%
Symmetric STAR	-3.89	-3.30	-3.02	-0.92	-0.48	0.24
Asymmetric STAR	-3.81	-3.23	-2.94	-1.02	-0.69	-0.11

Table 2: Asymptotic Critical Values

	$\rho = -0.5$			0			0.5		
	$t_{NL}^S$	$t_{NL}^{AS}$	$t_{DF}$	$t_{NL}^S$	$t_{NL}^{AS}$	$t_{DF}$	$t_{NL}^S$	$t_{NL}^{AS}$	$t_{DF}$
$k = 0$									
$T = 100$	0.4571	0.4306	0.3829	0.0643	0.0554	0.0556	0.0359	0.0359	0.0328
200	0.4660	0.4621	0.3977	0.0591	0.0509	0.0495	0.0336	0.0353	0.0324
300	0.4886	0.4660	0.3925	0.0622	0.0495	0.0512	0.0330	0.0307	0.0324
$k = 1$									
$T = 100$	0.0622	0.0491	0.0528	0.0625	0.0522	0.0552	0.0659	0.0543	0.0503
200	0.0536	0.0495	0.0508	0.0603	0.0510	0.0530	0.0608	0.0533	0.0520
300	0.0531	0.0499	0.0555	0.0547	0.0550	0.0548	0.0611	0.0492	0.0514
$k = 4$									
$T = 100$	0.0539	0.0462	0.0443	0.0556	0.0484	0.0457	0.0592	0.0494	0.0461
200	0.0516	0.0501	0.0533	0.0594	0.0464	0.0460	0.0591	0.0493	0.0437
300	0.0571	0.0452	0.0490	0.0588	0.0467	0.0461	0.0583	0.0519	0.0487

Table 3: Size of Symmetric and Asymmetric CMK-STAR

when the underlying DGP is a random walk process with serially correlated errors. Since the tests are based on demeaned data, we employ the same process here. To examine the power of the tests, we follow Park and Shintani (2005) and use the following DGP,

$$\Delta y_t = \beta S(y_{t-d}, \theta) y_{t-1} + \rho \Delta y_{t-1} + \varepsilon_t, \quad (10)$$

where  $u_t$  follows the standard normal distribution. We consider how the size is affected by the parameter  $\rho$  and consider the sample sizes 100, 200, and 300, where  $\beta = 0$  and  $\rho = \{-0.5, 0, 0.5\}$  respectively. For comparison we also report the size for the DF statistics  $t_{DF}$ . The  $t_{NL}^{AS}$  test is generally close to its nominal level at 5%. It is important to note what also reported in Sollis (2005), that is, under-fitting the number of lags lead to size distortions, while overfitting leads to smaller size distortions.

We now turn to the power analysis where use the GDP above in conjunction with the following equation

$$\Delta y_t = \phi y_{t-1} + \beta S(y_{t-d}, \theta) y_{t-1} + \varepsilon_t \quad (11)$$

where  $\phi = 0.1$  and  $\beta = -0.3$  with asymmetric parameters for  $c$  and  $\gamma$ . Overall the



Asymmetric DGP				$T = 100$			200			300		
$c_1$	$c_2$	$\gamma_1$	$\gamma_2$	$t_{NL}^S$	$t_{NL}^{AS}$	$t_{DF}$	$t_{NL}^S$	$t_{NL}^{AS}$	$t_{DF}$	$t_{NL}^S$	$t_{NL}^{AS}$	$t_{DF}$
-3.5	0.5	20	0.001	0.4340	0.5689	0.3074	0.8001	0.8803	0.5620	0.9571	0.9815	0.8502
	1.5			0.4337	0.5735	0.3141	0.7992	0.8835	0.5640	0.9554	0.9821	0.8499
	2..5			0.4359	0.5669	0.3114	0.8014	0.8831	0.5722	0.9566	0.9803	0.8401
0.5	0.1			0.1262	0.1268	0.1404	0.3271	0.3317	0.3754	0.6075	0.6447	0.7036
				0.1272	0.1348	0.1483	0.3153	0.3428	0.3806	0.6055	0.6357	0.6944
				0.1298	0.1256	0.1386	0.3178	0.3427	0.3792	0.6074	0.6495	0.7061

Table 4: Power of Symmetric and Asymmetric CMK-STAR

power of our  $t_{NL}^{AS}$  is good, and it is generally superior to the ADF test. On the other hand the ADF tests has a higher power when the time series are highly persistent.

## 5 Empirical Results

### 5.1 Linearity Test

The first step in estimating our proposed the model involves testing for linearity against STAR nonlinearity. Testing linearity against STAR-type nonlinearity implies testing the null hypothesis,  $H_0 : \beta = 0$  in equation (3). However, under the null, the parameter set,  $\theta$  is not identified. Alternatively, we could choose  $H'_0 : \gamma = 0$  as our null hypothesis in which case neither  $c$  nor  $\beta$  would be identified. A solution proposed by Luukkonen et al. (1988) and adopted by Terasvirta (1994) is to replace the transition function  $S(y_{t-d}, \theta)$  by the second order Taylor series approximation around  $\gamma = 0$ . With this linearized model, Harvey and Leybourne (2007) recently suggest a standard Wald test, denoted by  $W_T$ , which is shown to possess the usual  $\chi^2(2)$  distribution asymptotically. In this case testing for linearity is then performed by an auxiliary regression,

$$y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 y_{t-1}^2 + \beta_3 y_{t-1}^3 + \sum_{j=1}^p \beta_j \Delta y_{t-i} + \varepsilon_t, \quad (12)$$

which allow  $AR(p)$  structures.

Under the null hypothesis linearity is tested as

$$H_0 : \beta_2, \beta_3 = 0.$$

The alternative hypothesis of nonlinearity is then defined as

$$H_1 : \text{at least one of } \beta_2, \beta_3 \neq 0.$$

The test statistic is computed using the following procedure. First, estimate (12) under the null hypothesis by OLS and calculate the residual sum of squares,  $RSS_0$ . Second, using the residuals from the previous step, estimate a model that contains the regressors of (12) to compute the residual sum of squares  $RSS_1$ . The test of  $H_0$  against  $H_1$  can be then carried out using the  $W_T$ ,

$$W_T = \frac{RSS_1 - RSS_0}{RSS_0/T} \sim \chi^2(2)$$

The  $W_T$  will have an asymptotic  $\chi^2$  distribution with degree of freedom given by the number of parameter restrictions under the null hypothesis.

