# Extending conditional-autoregressive models for space-time disease mapping

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# Acknowledgements

Work in this talk part of a larger collaboration:

| | |
|---|---|
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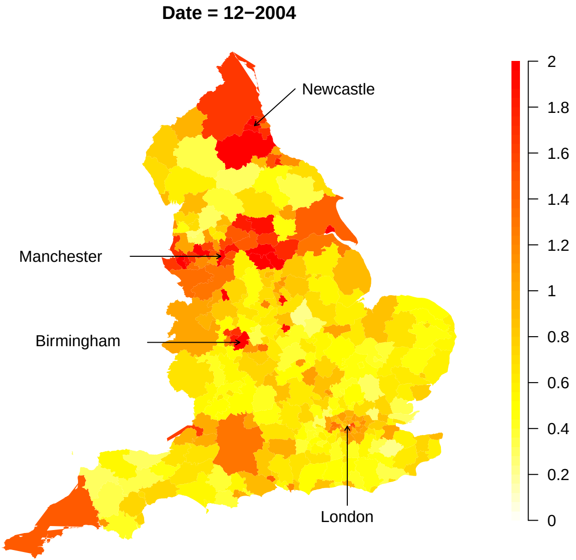
# Context: England LHAs - Respiratory admission SIR




- 123 months
- 323 Local Health Authorities (LHAs)
- January 2001 to December 2011
- 42636 data points in total

# Space-time disease mapping: why do it?

- Understanding if public health policies have an effect over time
- Understanding health inequality (spatial disease risk)
- Boundary (spatial discontinuity) detection
- Clustering of regions with similar (eg. temporal) risk attributes

# Modelling disease counts in space

Typical spatial model might look like

$$
\begin{aligned}
Y_i|E_i, R_i &\sim \mathrm{Poisson}(E_i R_i) \\
\ln(R_i) &= \beta_0 + \sum_{j=1}^{p} x_{ij}\beta_j + \phi_i, \qquad i = 1, \ldots, N
\end{aligned}
$$

Where

| | | | |
|---|---|---|---|
| $Y_i$ | disease counts | $R_i$ | disease risk |
| $E_i$ | expected cases | $\sum_{j=1}^{p} x_{ij}\beta_j$ | covariates |

$$
\pi(\phi) \propto \exp\left[-\frac{1}{2\sigma^2}\left(\sum_{i\sim j} w_{ij}(\phi_i - \phi_j)^2\right)\right]
$$

# Localised smoothing

Different strategies achieving this:

Treat $\phi$'s prior variance, $\sigma$, as spatially varying

- Brewer and Nolan (2007); Reich and Hodges (2008)

Treat the weights $w_{ij}$ as random variables

- Ma, Carlin and Banerjee (2010); Lee and Mitchell (2013)

Use a clustering or grouping prior for the random effects

- Richardson and Green (2002); Knorr-Held and Raßer (2000)

# Ma, Carlin and Banerjee (2010)

Starting with Poisson model and Intrinsic CAR for random effects:

$$
\begin{aligned}
Y_i | E_i, R_i &\sim Poisson(E_i R_i) \\
\log(R_i) &= \beta_0 + \sum_{j=1}^{p} x_{ij}\beta_j + \phi_i, \qquad i = 1, \ldots, N \\
p(\phi | \sigma, \{w_{ij}\}) &= C(\sigma, \{w_{ij}\}) \exp\left( -\frac{1}{2\sigma^2} \sum_{i \sim j} w_{ij}(\phi_i - \phi_j)^2 \right)
\end{aligned}
$$

Normalising term $C(\sigma, \{w_{ij}\})$ included to emphasise that $w_{ij}$ is now being treated as unknown.

# Ma, Carlin and Banerjee (2010); CAR2 model

**Idea:** Call the non-zero $w_{ij}$, $w_{ij}^{+}$, and treat as Bernoulli random variables with unknown probabilities, $p_{ij}$.

Transform and smooth $p_{ij}$ using a further CAR prior.

$$\begin{aligned} w_{ij}^{+}|p_{ij} &\sim Bernoulli(p_{ij}) \text{ and } \mathrm{logit}(p_{ij}) = \mathbf{z}_{ij}^{'}\boldsymbol{\gamma} + \theta_{ij} \\ p(\boldsymbol{\theta}|\zeta) &\propto \exp\left(-\frac{1}{2\zeta^2}\sum_{ij\sim kl}(\theta_{ij}-\theta_{kl})^2\right) \end{aligned}$$

**Pros:** Maintains binary nature of $w_{ij}$
**Cons:** Assumes relatively strong smoothness over the $w_{ij}$; a lot of parameters to estimate $\phi, w_{ij}, \boldsymbol{\theta}$ that can be hard to identify.