## 5.2 Data and Preliminary Tests

In this empirical application we use monthly data on black market nominal exchange rates and official nominal exchange rates for twenty-five and thirty-eight emerging market economies respectively. The former series are obtained from recent Cerrato and Sarantis (2007), which covers 1973:01-1998:10. The nominal exchange rate data set is retrieved from the International Monetary Fund's International Financial Statistics (IFS) over the free floating period 1980:1-2007:12. The data used are monthly nominal and black market exchange rate against US dollar and CPIs (Consumer's Price Index) for both series. We work with demeaned data measured in logs.

We begin with the official real exchange rates of thirty-eight emerging market economies, and use the standard DF test  $t_{DF}$ . The number of lags,  $p$  was determined using the general-to-specific testing strategy at the 10% level of significance, starting with  $p = 12$ . The results from the standard  $t_{DF}$  and the linearity test  $W_T$  for the real exchange rates are given in Table (5), along with the values of  $p$  for each series. The  $t_{DF}$  statistics in Table (5), suggests that the null hypothesis of a unit root is rejected only in seven out of thirty-eight countries, thus providing evidence against mean reversion.

To apply the linearity test,  $W_T$ , we select the  $AR$  order in the regression (12) using a general-to-specific methodology and a 10%-significance level, (4.605), with a maximum permitted  $AR$  order of four and a minimum order of two. We find evidence of nonlinearity for nineteen real exchange rates. Therefore half of the series analyzed exhibit evidence of nonlinearity and would suggest that nonlinear models may be

Country	Duration	$T$	$p$	$t_{DF}$	$W_T$
Asian emerging market					
India	1980:01-2007:10	334	2	-1.3941	16.355 <sup>†</sup>
Indonesia	1980:01-2007:10	334	1	-1.7208	16.931 <sup>†</sup>
Korea	1980:01-2007:10	334	2	-2.4181	38.366 <sup>†</sup>
Malaysia	1980:01-2007:10	334	1	-0.8961	6.472 <sup>†</sup>
Pakistan	1980:01-2007:10	334	2	-1.5478	3.445
Philippines	1980:01-2007:10	334	2	-1.7855	0.384
Singapore	1980:01-2007:10	334	12	-1.8375	0.199
Thailand	1980:01-2007:10	334	2	-1.3420	25.595 <sup>†</sup>
Other emerging market					
Algeria	1980:01-2007:10	334	4	-1.1741	31.469 <sup>†</sup>
Argentina	1980:01-2007:10	334	2	-2.7553 <sup>*</sup>	37.773 <sup>†</sup>
Bolivia	1980:01-2007:10	334	0	-4.1298 <sup>***</sup>	127.924 <sup>†</sup>
Botswana	1980:01-2007:10	334	2	-2.1014	3.457
Brazil	1980:01-2007:10	334	2	-2.2828	2.426
Burundi	1980:01-2007:10	334	0	-0.9976	16.158 <sup>†</sup>
Chile	1980:01-2007:10	334	2	-1.7540	0.642
Columbia	1980:01-2007:10	334	2	-1.7773	4.743 <sup>†</sup>
Costa Rica	1980:01-2007:10	334	2	-4.0042 <sup>***</sup>	3.579
Dominica Rep.	1980:01-2007:10	334	0	-2.3475	68.035 <sup>†</sup>
Egypt	1980:01-2007:10	334	12	-1.9443	1.696
El Salvador	1980:01-2007:10	334	1	-3.1679 <sup>***</sup>	27.660 <sup>†</sup>
Ethiopia	1980:01-2007:10	334	2	-1.1152	2.934
Guatemala	1980:01-2007:10	334	0	-2.0960	47.866 <sup>†</sup>
Haiti	1980:01-2007:10	334	0	-1.5733	3.171
Honduras	1980:01-2007:10	334	0	-2.5212	506.488 <sup>†</sup>
Jamaica	1980:01-2007:10	334	4	-2.0329	13.569 <sup>†</sup>
Jordan	1980:01-2007:10	334	1	-1.4372	1.812
Kenya	1980:01-2007:10	334	2	-2.3725	0.966
Madagascar	1980:01-2007:10	334	1	-1.9043	8.045 <sup>†</sup>
Malawi	1980:01-2007:10	334	2	-1.3723	3.857
Mauritius	1980:01-2007:10	334	12	-2.5105	1.741
Mexico	1980:01-2007:10	334	12	-3.6830 <sup>***</sup>	22.716 <sup>†</sup>
Morocco	1980:01-2007:10	334	1	-4.4510 <sup>***</sup>	0.236
Paraguay	1980:01-2007:10	334	1	-1.5376	0.096
Peru	1980:01-2007:10	334	0	-2.6784 <sup>*</sup>	40.029 <sup>†</sup>
South Africa	1980:01-2007:10	334	11	-2.1588	1.030
Turkey	1980:01-2007:10	334	2	-2.3503	8.755 <sup>†</sup>
Uruguay	1980:01-2007:10	334	12	-2.3618	1.616
Venezuela	1980:01-2007:10	334	0	-2.4544	3.066

Table 5: Estimated DF and Linearity Test Statistics for RER against the US Dollar

Country	Duration	$T$	$p$	$t_{DF}$	$W_T$
Asian emerging market					
India	1973:01-1998:10	307	3	-1.1732	1.057
Indonesia	1973:01-1998:10	307	5	-0.1700	17.322 <sup>†</sup>
Malaysia	1973:01-1998:10	307	0	1.3233	5.261 <sup>†</sup>
Pakistan	1973:01-1998:10	307	0	-0.9783	1.231
Philippines	1973:01-1998:10	307	0	-2.8242 <sup>*</sup>	26.798 <sup>†</sup>
Thailand	1973:01-1998:10	307	0	-1.8190	9.072 <sup>†</sup>
Other emerging market					
Argentina	1973:01-1998:10	307	0	-2.4285	69.890 <sup>†</sup>
Bolivia	1973:01-1998:10	307	0	-3.6346 <sup>***</sup>	49.978 <sup>†</sup>
Chile	1973:01-1998:10	307	2	-4.9681 <sup>***</sup>	36.599 <sup>†</sup>
Columbia	1973:01-1998:10	307	3	-1.1646	1.063
Cyprus	1973:01-1998:10	307	2	-2.6874 <sup>*</sup>	2.044
Dominica Rep.	1973:01-1998:10	307	1	-1.9126	2.633
Ecuador	1973:01-1998:10	307	1	-1.4390	4.652 <sup>†</sup>
Egypt	1973:01-1998:10	307	6	-4.9040 <sup>***</sup>	5.028 <sup>†</sup>
El Salvador	1973:01-1998:10	307	0	-1.6569	42.853 <sup>†</sup>
Ethiopia	1973:01-1998:10	307	0	-2.3821	0.271
Kenya	1973:01-1998:10	307	1	-2.5974 <sup>*</sup>	1.190
Mexico	1973:01-1998:10	307	0	-2.7611 <sup>*</sup>	14.028 <sup>†</sup>
Morocco	1973:01-1998:10	307	0	-1.4907	9.165 <sup>†</sup>
Paraguay	1973:01-1998:10	307	1	-1.3271	1.490
Peru	1973:01-1998:10	307	0	-1.6184	3.394
South Africa	1973:01-1998:10	307	0	-3.6084 <sup>***</sup>	14.228 <sup>†</sup>
Turkey	1973:01-1998:10	307	0	-2.2894	16.049 <sup>†</sup>
Uruguay	1973:01-1998:10	307	0	-1.8358	5.226 <sup>†</sup>
Venezuela	1973:01-1998:10	307	3	-1.6902	1.466

Table 6: Estimated DF and Linearity Test Statistics for BER against the US Dollar

appropriate.

Let us turn now to the black market exchange rate series. The results of the standard  $t_{DF}$  and the linearity test  $W_T$  are shown in Table (6). The standard  $t_{DF}$  rejects the null in eight series out of twenty-five countries. Furthermore the linearity test  $W_T$  shows the same results as in the previous case. Thus more than half of the series will be considered in the next section.

Note that hereafter, \*, \*\*, \*\*\* denote the 10%, 5% and 1% significance levels, respectively,  $T$  is the number of observations and  $p$  is the order of the autoregressive terms included to account for additional serial correlation in the data.

Country	Symmetric			Asymmetric				
	$ c $	$ \gamma $	$t_{NL}^S$	$c_1$	$c_2$	$\gamma_1$	$\gamma_2$	$t_{NL}^{AS}$
Asian emerging market								
India	0.2701	2886.3238	-1.7981	-0.3062	0.1498	457.4515	0.2886	-2.4539
Indonesia	0.4780	3.8123	-2.4427	-0.4780	0.1927	8.2800	0.2079	-3.0289*
Korea	0.1960	19.2553	-4.0005***	-0.1960	0.0519	25.7552	0.6469	-4.0325***
Malaysia	0.3470	3325.0853	-1.7474	-0.3470	0.1235	3325.0853	0.3325	-2.6176
Thailand	0.2998	11.8491	-1.9643	-0.2998	0.0959	15.8489	0.3981	-2.3189
Other emerging market								
Algeria	0.7045	872.9211	-2.5138	-0.2580	0.2048	1417.4289	0.1417	-1.7449
Argentina	0.4663	4.1841	-3.9073***	-0.4663	0.1180	9.0874	0.2283	-4.0130***
Bolivia	0.4265	1.6055	-6.0403***	-0.4265	0.1326	1.4571	1.4571	-5.6817***
Burundi	0.4826	4.4667	-1.5385	-0.4826	0.1568	9.7013	0.2437	-1.6170
Columbia	0.1551	3349.2495	-1.9954	-0.1615	0.1353	3349.2495	0.3349	-2.1558
Dominica Rep.	0.2264	2.1225	-2.9368	-0.1940	0.1377	3053.0825	1.9264	-2.8523
El Salvador	0.2131	3504.9674	-4.1643***	-0.2174	0.0484	0.5691	3.5909	-3.5780**
Guatemala	0.2304	4.1607	-2.7993	-0.2304	0.1129	14.6734	2.3256	-2.7500
Honduras	0.4188	5.7462	-5.0870***	-0.4188	0.0965	0.3134	1.9779	-3.9627***
Jamaica	0.2759	4.2589	-2.3892	-0.2759	0.0537	15.0197	2.3804	-2.3531
Madagascar	0.1595	558.9409	-1.9542	-0.1585	0.1936	379.2669	0.2393	-2.1119
Mexico	0.1913	5.8825	-4.2063***	-0.1913	0.0526	3.2880	3.2880	-4.1477***
Peru	0.5254	34.2456	-3.8879***	-0.5254	0.0971	10.6989	1.6956	-3.1809**
Turkey	0.1726	6.5206	-2.6542	-0.1482	0.0564	5776.4174	0.5776	-3.0397*

Table 7: Estimated Results for RER against the US Dollar

### 5.3 Application to The Real Exchange Rate

In this section we apply the symmetric and asymmetric nonlinear tests to the two data sets of exchange rates analyzed above. Table (7) reports the empirical results.

We note that now in addition to the seven rejections obtained by the  $t_{DF}$ , there are two additional rejection obtained by the  $t_{NL}^S$  test. All these rejections occur at the 1% level of significance. In particular, while for countries like Argentina and Peru rejections were at 10% level now all rejections are at the 1% significance level.

Looking at the empirical results when an asymmetric adjustment is considered, we note that there are now nine rejections. That is, there are two additional rejections that occur at the 10% significance level for Indonesia and Turkey. Thus this extension of the  $t_{NL}^S$  reveals evidence that supports long-run PPP that would not have been detected by the application of the  $t_{NL}^S$  alone.