# A localised model in space and time

Poisson model for counts in space and time (indexed by $i$ and $j$, respectively)

$$
\begin{aligned}
Y_{ij}|E_{ij}, R_{ij} &\sim Poisson(E_{ij}R_{ij}) \\
\log(R_{ij}) &= \beta_0 + \phi_{ij}, \quad i = 1, \ldots, N \text{ and } j = 1, \ldots, T
\end{aligned}
$$

Letting $\boldsymbol{\phi}_t = (\phi_{1t}, \ldots \phi_{Nt})$, prior for spatial random effects is

$$
p(\boldsymbol{\phi}_t|\sigma, \{w_{ij}\}) = C(\sigma, \{w_{ij}\}) \exp\left(-\frac{1}{2\sigma^2} \sum_{i\sim j} w_{ij}(\phi_{it} - \phi_{jt})^2\right)
$$

# A localised model in space and time

**Idea:** Treat the non-zero elements of $w_{ij}$, $w_{ij}^{+}$, as random variables on $[0, 1]$.

Then transform using $logit(w_{ij}^{+}) = z_{ij}$ and smooth these using a Leroux prior.

$$p(z_{ij}) \propto \exp\left[-\frac{1}{2\tau^2}\left(\rho\sum_{ij\sim kl}(z_{ij} - z_{kl})^2 + (1-\rho)\sum_{ij}(z_{ij})^2\right)\right]$$

**Pros:** Treating $w_{ij}$ as non-binary makes things slightly easier; Leroux prior is a bit more flexible than the Intrinsic CAR model.

**Cons:** Still a lot of parameters to estimate

# A localised model in space and time - computation

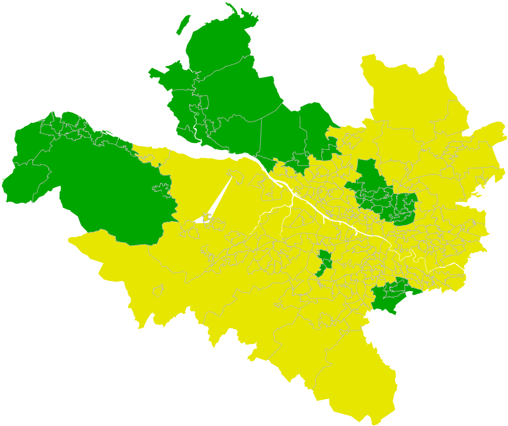
**Need to update** $\phi_{N \times T}$

- Can be done very quickly using one-at-a-time updating in C++ and exploiting matrix sparsity

**Need to update** $z_{ij}$

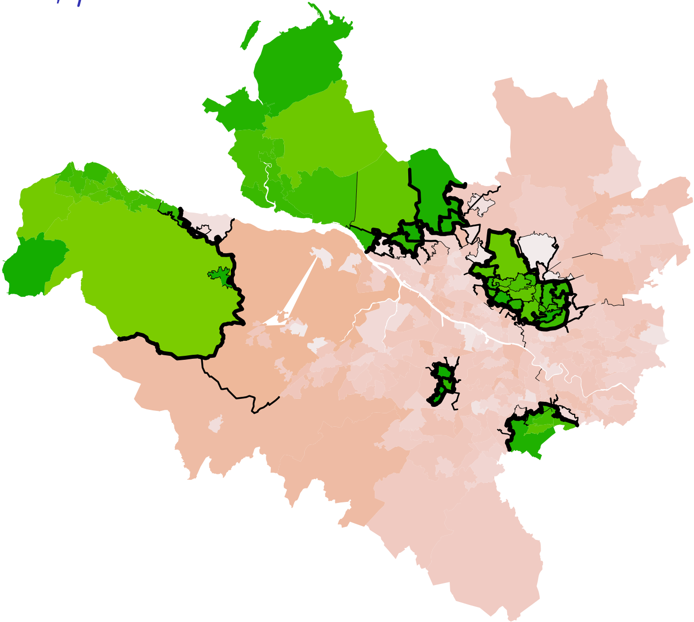
- Block updating using each blocks conditional prior variance
- Main bottleneck: recalculating determinant of $\phi$ prior precision for each block proposal
- 85% of MCMC time even with sparse Cholesky routine