For all emerging market countries that we have considered, Table (7) shows that the threshold range,  $c_1$ , is wider in absolute value and the speed of adjustment,  $\gamma_1$ , is greater the lower the threshold. For example, Argentina shows lower and upper thresholds of  $-0.4663$  and  $0.1180$  respectively. This indicates a higher threshold

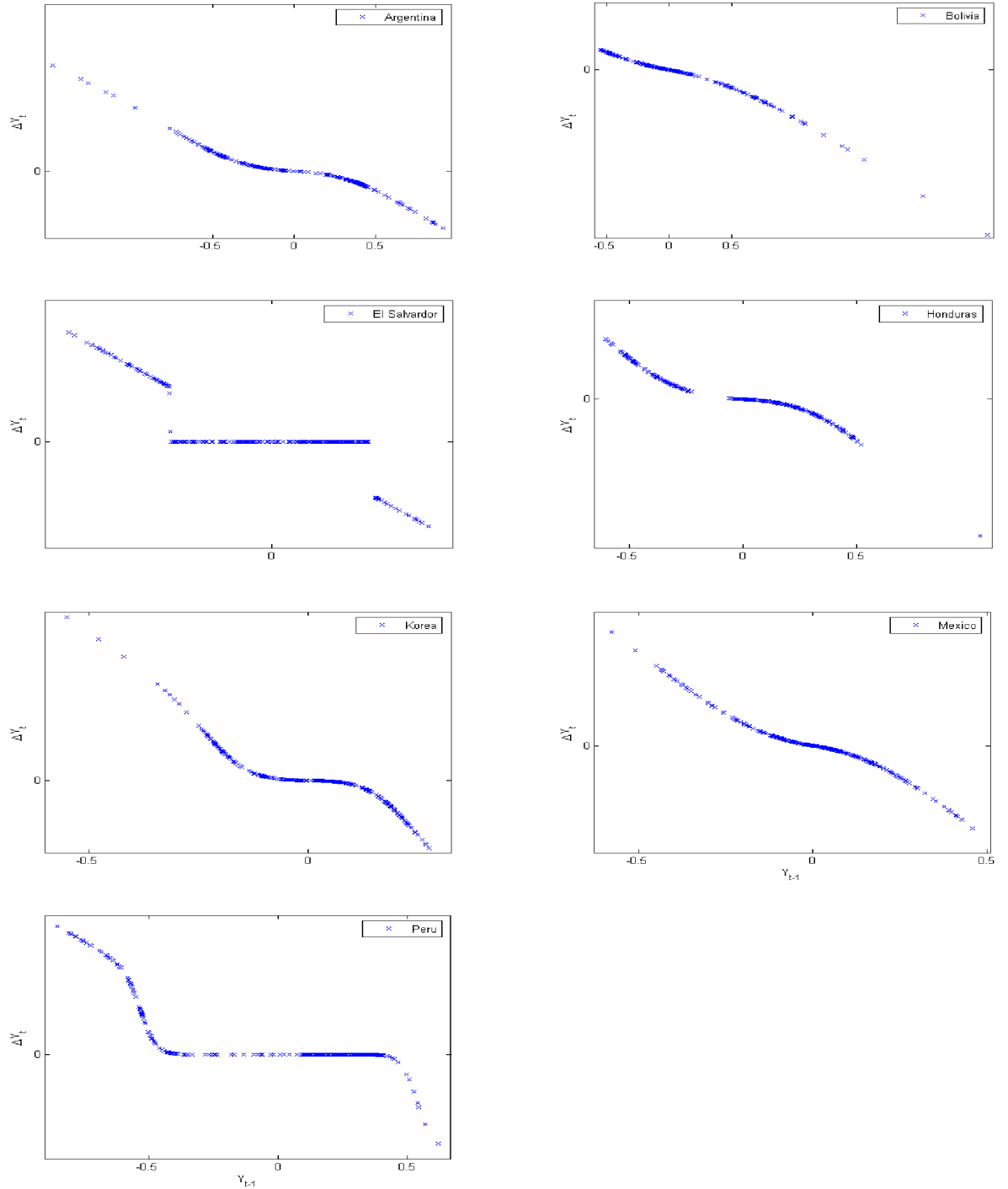


Figure 4: Symmetric CMK-STAR for RER

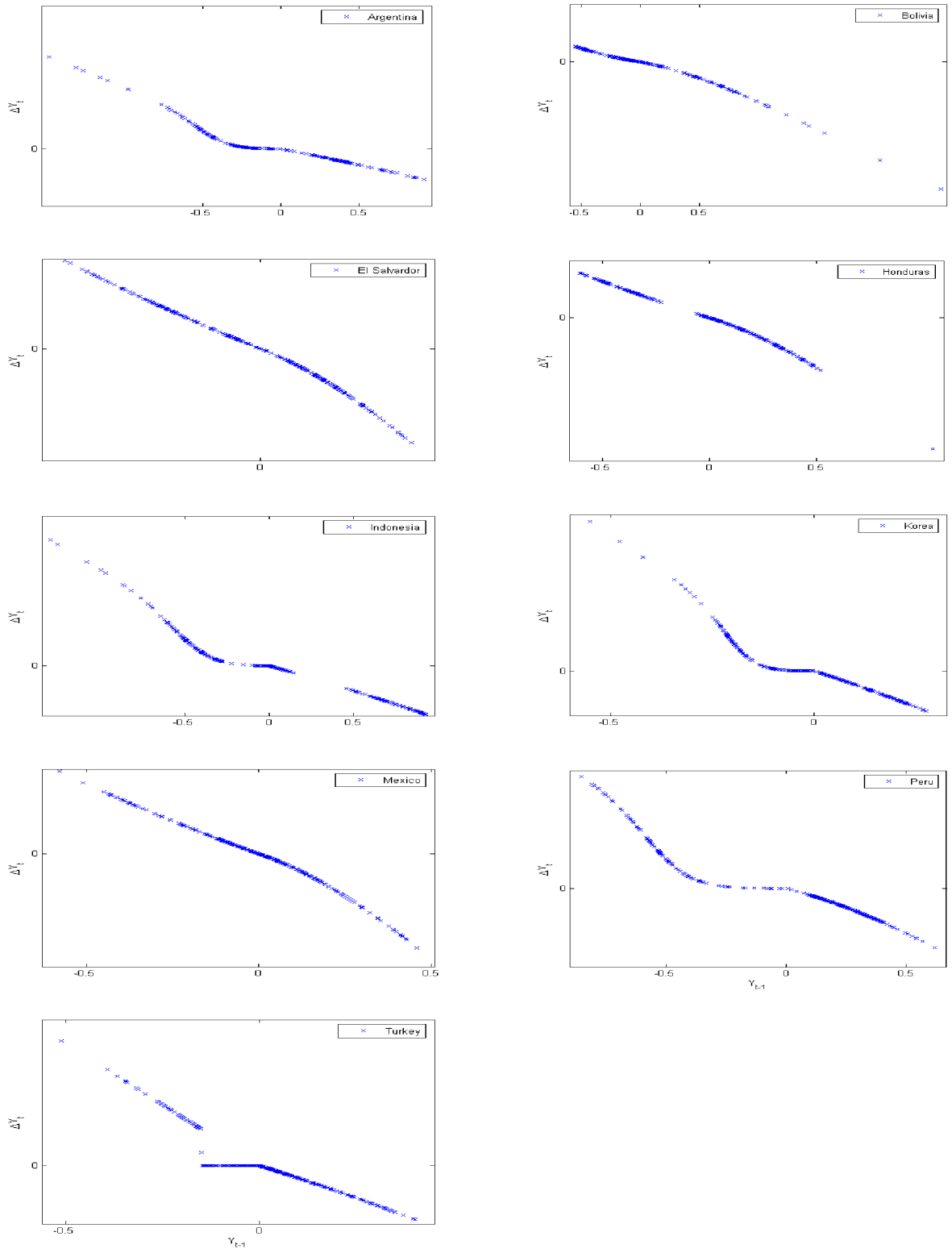


Figure 5: Asymmetric CMK-STAR for RER

Country	Symmetric			Asymmetric				
	$ c $	$ \gamma $	$t_{NL}^S$	$c_1$	$c_2$	$\gamma_1$	$\gamma_2$	$t_{NL}^{AS}$
Asian emerging market								
Indonesia	0.4097	1.5251	-0.7438	-0.4098	0.4117	1.3841	1.3841	-0.7286
Malaysia	0.0968	5202.3604	-1.2444	-0.1849	0.0598	0.5202	130.6773	0.5316
Philippines	0.1596	7.3134	-3.9706***	-0.1595	0.0454	4.0877	0.6478	-3.8151***
Thailand	0.0754	7534.9477	-2.0549	-0.0785	0.0388	1194.2087	29.9972	-2.1116
Other emerging market								
Argentina	0.6033	5.0994	-4.2822***	-0.6033	0.1836	1713.3364	0.1713	-4.3831***
Bolivia	0.2722	3.4464	-4.6984***	-0.2721	0.0865	1.9264	0.3053	-4.6503***
Chile	0.3280	4.3052	-5.8482***	-0.3280	0.1243	2348.7630	0.2349	-6.3066***
Equador	0.3505	1105.9413	-1.9414	-0.3575	0.1195	11.6087	0.2916	-2.0263
Egypt	0.1335	3.4743	-5.8779***	-0.1335	0.0499	125.5331	0.4998	-6.4986***
El Salvador	0.3772	134.2111	-2.8114	-0.3772	0.1397	61.7940	0.2460	-3.7975***
Mexico	0.3262	10.8116	-3.5277**	-0.3262	0.0814	14.4614	2.2919	-3.1889**
Morocco	0.2092	22.3533	-2.2342	-0.1493	0.0782	733.0452	2.9183	-2.1309
South Africa	0.1249	5968.2076	-4.2752***	-0.1449	0.0478	3.7656	3.7656	-4.0822***
Turkey	0.2407	4282.1032	-3.4631**	-0.2238	0.0883	4282.1032	0.4282	-3.4306**
Uruguay	0.2153	3285.5944	-2.1868	-0.2241	0.1178	82.5304	0.3286	-2.5256

Table 8: Estimated Results for BER against the US Dollar

tolerance for depreciations. The speed of adjustment is 0.2283 between the middle and upper regimes and 9.0874 from the lower to the middle regime. This indicates a quicker move between the corridor and the depreciation regimes than between the appreciation regime and the corridor. This is consistent with previous results in (e.g. Sollis et al. (2002)).

As shown in Figure (4) and Figure (5), the nature of symmetry and asymmetry from estimated results can be best illustrated by plotting the values  $y_{t-1}$  against  $\Delta y_t$  for the symmetric and asymmetric models respectively. In particular, all figures consistently show that when the rate is below the mean it shows rather faster mean reversion than when the rate is above the mean.

## 5.4 Application to Black Market Exchange Rates

To further investigate nonlinearity and asymmetry in exchange rate dynamics, we now use the black market exchange rate data set. Since non-linearity was detected in six out of eight series, in this application we have also included them. We now, additionally, reject the unit root null hypothesis in two countries, Argentina and Turkey.

We turn now to the asymmetric test. We note that in addition to the eight rejections obtained by the  $t_{NL}^S$  test, there is one additional rejection obtained by the