# Some illustrative simulations

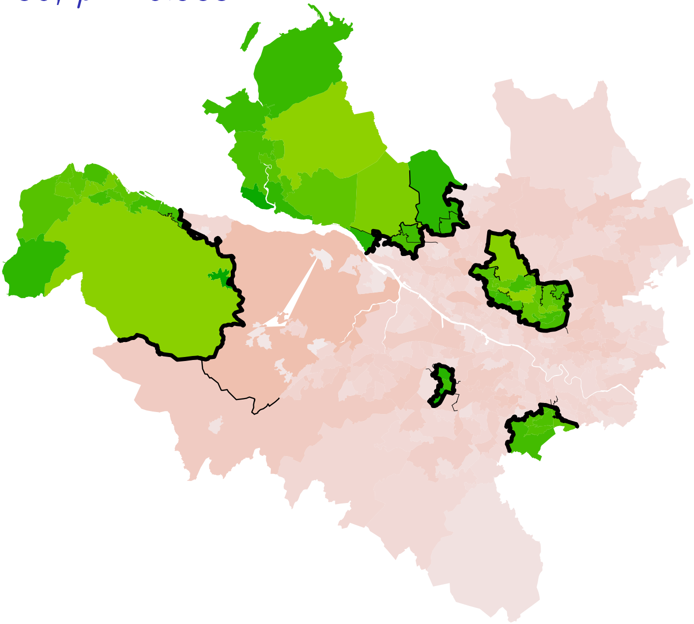


- Using Glasgow intermediate geographies as a spatial template
- To investigate effect of increasing time points, generate 10 or 50 repeated realisations of this surface
- Simulate spatial counts step-function using Poisson with mean of either 100 or 200.
- Interested in effect of $\rho$: fix at 0, or treat as unknown.