$t_{NL}^{AS}$ . This rejection occurs at the 1% level of significance (El Salvador). Thus this extension of the  $t_{NL}^S$  test reveals evidence that support long-run PPP that would not have been revealed by the application of the  $t_{NL}^S$  test alone.

Table (8) shows that the threshold range  $c_1$  is wider in absolute value and the speed of adjustment  $\gamma_1$  is greater in the lower threshold. As an example of this, El Salvador has lower and upper thresholds of  $-0.3575$  and  $0.1195$ , respectively. This result implies a higher threshold tolerance for depreciations. The speed of adjustment is  $0.2916$  between the middle and upper regimes, and  $11.6087$  from the lower to the middle regime. This indicates a quicker movement between the corridor and the depreciation regimes than between the appreciation regime and the corridor. These results are consistent with the RER models suggested in the literature.

Figure (6) and (7) confirm that when exchange rates are below their mean, the value of  $\Delta y_t$  is higher than when they are above their mean. Interestingly, the applications of asymmetric models to both the data sets consistently supports the argument that when the exchange rate is depreciated tend to defend the currency more vigorously.

## 5.5 Application to OECD data

To compare emerging market with developed countries, we now test the quarterly OECD countries data set. In this application, there are four rejections obtained by the  $t_{NL}^S$  test.<sup>4</sup> We note that there are only one additional rejection obtained by the  $t_{NL}^S$  test. All these rejections occur at the 5% level of significance.

In the asymmetric test, we note that in addition to the four rejections obtained by the  $t_{NL}^S$  test, there are seven additional rejections obtained by the  $t_{NL}^{AS}$  test. Most of these rejections occur at the 5% level of significance and only Netherland rejects the hypothesis at the 10% level. The additional seven countries would not have been shown by the application of the linear test,  $t_{DF}$  or symmetric test,  $t_{NL}^S$ . In particular, this extension of the  $t_{NL}^{AS}$  test reveals evidence that supports long-run PPP more than half of the data set.

Looking at the Table (9) when asymmetric test is considered, as shown in previous tests, the results show that the threshold range  $c_1$  is generally wider in absolute value except Finland and the speed of adjustment  $\gamma_1$  is consistently greater in the lower threshold. For example, while only Finland has lower and upper thresholds of  $-0.0539$  and  $0.0912$ , respectively and upper threshold  $c_2$  is slightly wider in absolute value, other results are consistently wider in lower threshold, which implies higher

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<sup>4</sup>We use quarterly data for twenty OECD economies, which covers 1973:1-1998:2. In a preliminary test, three rejections obtained by the Dickey-Fuller test.