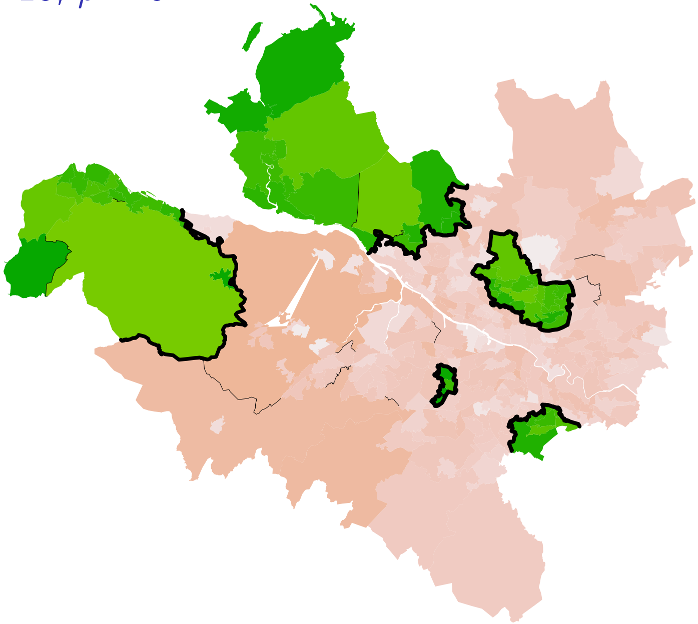
$T = 10; \rho = 0.993$



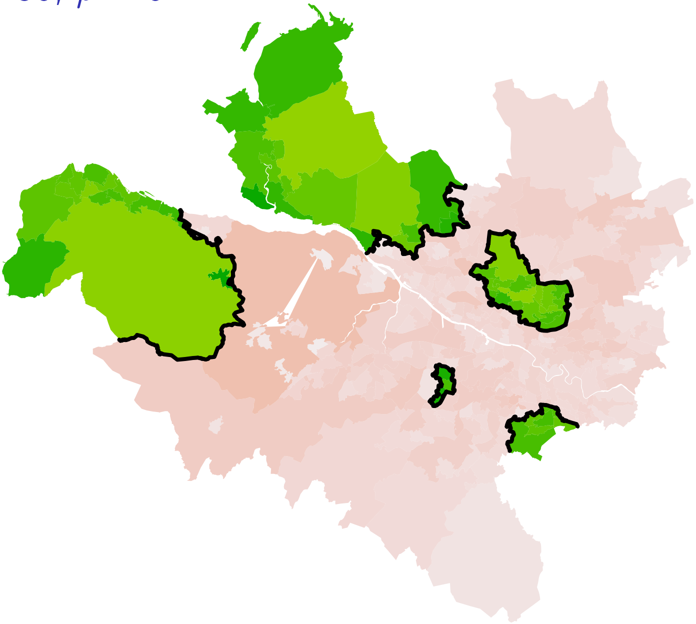
T $= 50; \rho = 0.983$



$\mathsf{T} = 10; \rho = 0$



$T = 50; \rho = 0$



# DIC and parameter estimates for each model

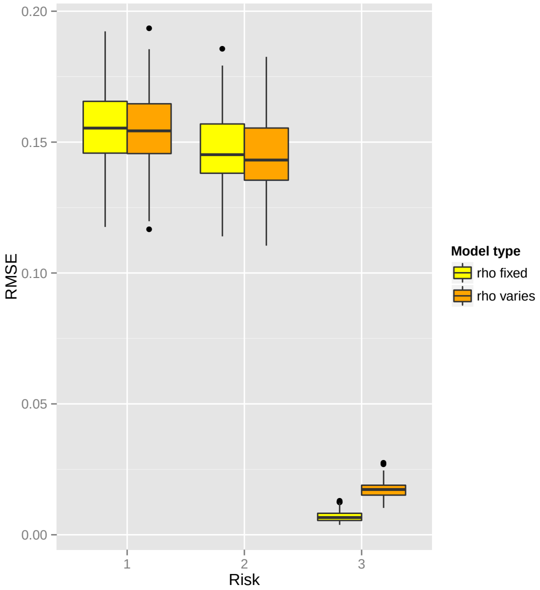
**DIC table for $T = 10$**

| $\rho$ | DIC |
|---|---|
| 0.993 | 25093.07 |
| 0 | 25331.94 |

**DIC table for $T = 50$**

| $\rho$ | DIC |
|---|---|
| 0.983 | 127717.5 |
| 0 | 128425.7 |

# Some more (simulation) results



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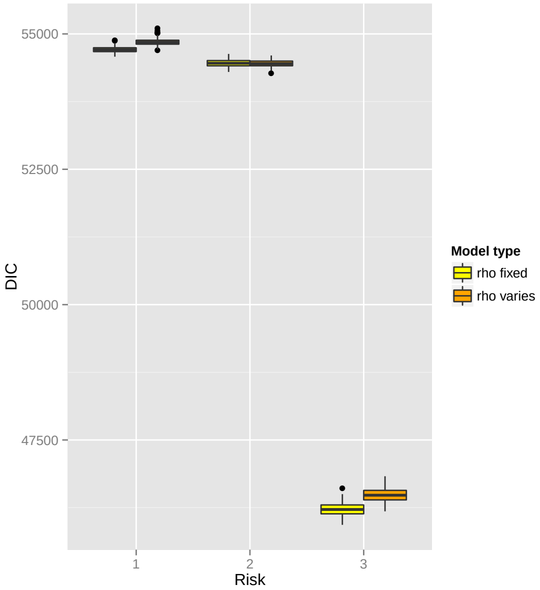
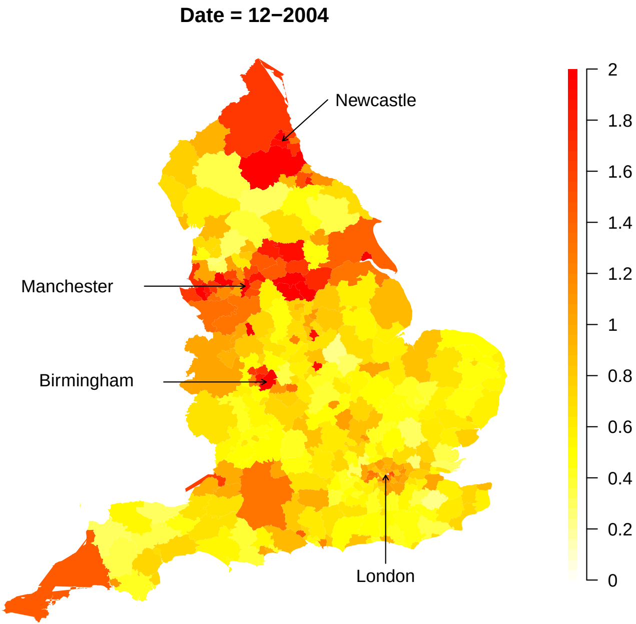
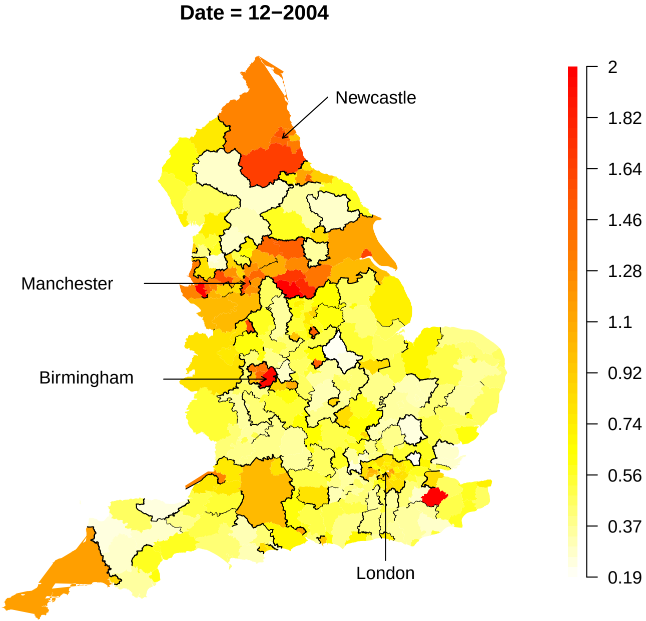