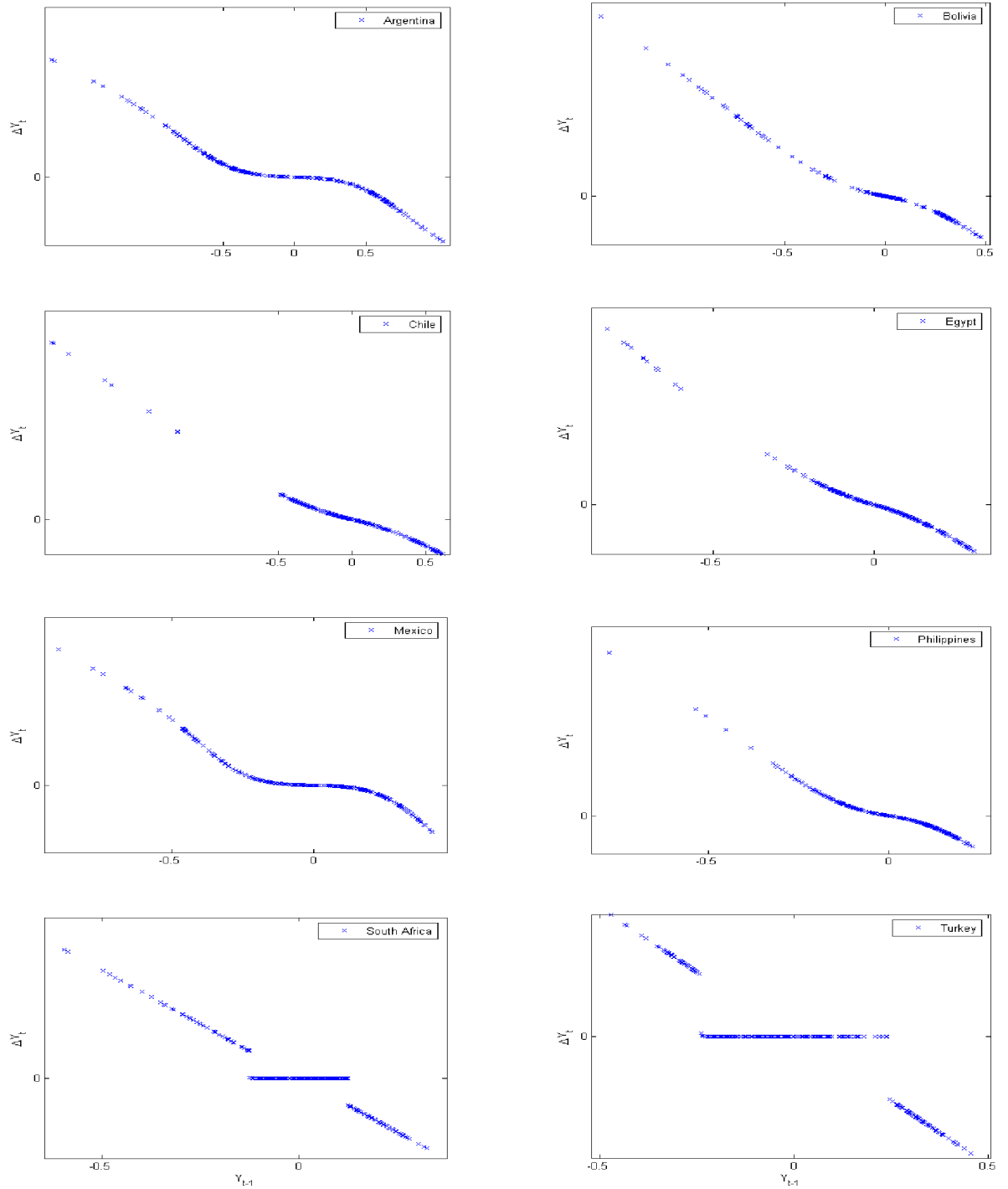


Figure 6: Symmetric CMK-STAR for PER

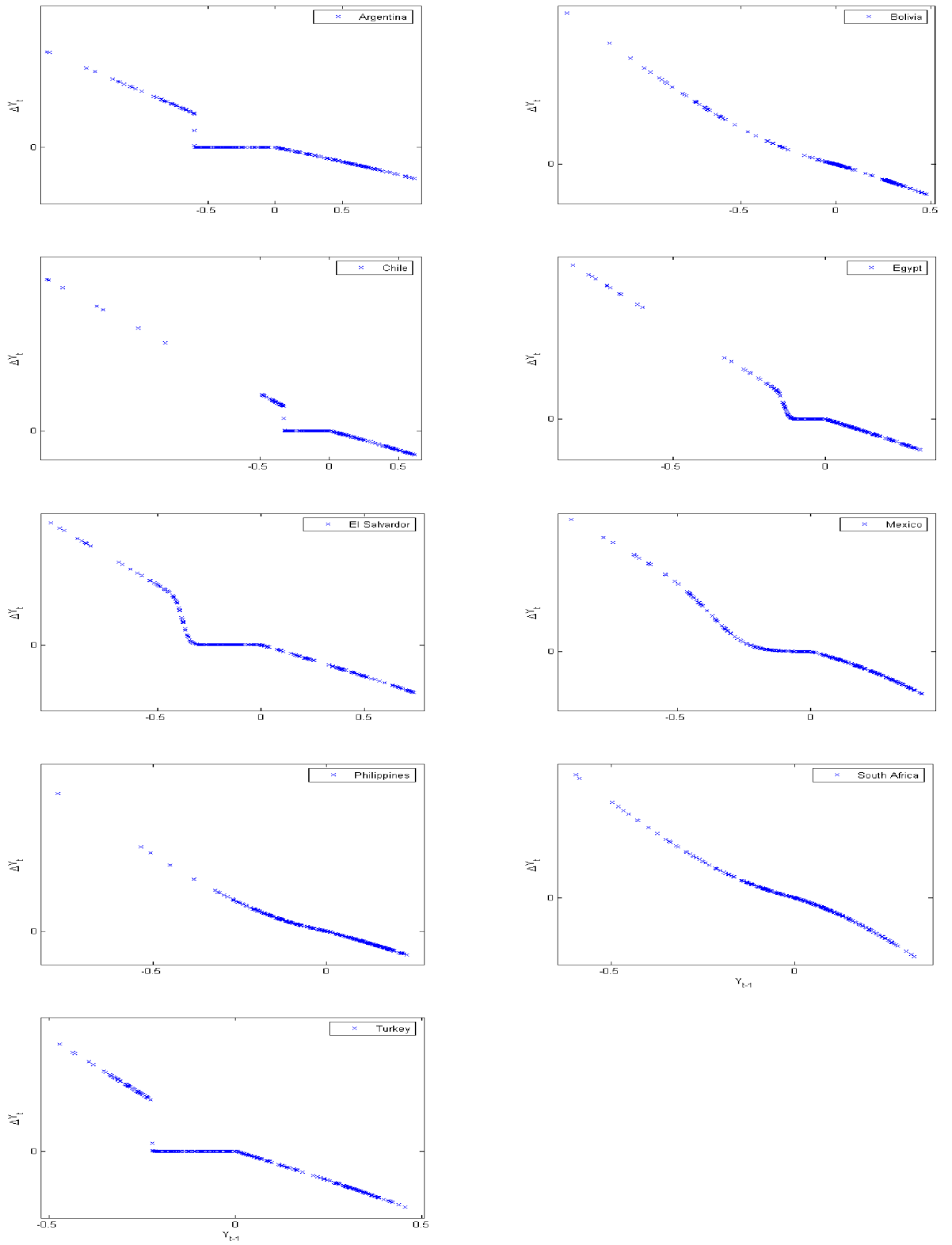


Figure 7: Asymmetric CMK-STAR for PER

Country	Symmetric			Asymmetric				
	$ c $	$ \gamma $	$t_{NL}^S$	$c_1$	$c_2$	$\gamma_1$	$\gamma_2$	$t_{NL}^{AS}$
Australia	0.1393	7665.5241	-2.1256	-0.1393	0.0331	192.5492	0.7665	-2.0108
Austria	0.1009	5916.4187	-2.8744	-0.1185	0.0552	5916.4187	0.5916	-3.3511**
Belgium	0.1033	5319.4397	-2.4927	-0.1212	0.0686	5319.4397	0.5319	-2.7931
Canada	0.0379	10418.4460	-1.7350	-0.0511	0.0303	1651.2124	1.0418	-1.7691
Denmark	0.0463	5909.9979	-2.4605	-0.0482	0.0589	936.6715	0.5910	-2.9252
Finland	0.0591	7013.7319	-3.3056**	-0.0539	0.0912	7013.7319	4.4253	-3.2300**
France	0.1389	6467.9852	-2.6823	-0.1416	0.0382	6467.9852	0.6467	-3.3699**
Germany	0.0348	6092.5821	-2.7250	-0.1133	0.0509	965.6091	0.6092	-3.2510**
Greece	0.0882	6287.4421	-2.5335	-0.1246	0.0453	6287.4421	0.6287	-2.8086
Ireland	0.0987	7806.6516	-3.0078	-0.1323	0.0386	31.0788	0.7806	-3.3801**
Italy	0.1331	7.6023	-2.6402	-0.1142	0.0465	6734.6729	0.6734	-2.8704
Japan	0.1531	4237.2333	-2.4163	-0.1439	0.1175	106.4344	2.6735	-2.2747
Netherland	0.0283	6222.7671	-2.7111	-0.0896	0.0424	6222.7671	0.6222	-3.0821*
New Zealand	0.1451	1.8134	-3.3679**	-0.0423	0.0392	1090.0543	4.3395	-3.4825**
Norway	0.0898	5096.0317	-2.6771	-0.0934	0.0322	207.8540	0.8275	-3.3408**
Portugal	0.1961	3142.2792	-2.1374	-0.2001	0.0642	5102.3599	0.5102	-2.5451
Spain	0.0508	5328.2922	-2.5063	-0.1837	0.0488	21.2123	0.5328	-2.7186
Sweden	0.1731	846.2946	-2.9879	-0.1731	0.0444	147.7838	0.5883	-3.6004**
Swiss	0.1684	5244.4175	-3.6096**	-0.1909	0.0620	131.7338	3.3090	-3.4299**
U.K.	0.0873	7315.9827	-3.5523**	-0.1356	0.0395	29.1254	0.7315	-3.7958**

Table 9: Estimated Results for OECD RER agianst the US Dollar

tolerance for depreciations and quicker movement between the corridor and the depreciation regimes. These results are the same as the RER and BER for emerging market provided in previous tests.

In Figure (8) and Figure (9), the properties of symmetry and asymmetry are graphically shown when exchange rates are appreciated or depreciated. Particularly, as shown in emerging market cases, all figures in Figure (9) except Finland show that when the rates of OECD countries are below the mean it shows rather faster mean reversion than the rate is above mean. This implies that the "dread of depreciation" is also applicable in OECD countries and not just in emerging market economies.

## 6 Conclusion

In this paper we have re-examined the PPP hypothesis using non-linear modelling methods. Although such modelling has become increasingly popular of late, we offer a number of novel features in our own work. First, we use more general STAR transition functions than have been used hitherto in the literature and these functions encompass both threshold nonlinearity and asymmetric effects. Our framework al-