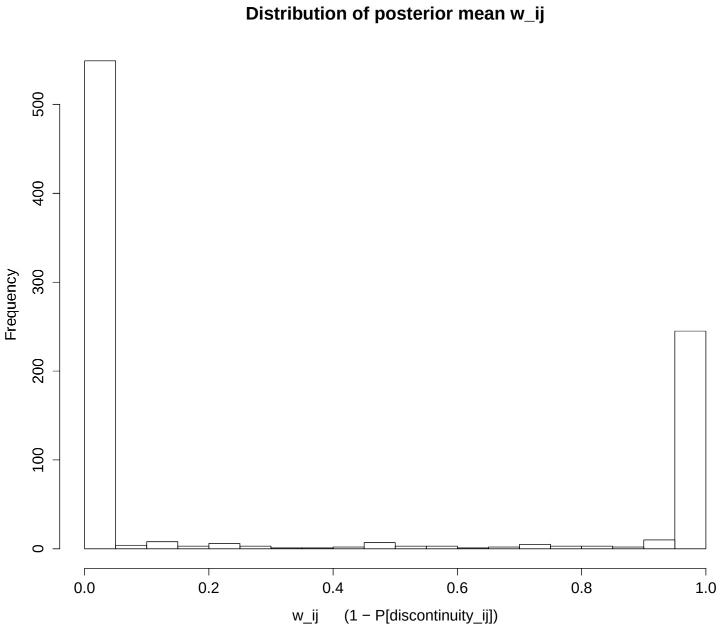
# Application to England LHAs - Data



# Application to England LHAs - Results



# Application to England LHAs - Results



# Current work

Model is obviously overparameterised

- Set of spatial random effects for each time and region
- Doesn't take account of temporal structure

Letting $\tilde{\phi}_t = (\phi_{t1}, \ldots, \phi_{tN})$, where $t = 1, \ldots, T$. Assume a model of the form

$$f(\tilde{\phi}_1, \ldots, \tilde{\phi}_T) = f(\tilde{\phi}_1) \prod_{t=2}^{T} f(\tilde{\phi}_t | \tilde{\phi}_{t-1}),$$

where

$$\begin{aligned} \tilde{\phi}_1 &\sim ICAR\left(\mathbf{0}, \sigma^2 Q^{-1}\right) \\ \tilde{\phi}_t | \tilde{\phi}_{t-1} &\sim ICAR\left(\alpha \tilde{\phi}_{t-1}, \sigma^2 Q^{-1}\right). \end{aligned}$$

# Comments

More work required to understand properties of these models

- Discontinuity detection rates under realistic surfaces
- How many replications make this analysis worthwhile?
- Simulations in progress...

Difficult to incorporate information about likely discontinuity structure into prior - compromise sometimes is required due to computational constraints:

- This is the reason CAR models are so nice: (matrix) sparsity; well established tricks for model-fitting; easy to write very efficient code.

- Knorr-Held and Raßer (2000) use a nice clustering prior that can induce smoothness; but requires RJMCMC.

# References

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# Thanks for listening!