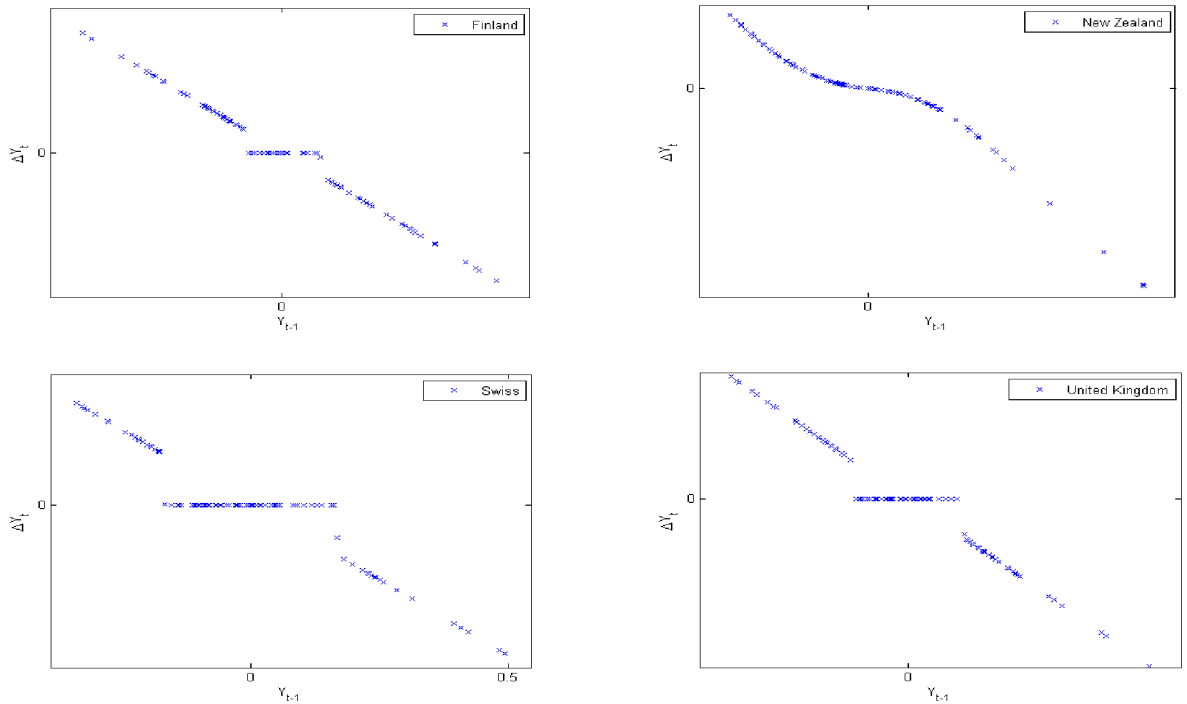


Figure 8: Symmetric CMK-STAR for OECD RER

allows for a gradual adjustment from one regime to another, and considers threshold effects by encompassing other existing models, such as TAR models. Second, we present Monte Carlo simulations which show that our test has good size and power properties. Finally, we apply the proposed test to three different exchange rate datasets, one for developing countries, using official nominal exchange rates, the second consisting of a unique data set of emerging market economies using a black market exchange rates, and the third consisting of quarterly OECD data.

Our results provide evidence suggesting that for the majority of currencies, the asymmetric STAR model characterizes well deviations from PPP. Also our empirical results supports what Dutta and Leon (2002) call the "dread of depreciation" in emerging markets. Our results are consistent with previous studies that consider the role of transaction costs in international market arbitrage, although we have used a less restrictive framework than other researchers to obtain our results.

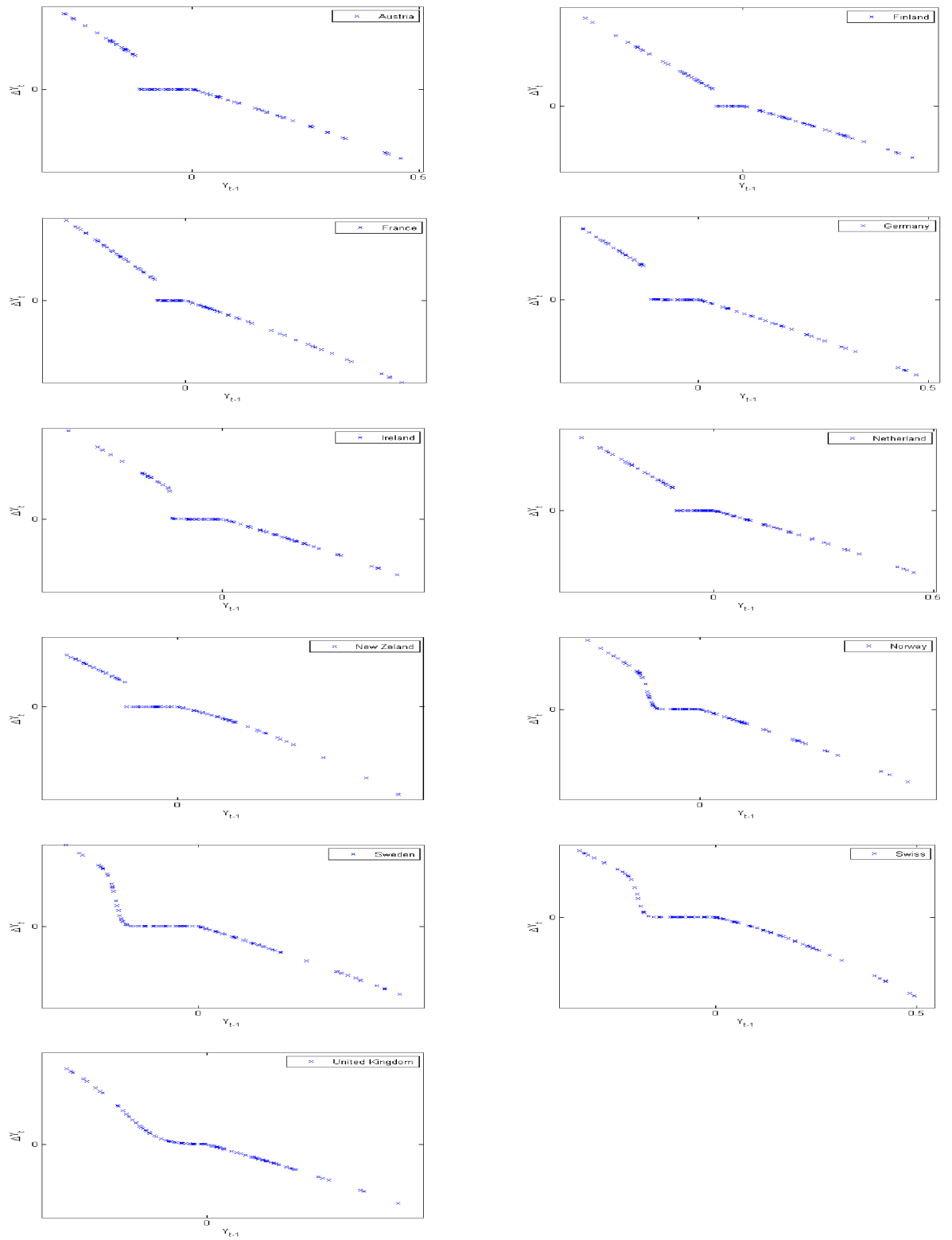


Figure 9: Asymmetric CMK-STAR for OECD RER

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